POTENTIAL SWEEP METHODS

Potential Sweep Methods

 \rightarrow The potential is varied linearly with time

(i.e., the applied signal is a voltage ramp)

→ with sweep rates v ranging from 10 mV/s to about 1000 V/s with conventional electrodes



- In this experiment,
- \rightarrow it is customary to record the current as a function of potential,
- \rightarrow obviously equivalent to recording current versus time.
- The formal name for the method is linear potential sweep chronoamperometry,
- \rightarrow but most workers refer to it as linear sweep voltammetry (LSV)



- A typical LSV response curve for the anthracene (A) system considered in Ch. 5
- 1) If the scan is begun at a potential well positive of E⁰' for the reduction,
 - \rightarrow only nonfaradaic currents flow for a while.
- 2) When the electrode potential reaches the vicinity of $E^{0'}$,
 - \rightarrow the reduction begins and current starts to flow.
- 3) As the potential continues to grow more negative,
 - \rightarrow the surface concentration of anthracene must drop;
 - \rightarrow hence the flux to the surface (and the current) increases.



- 4) As the potential moves past $E^{0'}$,
 - \rightarrow the surface concentration drops nearly to zero
 - \rightarrow mass transfer of anthracene to the surface reaches a maximum rate
- 5) Then it declines as the depletion effect sets in.
 - \rightarrow the observation is therefore a peaked current-potential curve



- Let us consider what happens if we reverse the potential scan
- Suddenly the potential is sweeping in a positive direction,
 - → in the electrode's vicinity there is a large concentration of the oxidizable anion radical of anthracene.
- As the potential approaches, then passes, E⁰',
 - → the electrochemical balance at the surface grows more and more favorable toward the neutral anthracene species.
 - \rightarrow Thus the anion radical becomes reoxidized and an anodic current flows.



Figure 6.1.3 (a) Cyclic potential sweep. (b) Resulting cyclic voltammogram.

- This reversal current has a shape much like that of the forward peak for essentially the same reasons.
- This a reversal technique is called cyclic voltammetry (CV)



Figure 6.1.3 (a) Cyclic potential sweep. (b) Resulting cyclic voltammogram.

- Consider the reaction O + ne \rightleftharpoons R
- Assume semi-infinite linear diffusion and a solution initially containing only species O,
 with the electrode held initially at a potential E_i, where no electrode reaction occurs.
- The potential is swept linearly at v (V/s) so that the potential at any time is

$$E(t) = E_{\rm i} - v t$$

- If we can assume that the rate of electron transfer is rapid at the electrode surface,
- → species O and R immediately adjust to the ratio dictated by the Nernst equation

$$\begin{aligned} \frac{\partial C_{O}(x,t)}{\partial t} &= D_{O} \frac{\partial^{2} C_{O}(x,t)}{\partial x^{2}} & \frac{\partial C_{R}(x,t)}{\partial t} &= D_{R} \frac{\partial^{2} C_{R}(x,t)}{\partial x^{2}} \\ C_{O}(x,0) &= C_{O}^{*} & C_{R}(x,0) &= 0 \\ \lim_{x \to \infty} C_{O}(x,t) &= C_{O}^{*} & \lim_{x \to \infty} C_{R}(x,t) &= 0 & D_{O} \left(\frac{\partial C_{O}(x,t)}{\partial x} \right)_{x=0} + D_{R} \left(\frac{\partial C_{R}(x,t)}{\partial x} \right)_{x=0} &= 0 \end{aligned}$$

 However, the surface concentration relation must be recognized as having a timedependent form:

- Because of the time dependence, the Laplace transformation cannot be performed
- \rightarrow the mathematics for sweep experiments are greatly complicated as a consequence.

