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457.562 Special Issue on River Mechanics (Sediment Transport) .14 Suspended Load

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Modification of the Rousean Formulation for stratification effects

- River flows carrying suspended sediment are self-stratifying.
- The Rousean profile predicts a concentration of suspended sediment that decreases with increasing elevation above the bed.
- The stable stratification inhibits turbulent mixing of both flow momentum and suspended sediment concentration..
- Stratification effects lead to a streawise velocity profile that increases more rapidly in the vertical than the logarithmic profile,
- and a suspended sediment profile that decreases more rapidly in the vertical than the Rousean profile

Modification of the Rousean Formulation for stratification effects

- Stratification effects consideration.
- The kinematic eddy diffusivity D_d is not denoted D_{do}, where th e subscript “o” denotes the absence of stratification effects.
- The values of D_d in the presence of stratification effects is give n as

$$D_d = D_{do} F_{strat}\left(Ri_g\right)$$

$$D_{do} = \kappa u_* z\left(1 - \frac{z}{H}\right)$$

$$Ri_g = \frac{-Rg\left(d\bar{c}/dz\right)}{\left(d\bar{u}/dz\right)^2}$$

- Ri_g denotes a gradient Richardson number

Modification of the Rousean Formulation for stratification effects

- F_{strat} is a function that decreases with increasing gradient Rich ardson number, thus capturing the effect of damping of the tur bulence due to flow stratification.

$$F_{strat}\left(Ri_g\right) = 1 - 4.7 Ri_g$$

- Note that F_{strat} equals unity for a gradient Richardson number of zero (no stratification effects) and decreases to zero as Ri_g i ncreases to a value of 0.21, at which turbulent mixing is exting uished.

Modification of the Rousean Formulation for stratification effects

- Smith and McLean (1977) approximate the equation of stream wise momentum balance for the case of equilibrium flow in a wide channel to the form

$$D_d \frac{d\bar{u}}{dz} = u_*^2 \left(1 - \frac{z}{H} \right)$$

- An appropriate near-bed boundary condition at z=b is obtaine d matching the velocity profile to the logarithmic law

$$\frac{\bar{u}\big|_{z=b}}{u_*} = \frac{1}{\kappa} \ln\left(30 \frac{b}{k_c} \right)$$

- The corresponding boundary condition

$$-D_d \frac{d\bar{c}}{dz}\bigg|_b = v_s E_s$$

Modification of the Rousean Formulation for stratification effects

- Therefore

$$D_d = D_{do} F_{strat}\left(Ri_g\right) = \kappa u_* z\left(1 - \frac{z}{H}\right) F_{strat}\left(Ri_g\right)$$

$$\frac{d\bar{c}}{dz} = -\frac{v_s E_s}{D_d}$$

$$\bar{c} = E_s \exp\int_b^z \left[-\frac{v_s}{\kappa u_* z\left(1 - \frac{z}{H}\right) F_{strat}\left(Ri_g\right)} dz \right]$$

$$\bar{u} = \frac{u_*}{\kappa}\left[\ln\left(\frac{b}{z_0}\right) + \int_b^z \frac{1}{z F_{st}\left(Ri_g\right)} dz \right]$$

$$D_d \frac{d\bar{u}}{dz} = u_*^2 \left(1 - \frac{z}{H}\right)$$

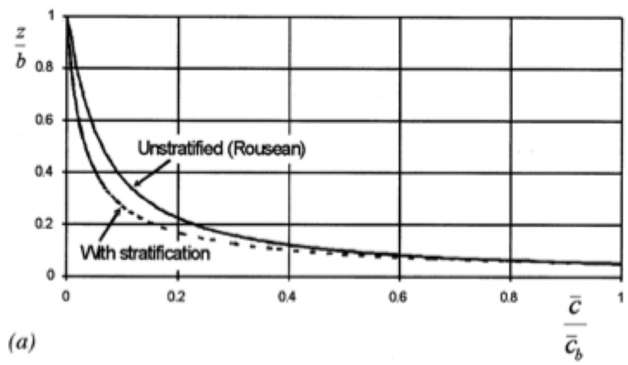
$$\frac{d\bar{u}}{dz} = \frac{u_*^2}{D_d}\left(1 - \frac{z}{H}\right)$$

