Innovative Ship and Offshore Plant Design

Part II. Offshore Plant Design

Ch. 2 Sizing and Configuration of Topside Systems

Spring 2016

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Ch. 2 Sizing and Configuration of Topside Systems

1. Determination of Optimal Operating Condition
2. Optimal Synthesis of Liquefaction Cycle
3. Determination of Optimal Operating Condition of the Liquefaction Cycle for LNG FPSO

Procedures of Process FEED of Liquefaction System of LNG FPSO and Importance of Optimal Operating Condition

Procedure of Construction of LNG FPSO

Exploration & Feasibility Study → Pre-FEED → **FEED** → EPCI → Commissioning

Engineering (Detail Design) → Procurement → Construction → Installation

① **Design Criteria**

➲ Well Components, Well Scale, Required Daily Production, Environment & Geographical Factor, etc.

② **Utility Consideration**

➲ Configuration of the process system and **operating conditions of each stream of the refrigerant and natural gas such as** temperature, pressure, specific volume, flow rate and mole fraction[1].

1) Mole fraction: Components of the mixed refrigerant and natural gas

③ **Process & Utility Hydraulic Calculations**

➲ Diameter of the pipe for each stream

④ **PFD** (Process Flow Diagram), **UFD** (Utility Flow Diagram)

➲ Diagram to show the safety & control logic of the topside systems and heat & material balance tables2)

⑤ **PED** (Process Equipment Datasheet), **UED** (Utility Equipment Datasheet)
PID (Process Instrument Datasheet), **UID** (Utility Instrument Datasheet)

➲ **Datasheets to show the operating conditions** and diameter of the inlet and outlet of each equipment for **performing procurement, construction, and operation of the topside process systems**

⑥ **P&ID** (Pipe & Instrument Diagram), SAC(Safety Analysis Checklist)

➲ Diagram that shows all data about the operating conditions, process control logic, safety and maintenance for the equipment and instruments, and vendor data about the equipment.

⑦ Plant Layout for Liquefaction Process

➲ For the compactness, the plant layout for the liquefaction process system of the LNG FPSO is multi-floor plant layout!

- Development of the optimization program corresponding to ASPEN HYSYS* for determining optimal operating conditions of the liquefaction cycle of LNG FPSO

* ASPEN HYHSYS: commercial software for solving non-linear equation in domain of process engineering

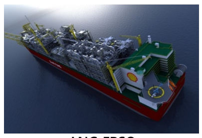
Major Considerations for the Selection of the Liquefaction Cycle in Offshore Application (1/2)


<Liquefaction process system>

+


<Exploration and Production of the Natural Gas>

=


<LNG FPSO>

- All major oil companies required that liquefaction cycles shall have reliability based on the results from previous onshore projects.
- Dual Mixed Refrigerant(DMR) cycle was verified from the SAKHALIN onshore liquefaction cycle in 2005.

- Safety studies: HAZard and Operability(HAZOP), HAZard Identification(HAZID), Failure Modes and Effects Analysis(FMEA), Fault Tree Analysis(FTA), Event Tree Analysis(ETA), CFD Exhausts Dispersion Study - Helideck Study Report, Dropped Object Study, Explosion Risk Analysis, Failure, etc.

* HAZOP: Structured and systematic examination of a planned or existing process or operation in order to identify and evaluate problems that may represent risks to personnel or equipment, or prevent efficient operation
* HAZID: The process of identifying hazards, which forms the essential first step of a risk assessment
* FMEA: This analysis technique is used to systematically analyze postulated component failures and identify the resultant effects on system operations.
* FTA: This analysis technique is used to understand how systems can fail, to identify the best ways to reduce risk or to determine event rates of a safety accident or a particular system level failure.
* ETA: This analysis technique is used to analyze the effects of functioning or failed systems given that an event has occurred.

Major Considerations for the Selection of the Liquefaction Cycle in Offshore Application (2/2)


<Liquefaction process system>

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<Exploration and Production of the Natural Gas>

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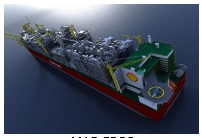

<LNG FPSO>

- If the LNG FPSO is inclined more than 1.5 degrees, the capacity of LNG production can be reduced by 10%.
- Therefore, the liquefaction cycle in the LNG FPSO has to be designed by considering mechanical damping devices, internal turret system, and dynamic positioning system.

- Available area for the liquefaction cycle in offshore application is smaller than that of onshore plant.
- By determining the optimal operating conditions and doing the optimal synthesis of the liquefaction cycle, the required power for the compressors can be reduced which will result in the reduction of the compressor size and the flow rate of the refrigerant. Thus, the overall sizes of the liquefaction cycle including the pipe diameter, equipment and instrument can be reduced.
- Therefore, the compactness can be achieved by optimization studies such as determination of the optimal operating condition or optimal synthesis of the liquefaction cycle.

➡ **Optimum design can be used!**

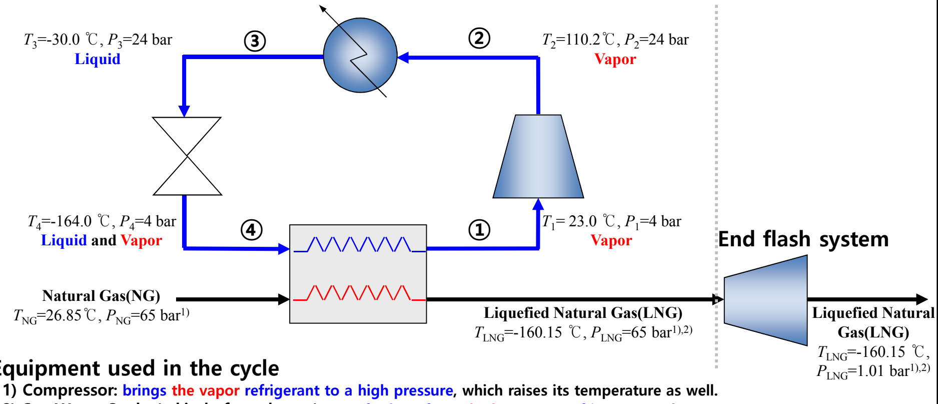
1. Determination of Optimal Operating Condition

Introduction to Liquefaction Cycle

- **Goal of the LNG Liquefaction Cycle**

  To liquefy NG to LNG for decreasing the volume of the NG

- **An example of Simplified LNG Liquefaction Cycle**

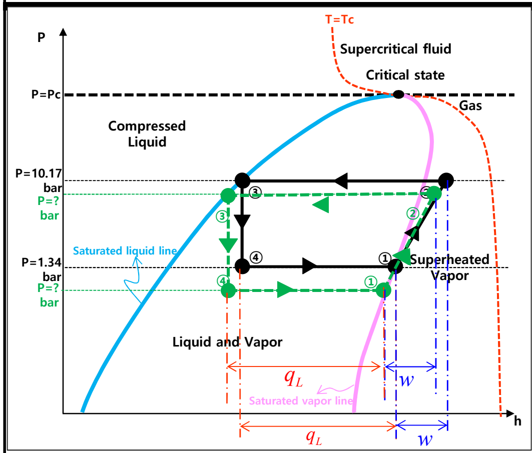


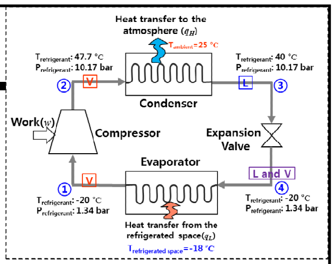
Equipment used in the cycle

1) **Compressor: brings the vapor refrigerant to a high pressure, which raises its temperature as well.**
2) **Sea Water Cooler(a kind of condenser): transfer heat from the hot vapor refrigerant to the sea water.**
3) **Valve: decreases the pressure of the liquid refrigerant, which decreases its temperature as well.**
4) **Heat Exchanger(a kind of evaporator): absorbs heat from the natural gas to cool down the NG, while the refrigerant is vaporized.**

1) The temperature and pressure of the natural gas and liquefied natural gas are the values of the general case.
2) In the end flash system, the pressure of LNG expanded to the atmospheric pressure (1.01 bar) to be stored in the LNG tank.

Optimal Design of Liquefaction Cycle for LNG FPSO





- q_L: specific heat transfer from the refrigerated space to the refrigerant(Given)
- w: work provided to the compressor(Minimizing)

- **Given:** The quantity of the specific heat transfer from the refrigerated space to the refrigerant(q_L) in the evaporator.

Constraint

Design variables

1. Determination of the optimal operating condition
   - **Find:** The operating conditions such as the pressure, temperature, and specific volume minimizing the work provided to the compressor. → Objective function

2. Optimal synthesis on liquefaction cycles (Design variables)
   - Find: The number of the combination of equipment that make up the liquefaction cycle minimizing the work provided to the compressor. → Objective function

Mathematical Model of the Liquefaction Cycle - Calculation of Specific Enthalpy (h)

$h = u + P \cdot v$

- **Physical Constraint based on Thermodynamics #1**

Energy conservation

Calculation of

Many tables of thermodynamics properties does not give values for internal energy. To allow calculation of enthalpy from the pressure, specific volume, and temperature, the following equation is derived by using the definition($h=u+Pv$), the equation of state, and the experiment.

$$h = h^{IG} + h^{R}$$

h^{IG}: **Ideal gas value of the specific enthalpy**
h^{R}: Residual specific enthalpy(correction of the ideal gas state values to the real gas values)

* Enthalpy: Internal energy plus the product of pressure and volume. Meaning of thermodynamic potential. Having the same dimension with the amount of heat or energy
* Reference: Smith, J.M., Introduction to Chemical Engineering Thermodynamics, 7th edition, McGraw-Hill, 2005, pp.199-253

Mathematical Model of the Liquefaction Cycle - Equations of State

- **Physical Constraint based on Thermodynamics #2**

Equations of state

$$P_1 v_1 = RT_1$$

[Equation of state for an ideal gas(Boyle-Charles' Law)]

To improve the equation of state for the liquids and vapors, the equation of state for an ideal gas is modified by using the experiment and experience.

Example) Soave, Redlich, Kwong(SRK) equation

$$v = \frac{RT}{P} + b - \frac{a(T)}{P}\frac{v-b}{(v-\varepsilon b)(v-\sigma b)}$$

$$v_1 = \frac{RT_1}{P} + b - \frac{a(T_1)}{P_1}\frac{v_1-b}{(v_1-\varepsilon b)(v_1-\sigma b)}$$

$$v_{4,v} = \frac{RT_4}{P} + b - \frac{a(T_4)}{P_4}\frac{v_{4,v}-b}{(v_{4,v}-\varepsilon b)(v_{4,v}-\sigma b)}$$

$$v_{4,l} = \frac{RT_4}{P} + b - \frac{a(T_4)}{P_4}\frac{v_{4,l}-b}{(v_{4,l}-\varepsilon b)(v_{4,l}-\sigma b)}$$

Example) Ammonia:
$\omega = 0.253$, P_c=112.80 (bar), T_c=405.7 (K)

s: specific entropy

$a(T) = \psi \frac{\alpha(T_r) R^2 T_C^2}{P_C}$

$\psi = 0.42748$ for SRK equation

R: gas constant (=8.314 Jmol⁻¹K⁻¹)

P_c: critical pressure of the refrigerant

T_c: critical temperature of the refrigerant

$b = \Omega \frac{RT_C}{P_C}$ $\Omega = 0.08664$ for SRK equation, $\varepsilon = 0$ for SRK equation, $\sigma = 1$ for SRK equation

* Equations of state: Thermodynamic equation describing the state of matter under a given set of physical conditions
* Classical ideal gas law (Boyle-Charles's law) : Combination of Boyle's law (gas volume varies inversely with the pressure.) and Charles' law (linear relationship between volume and temperature)

Mathematical Model of the Liquefaction Cycle - Calculation of Specific Entropy (s)

$$ds = \frac{dq}{T}$$

- **Physical Constraint based on Thermodynamics #3**

Criteria for quality of the energy

Calculation of

To allow calculation of entropy from the pressure, specific volume, and temperature, the following equation is derived by using the definition($ds=dq/T$), the equation of state, and the experiment.

$$s = s^{IG} + s^{R}$$

s^{ig}: **Ideal gas value of the entropy**
s^{R}: Residual entropy(correction of the ideal gas state values to the real gas values)

* Entropy: Unavailable energy. Total energy = Available energy (enthalpy) + Unavailable energy (Entropy).
In a natural thermodynamic process, there is an increase in the sum of the entropies of the participating systems (The second law of thermodynamics)
* Reference: Smith, J.M., Introduction to Chemical Engineering Thermodynamics, 7th edition, McGraw-Hill, 2005, pp.199-253

Mathematical Model of the Liquefaction Cycle - Physical Assumptions

- **Physical Constraint based on Thermodynamics #4**

Physical assumptions for the liquefaction process

"Isobaric process"
- There is no pressure drop.