 $C_{\rm O}(0, t) = \theta C_{\rm R}(0, t)$ $\Rightarrow \overline{C}_{\rm O}(0, s) = \theta \overline{C}_{\rm R}(0, s)$ only when θ is not a function of time

The boundary condition can be written

$$\frac{C_{\rm O}(0,t)}{C_{\rm R}(0,t)} = \theta \ e^{-\sigma t} = \theta S(t) \qquad S(t) = e^{-\sigma t}, \ \theta = \exp[(nF/RT)(E_{\rm i} - E^{0'})] \qquad \sigma = (nF/RT)v$$

• Laplace transformation of the diffusion equations and application of the initial and semi-infinite conditions leads to

$$\overline{C}_{O}(x,s) = \frac{C_{O}^{*}}{s} + A(s) \exp\left[-\left(\frac{s}{D_{O}}\right)^{1/2}x\right]$$

• The transform of the current is given by

$$\overline{i}(s) = nFAD_{O}\left[\frac{\partial\overline{C}_{O}(x,s)}{\partial x}\right]_{x=0} = -A(s)\left(\frac{s}{D_{O}}\right)^{1/2}$$

$$\overline{i}(s) = -nFAD_{O}A(s)\left(\frac{s}{D_{O}}\right)^{1/2}$$

Thus,

$$A(s) = -\frac{\overline{i}(s)}{nFAD_0} (\frac{s}{D_0})^{-1/2}$$

$$\overline{C}_0(x,s) = \frac{C_0^*}{s} - \frac{\overline{i}(s)}{nFAD_0} (\frac{s}{D_0})^{-1/2} \exp(-(\frac{s}{D_0})^{\frac{1}{2}x})$$

$$\overline{C_0}(x,s) = \frac{C_0^*}{s} - \frac{\overline{i}(s)}{nFAD_0} (\frac{s}{D_0})^{-1/2} \exp(-(\frac{s}{D_0})^{\frac{1}{2}}x)$$

$$\overline{C_0}(0,s) = \frac{C_0^*}{s} - \frac{\overline{i}(s)}{nFAD_0} (\frac{s}{D_0})^{-1/2} = \frac{C_0^*}{s} - \frac{1}{nFAD_0} \frac{1}{1/2} \overline{i}(s)s^{-1/2}$$

$$L^{-1}[f(s)g(s)] = F(t)^*G(t)$$

$$= \int_0^t F(t - \tau)G(\tau)d\tau$$

• By inverting with the convolution theorem, we obtain

$$C_{\rm O}(0,t) = C_{\rm O}^* - [nFA(\pi D_{\rm O})^{1/2}]^{-1} \int_0^t i(\tau)(t-\tau)^{-1/2} d\tau$$

: τ in the integral is a dummy variable that is lost when the definite integral is evaluated.

• By letting

$$f(\tau) = \frac{i(\tau)}{nFA}$$

$$C_{\rm O}(0,t) = C_{\rm O}^* - [nFA(\pi D_{\rm O})^{1/2}]^{-1} \int_0^t i(\tau)(t-\tau)^{-1/2} d\tau$$

$$C_{\rm O}(0,t) = C_{\rm O}^* - (\pi D_{\rm O})^{-1/2} \int_0^t f(\tau)(t-\tau)^{-1/2} d\tau$$

• Similarly, an expression for $C_R(0,t)$ can be obtained (assuming R is initially absent):

$$C_{\rm R}(0,t) = (\pi D_{\rm R})^{-1/2} \int_0^t f(\tau)(t-\tau)^{-1/2} d\tau$$

$$C_{O}(0, t) = C_{O}^{*} - (\pi D_{O})^{-1/2} \int_{0}^{t} f(\tau)(t - \tau)^{-1/2} d\tau$$

$$C_{R}(0, t) = (\pi D_{R})^{-1/2} \int_{0}^{t} f(\tau)(t - \tau)^{-1/2} d\tau$$

$$\frac{C_{O}(0, t)}{C_{R}(0, t)} = \theta \ e^{-\sigma t} = \theta S(t)$$

$$\int_{0}^{t} f(\tau)(t - \tau)^{-1/2} d\tau = \frac{C_{O}^{*}}{[\theta S(t)(\pi D_{R})^{-1/2} + (\pi D_{O})^{-1/2}]}$$

$$\int_{0}^{t} i(\tau)(t - \tau)^{-1/2} d\tau = \frac{nFA\pi^{1/2}D_{O}^{1/2}C_{O}^{*}}{[\theta S(t)\xi + 1]}$$

$$\xi = (D_{O}/D_{R})^{1/2}$$

• An analytical solution cannot be obtained, and a numerical method must be employed.