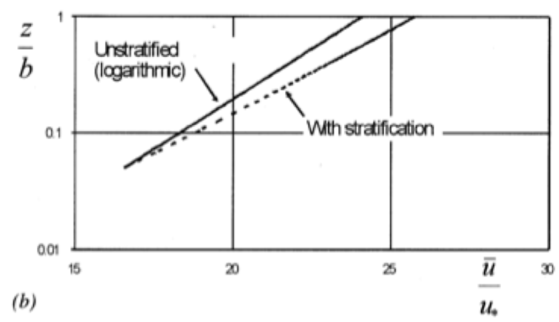
$$= \frac{u_*^2}{D_{do} F_{strat}(Ri)}\left(1 - \frac{z}{H}\right)$$

$$= \frac{u_*^2}{\kappa u_* z F_{strat}(Ri)}$$

Modification of the Rousean Formulation for stratification effects

- The previous two equations do not constitute an explicit soluti on for the concentration and velocity profiles, because Ri is a f unction of the concentration gradient.
- In the limit, as Ri goes to zero, the concentration equation con verges to the Rousian solution and velocity equation converge s to the logarithmic profile.
- These two unstratified profiles can be used as base forms for an iterative solution.





Vertically Averaged Concentration Suspended load

- Assuming that a value of near-bed elevation b is selected can be used to evaluate a depth-averaged volume sediment conc entration C defined by

$$\bar{C} = \frac{1}{H}\int_b^H \bar{c}(z)\,dz$$

- Using Rousean distribution

$$\bar{c} = \bar{c}_b\left[\frac{(H-z)/z}{(H-b)/b}\right]^{Z_R}$$

$$\bar{C} = \bar{c}_b J_1 = \bar{c}_b\int_{\delta_b}^{1}\left[\frac{(1-\delta)/\delta}{(1-\delta_b)/\delta_b}\right]^{Z_R} d\delta \qquad (where\ \delta = \frac{z}{H},\ \delta_b = \frac{b}{H})$$

Vertically Averaged Concentration Suspended load

- Einstein (1950) proposed a relation for the depth-averaging se diment concentration as

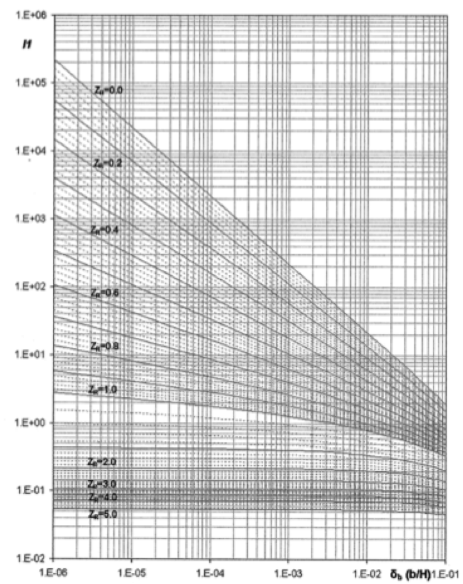
$$\bar{C} = \bar{c}_b \frac{\delta_b}{0.216} I_1$$

$$J_1 = \frac{\delta_b}{0.216} I_1 = \int_{\delta_b}^{1} \left[\frac{(1-\delta)/\delta}{(1-\delta_b)/\delta_b} \right]^{Z_R} d\delta$$

- The steamwise suspended load q,

$$q_s = \int_b^H \bar{c}_b \bar{u}\, dz$$

$$= \frac{1}{\kappa} \bar{c}_b u_* H \left[J_1 \ln\left(30 \frac{H}{k_c} \right) + J_2 \right]$$



- This equation indicates that to compute the rate of volumetric suspended sediment transport per unit width under uniform flo w conditions.

Vertically Averaged Concentration Suspended load

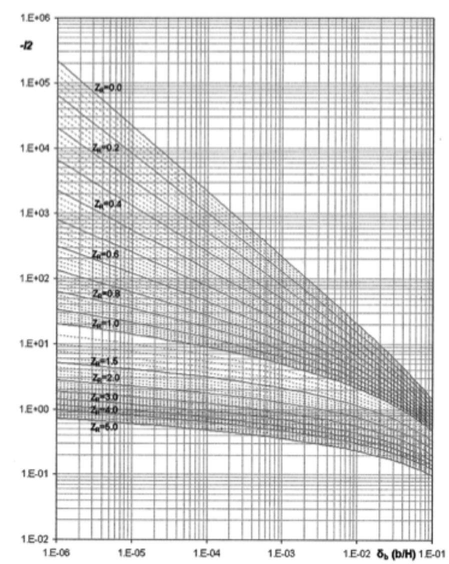
- Therefore, we need to know the near-bed concentration, the t otal friction velocity, the flow depth, the value of the composite roughness which can be computed as

$$k_c = 11H\exp\left(-\frac{\kappa U}{u_*}\right) \qquad ,\bar{c}_b\,, u_* = \sqrt{(\tau_{bs}+\tau_{bf})/\rho}\,, H$$