"Adiabatic process"
- There is no sufficient time to transfer much heat.

"Isentropic process"
- "Entropy" does not change.
- "Adiabatic process" and "Reversible"

[Summary] Mathematical Model for the Determination of Optimal Operating Condition for the Refrigerator (1/2)

T: Temperature, h: specific enthalpy, s: specific entropy
P: Pressure
v: Specific volume
w: Power provided to the compressor per mass
q_H: Specific heat transfer from the refrigerant to the atmosphere
q_L: Specific heat transfer from the refrigerated space to the refrigerant(Given)
V_f: Vapor fraction

1. Compressor

1) Design Variables: $P_1, v_1, T_1, P_2, v_2, T_2, T_S, w$

2) Constraint:

$$h_1(P_1,v_1,T_1)+w=h_2(P_2,v_2,T_2)$$ [The first law of the thermodynamics]

$$\eta=\frac{h_2(P_2,v_2,T_2)-h_1(P_1,v_1,T_1)}{h_S(P_2,v_S,T_S)-h_1(P_1,v_1,T_1)}$$ [Efficiency of the compressor]

$$s_1(P_1,v_1,T_1)=s_2(P_2,v_S,T_S)$$ [The second law of the thermodynamics]

$$v_1=\frac{RT_1}{P}+b-\frac{a(T_1)}{P_1}\frac{v_1-b}{(v_1-\varepsilon b)(v_1-\sigma b)}$$ [Equation of state]

$$v_2=\frac{RT_2}{P}+b-\frac{a(T_2)}{P_2}\frac{v_2-b}{(v_2-\varepsilon b)(v_2-\sigma b)}$$ [Equation of state]

2. Condenser

1) Design Variables: $P_2, v_2, T_2, P_3, v_3, T_3, q_H$

2) Constraint:

$$h_2(P_2,v_2,T_2)=q_H+h_3(P_3,v_3,T_3)$$ [The first law of the thermodynamics]

$$P_2=P_3$$ [Isobaric process]

$$v_2=\frac{RT_2}{P}+b-\frac{a(T_2)}{P_2}\frac{v_2-b}{(v_2-\varepsilon b)(v_2-\sigma b)}$$ [Equation of state]

$$v_3=\frac{RT_3}{P}+b-\frac{a(T_3)}{P_3}\frac{v_3-b}{(v_3-\varepsilon b)(v_3-\sigma b)}$$ [Equation of state]

$$P_3=10^{A-\frac{B}{T_3+C-273.15}}$$ [Saturated pressure and temperature]

$$T_3>T_{amb}+\Delta T_{\min}$$ [Outlet temperature of the condenser]

3. Expansion Valve

1) Design Variables: $P_3, v_3, T_3, P_4, v_4, T_4, v_{4,l}, v_{4,v}, v\_f$

2) Constraint:

$$h_3(P_3,v_3,T_3)=(1-v\_f)\cdot h_{4,l}(P_4,v_{4,l},T_4)+v\_f\cdot h_{4,v}(P_4,v_{4,v},T_4)$$ [The first law of the thermodynamics]

$$P_4=10^{A-\frac{B}{T_4+C-273.15}}$$ [Saturated pressure and temperature]

$$v_3=\frac{RT_3}{P}+b-\frac{a(T_3)}{P_3}\frac{v_3-b}{(v_3-\varepsilon b)(v_3-\sigma b)}$$ [Equation of state]

$$v_4=(1-v\_f)\cdot v_{4,l}+v\_f\cdot v_{4,v}$$

$$v_{4,l}=\frac{RT_4}{P}+b-\frac{a(T_4)}{P_4}\frac{v_{4,l}-b}{(v_{4,l}-\varepsilon b)(v_{4,l}-\sigma b)}$$ [Equation of state]

$$v_{4,v}=\frac{RT_4}{P}+b-\frac{a(T_4)}{P_4}\frac{v_{4,v}-b}{(v_{4,v}-\varepsilon b)(v_{4,v}-\sigma b)}$$ [Equation of state]

4. Evaporator

1) Design Variables: $P_4, v_4, T_4, P_1, v_1, T_1, v_{4,l}, v_{4,v}, v\_f, M, q_L$

2) Constraint:

$$M\cdot(1-v\_f)\cdot h_{4,l}(P_4,v_{4,l},T_4)+M\cdot v\_f\cdot h_{4,v}(P_4,v_{4,v},T_4)+M\cdot q_L=M\cdot h_1(P_1,v_1,T_1)$$ [The first law of the thermodynamics]

$$P_4=P_1$$ [Isobaric process]

$$v_1=\frac{RT_1}{P}+b-\frac{a(T_1)}{P_1}\frac{v_1-b}{(v_1-\varepsilon b)(v_1-\sigma b)}$$ [Equation of state]

$$v_4=(1-v\_f)\cdot v_{4,l}+v\_f\cdot v_{4,v}$$

$$v_{4,l}=\frac{RT_4}{P}+b-\frac{a(T_4)}{P_4}\frac{v_{4,l}-b}{(v_{4,l}-\varepsilon b)(v_{4,l}-\sigma b)}$$ [Equation of state]

$$v_{4,v}=\frac{RT_4}{P}+b-\frac{a(T_4)}{P_4}\frac{v_{4,v}-b}{(v_{4,v}-\varepsilon b)(v_{4,v}-\sigma b)}$$ [Equation of state]

$$P_1=10^{A-\frac{B}{T_1+C-273.15}}$$ [Saturated pressure and temperature]

$$M\cdot q_L=20[kW]$$ [Given]

[Summary] Mathematical Model for the Determination of Optimal Operating Condition for the Refrigerator (2/2)

T: Temperature, h: specific enthalpy, s: specific entropy
P: Pressure
v: Specific volume
w: Power provided to the compressor per mass
q_H: Specific heat transfer from the refrigerant to the atmosphere
q_L: Specific heat transfer from the refrigerated space to the refrigerant(Given)
U: Heat transfer coefficient of the evaporator
A: Area of the evaporator

1. Design variables (Operating conditions) [21]: $P_i, T_i, v_i, T_s, v_S, v_{4,l}, v_{4,v}, v\_f, w, M, q_L, q_H$ (i=1,2,3,4)

2. Equality constraints [19]

1) Compressor [6]

$\dot{m}\cdot h_1(P_1,v_1,T_1)+\dot{m}\cdot w=\dot{m}h_2(P_2,v_2,T_2)$ **[The first law of the thermodynamics]**

$\eta=\dfrac{h_S(P_2,v_S,T_S)-h_1(P_1,v_1,T_1)}{h_2(P_2,v_2,T_2)-h_1(P_1,v_1,T_1)}$ **[The second law of the thermodynamics]**

$s_1(P_1,v_1,T_1)=s_2(P_2,v_S,T_S)$

$v_1=\dfrac{RT_1}{P_1}+b-\dfrac{a(T_1)}{P_1}\dfrac{v_1-b}{(v_1-\varepsilon b)(v_1-\sigma b)}$ **[Equation of state]**

$v_2=\dfrac{RT_2}{P_2}+b-\dfrac{a(T_2)}{P_2}\dfrac{v_2-b}{(v_2-\varepsilon b)(v_2-\sigma b)}$ $\quad v_S=\dfrac{RT_S}{P_2}+b-\dfrac{a(T_S)}{P_2}\dfrac{v_S-b}{(v_S-\varepsilon b)(v_S-\sigma b)}$

2) Condenser [4]

$h_2(P_2,v_2,T_2)=q_H+h_3(P_3,v_3,T_3)$ **[The first law of the thermodynamics]**

$P_2=P_3$ **[Isobaric process]**

$v_3=\dfrac{RT_3}{P_3}+b-\dfrac{a(T_3)}{P_3}\dfrac{v_3-b}{(v_3-\varepsilon b)(v_3-\sigma b)}$ **[Equation of state]**

$\dfrac{P_3}{10^5}=10^{A-\frac{B}{T_3+C-273.15}}$ **[Saturated pressure and temperature]**

3) Expansion valve [5]

$h_3(P_3,v_3,T_3)$
$=(1-v\_f)\cdot h_{4,l}(P_4,v_{4,l},T_4)+v\_f\cdot h_{4,v}(P_4,v_{4,v},T_4)$ **[The first law of the thermodynamics]**

$\dfrac{P_4}{10^5}=10^{A-\frac{B}{T_4+C-273.15}}$ **[Saturated pressure and temperature]**

$v_{4,l}=\dfrac{RT_4}{P_4}+b-\dfrac{a(T_4)}{P_4}\dfrac{v_{4,l}-b}{(v_{4,l}-\varepsilon b)(v_{4,l}-\sigma b)}$ $\quad v_{4,v}=\dfrac{RT_4}{P_4}+b-\dfrac{a(T_4)}{P_4}\dfrac{v_{4,v}-b}{(v_{4,v}-\varepsilon b)(v_{4,v}-\sigma b)}$

$v_4=(1-v\_f)\cdot v_{4,l}+v\_f\cdot v_{4,v}$

4) Evaporator [4]

$M\cdot(1-v\_f)\cdot h_{4,l}(P_4,v_{4,l},T_4)+M\cdot v\_f\cdot h_{4,v}(P_4,v_{4,v},T_4)+M\cdot q_L$
$=M\cdot h_1(P_1,v_1,T_1)$ **[The first law of the thermodynamics]**

$P_4=P_1$ **[Isobaric process]**

$\dfrac{P_1}{10^5}=10^{A-\frac{B}{T_1+C-273.15}}$ $\quad M\cdot q_L=20[kW]$ **[Saturated pressure and temperature]** **[Heat transfer in the evaporator]**

3. Inequality constraint [1]

$T_3>T_{amb}+\Delta T_{\min}$ **[Outlet temperature of the condenser]**

Since the number of equality constraints is less than the number of design variables, these equations form an indeterminate problem.

We need a certain criteria to determine the proper solution. By introducing the criteria(objective function), this problem can be formulated as an optimization problem.