- Before solving the equation numerically, it is convenient
- → (a) to change from i(t) to i(E), since that is the way in which the data are usually considered
- → (b) to put the equation in a dimensionless form so that a single numerical solution will give results that will be useful under any experimental conditions.
- The dimensionless form is accomplished by using the following substitution:

$$S(t) = e^{-\sigma t}, \qquad \sigma t = \frac{nF}{RT} vt = \left(\frac{nF}{RT}\right)(E_{i} - E) \qquad \tau = z/\sigma$$

$$\sigma = (nF/RT)v \qquad f(\tau) = g(\sigma\tau) \qquad z = \sigma\tau \qquad \left\{ \begin{array}{c} \tau = z/\sigma \\ d\tau = dz/\sigma \\ z = 0 \text{ at } \tau = 0 \\ z = \sigma t \text{ at } \tau = t \end{array} \right.$$

$$\int_0^t f(\tau)(t-\tau)^{-1/2} d\tau = \int_0^{\sigma t} g(z) \left(t - \frac{z}{\sigma}\right)^{-1/2} \frac{dz}{\sigma}$$

• Rearrangement:

- At any given point,
- $\rightarrow \chi(\sigma t)$ is a pure number
- The functional relationship between the current at any point on the LSV curve and the variables.
- → i is proportional to C_0^* and $v^{1/2}$. $\sigma t = \frac{nF}{RT}vt$

Solution:	THEEL OLD CONTENT TUNCTIONS FOR REVERSION CONTROL (C)							
$\rightarrow \chi(\sigma t)$	$\frac{n(E-E_{1/2})}{RT/F}$	$n(E - E_{1/2})$ mV at 25°C	$\pi^{1/2}\chi(\sigma t)$	$\phi(\sigma t)$	$\frac{n(E-E_{1/2})}{RT/F}$	$n(E - E_{1/2})$ mV at 25°C	$\pi^{1/2}\chi(\sigma t)$	$\phi(\sigma t)$
as a function of	4.67	120	0.009	0.008	-0.19	-5	0.400	0.548
	3.89	100	0.020	0.019	-0.39	-10	0.418	0.596
σt	3.11	80	0.042	0.041	-0.58	-15	0.432	0.641
	2.34	60	0.084	0.087	-0.78	-20	0.441	0.685
or	1.95	50	0.117	0.124	-0.97	-25	0.445	0.725
$n(E-E_{1/2})$	1.75	45	0.138	0.146	-1.109	-28.50	0.4463	0.7516
	1.56	40	0.160	0.173	-1.17	-30	0.446	0.763
	1.36	35	0.185	0.208	-1.36	-35	0.443	0.796
	1.17	30	0.211	0.236	-1.56	-40	0.438	0.826
	0.97	25	0.240	0.273	-1.95	-50	0.421	0.875
	0.78	20	0.269	0.314	-2.34	-60	0.399	0.912
	0.58	15	0.298	0.357	-3.11	-80	0.353	0.957
	0.39	10	0.328	0.403	-3.89	-100	0.312	0.980
	0.19	5	0.355	0.451	-4.67	-120	0.280	0.991
	0.00	0	0.380	0.499	-5.84	-150	0.245	0.997

TABLE 6.2.1 Current Functions for Reversible Charge Transfer $(3)^{a,b}$

^{*a*}To calculate the current:

1. i = i(plane) + i(spherical correction).

- 2. $i = nFAD_{O}^{1/2}C_{O}^{*}\sigma^{1/2}\pi^{1/2}\chi(\sigma t) + nFAD_{O}C_{O}^{*}(1/r_{0})\phi(\sigma t)$
- 3. $i = 602n^{3/2}AD_0^{1/2}C_0^*v^{1/2}\{\pi^{1/2}\chi(\sigma t) + 0.160[D_0^{1/2}/(r_0n^{1/2}v^{1/2})]\phi(\sigma t)\}$ at 25°C with quantities in the following units: *i*, amperes; A, cm²; D_0 , cm²/s; v, V/s; C_0^* , M; r_0 , cm.

$${}^{b}E_{1/2} = E^{0'} + (RT/nF) \ln (D_{\rm R}/D_{\rm O})^{1/2}$$

⁴Note that $\ln \xi \theta S(\sigma t) = nf(E - E_{1/2})$, where $E_{1/2} \equiv E^{0'} + (RT/nF) \ln (D_R/D_O)^{1/2}$.