- And the values of the integral parameter J_1 and J_2.

$$J_2 = \int_{\delta_b}^{1}\left[\frac{(1-\delta)/\delta}{(1-\delta_b)/\delta_b}\right]\ln(\delta)\,d\delta$$

$$J_2 = \frac{\delta_b}{0.216} I_2$$



Relation for Sediment Entrainment

- The entrainment rate of sediment into suspension E.
- The equations are in text book, can be selected for determinin g entrainment.
- Measuring sediment concentration directly at some near-bed elevation z=b, and to equate the result to E.
- The data consisted of some 64 sets from ten different sources , all pertaining to laboratory suspensions of uniform sand with a submerged specific gravity R near 1.65.
- Information about the bedforms was typically not sufficient to allow for a partition of boundary shear stress in accordance wi th Nelson and Smith.

Relation for Sediment Entrainment

- As a result, the shear stress due to skin friction alone and the associated shear velocity due to skin friction, given by

$$\tau_{bs} = \rho u_{*s}^2$$

- Were computed using with

$$C_{fs} = \left[\frac{1}{\kappa} \ln \left(11 \frac{H_s}{k_s} \right) \right]$$

- The roughness height $k_s = 2D_{50}$.
- The data covered the following ranges:

E_S : 0.0002 to 0.06

u_s / v_s : 0.70 to 7.50

H / D : 240 to 2400

R_{ep} : 3.50 to 37.00 (Grain size 0.09~0.44)

Relation for Sediment Entrainment

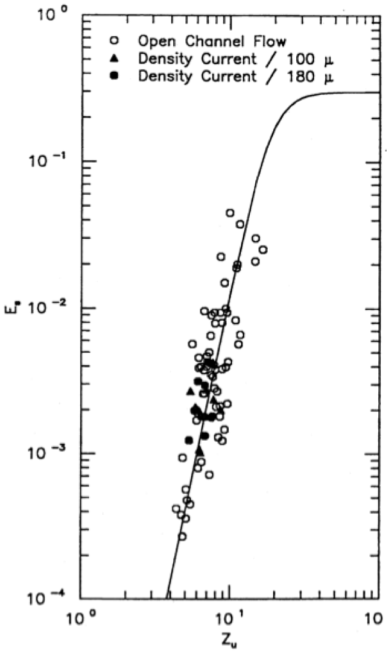
- The range of values of Reynolds number corresponds to a grain size range from 0.09 mm to 0.44 mm. Except for the somewhat small values of H/D, the values cover a range that includes typical field sand-bed streams.
- Garcia & Parkers (1991)
  - The reference level is taken to be five percent of the depth; that is,

$$\frac{b}{H} = \zeta_b = 0.05$$

  - The relation takes the form

$$E_s = \frac{AZ_u^5}{\left(1 + \frac{A}{0.3} Z_u^5\right)}$$

$$A = 1.3 \cdot 10^{-7} \qquad Z_u = \frac{u_{*s}}{v_s} \mathrm{R}_{ep}^{0.6}$$



Relation for Sediment Entrainment

- Van Rijin (1984)

$$E = 0.015\frac{D}{b}\left(\frac{\tau_s^*}{\tau_c^*} - 1\right)^{1.5} D_*^{-0.3}$$

$$where\ D_* = D\left(\frac{gR}{v^2}\right)^{1/3}$$

- Where τ_s^* denotes the Shields stress due to skin friction. And Van Rijin's relations are

$$C_{fs} = \frac{1}{\kappa}\ln\left(12\frac{H}{k_s}\right)^{-2}$$

- Where for uniform material

$$k_s = 3 \cdot D$$

- The above equation use *H* not H_s.