4. Objective function (f)

Minimize the power provided to the compressor.

$$f=M\cdot w$$

Mathematical Model for the Determination of Optimal Operating Condition for the Refrigerator

- Optimization Method

Optimization Problem

1. Design variables (Operating conditions) [21]
   : $P_i, T_i, v_i, T_s, v_S, v_{4,l}, v_{4,v}, v\_f, w, M, q_L, q_H$ (i=1,2,3,4)

2. Equality constraints [19]
   1) Compressor (6)
   2) Condenser (4)
   3) Expansion valve (5)
   4) Evaporator (4)

3. Inequality constraints [1]

4. Objective function: **Minimize the compressors power**

Minize $f=M\cdot w$

T: Temperature, h: specific enthalpy, s: specific entropy
P: Pressure
v: Specific volume
w: Power provided to the compressor per mass
q_H: Specific heat transfer from the refrigerant to the atmosphere
q_L: Specific heat transfer from the refrigerated space to the refrigerant(Given)

Modified optimization problem

- **Free variables** [2 = 21 - 19]
  : T_1, P_2
- **Inequality constraints** [1]

④ Calculation of objective function

Minize $f=M\cdot w$

⑤ Assume the free variable to calculate smaller value of the objective function using sequential quadratic programming(SQP) method

① Assume the free variable T_1, P_2

③ Determined 19 variables

System of nonlinear equations

- **Design variables** [19]
- **Equality constraints** [19]

② ➡ Determine the 19 variables using Newton-Raphson method

Mathematical Model for the Determination of Optimal Operating Condition for the Refrigerator

- Optimization Result

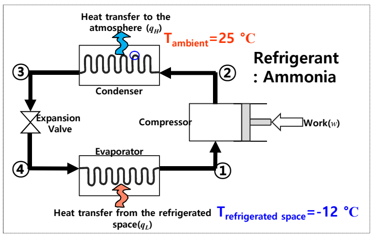
Problem[1)]

Given: $M \cdot q_L = 20[kW], U = 1,000[Wm^2/K],\ A = 4.0\ [m^2],$
$T_C = -12°C, \eta = 95\%,\ T_{amb} = 25°C, \Delta T_{\min} = 5°C$

where $\dot{m} \cdot q_L$: Rate of heat transfer from the refrigerated space to the refrigerant
Tc: temperature of the refrigerated space
η: efficiency of the compressorwhere
T_{amb}: ambient temperature
ΔT_{min}: minimum value of the difference between the ambient temperature and outlet temperature in the condenser

Find: Operating condition

Minimize $f = M \cdot w$: Power provided to the compressor$[kW]$



Optimization result

Items	Result obtained in this study
P_1[bar]	2.115
T_1[K]	256.152
v_1[m³/mol]	0.0098128
P_2[bar]	11.717
T_2[K]	390.278
v_2[m³/mol]	0.0026551
P_3[bar]	11.717
T_3[K]	303.273
v_3[m³/mol]	0.0000365
P_4[bar]	2.114
T_4[K]	256.151
v_4[m³/mol]	0.0017003
v_f	0.1705
v4v[m³/mol]	0.0098130
v4l[m³/mol]	0.0000327
T_s[K]	384.793
v_s[m³/mol]	0.0026119
M[kg/s]	0.0176
w[J/g]	265.698
q_L[J/g]	1,136.364
q_H[J/g]	1,402.676
Objective function(W)	4.672

* Reference: 1) Jensen, J.B., 2008, Optimal Operation of Refrigeration Cycles, Ph.D. thesis, Norwegian University of Science and Technology.

Mathematical Model for the Determination of Optimal Operating Condition for the Refrigerator

- Verification of the Mathematical Model and Optimization Result (1/3)

How can we verify the mathematical model of this research for refrigerator?

<Input>

Assume the 2 design variables

Mathematical model:
- Design variables [21]
- Equality constraints [19]

<Output>

The values of the other design variables

	Mathematical Model
P_1[bar]	2.113
T_1[K]	**256.150**
v_1[m³/mol]	0.0098157
P_2[bar]	**11.698**
T_2[K]	390.001
v_2[m³/mol]	0.0026570
P_3[bar]	11.698
T_3[K]	303.180
v_3[m³/mol]	0.0000365
P_4[bar]	2.113
T_4[K]	256.150
v_4[m³/mol]	0.0017269
v_f	0.1732
v4v[m³/mol]	0.0098157
v4l[m³/mol]	0.0000327
T_s[K]	384.296
v_s[m³/mol]	0.0026153
M[kg/s]	0.0176
w[J/g]	268.657
q_L[J/g]	1,136.364
q_H[J/g]	1,402.189

- **Mathematical Model of this research**
  1. Design variables (Operating conditions) [21]
  2. Equality constraints [19]

  ➡ Indeterminate problem

To verify the mathematical model this research, we assume the values of the two design variables, solve and compare the result with that of the Aspen HYSYS.

Mathematical Model for the Determination of Optimal Operating Condition for the Refrigerator

- Verification of the Mathematical Model and Optimization Result (2/3)

How can we verify the mathematical model of this research for refrigerator?

<Input>

Assume the 2 design variables

Mathematical model:
- Design variables (21)
- Equality constraints (19)

<Output>

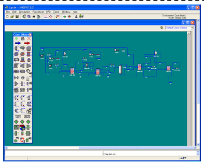
The values of the other design variables

	Mathematical Model
P_1[bar]	2.113
T_1[K]	**256.150**
v_1[m^3/mol]	0.0098157
P_2[bar]	**11.698**
T_2[K]	390.001
v_2[m^3/mol]	0.0026570
P_3[bar]	11.698
T_3[K]	303.180
v_3[m^3/mol]	0.0000365
P_4[bar]	2.113
T_4[K]	256.150
v_4[m^3/mol]	0.0017269
v_f	0.1732
v4v[m^3/mol]	0.0098157
v4l[m^3/mol]	0.0000327
T_s[K]	384.296
v_s[m^3/mol]	0.0026153
M[kg/s]	0.0176
w[J/g]	268.657
q_L[J/g]	1,136.364
q_H[J/g]	1,402.189

Comparison for verifying the mathematical model

<Input>

Assume the 2 design variables



Aspen HYSYS[1)]

There are equality constraints, but it is difficult to find the equality constraints(Black box).

<Output>

The values of the other design variables

	Aspen HYSYS	Difference[%]
P_1[bar]	2.117	0.16%
T_1[K]	**256.150**	0.00%
v_1[m^3/mol]	0.0098003	0.16%
P_2[bar]	**11.698**	0.00%
T_2[K]	389.863	0.04%
v_2[m^3/mol]	0.0026562	0.03%
P_3[bar]	11.698	0.00%
T_3[K]	303.150	0.01%
v_3[m^3/mol]	0.0000365	0.00%
P_4[bar]	2.117	0.16%
T_4[K]	256.151	0.00%
v_4[m^3/mol]	0.0017234	0.20%
v_f	0.1731	0.04%
v4v[m^3/mol]	0.0098003	0.16%
v4l[m^3/mol]	0.0000327	0.01%
T_s[K]	384.165	0.03%
v_s[m^3/mol]	0.0026116	0.14%
M[kg/s]	0.0176	0.05%
w[J/g]	268.353	0.11%
q_L[J/g]	1,136.364	0.00%
q_H[J/g]	1,401.532	0.05%
Max difference		0.20%

1) It is a kind of simulation program to solve the simultaneous equations by considering equality constraint related with thermodynamics and made by Aspentech.

The mathematical model can be verified by comparing between the values of the other design variables obtained by mathematical model of this research and Aspen HYSYS.

Mathematical Model for the Determination of Optimal Operating Condition for the Refrigerator

- Verification of the Mathematical Model and Optimization Result (3/3)

Indeterminate problem

1. Design variables (Operating conditions) [21]
   : P_i, T_i, v_i, T_s, v_S, $v_{4,l}$, $v_{4,v}$, v_f, w, M, q_L, q_H (i=1,2,3,4)
2. Equality constraints [19]
   1) Compressor [6]
   2) Condenser [4]
   3) Expansion valve [5]
   4) Evaporator [4]

- **Assumption of the values of the free variables** [2 = 21 - 19]
  : $T_1 = 256.150[K]$
  $P_2 = 11.698[bar]$

Problem of nonlinear equations

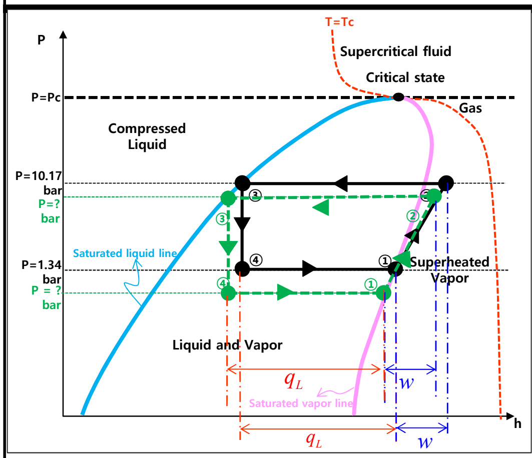
- Design variables [19]
- Equality constraints [19]

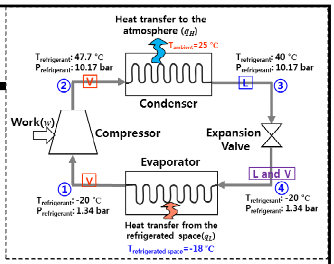
➡ **Use of Newton-Raphson method**

	Mathematical Model
P_1[bar]	2.113
T_1[K]	**256.150**
v_1[m^3/mol]	0.0098157
P_2[bar]	**11.698**
T_2[K]	390.001
v_2[m^3/mol]	0.0026570
P_3[bar]	11.698
T_3[K]	303.180
v_3[m^3/mol]	0.0000365
P_4[bar]	2.113
T_4[K]	256.150
v_4[m^3/mol]	0.0017269
v_f	0.1732
v4v[m^3/mol]	0.0098157
v4l[m^3/mol]	0.0000327
T_s[K]	384.296
v_s[m^3/mol]	0.0026153
M[kg/s]	0.0176
w[J/g]	268.657
q_L[J/g]	1,136.364
q_H[J/g]	1,402.189

2. Optimal Synthesis of Liquefaction Cycle

Optimal Design of Liquefaction Cycle for LNG FPSO





- q_L: specific heat transfer from the refrigerated space to the refrigerant(Given)
- w: work provided to the compressor(Minimizing)

- **Given:** The quantity of the specific heat transfer from the refrigerated space to the refrigerant(q_L) in the evaporator.

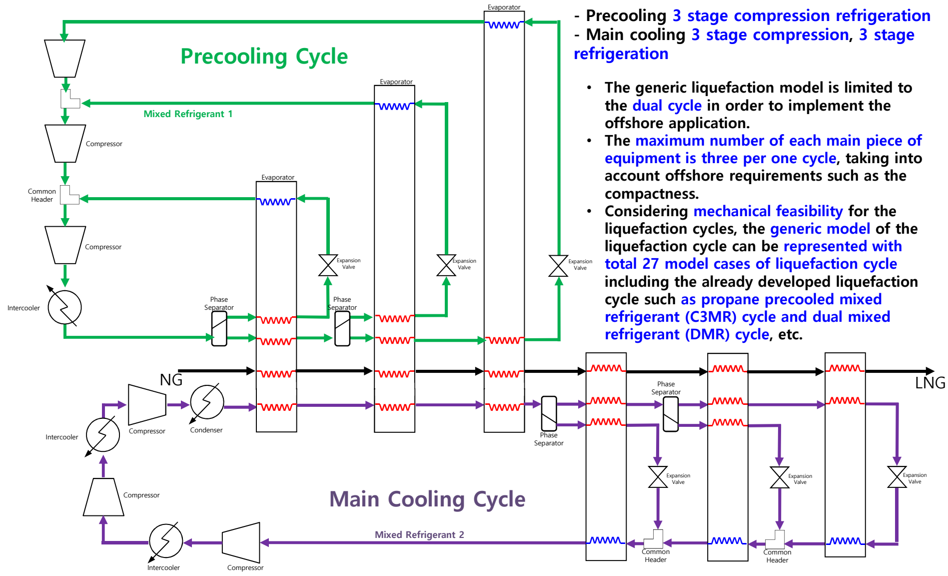
⇩ **Constraint**

1. Determination of the optimal operating condition
   - **Find:** The operating conditions such as the pressure, temperature and specific volume (Design variables) minimizing the work provided to the compressor. ⇨ Objective function

2. Optimal synthesis on liquefaction cycles
   - **Find:** The number of the combination of equipment that make up the liquefaction cycle (Design variables) minimizing the work provided to the compressor. ⇨ Objective function

Optimal Synthesis of the Liquefaction Cycle
- Generic Model of the Liquefaction Cycle of LNG FPSO

Generic Model: Dual Cycle with Regeneration + Multistage Compression with Intercooling + Multistage Compression Refrigeration + Multistage Refrigeration

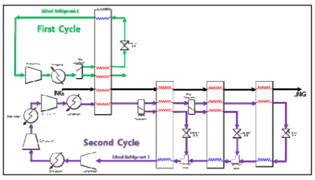


- Precooling 3 stage compression refrigeration
- Main cooling 3 stage compression, 3 stage refrigeration

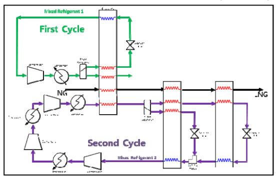
- The generic liquefaction model is limited to the dual cycle in order to implement the offshore application.
- The maximum number of each main piece of equipment is three per one cycle, taking into account offshore requirements such as the compactness.
- Considering mechanical feasibility for the liquefaction cycles, the generic model of the liquefaction cycle can be represented with total 27 model cases of liquefaction cycle including the already developed liquefaction cycle such as propane precooled mixed refrigerant (C3MR) cycle and dual mixed refrigerant (DMR) cycle, etc.