• Linear potential sweep voltammogram in terms of dimensionless current function



Figure 6.2.1 Linear potential sweep voltammogram in terms of dimensionless current function. Values on the potential axis are for 25°C

6.2.2 Peak Current and Potential

- The function $\pi^{1/2}\chi(\sigma t)$ and hence the current \rightarrow reaches a maximum where $\pi^{1/2}\chi(\sigma t) = 0.4463$ (Table 6.2.1)
- So, the peak current, i_p, is

• At 25°C,

$$i_{\rm p} = (2.69 \times 10^5) n^{3/2} A D_{\rm O}^{1/2} C_{\rm O}^* v^{1/2}$$
 : a plot of $i_{\rm P}$ vs. $v^{1/2} \rightarrow$ n or $D_{\rm O}$

: i_P in amperes, A in cm², D₀ in cm²/s, C₀* in mol/cm³, and v in V/s

• The peak potential, E_{p} , is (Table 6.2.1)

$$E_{\rm p} = E_{1/2} - 1.109 \frac{RT}{nF} = E_{1/2} - 28.5/n \,\text{mV} \text{ at } 25^{\circ}\text{C}$$
 $E_{1/2} \equiv E^{0'} + (RT/nF) \ln (D_{\rm R}/D_{\rm O})^{1/2}$

- Because the peak is somewhat broad, so that the peak potential may be difficult to determine
- \rightarrow it is sometimes convenient to report the potential at i_P/2, called the half-peak potential, $E_{P/2}$, which is

$$E_{p/2} = E_{1/2} + 1.09 \frac{RT}{nF} = E_{1/2} + 28.0/n \text{ mV at } 25^{\circ}\text{C}$$

$$E_p = E_{1/2} - 1.109 \frac{RT}{nF} = E_{1/2} - 28.5/n \text{ mV at } 25^{\circ}\text{C}$$

$$|E_p - E_{p/2}| = 2.20 \frac{RT}{nF} = 56.5/n \text{ mV at } 25^{\circ}\text{C}$$

• Thus for a reversible wave, E_P is independent of scan rate, and i_P (as well as the current at any other point on the wave) is proportional to $v^{1/2}$.

Summary



Reversible

 $K_3Fe(CN)_6$: Fe(CN)₆³⁻ + e \Rightarrow Fe(CN)₆⁴⁻



Summary



- For a totally irreversible one-step, one-electron reaction $(O + e \xrightarrow{k_{\rm f}} R)$
- \rightarrow The nernstian boundary condition is replaced by

$$\frac{C_{O}(0,t)}{C_{R}(0,t)} = f(t) = \exp\left[\frac{nF}{RT} \left(E_{i} - vt - E^{0'}\right)\right] \qquad \Longrightarrow \qquad \frac{i}{FA} = D_{O}\left[\frac{\partial C_{O}(x,t)}{\partial x}\right]_{x=0} = k_{f}(t)C_{O}(0,t)$$
$$k_{f}(t) = k^{0} \exp\{-\alpha f[E(t) - E^{0'}]\}$$
$$E(t) = E_{i} - vt$$

$$k_{\rm f}(t)C_{\rm O}(0,t) = k_{\rm fi}C_{\rm O}(0,t)e^{bt} \qquad \begin{cases} b = \alpha fv \\ k_{\rm fi} = k^0 \exp[-\alpha f(E_{\rm i} - E^{0'})] \end{cases}$$

• When we solve using the same numerical method as the reversible case,

$$i = FAC_{\rm O}^*D_{\rm O}^{1/2}v^{1/2}\left(\frac{\alpha F}{RT}\right)^{1/2}\pi^{1/2}\chi(bt)$$