Relation for Sediment Entrainment

- Smith and McLean

$$\bar{c}_b = E = 0.65\frac{\gamma_0\left(\tau_s^*/\tau_c^* - 1\right)}{1+\gamma_0\left(\tau_s^*/\tau_c^* - 1\right)} \quad where \quad \gamma_0 = 0.0024$$

- The value b at which E is to be evaluated is given by the relation bel ow:

$$b = 26.3\left(\tau_s^*/\tau_c^* - 1\right)D + k_s$$

Example of Depth-Discharge and Sediment Load

- At bankfull flow, the stream width is 75m. For flows below bankfull, the following sample relation is assumed

$$\frac{B}{B_{bf}} = \left(\frac{Q}{Q_{bf}} \right)^{0.1}$$

  - Assume that the stream is wide enough to equate the hydraulic radius Rh with the cross-sectionally averaged depth H.
  - Compute depth-discharge relations for flows up to bank-full (lower regime only) using the Engelund-Hansen method. Plot H versus Q. Use the results of the Engelund-Hansen method to compute values of skin shield shear stress.
  - Use the value of skin shield stress to compute the bed load discharge $Q_b = q_b B$ using the Ashida-Michiue formula. For each value of H and U, back calculate the composite roughness k_c.
  - Then compute the suspended load $Q_s = q_s B$ from the Einstein formulation and the relation for Es due to Garcia & Parker. Plot Q_s, Q_b, and $Q_T = Q_b + Q_s$ as functions of water discharge Q.

Example of Depth-Discharge and Sediment Load

- S= 0.0004, D_s=0.35mm, R=1.65, B=75m at bankfull, H=2.9m at ban kfull.
- For flows below bank-full, the following relation is used to calculated the stream width:
- Depth-Discharge calculation
  - The first step is to compute the resistance coefficient

$$C_{fs} = \left[\frac{1}{\kappa} \ln\left(11 \frac{H_s}{k_s} \right) \right]^{-2} \qquad k_s = 2.5 D_s = 2.5 \times 3.5 \cdot 10^{-4} = 8.75 \times 10^{-4}$$

  - U can be found

$$U = \sqrt{\frac{g H_s S}{C_{fs}}}$$

Example of Depth-Discharge and Sediment Load

– The shields stress due to skin friction is

$$\tau_s^* = \frac{\tau_{bs}}{\rho g R D_s} = \frac{H_s S}{R D_s}$$

– According to Engelund-Hansen, the total Shields stress for the lower re gime can be found from the following relations

$$\tau^* = \sqrt{\frac{\tau_s^* - 6}{0.4}}$$

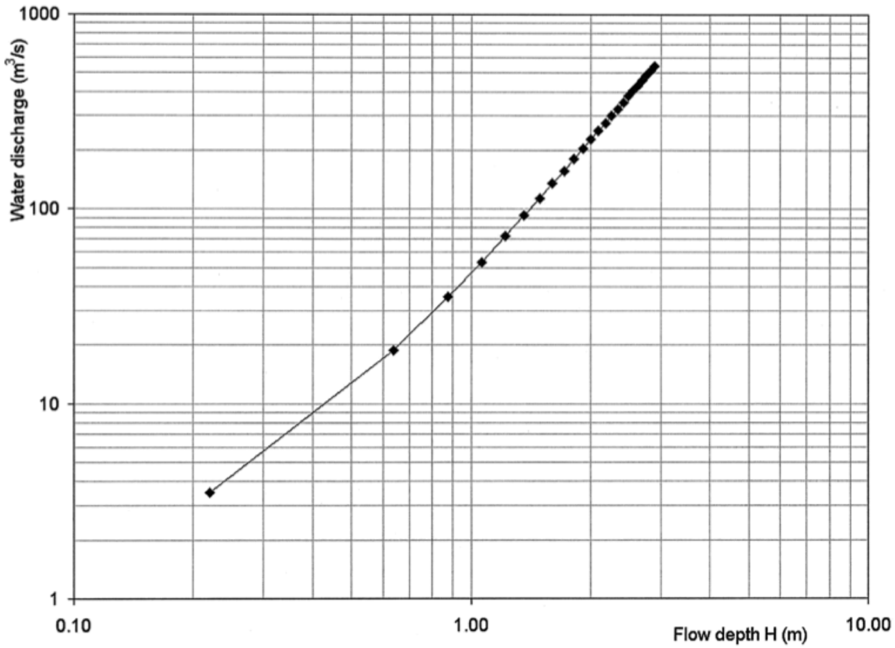
– The flow depth can be calculated from the total Shields stress as

$$H = \frac{\tau^* R D_s}{S}$$

– Finally, the discharge can be calculated from the results as

$$Q = UHB \qquad \frac{B}{B_{bf}} = \left(\frac{Q}{Q_{bf}}\right)^{0.1} \Rightarrow B = \left[B_{bf}\left(\frac{UH}{Q_{bf}}\right)^{0.1}\right]^{1/0.9}$$