Optimal Synthesis of the Liquefaction Cycle
- 27 Feasible Liquefaction Cycles (Case 1 ~ Case 9)

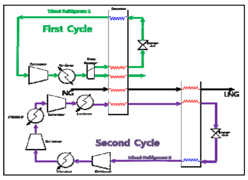
FEASIBLE LIQUEFACTION MODEL (CASE 1)



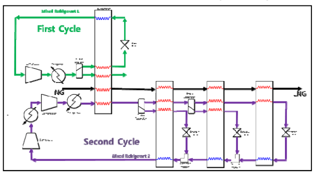
FEASIBLE LIQUEFACTION MODEL (CASE 2)



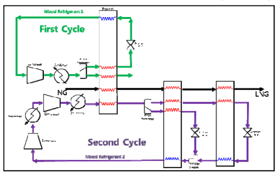
FEASIBLE LIQUEFACTION MODEL (CASE 3)



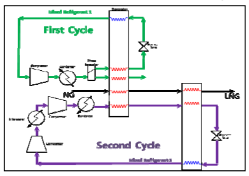
FEASIBLE LIQUEFACTION MODEL (CASE 4)



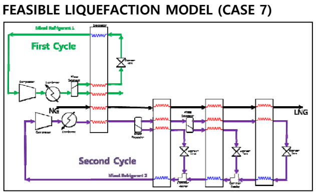
FEASIBLE LIQUEFACTION MODEL (CASE 5)



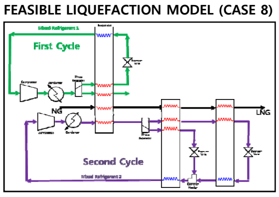
FEASIBLE LIQUEFACTION MODEL (CASE 6)



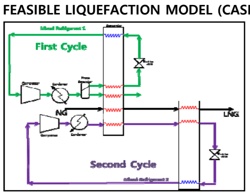
FEASIBLE LIQUEFACTION MODEL (CASE 7)



FEASIBLE LIQUEFACTION MODEL (CASE 8)

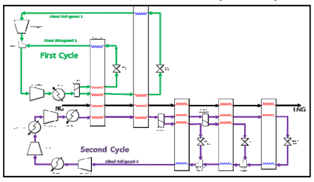


FEASIBLE LIQUEFACTION MODEL (CASE 9)

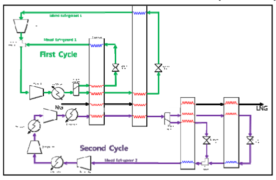


Optimal Synthesis of the Liquefaction Cycle
- 27 Feasible Liquefaction Cycles (Case 10 ~ Case 18)

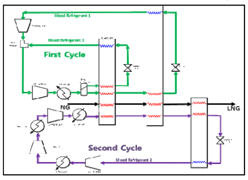
FEASIBLE LIQUEFACTION MODEL (CASE 10)



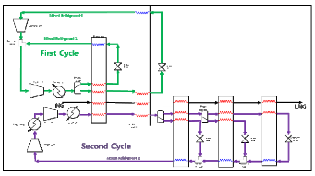
FEASIBLE LIQUEFACTION MODEL (CASE 11)



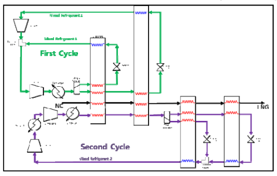
FEASIBLE LIQUEFACTION MODEL (CASE 12)



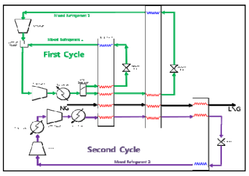
FEASIBLE LIQUEFACTION MODEL (CASE 13)



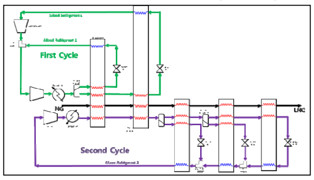
FEASIBLE LIQUEFACTION MODEL (CASE 14)



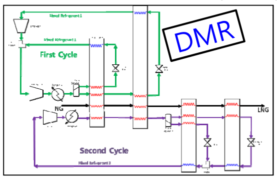
FEASIBLE LIQUEFACTION MODEL (CASE 15)



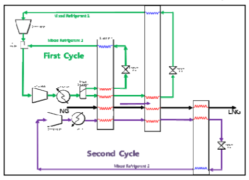
FEASIBLE LIQUEFACTION MODEL (CASE 16)



FEASIBLE LIQUEFACTION MODEL (CASE 17)



FEASIBLE LIQUEFACTION MODEL (CASE 18)



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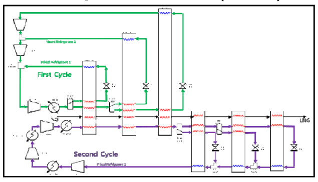
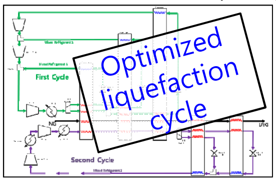
Optimal Synthesis of the Liquefaction Cycle
- 27 Feasible Liquefaction Cycles (Case 19 ~ Case 27)

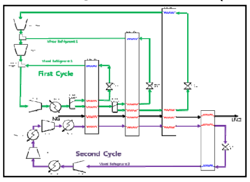
FEASIBLE LIQUEFACTION MODEL (CASE 19)



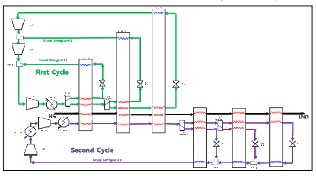
FEASIBLE LIQUEFACTION MODEL (CASE 20)



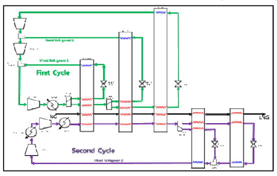
FEASIBLE LIQUEFACTION MODEL (CASE21)



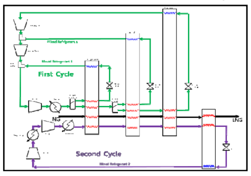
FEASIBLE LIQUEFACTION MODEL (CASE 22)



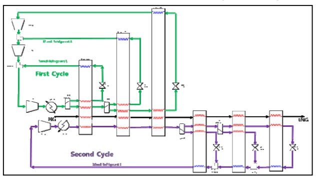
FEASIBLE LIQUEFACTION MODEL (CASE 23)



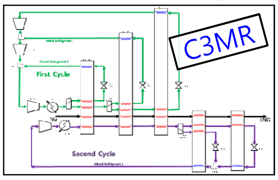
FEASIBLE LIQUEFACTION MODEL (CASE 24)



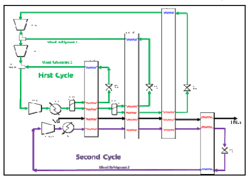
FEASIBLE LIQUEFACTION MODEL (CASE 25)



FEASIBLE LIQUEFACTION MODEL (CASE 26)



FEASIBLE LIQUEFACTION MODEL (CASE 27)



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Mathematical Model for the Determination of Optimal Operating Condition of the Generic Liquefaction Cycle (1/6)

1. Design variables (Operating conditions) [187]

T: Temperature / P: Pressure / v: Specific volume / $z_{j,pre}$: mole fraction of the component j at the precooling part / w: work input to the compressor per mass / c: flow rate ratio between inlet and outlet 4 / m_{pre}: mass flow rate at the precooling refrigerant
*Subscript 'NG': natural gas, Subscript 'main': main cooling refrigerant

$$: P_i,\ T_i,\ v_i\left(i=1_p,\ldots,21_p,1_m,\ldots,26_m,1_{NG},\ldots,5_{NG}\right),$$
$$T_{S,2p},T_{S,19p},T_{S,21p},T_{S,2m},T_{S,4m},T_{S,6m},v_{S,2p},v_{S,19p},v_{S,21p},v_{S,2m},v_{S,4m},v_{S,6m},$$
$$w_1,\ w_2,\ w_3,w_4,\ w_5,\ w_6,c_1,c_2,\dot{m}_{pre},\ \dot{m}_{main},v\_f_{10},v\_f_{15},z_{j,pre}\left(j=1,2,3\right),z_{k,main}\left(k=1,2,3,4\right)$$

2. Equality constraints [165]

2.1 Equality constraints of precooling part [83]

1) Compressor 1: [6]

$$h_{1p}\left(P_{1p},T_{1p},v_{1p},z_{j,pre}\right)+w_1=h_{2p}\left(P_{2p},T_{2p},v_{2p},z_{j,pre}\right)$$
$$\eta=\frac{h_{S,2p}\left(P_{2p},T_{S,2p},v_{S,2p},z_{j,pre}\right)-h_{1p}\left(P_{1p},T_{1p},v_{1p},z_{j,pre}\right)}{h_{2p}\left(P_{2p},T_{2p},v_{2p},z_{j,pre}\right)-h_{1p}\left(P_{1p},T_{1p},v_{1p},z_{j,pre}\right)}$$
$$s_{1p}\left(P_{1p},T_{1p},v_{1p},z_{j,pre}\right)=s_{2p}\left(P_{2p},T_{S,2p},v_{S,2p},z_{j,pre}\right)$$
$$v_{1p}=v_{1p}\left(P_{1p},T_{1p},z_{j,pre}\right)$$
$$v_{S,2p}=v_{S,2p}\left(P_{2p},T_{S,2p},z_{j,pre}\right)$$
$$v_{2p}=v_{2p}\left(P_{2p},T_{2p},z_{j,pre}\right)$$

2) Condenser 1: [3]

The temperature of the outlet of the sea water cooler is usually given. T=310K

$$P_{2p}=P_{3p}$$
$$v_{3p}=v_{3p}(T_{3p},P_{3p},z_{j,pre})$$

3) Tee 1: [6]

$$h_{3p}\left(P_{3p},T_{3p},v_{3p},z_{j,pre}\right)=c_1\cdot h_{4p}\left(P_{4p},T_{4p},v_{4p},z_{j,pre}\right)+(1-c_1)\cdot h_{8p}\left(P_{8p},T_{8p},v_{8p},z_{j,pre}\right)$$
$$P_{3p}=P_{4p},\ \ P_{3p}=P_{8p}$$
$$T_{4p}=T_{8p}$$
$$v_{4p}=v_{4p}\left(T_{4p},P_{4p},z_{j,pre}\right),\ v_{8p}=v_{8p}\left(T_{8p},P_{8p},z_{j,pre}\right)$$