Dimensionless potential

$$(\alpha F/RT)(E - E^{0'}) + \ln [(\pi D_0 b)^{1/2}/k^0].$$

TABLE 6.3.1	Current Functions fo	r Irreversible Charge	Transfer $(3)^a$
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Dimensionless Potential ^b	Potential ^c mV at 25°C	$\pi^{1/2}\chi(bt)$	$\phi(bt)$	Dimensionless Potential ^b	Potential ^c mV at 25°C	$\pi^{1/2}\chi(bt)$	$\phi(bt)$
6.23	160	0.003		0.58	15	0.437	0.323
5.45	140	0.008		0.39	10	0.462	0.396
4.67	120	0.016		0.19	5	0.480	0.482
4.28	110	0.024		0.00	0	0.492	0.600
3.89	100	0.035		-0.19	-5	0.496	0.685
3.50	90	0.050		-0.21	-5.34	0.4958	0.694
3.11	80	0.073	0.004	-0.39	-10	0.493	0.755
2.72	70	0.104	0.010	-0.58	-15	0.485	0.823
2.34	60	0.145	0.021	-0.78	-20	0.472	0.895
1.95	50	0.199	0.042	-0.97	-25	0.457	0.952
1.56	40	0.264	0.083	-1.17	-30	0.441	0.992
1.36	35	0.300	0.115	-1.36	-35	0.423	1.000
1.17	30	0.337	0.154	-1.56	-40	0.406	
0.97	25	0.372	0.199	-1.95	-50	0.374	
0.78	20	0.406	0.253	-2.72	-70	0.323	

^{*a*}To calculate the current:

1. i = i(plane) + i(spherical correction).

2. $i = FAD_{O}^{1/2}C_{O}^{*}b^{1/2}\pi^{1/2}\chi(bt) + FAD_{O}C_{O}^{*}(1/r_{0})\phi(bt)$

3. $i = 602AD_O^{1/2}C_O^*\alpha^{1/2}v^{1/2}\{\pi^{1/2}\chi(bt) + 0.160[D_O^{1/2}v^{1/2}]\phi(bt)\}$. Units for step 3 are the same as in Table 6.2.1.

^bDimensionless potential is $(\alpha F/RT)(E - E^{0'}) + \ln [(\pi D_0 b)^{1/2}/k^0]$.

^cPotential scale in mV for 25°C is $\alpha(E - E^{0'}) + (59.1) \ln [(\pi D_0 b)^{1/2} / k^0]$.

6.3 TOTALLY IRREVERSIBLE SYSTEMS

• The function $\chi(bt)$ and hence the current

 \rightarrow reaches a maximum where $\pi^{1/2}\chi(bt) = 0.4958$ (Table 6.3.1)

• So, the peak current, i_p, is

• The peak potential, E_p, is (Table 6.3.1)

$$\alpha(E_{\rm p} - E^{0'}) + \frac{RT}{F} \ln\left[\frac{(\pi D_{\rm O}b)^{1/2}}{k^0}\right] = -0.21 \frac{RT}{F} = -5.34 \text{ mV at } 25^{\circ}\text{C}$$

$$E_{\rm p} = E^{0'} - \frac{RT}{\alpha F} \left[0.780 + \ln\left(\frac{D_{\rm O}^{1/2}}{k^0}\right) + \ln\left(\frac{\alpha Fv}{RT}\right)^{1/2} \right]$$

$$|E_{\rm p} - E_{{\rm p}/2}| = \frac{1.857RT}{\alpha F} = \frac{47.7}{\alpha} \,\mathrm{mV} \text{ at } 25^{\circ}\mathrm{C}$$

- : For a totally irreversible wave,
 - \rightarrow E_P is a function of scan rate

Summary



Irreversible

- Consider the potential sweep experiment
- → in which nonfaradaic (capacitive) currents are comparable to faradaic currents

- For a potential step experiment at a stationary, constant-area electrode,
- \rightarrow the charging current (nonfaradaic current) decays with time
- For a potential sweep experiment, a charging current (nonfaradaic current), \rightarrow i_c is a steady state current (Ch. 1)

$$|i_{\rm c}| = AC_{\rm d}v$$





Current-time behavior resulting from a linear potential sweep applied to an *RC* circuit.