Example of Depth-Discharge and Sediment Load



- Bankfull Discharge curve

Example of Depth-Discharge and Sediment Load

- Bed load discharge calculations
  - The dimensionless bed load transport rate is found from the Ahida-Michi ue formulation

$$q^* = 17\left(\tau_s^* - \tau_c^*\right)\left[\left(\tau_s^*\right)^{0.5} - \left(\tau_c^*\right)^{0.5}\right] \qquad where \;\; \tau_c^* = 0.05$$

  - The bed load transport rate per unit width is

$$q_b = q^* \sqrt{gRD_s} D_s$$

  - Therefore, the bedload transport rate is given by

$$Q_b = q_b B$$

Example of Depth-Discharge and Sediment Load

- Sediment load discharge calculations
  - The Einstein formulation is used to compute the suspended load transp ort rate per unit width q_s.

$$q_s = \frac{1}{\kappa} \bar{c}_b u_* H \left[J_1 \ln\left(30 \frac{H}{k_c} \right) + J_2 \right]$$

$$u_* = \sqrt{gHS}$$

  - If the suspension is assumed to be at equilibrium,

$$\bar{c}_b = E_s = \frac{A Z_u^5}{\left(1 + \frac{A}{0.3} Z_u^5 \right)} \qquad where \;\; A = 1.3 \times 10^{-7}$$

$$Z_u = \frac{u_{*s}}{v_s} R_{ep}^{0.6}, \quad u_{*s} = \sqrt{gH_s S}, \quad R_{ep} = \frac{\sqrt{RgD_s} D_s}{\nu}$$

Example of Depth-Discharge and Sediment Load

- Determine the fall velocity and etc. The composite roughness is

$$k_c = 11H \exp\left[-\frac{\kappa U}{u_*}\right]$$

- And find some values of J_1 and J_2 with $\delta_b = 0.05$
- Finally

$$Q_s = q_s B$$

- Determination of Bank full flow discharge
  - The flow discharge at bankfull is determined by assuming that up to ban k-full flow, lower regime conditions exist.
  - The bankfull flow depth for this stream is assumed to be 2.9m.

$$\tau^* = \frac{HS}{RD_s} = \frac{2.9 \cdot 0.0004}{1.65 \cdot 3.5 \times 10^{-4}} = 2.01$$

Example of Depth-Discharge and Sediment Load

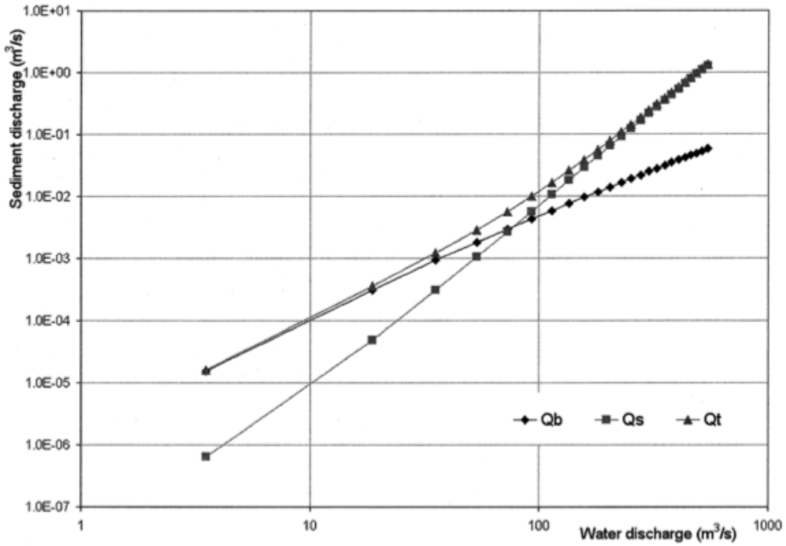
– From Engelund and Hansen

$$\tau_s^* = 0.06 + 0.4\left(\tau^*\right)^2 = 1.67$$

$$H_s = \frac{\tau_s^* R D_s}{S} = 2.42m$$

$$C_{fs} = \left[\frac{1}{\kappa}\ln\left(11\frac{H_s}{k_s}\right)\right]^{-2} = 1.5\times 10^{-3}$$

$$U = \sqrt{\frac{gH_s S}{C_{fs}}} = 2.51m/\sec$$



– Upto 100cms, the bed load discharge is larger than the suspended load discharge. As the flow discharge increases, the suspended load become s much larger than the bed load all the way up to bank-full conditions.

– Notice that the composite roughness first increases with flow discharge f or low flows, but from then on it decreases monotonically as the bedform s begin washed out