4) Evaporator 1: [14]

$$c_1\cdot\dot{m}_{pre}\cdot h_{4p}\left(P_{4p},T_{4p},v_{4p},z_{j,pre}\right)+c_1\cdot\dot{m}_{pre}\cdot h_{6p}\left(P_{6p},T_{6p},v_{6p},z_{j,pre}\right)$$
$$+(1-c_1)\cdot\dot{m}_{pre}\cdot h_{8p}\left(P_{8p},T_{8p},v_{8p},z_{j,pre}\right)$$
$$+\dot{m}_{main}\cdot h_{7m}\left(P_{7m},T_{7m},v_{7m},z_{k,main}\right)+\dot{m}_{NG}\cdot h_{NG}\left(P_{NG},T_{NG},v_{NG},z_{l,NG}\right)$$
$$=c_1\cdot\dot{m}_{pre}\cdot h_{5p}\left(P_{5p},T_{5p},v_{5p},z_{j,pre}\right)+c_1\cdot\dot{m}_{pre}\cdot h_{7p}\left(P_{7p},T_{7p},v_{7p},z_{j,pre}\right)$$
$$+(1-c_1)\cdot\dot{m}_{pre}\cdot h_{9p}\left(P_{9p},T_{9p},v_{9p},z_{j,pre}\right)+\dot{m}_{main}\cdot h_{8m}\left(P_{8m},T_{8m},v_{8m},z_{k,main}\right)$$
$$+\dot{m}_{NG}\cdot h_{1NG}\left(P_{1NG},T_{1NG},v_{1NG},z_{l,NG}\right)$$
$$P_{4p}=P_{5p},\ P_{6p}=P_{7p},P_{8p}=P_{9p},P_{7m}=P_{8m},P_{NG}=P_{1NG}$$
$$T_{5p}=T_{9p},T_{5p}=T_{8m},T_{5p}=T_{1NG}$$
$$v_{5p}=v_{5p}\left(T_{5p},P_{5p},z_{j,pre}\right),\ v_{7p}=v_{7p}\left(T_{7p},P_{7p},z_{j,pre}\right),$$
$$v_{9p}=v_{9p}\left(T_{9p},P_{9p},z_{j,pre}\right),\ v_{8m}=v_{8m}\left(T_{8m},P_{8m},z_{k,main}\right),$$
$$v_{1NG}=v_{1NG}\left(T_{1NG},P_{1NG},z_{l,NG}\right)$$

5) Expansion valve 1: [2]

$$h_{5p}\left(P_{5p},T_{5p},v_{5p},z_{j,pre}\right)=h_{6p}\left(P_{6p},T_{6p},v_{6p},z_{j,pre}\right)$$
$$v_{6p}=v_{6p}\left(T_{6p},P_{6p},z_{j,pre}\right)$$

6) Tee 2: [6]

$$(1-c_1)\cdot h_{9p}\left(P_{9p},T_{9p},v_{9p},z_{j,pre}\right)=c_2\cdot(1-c_1)\cdot h_{10p}\left(P_{10p},T_{10p},v_{10p},z_{j,pre}\right)+(1-c_2)\cdot(1-c_1)\cdot h_{14p}\left(P_{14p},T_{14p},v_{14p},z_{j,pre}\right)$$
$$P_{9p}=P_{10p},\ \ P_{9p}=P_{14p}$$
$$T_{10p}=T_{14p}$$
$$v_{10p}=v_{10p}\left(T_{10p},P_{10p},z_{j,pre}\right),$$
$$v_{14p}=v_{14p}\left(T_{14p},P_{14p},z_{j,pre}\right)$$

Mathematical Model for the Determination of Optimal Operating Condition of the Generic Liquefaction Cycle (2/6)

1. Design variables (Operating conditions) [187]

T: Temperature / P: Pressure / v: Specific volume / $z_{j,pre}$: mole fraction of the component j at the precooling part / w: work input to the compressor per mass / c: flow rate ratio between inlet and outlet 4 / m_{pre}: mass flow rate at the precooling refrigerant
*Subscript 'NG': natural gas, Subscript 'main': main cooling refrigerant

$$: P_i,\ T_i,\ v_i\left(i=1_p,\ldots,21_p,1_m,\ldots,26_m,1_{NG},\ldots,5_{NG}\right),$$
$$T_{S,2p},T_{S,19p},T_{S,21p},T_{S,2m},T_{S,4m},T_{S,6m},v_{S,2p},v_{S,19p},v_{S,21p},v_{S,2m},v_{S,4m},v_{S,6m},$$
$$w_1,\ w_2,\ w_3,w_4,\ w_5,\ w_6,c_1,c_2,\dot{m}_{pre},\ \dot{m}_{main},v\_f_{10},v\_f_{15},z_{j,pre}\left(j=1,2,3\right),z_{k,main}\left(k=1,2,3,4\right)$$

2. Equality constraints [165]

2.1 Equality constraints of precooling part [83]

7) Evaporator 2: [14]

$$c_2\cdot(1-c_1)\cdot\dot{m}_{pre}\cdot h_{10p}\left(P_{10p},T_{10p},v_{10p},z_{j,pre}\right)+c_2\cdot(1-c_1)\cdot\dot{m}_{pre}\cdot h_{12p}\left(P_{12p},T_{12p},v_{12p},z_{j,pre}\right)$$
$$+(1-c_2)\cdot(1-c_1)\cdot\dot{m}_{pre}\cdot h_{14p}\left(P_{14p},T_{14p},v_{14p},z_{j,pre}\right)+$$
$$+\dot{m}_{main}\cdot h_{8m}\left(P_{8m},T_{8m},v_{8m},z_{k,main}\right)+\dot{m}_{NG}\cdot h_{1NG}\left(P_{1NG},T_{1NG},v_{1NG},z_{l,NG}\right)$$
$$=c_2\cdot(1-c_1)\cdot\dot{m}_{pre}\cdot h_{11p}\left(P_{11p},T_{11p},v_{11p},z_{j,pre}\right)+c_2\cdot(1-c_1)\cdot\dot{m}_{pre}\cdot h_{13p}\left(P_{13p},T_{13p},v_{13p},z_{j,pre}\right)$$
$$+(1-c_2)\cdot(1-c_1)\cdot\dot{m}_{pre}\cdot h_{15p}\left(P_{15p},T_{15p},v_{15p},z_{j,pre}\right)+$$
$$+\dot{m}_{main}\cdot h_{9m}\left(P_{9m},T_{9m},v_{9m},z_{k,main}\right)+\dot{m}_{NG}\cdot h_{2NG}\left(P_{2NG},T_{2NG},v_{2NG},z_{l,NG}\right)$$
$$P_{10p}=P_{11p},\ P_{12p}=P_{13p},P_{14p}=P_{15p},P_{8m}=P_{9m},P_{1NG}=P_{2NG}$$
$$T_{11p}=T_{15p},T_{11p}=T_{9m},T_{11p}=T_{2NG}$$
$$v_{11p}=v_{11p}\left(T_{11p},P_{11p},z_{j,pre}\right),\ v_{13p}=v_{13p}\left(T_{13p},P_{13p},z_{j,pre}\right),$$
$$v_{15p}=v_{15p}\left(T_{15p},P_{15p},z_{j,pre}\right),\ v_{9m}=v_{9m}\left(T_{9m},P_{9m},z_{k,main}\right),$$
$$v_{2NG}=v_{2NG}\left(T_{2NG},P_{2NG},z_{l,NG}\right)$$

8) Expansion valve 2: [2]

$$h_{11p}\left(P_{11p},T_{11p},v_{11p},z_{j,pre}\right)=h_{12p}\left(P_{12p},T_{12p},v_{12p},z_{j,pre}\right)$$
$$v_{12p}=v_{12p}\left(T_{12p},P_{12p},z_{j,pre}\right)$$

9) Evaporator 3: [11]

$$(1-c_2)\cdot(1-c_1)\cdot\dot{m}_{pre}\cdot h_{15p}\left(P_{15p},T_{15p},v_{15p},z_{j,pre}\right)$$
$$+(1-c_2)\cdot(1-c_1)\cdot\dot{m}_{pre}\cdot h_{17p}\left(P_{17p},T_{17p},v_{17p},z_{j,pre}\right)$$
$$+\dot{m}_{main}\cdot h_{9m}\left(P_{9m},T_{9m},v_{9m},z_{k,main}\right)+\dot{m}_{NG}\cdot h_{2NG}\left(P_{2NG},T_{2NG},v_{2NG},z_{l,NG}\right)$$
$$=(1-c_2)\cdot(1-c_1)\cdot\dot{m}_{pre}\cdot h_{16p}\left(P_{16p},T_{16p},v_{16p},z_{j,pre}\right)$$
$$+(1-c_2)\cdot(1-c_1)\cdot\dot{m}_{pre}\cdot h_{18p}\left(P_{18p},T_{18p},v_{18p},z_{j,pre}\right)$$
$$+\dot{m}_{main}\cdot h_{10m}\left(P_{10m},T_{10m},v_{10m},z_{k,main}\right)+\dot{m}_{NG}\cdot h_{3NG}\left(P_{3NG},T_{3NG},v_{3NG},z_{l,NG}\right)$$
$$P_{15p}=P_{16p},P_{17p}=P_{18p},P_{9m}=P_{10m},P_{2NG}=P_{3NG}$$
$$T_{16p}=T_{10m},T_{16p}=T_{3NG}$$
$$v_{16p}=v_{16p}\left(T_{16p},P_{16p},z_{j,pre}\right),\ v_{18p}=v_{18p}\left(T_{18p},P_{18p},z_{j,pre}\right),$$
$$v_{10m}=v_{10m}\left(T_{10m},P_{10m},z_{k,main}\right),\ v_{3NG}=v_{3NG}\left(T_{3NG},P_{3NG},z_{l,NG}\right)$$

10) Expansion valve 3: [2]

$$h_{16p}\left(P_{16p},T_{16p},v_{16p},z_{j,pre}\right)=h_{17p}\left(P_{17p},T_{17p},v_{17p},z_{j,pre}\right)$$
$$v_{17p}=v_{17p}\left(T_{17p},P_{17p},z_{j,pre}\right)$$

Mathematical Model for the Determination of Optimal Operating Condition of the Generic Liquefaction Cycle (3/6)

1. Design variables (Operating conditions) [187]

T: Temperature / P: Pressure / v: Specific volume / $z_{j,pre}$: mole fraction of the component j at the precooling part / w: work input to the compressor per mass / c: flow rate ratio between inlet and outlet 4 / m_{pre}: mass flow rate at the precooling refrigerant
*Subscript 'NG': natural gas, Subscript 'main': main cooling refrigerant

: $P_i,\ T_i,\ v_i\left(i=1_p,...,21_p,1_m,...,26_m,1_{NG},...,5_{NG}\right),$

$T_{S,2p}, T_{S,19p}, T_{S,21p}, T_{S,2m}, T_{S,4m}, T_{S,6m}, v_{S,2p}, v_{S,19p}, v_{S,21p}, v_{S,2m}, v_{S,4m}, v_{S,6m},$

$w_1,\ w_2,\ w_3, w_4,\ w_5,\ w_6, c_1, c_2, \dot{m}_{pre},\ \dot{m}_{main}, v\_f_{10}, v\_f_{15}, z_{j,pre}\left(j=1,2,3\right), z_{k,main}\left(k=1,2,3,4\right)$