- A voltage ramp or linear potential sweep
- : a potential increases linearly with time

• Let us derive the governing equation of the current transient, i, with time, t, when a potential increases linearly with time starting at some initial value (here assumed to be zero) at a sweep rate v (in V s⁻¹)

$$E = vt$$

1) If such a ramp is applied to the R_sC_d circuit, the following equation still applies; hence

$$E = E_{\rm R} + E_{\rm C} = iR_{\rm s} + \frac{q}{C_{\rm d}}$$

$$vt = R_{\rm s}(dq/dt) + q/C_{\rm d}$$
2) If q = 0 at t = 0,
$$i = vC_{\rm d} \left[1 - \exp(-t/R_{\rm s}C_{\rm d})\right]$$

t

(b)

- The current rises from zero as the scan starts and attains a steady-state value, νC_d
- This steady-state current can then be used to estimate C_d because v is known



- For the potential sweep experiments,
 - \rightarrow the measured current = the sum of faradaic and nonfaradaic currents
 - \rightarrow the faradaic current must be measured from a baseline of charging current,
- While faradaic peak current, $i_{p'}$ varies with $v^{1/2}$ for linear diffusion,

 \rightarrow i_c varies with v

 \rightarrow i_c becomes relatively more important at faster scan rates.

$$i_{\rm p} = (2.69 \times 10^5) n^{3/2} A D_{\rm O}^{1/2} C_{\rm O}^* v^{1/2}$$
$$\frac{|i_{\rm c}|}{i_{\rm p}} = \frac{C_{\rm d} v^{1/2} (10^{-5})}{2.69 n^{3/2} D_{\rm O}^{1/2} C_{\rm O}^*}$$

- Thus at high v and low C₀* values,
 - \rightarrow severe distortion of the LSV wave occurs.
 - → This effect often sets the limits of maximum useful scan rate and minimum useful concentration.

6.2.4 Effect of Double-Layer Capacitance



Figure 6.2.3 Effect of double-layer charging at different sweep rates on a linear potential sweep voltammogram. Curves are plotted with the assumption that C_d is independent of *E*. The magnitudes of the charging current, i_c , and the faradaic peak current, i_p , are shown. Note that the current scale in (*c*) is 10× and in (*d*) is 100× that in (*a*) and (*b*).

- Consider the consecutive reduction of two substances O and O' in a potential scan experiment (multicomponent systems)
 : O + ne → R and O' + n'e → R' occur.
- If the diffusion of O and O' takes place independently,
- \rightarrow the fluxes are additive
- \rightarrow the i-E curve for the mixture: the sum of the individual i-E curves of O and O'



- The measurement of i'_P
- \rightarrow must be made using the decaying current of the first wave as the baseline.
- → Usually this baseline is obtained by assuming that the current past the peak potential follows that for the large-amplitude potential step and decays as $t^{1/2}$.



- An experimental approach to obtaining the baseline
- → Since the concentration of O at the electrode falls essentially to zero at potentials just beyond E_p
 - \rightarrow the current beyond E_p : independent of potential.
- → Thus if the voltammogram of a single-component system is recorded on a time base and the potential scan is held at about 60/n mV beyond E_p while the time base continues,
 - → the current-time curve: the same as that obtained with the potential sweep continuing



Multicomponent Systems



Multicomponent Systems



Figure 6.6.3 Method of allowing current of first wave to decay before scanning second wave. *Upper curve:* potential program. *Lower curve:* resulting voltammogram. System as in Figure 6.6.1.

- For the stepwise reduction of a single substance O (multistep charge transfer) $\rightarrow O + n_1 e \rightarrow R_1 (E_1^0)$ and $R_1 + n_2 e \rightarrow R_2 (E_2^0)$
 - \rightarrow similar to the two-component case, but more complicated.
- If E_1^0 and E_2^0 are well separated, with $E_1^0 > E_2^0$ (i.e., O reduces before R_1),
 - \rightarrow observe two separate waves
 - 1) first wave: reduction of O to R_1 with R_1 diffusing into the solution
 - 2) second wave: reduction of R_1 to R_2
 - \rightarrow At the second wave,
 - : O continues to be reduced, either
 - i) directly at the electrode or

ii) by reaction with R_{2} diffusing away from the electrode

 $(O + R_2 \rightarrow 2R_1)$, and R_1 diffuses back toward the

electrode to be reduced





The i-E curve depends on ΔE^0 (= $E_2^0 - E_1^0$)

- (b) When ΔE^0 is between 0 and -100 mV
- → the individual waves are merged into a broad wave whose E_P is independent of scan rate.

(c) When $\Delta E^0 = 0$, \rightarrow a single peak