2. Equality constraints [165]

2.1 Equality constraints of precooling part [83]

11) Compressor 3: [5]

$$(1-c_2)\cdot(1-c_1)\cdot\dot{m}_{pre}\cdot h_{18p}\left(P_{18p},T_{18p},v_{18p},z_{j,pre}\right)+w_3$$
$$=(1-c_2)\cdot(1-c_1)\cdot\dot{m}_{pre}\cdot h_{19p}\left(P_{19p},T_{19p},v_{19p},z_{j,pre}\right)$$

$$\eta=\frac{h_{S,19p}\left(P_{19p},T_{S,19p},v_{S,19p},z_{j,pre}\right)-h_{18p}\left(P_{18p},T_{18p},v_{18p},z_{j,pre}\right)}{h_{19p}\left(P_{19p},T_{19p},v_{19p},z_{j,pre}\right)-h_{18p}\left(P_{18p},T_{18p},v_{18p},z_{j,pre}\right)}$$

$$s_{18p}\left(P_{18p},T_{18p},v_{18p},z_{j,pre}\right)=s_{19p}\left(P_{19p},T_{S,19p},v_{S,19p},z_{j,pre}\right)$$

$$v_{19p}=v_{19p}\left(P_{19p},T_{19p},z_{j,pre}\right)$$

$$v_{S,19p}=v_{S,19p}\left(P_{19p},T_{S,19p},z_{j,pre}\right)$$

12) Common Header 2: [4]

$$c_2\cdot(1-c_1)\cdot h_{13p}\left(P_{13p},T_{13p},v_{13p},z_{j,pre}\right)+(1-c_2)\cdot(1-c_1)\cdot h_{19p}\left(P_{19p},T_{19p},v_{19p},z_{j,pre}\right)$$
$$=(1-c_1)\cdot h_{20p}\left(P_{20p},T_{20p},v_{20p},z_{j,pre}\right)$$

$$P_{13p}=P_{19p},\ P_{13p}=P_{20p}$$

$$v_{20p}=v_{20p}\left(P_{20p},T_{20p},z_{j,pre}\right)$$

13) Compressor 2: [5]

$$(1-c_1)\cdot\dot{m}_{pre}\cdot h_{20p}\left(P_{20p},T_{20p},v_{20p},z_{j,pre}\right)+w_2$$
$$=(1-c_1)\cdot\dot{m}_{pre}\cdot h_{21p}\left(P_{21p},T_{21p},v_{21p},z_{j,pre}\right)$$

$$\eta=\frac{h_{S,21p}\left(P_{21p},T_{S,21p},v_{S,21p},z_{j,pre}\right)-h_{20p}\left(P_{20p},T_{20p},v_{20p},z_{j,pre}\right)}{h_{21p}\left(P_{21p},T_{21p},v_{21p},z_{j,pre}\right)-h_{20p}\left(P_{20p},T_{20p},v_{20p},z_{j,pre}\right)}$$

$$s_{20p}\left(P_{20p},T_{20p},v_{20p},z_{j,pre}\right)=s_{21p}\left(P_{21p},T_{S,21p},v_{S,21p},z_{j,pre}\right)$$

$$v_{21p}=v_{21p}\left(P_{21p},T_{21p},z_{j,pre}\right)$$

$$v_{S,21p}=v_{S,21p}\left(P_{21p},T_{S,21p},z_{j,pre}\right)$$

14) Common Header 1: [3]

$$(1-c_1)\cdot h_{21p}\left(P_{21p},T_{21p},v_{21p},z_{j,pre}\right)+c_1\cdot h_{7p}\left(P_{7p},T_{7p},v_{7p},z_{j,pre}\right)$$
$$=h_{1p}\left(P_{1p},T_{1p},v_{1p},z_{j,pre}\right)$$

$$P_{7p}=P_{21p},\ P_{7p}=P_{1p}$$

Mathematical Model for the Determination of Optimal Operating Condition of the Generic Liquefaction Cycle (4/6)

1. Design variables (Operating conditions) [187]

T: Temperature / P: Pressure / v: Specific volume / $z_{j,pre}$: mole fraction of the component j at the precooling part / w: work input to the compressor per mass / c: flow rate ratio between inlet and outlet 4 / m_{pre}: mass flow rate at the precooling refrigerant
*Subscript 'NG': natural gas, Subscript 'main': main cooling refrigerant

: $P_i,\ T_i,\ v_i\left(i=1_p,...,21_p,1_m,...,26_m,1_{NG},...,5_{NG}\right),$

$T_{S,2p}, T_{S,19p}, T_{S,21p}, T_{S,2m}, T_{S,4m}, T_{S,6m}, v_{S,2p}, v_{S,19p}, v_{S,21p}, v_{S,2m}, v_{S,4m}, v_{S,6m},$

$w_1,\ w_2,\ w_3, w_4,\ w_5,\ w_6, c_1, c_2, \dot{m}_{pre},\ \dot{m}_{main}, v\_f_{10}, v\_f_{15}, z_{j,pre}\left(j=1,2,3\right), z_{k,main}\left(k=1,2,3,4\right)$

2. Equality constraints [165]

2.2 Equality constraints of main cooling part [80]

1) Compressor 6: [6]

$$h_{1m}\left(P_{1m},T_{1m},v_{1m},z_{k,main}\right)+w_6=h_{2m}\left(P_{2m},T_{2m},v_{2m},z_{k,main}\right)$$

$$\eta=\frac{h_{S,2m}\left(P_{2m},T_{S,2m},v_{S,2m},z_{k,main}\right)-h_{1m}\left(P_{1m},T_{1m},v_{1m},z_{k,main}\right)}{h_{2m}\left(P_{2m},T_{2m},v_{2m},z_{k,main}\right)-h_{1m}\left(P_{1m},T_{1m},v_{1m},z_{k,main}\right)}$$

$$s_{1m}\left(P_{1m},T_{1m},v_{1m},z_{k,main}\right)=s_{2m}\left(P_{2m},T_{S,2m},v_{S,2m},z_{k,main}\right)$$

$$v_{1m}=v_{1m}\left(P_{1m},T_{1m},z_{k,main}\right)$$

$$v_{S,2m}=v_{S,2m}\left(P_{2m},T_{S,2m},z_{k,main}\right)$$

$$v_{2m}=v_{2m}\left(P_{2m},T_{2m},z_{k,main}\right)$$

2) Intercooler 2: [3]

The temperature of the outlet of the sea water cooler is usually given.
T=305K

$$P_{2m}=P_{3m}$$

$$v_{3m}=v_{3m}(T_{3m},P_{3m},z_{k,main})$$

3) Compressor 5: [5]

$$h_{3m}\left(P_{3m},T_{3m},v_{3m},z_{k,main}\right)+w_5=h_{4m}\left(P_{4m},T_{4m},v_{4m},z_{k,main}\right)$$

$$\eta=\frac{h_{S,4m}\left(P_{4m},T_{S,4m},v_{S,4m},z_{k,main}\right)-h_{3m}\left(P_{3m},T_{3m},v_{3m},z_{k,main}\right)}{h_{4m}\left(P_{4m},T_{4m},v_{4m},z_{k,main}\right)-h_{3m}\left(P_{3m},T_{3m},v_{3m},z_{k,main}\right)}$$

$$s_{3m}\left(P_{3m},T_{3m},v_{3m},z_{k,main}\right)=s_{4m}\left(P_{4m},T_{S,4m},v_{S,4m},z_{k,main}\right)$$

$$v_{S,4m}=v_{S,4m}\left(P_{4m},T_{S,4m},z_{k,main}\right)$$

$$v_{4m}=v_{4m}\left(P_{4m},T_{4m},z_{k,main}\right)$$

4) Intercooler 1: [3]

The temperature of the outlet of the sea water cooler is usually given.
T=305K

$$P_{4m}=P_{5m}$$

$$v_{5m}=v_{5m}(T_{5m},P_{5m},z_{k,main})$$

5) Compressor 4: [5]

$$h_{5m}\left(P_{5m},T_{5m},v_{5m},z_{k,main}\right)+w_4=h_{6m}\left(P_{6m},T_{6m},v_{6m},z_{k,main}\right)$$

$$\eta=\frac{h_{S,6m}\left(P_{6m},T_{S,6m},v_{S,6m},z_{k,main}\right)-h_{5m}\left(P_{5m},T_{5m},v_{5m},z_{k,main}\right)}{h_{6m}\left(P_{6m},T_{6m},v_{6m},z_{k,main}\right)-h_{5m}\left(P_{5m},T_{5m},v_{5m},z_{k,main}\right)}$$

$$s_{5m}\left(P_{5m},T_{5m},v_{5m},z_{k,main}\right)=s_{6m}\left(P_{6m},T_{S,6m},v_{S,6m},z_{k,main}\right)$$

$$v_{S,6m}=v_{S,6m}\left(P_{6m},T_{S,6m},z_{k,main}\right)$$

$$v_{6m}=v_{6m}\left(P_{6m},T_{6m},z_{k,main}\right)$$

6) Condenser 2: [3]

The temperature of the outlet of the sea water cooler is usually given.
T=305K

$$P_{6m}=P_{7m}$$

$$v_{7m}=v_{7m}(T_{7m},P_{7m},z_{k,main})$$

Mathematical Model for the Determination of Optimal Operating Condition of the Generic Liquefaction Cycle (5/6)

1. Design variables (Operating conditions) [187]

T: Temperature / P: Pressure / v: Specific volume / $z_{j,pre}$: mole fraction of the component j at the precooling part / w: work input to the compressor per mass / c: flow rate ratio between inlet and outlet 4 / $\dot{m}_{pre}$: mass flow rate at the precooling refrigerant
*Subscript 'NG': natural gas, Subscript 'main': main cooling refrigerant

$$: P_i,\ T_i,\ v_i\left(i = 1_p,\dots,21_p,1_m,\dots,26_m,1_{NG},\dots,5_{NG}\right),$$
$$T_{S,2p}, T_{S,19p}, T_{S,21p}, T_{S,2m}, T_{S,4m}, T_{S,6m}, v_{S,2p}, v_{S,19p}, v_{S,21p}, v_{S,2m}, v_{S,4m}, v_{S,6m},$$
$$w_1,\ w_2,\ w_3, w_4,\ w_5,\ w_6, c_1, c_2, \dot{m}_{pre},\ \dot{m}_{main}, v\_f_{10}, v\_f_{15}, z_{j,pre}\left(j=1,2,3\right), z_{k,main}\left(k=1,2,3,4\right)$$

2. Equality constraints [165]

2.2 Equality constraints of main cooling part [80]

7) Phase Separator 1: [7]

$$h_{10m}\left(P_{10m},T_{10m},v_{10m},z_{k,main}\right)$$
$$= v\_f_{10}\cdot h_{14m}\left(P_{14m},T_{14m},v_{14m},v\_f_{10}\cdot z_{k,main}\right)$$
$$+\left(1-v\_f_{10}\right)\cdot h_{11m}\left(P_{11m},T_{11m},v_{11m},\left(1-v\_f_{10}\right)\cdot z_{k,main}\right)$$
$$P_{10m} = P_{11m},\ P_{10m} = P_{14m}$$
$$T_{10m} = T_{11m},\ T_{11m} = T_{14m}$$
$$v_{11m} = v_{11m}\left(P_{11m},T_{11m},\left(1-v\_f_{10}\right)\cdot z_{k,main}\right),\ v_{14m} = v_{14m}\left(P_{14m},T_{14m},v\_f_{10}\cdot z_{k,main}\right)$$

8) Evaporator 4: [10]

$$\left(1-v\_f_{10}\right)\cdot \dot{m}_{main}\cdot h_{11m}\left(P_{11m},T_{11m},v_{11m},\left(1-v\_f_{10}\right)\cdot z_{k,main}\right)$$
$$+v\_f_{10}\cdot \dot{m}_{main}\cdot h_{14m}\left(P_{14m},T_{14m},v_{14m},v\_f_{10}\cdot z_{k,main}\right)+\dot{m}_{main}\cdot h_{26m}\left(P_{26m},T_{26m},v_{26m},z_{k,main}\right)$$
$$+\dot{m}_{NG}\cdot h_{3NG}\left(P_{3NG},T_{3NG},v_{3NG},z_{l,NG}\right)$$
$$=\left(1-v\_f_{10}\right)\cdot \dot{m}_{main}\cdot h_{12m}\left(P_{12m},T_{12m},v_{12m},\left(1-v\_f_{10}\right)\cdot z_{k,main}\right)$$
$$+v\_f_{10}\cdot \dot{m}_{main}\cdot h_{15m}\left(P_{15m},T_{15m},v_{15m},v\_f_{10}\cdot z_{k,main}\right)+h_{1m}\left(P_{1m},T_{1m},v_{1m},z_{k,main}\right)$$
$$+\dot{m}_{NG}\cdot h_{4NG}\left(P_{4NG},T_{4NG},v_{4NG},z_{l,NG}\right)$$
$$P_{11m} = P_{12m}, P_{14m} = P_{15m}, P_{26m} = P_{1m}, P_{3NG} = P_{4NG}$$
$$T_{12m} = T_{15m}, T_{12m} = T_{4NG}$$
$$v_{12m} = v_{12m}\left(T_{12m},P_{12m},\left(1-v\_f_{10}\right)\cdot z_{k,main}\right),\ v_{15m} = v_{15m}\left(T_{15m},P_{15m},v\_f_{10}\cdot z_{k,main}\right),$$
$$v_{4NG} = v_{4NG}\left(T_{4NG},P_{4NG},z_{l,NG}\right)$$

9) Phase Separator 2: [7]

$$v\_f_{10}\cdot h_{15m}\left(P_{15m},T_{15m},v_{15m},v\_f_{10}\cdot z_{k,main}\right)$$
$$= v\_f_{15}\cdot v\_f_{10}\cdot h_{19m}\left(P_{19m},T_{19m},v_{19m},v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right)$$
$$+\left(1-v\_f_{15}\right)\cdot v\_f_{10}\cdot h_{16m}\left(P_{16m},T_{16m},v_{16m},\left(1-v\_f_{15}\right)\cdot v\_f_{10}\cdot z_{k,main}\right)$$
$$P_{15m} = P_{16m},\ P_{15m} = P_{19m}$$
$$T_{15m} = T_{16m},\ T_{16m} = T_{19m}$$
$$v_{16m} = v_{16m}\left(P_{16m},T_{16m},\left(1-v\_f_{15}\right)\cdot v\_f_{10}\cdot z_{k,main}\right),\ v_{19m} = v_{19m}\left(P_{19m},T_{19m},v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right)$$

10) Evaporator 5: [11]

$$\left(1-v\_f_{15}\right)\cdot v\_f_{10}\cdot \dot{m}_{main}\cdot h_{16m}\left(P_{16m},T_{16m},v_{16m},\left(1-v\_f_{15}\right)\cdot v\_f_{10}\cdot z_{k,main}\right)$$
$$+v\_f_{15}\cdot v\_f_{10}\cdot \dot{m}_{main}\cdot h_{19m}\left(P_{19m},T_{19m},v_{19m},v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right)$$
$$+v\_f_{10}\cdot \dot{m}_{main}\cdot h_{24m}\left(P_{24m},T_{24m},v_{24m},v\_f_{10}\cdot z_{k,main}\right)+\dot{m}_{NG}\cdot h_{4NG}\left(P_{4NG},T_{4NG},v_{4NG},z_{l,NG}\right)$$
$$=\left(1-v\_f_{15}\right)\cdot v\_f_{10}\cdot \dot{m}_{main}\cdot h_{17m}\left(P_{17m},T_{17m},v_{17m},\left(1-v\_f_{15}\right)\cdot v\_f_{10}\cdot z_{k,main}\right)$$
$$+v\_f_{15}\cdot v\_f_{10}\cdot \dot{m}_{main}\cdot h_{20m}\left(P_{20m},T_{20m},v_{20m},v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right)$$
$$+v\_f_{10}\cdot \dot{m}_{main}\cdot h_{25m}\left(P_{25m},T_{25m},v_{25m},v\_f_{10}\cdot z_{k,main}\right)+\dot{m}_{NG}\cdot h_{5NG}\left(P_{5NG},T_{5NG},v_{5NG},z_{l,NG}\right)$$
$$P_{16m} = P_{17m}, P_{19m} = P_{20m}, P_{24m} = P_{25m}, P_{4NG} = P_{5NG}$$
$$T_{17m} = T_{20m}, T_{17m} = T_{5NG}$$
$$v_{17m} = v_{17m}\left(P_{17m},T_{17m},\left(1-v\_f_{15}\right)\cdot v\_f_{10}\cdot z_{k,main}\right),\ v_{20m} = v_{20m}\left(P_{20m},T_{20m},v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right),$$
$$v_{25m} = v_{25m}\left(P_{25m},T_{25m},v\_f_{10}\cdot z_{k,main}\right),\ v_{5NG} = v_{5NG}\left(T_{5NG},P_{5NG},z_{l,NG}\right)$$

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Mathematical Model for the Determination of Optimal Operating Condition of the Generic Liquefaction Cycle (6/6)

1. Design variables (Operating conditions) [187]

T: Temperature / P: Pressure / v: Specific volume / $z_{j,pre}$: mole fraction of the component j at the precooling part / w: work input to the compressor per mass / c: flow rate ratio between inlet and outlet 4 / $\dot{m}_{pre}$: mass flow rate at the precooling refrigerant
*Subscript 'NG': natural gas, Subscript 'main': main cooling refrigerant

$$: P_i,\ T_i,\ v_i\left(i = 1_p,\dots,21_p,1_m,\dots,26_m,1_{NG},\dots,5_{NG}\right),$$
$$T_{S,2p}, T_{S,19p}, T_{S,21p}, T_{S,2m}, T_{S,4m}, T_{S,6m}, v_{S,2p}, v_{S,19p}, v_{S,21p}, v_{S,2m}, v_{S,4m}, v_{S,6m},$$
$$w_1,\ w_2,\ w_3, w_4,\ w_5,\ w_6, c_1, c_2, \dot{m}_{pre},\ \dot{m}_{main}, v\_f_{10}, v\_f_{15}, z_{j,pre}\left(j=1,2,3\right), z_{k,main}\left(k=1,2,3,4\right)$$

2. Equality constraints [165]

2.2 Equality constraints of main cooling part [80]

11) Evaporator 6: [6]

$$v\_f_{15}\cdot v\_f_{10}\cdot \dot{m}_{main}\cdot h_{20m}\left(P_{20m},T_{20m},v_{20m},v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right)$$
$$+v\_f_{15}\cdot v\_f_{10}\cdot \dot{m}_{main}\cdot h_{22m}\left(P_{22m},T_{22m},v_{22m},v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right)$$
$$+\dot{m}_{NG}\cdot h_{5NG}\left(P_{5NG},T_{5NG},v_{5NG},z_{l,NG}\right)$$
$$= v\_f_{15}\cdot v\_f_{10}\cdot \dot{m}_{main}\cdot h_{21m}\left(P_{21m},T_{21m},v_{21m},v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right)$$
$$+v\_f_{15}\cdot v\_f_{10}\cdot \dot{m}_{main}\cdot h_{23m}\left(P_{23m},T_{23m},v_{23m},v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right)$$
$$+\dot{m}_{NG}\cdot h_{LNG}\left(P_{LNG},T_{LNG},v_{LNG},z_{l,NG}\right)$$
$$P_{20m} = P_{21m}, P_{22m} = P_{23m}$$
$$T_{21m} = T_{LNG}$$
$$v_{21m} = v_{21m}\left(P_{21m},T_{21m},v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right),$$
$$v_{23m} = v_{23m}\left(P_{23m},T_{23m},v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right)$$

12) Expansion valve 4: [2]

$$h_{12m}\left(P_{12m},T_{12m},v_{12m},\left(1-v\_f_{10}\right)\cdot z_{k,main}\right)$$
$$= h_{13m}\left(P_{13m},T_{13m},v_{13m},\left(1-v\_f_{10}\right)\cdot z_{k,main}\right)$$
$$v_{13m} = v_{13m}\left(P_{13m},T_{13m},\left(1-v\_f_{10}\right)\cdot z_{k,main}\right)$$

13) Expansion valve 5: [2]

$$h_{17m}\left(P_{17m},T_{17m},v_{17m},\left(1-v\_f_{15}\right)\cdot v\_f_{10}\cdot z_{k,main}\right) = h_{18m}\left(P_{18m},T_{18m},v_{18m},\left(1-v\_f_{15}\right)\cdot v\_f_{10}\cdot z_{k,main}\right)$$
$$v_{18m} = v_{18m}\left(P_{18m},T_{18m},\left(1-v\_f_{15}\right)\cdot v\_f_{10}\cdot z_{k,main}\right)$$

14) Expansion valve 6: [2]

$$h_{21m}\left(P_{21m},T_{21m},v_{21m},v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right) = h_{22m}\left(P_{22m},T_{22m},v_{22m},v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right)$$
$$v_{22m} = v_{22m}\left(P_{22m},T_{22m}v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right)$$

15) Common Header 3: [4]

$$\left(1-v\_f_{10}\right)\cdot h_{13m}\left(P_{13m},T_{13m},v_{13m},\left(1-v\_f_{10}\right)\cdot z_{k,main}\right)+v\_f_{10}\cdot h_{25m}\left(P_{25m},T_{25m},v_{25m},v\_f_{10}\cdot z_{k,main}\right)$$
$$= h_{26m}\left(P_{26m},T_{26m},v_{26m},z_{k,main}\right)$$
$$P_{13m} = P_{25m},\ P_{13m} = P_{26m}\quad v_{26m} = v_{26m}\left(P_{26m},T_{26m},z_{k,main}\right)$$

16) Common Header 4: [4]

$$\left(1-v\_f_{15}\right)\cdot v\_f_{10}\cdot h_{18m}\left(P_{18m},T_{18m},v_{18m},\left(1-v\_f_{15}\right)\cdot v\_f_{10}\cdot z_{k,main}\right)$$
$$+v\_f_{15}\cdot v\_f_{10}\cdot h_{23m}\left(P_{23m},T_{23m},v_{23m},v\_f_{15}\cdot v\_f_{10}\cdot z_{k,main}\right)$$
$$= v\_f_{10}\cdot h_{24m}\left(P_{24m},T_{24m},v_{24m},v\_f_{10}\cdot z_{k,main}\right)$$
$$P_{18m} = P_{23m},\ P_{18m} = P_{24m}\quad v_{24m} = v_{24m}\left(T_{24m},P_{24m},v\_f_{10}\cdot z_{k,main}\right),$$

$$\sum_{j=1}^{3} z_{j,pre} = 1,\quad \sum_{k=1}^{4} z_{k,main} = 1$$

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[Summary] Mathematical Model for the Determination of Optimal Operating Condition of the Generic Liquefaction Cycle

1. Design variables (Operating conditions) [187]

: $P_i,\ T_i,\ v_i\left(i=1_p,\ldots,21_p,1_m,\ldots,26_m,1_{NG},\ldots,5_{NG}\right),$

$T_{S,2p}, T_{S,19p}, T_{S,21p}, T_{S,2m}, T_{S,4m}, T_{S,6m}, v_{S,2p}, v_{S,19p}, v_{S,21p}, v_{S,2m}, v_{S,4m}, v_{S,6m},$

$w_1,\ w_2,\ w_3, w_4,\ w_5,\ w_6, c_1, c_2, \dot{m}_{pre},\ \dot{m}_{main}, v\_f_{10}, v\_f_{15}, z_{j,pre}\left(j=1,2,3\right), z_{k,main}\left(k=1,2,3,4\right)$

T: Temperature / *P*: Pressure / *v*: Specific volume / $z_{j,pre}$: mole fraction of the component *j* at the precooling part / w: work input to the compressor per mass / c: flow rate ratio between inlet and outlet 4 / m_{pre}: mass flow rate at the precooling refrigerant
*Subscript 'NG': natural gas, Subscript 'main': main cooling refrigerant

2. Equality constraints [165]

2.1 Equality constraints of precooling part [83]

2.2 Equality constraints of main cooling part [80]

3. Objective Function: Minimize the compressors power

$$\text{Minize}\ \ \dot{m}_{pre}\cdot w_1+\dot{m}_{pre}\cdot w_2+\dot{m}_{pre}\cdot w_3+\dot{m}_{main}\cdot w_4+\dot{m}_{main}\cdot w_5+\dot{m}_{main}\cdot w_6$$

4. Free variables [22 = 187 – 165]

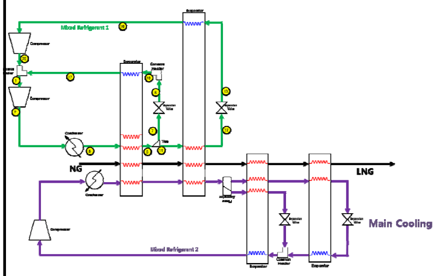
: $P_{1p},\ P_{2p},\ P_{12p},\ P_{17p},\ T_{5p},\ T_{11p},\ T_{16p},\ c_1,\ c_2,\ z_{1,pre},\ z_{2,pre},\ \dot{m}_{pre},$

$P_{1m},\ P_{2m},\ P_{4m},\ P_{6m},\ T_{12m},\ T_{17m},\ z_{1,main},\ z_{2,main},\ z_{3,main},\ \dot{m}_{main}$

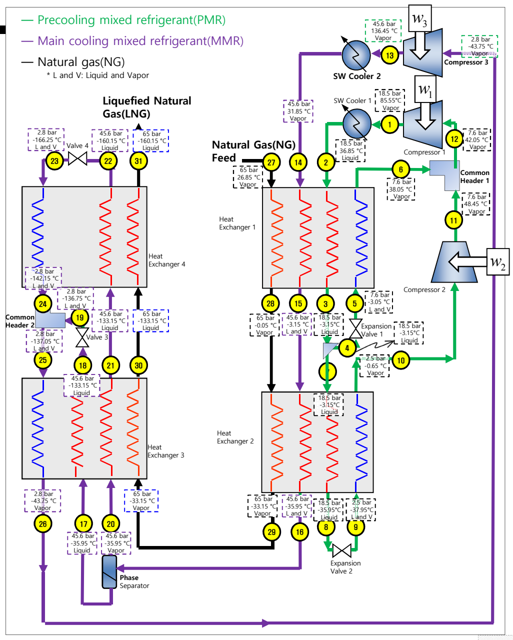
3. Determination of Optimal Operating Condition of the Liquefaction Cycle for LNG FPSO

Configuration of the Dual Mixed Refrigerant (DMR) Cycle (1/2)

FEASIBLE LIQUEFACTION MODEL (CASE 17)

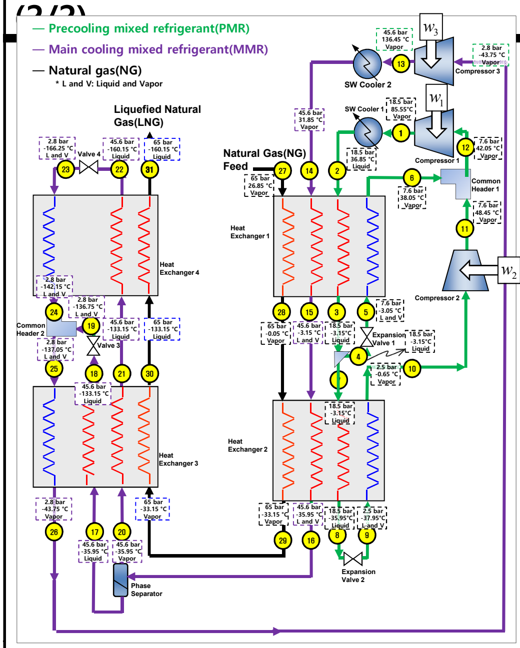


- Precooling 2 stage compression refrigeration
- Main cooling 1 stage compression, 2 stage refrigeration



Configuration of the Dual Mixed Refrigerant Cycle

Configuration of the Dual Mixed Refrigerant (DMR) Cycle (2/2)



Configuration of the Dual Mixed Refrigerant Cycle

- **Purpose:** Liquefying the natural gas by using two kind of mixed refrigerants
- **Refrigerant:**
  - Mixed refrigerant composed of Ethane(C_2H_6), Propane(C_3H_8), n-Butane(C_4H_{10}) for precooling
  - Mixed refrigerant composed of Nitrogen(N_2[1]), Methane(C_1H_4), Ethane(C_2H_6), Propane(C_3H_8) for main cooling
- **Problem Statement:**

  [Given]:

  NG(27) T=26.85°C, P=65bar,
  LNG(31) T=-160.15°C, P=65bar

  $$M_{NG} = 49.21\ kg/h$$
  $$(= 0.0004\ MMTA)$$

  [Find]:

  The operating conditions such as the pressure, temperature, specific volume, mass flow rate, and composition of the refrigerants minimizing the work provided to the compressor.

* Reference: 1) Venkatarathnam, G., 2008, Cryogenic Mixed Refrigerant Processes, Springer, New York
* N2: boiling point -196°

Mathematical Model for the Determination of Optimal Operating Condition of the DMR Cycle (1/2)

Composition of the refrigerant
Precooling: Ethane, Propane, n-Butane
Main cooling: Nitrogen, Methane, Ethane, Propane
Natural Gas: Methane(87.5%), Ethane(5.5%), Nitrogen(4.0%), Propane(2.1%), n-Butane(0.5%), i-Butane(0.3%), i-Pentane(0.1%)

1. Design variables (Operating conditions) [107]

: $P_i, T_i, v_i\left(i=1,...,26,28,29,30\right), T_{S,1}, T_{S,11}, T_{S,13}, v_{S,1}, v_{S,11}, v_{S,13}, w_1, w_2, w_3, c, M_{pre}, M_{main}, v\_f, z_{j,pre}\left(j=1,2,3\right), z_{k,main}\left(k=1,2,3,4\right)$

2. Equality constraints [91] $\sum_{j=1}^{3} z_{j,pre}=1,\quad \sum_{k=1}^{4} z_{k,main}=1$

T: Temperature / *P*: Pressure / *v*: Specific volume / $z_{j,pre}$: mole fraction of the component *j* at the precooling part / w: work input to the compressor per mass / c: flow rate ratio between inlet and outlet 4 / m_{pre}: mass flow rate at the precooling refrigerant
*Subscript 'NG': natural gas, Subscript 'main': main cooling refrigerant

2.1 Equality constraints of precooling part [49]

1) Compressor 1: [6]

$$h_{12}\left(P_{12},T_{12},v_{12},z_{j,pre}\right)+w_1=h_1\left(P_1,T_1,v_1,z_{j,pre}\right)$$
$$\eta=\frac{h_S\left(P_1,T_{S,1},v_{S,1},z_{j,pre}\right)-h_{12}\left(P_{12},T_{12},v_{12},z_{j,pre}\right)}{h_1\left(P_1,T_1,v_1,z_{j,pre}\right)-h_{12}\left(P_{12},T_{12},v_{12},z_{j,pre}\right)}$$
$$s_{12}\left(P_{12},T_{12},v_{12},z_{j,pre}\right)=s_1\left(P_1,T_{S,1},v_{S,1},z_{j,pre}\right)$$
$$v_{12}=v_{12}(T_{12},P_{12},z_{j,pre})$$
$$v_{S,1}=v_{S,1}(T_{S,1},P_1,z_{j,pre})$$
$$v_1=v_1(T_1,P_1,z_{j,pre})$$

4) Tee: [6]

$$h_3\left(P_3,T_3,v_3,z_{j,pre}\right)=c\cdot h_4\left(P_4,T_4,v_4,z_{j,pre}\right)+(1-c)\cdot h_7\left(P_7,T_7,v_7,z_{j,pre}\right)$$
$$P_3=P_4,\ P_3=P_7$$
$$T_4=T_7$$
$$v_4=v_4\left(T_4,P_4,z_{j,pre}\right),\ v_7=v_7\left(T_7,P_7,z_{j,pre}\right)$$

5) Expansion Valve 1: [2]

$$h_4\left(P_4,T_4,v_4\right)=h_5\left(P_5,T_5,v_5\right)$$
$$v_5=v_5\left(T_5,P_5\right)$$

8) Compressor 2: [5]

$$(1-c)\cdot h_{10}\left(P_{10},T_{10},v_{10},z_{j,pre}\right)+w_2=(1-c)\cdot h\left(P_{11},T_{11},v_{11},z_{j,pre}\right)$$
$$\eta=\frac{h_S\left(P_{11},T_{S,11},v_{S,11},z_{j,pre}\right)-h_{10}\left(P_{10},T_{10},v_{10},z_{j,pre}\right)}{h_{11}\left(P_{11},T_{11},v_{11},z_{j,pre}\right)-h_{10}\left(P_{10},T_{10},v_{10},z_{j,pre}\right)}$$
$$s_{10}\left(P_{10},T_{10},v_{10},z_{j,pre}\right)=s_{11}\left(P_{11},T_{S,11},v_{S,11},z_{j,pre}\right)$$
$$v_{S,11}=v_{S,11}\left(P_{11},T_{S,11},z_{j,pre}\right)$$
$$v_{11}=v_{11}(T_{11},P_{11},z_{j,pre})$$

2) Sea water cooler 1: [3]

The temperature of the outlet of the sea water cooler is usually given. T=310K

$$P_1=P_2$$
$$v_2=v_2(T_2,P_2,z_{j,pre})$$

6) Heat exchanger 2: [11]

$$(1-c)\cdot M_{pre}\cdot h_7\left(P_7,T_7,v_7,z_{j,pre}\right)+(1-c)\cdot M_{pre}\cdot h_9\left(P_9,T_9,v_9,z_{j,pre}\right)+M_{main}\cdot h_{15}\left(P_{15},T_{15},v_{15},z_{k,main}\right)+M_{NG}\cdot h_{28}\left(P_{28},T_{28},v_{28},z_{l,NG}\right)$$
$$=(1-c)\cdot M_{pre}\cdot h_8\left(P_8,T_8,v_8,z_{j,pre}\right)+(1-c)\cdot M_{pre}\cdot h_{10}\left(P_{10},T_{10},v_{10},z_{j,pre}\right)+M_{main}\cdot h_{16}\left(P_{16},T_{16},v_{16},z_{k,main}\right)+M_{NG}\cdot h_{29}\left(P_{29},T_{29},v_{29},z_{l,NG}\right)$$
$$P_7=P_8,\ P_9=P_{10},P_{15}=P_{16},P_{28}=P_{29}\qquad T_8=T_{16},\ T_8=T_{29}$$
$$v_8=v_8\left(T_8,P_8,z_{j,pre}\right),\ v_{10}=v_{10}\left(T_{10},P_{10},z_{j,pre}\right),\ v_{16}=v_{16}\left(T_{16},P_{16},z_{k,main}\right),\ v_{29}=v_{29}\left(T_{29},P_{29},z_{l,NG}\right)$$

3) Heat exchanger 1: [11]

$$M_{pre}\cdot h_2\left(P_2,T_2,v_2,z_{j,pre}\right)+c\cdot M_{pre}\cdot h_5\left(P_5,T_5,v_5,z_{j,pre}\right)+M_{main}\cdot h_{14}\left(P_{14},T_{14},v_{14},z_{k,main}\right)+M_{NG}\cdot h_{NG}\left(P_{NG},T_{NG},v_{NG},z_{l,NG}\right)$$
$$=M_{pre}\cdot h_3\left(P_3,T_3,v_3,z_{j,pre}\right)+c\cdot M_{pre}\cdot h_6\left(P_6,T_6,v_6,z_{j,pre}\right)+M_{main}\cdot h_{15}\left(P_{15},T_{15},v_{15},z_{k,main}\right)+M_{NG}\cdot h_{28}\left(P_{28},T_{28},v_{28},z_{l,NG}\right)$$
$$P_2=P_3,\ \ P_5=P_6,\ \ P_{14}=P_{15},\ \ P_{NG}=P_{28}\qquad T_3=T_{15},\ \ T_3=T_{28}$$
$$v_3=v_3\left(T_3,P_3,z_{j,pre}\right),\ v_6=v_6\left(T_6,P_6,z_{j,pre}\right),\ v_{15}=v_{15}\left(T_{15},P_{15},z_{k,main}\right),\ v_{28}=v_{28}\left(T_{28},P_{28},z_{l,NG}\right)$$

7) Expansion Valve 2: [2]

$$h_8\left(P_8,T_8,v_8,z_{j,pre}\right)=h_9\left(P_9,T_9,v_9,z_{j,pre}\right)$$
$$v_9=v_9\left(T_9,P_9,z_{j,pre}\right)$$

9) Common header 1: [3]

$$c\cdot h_6\left(P_6,T_6,v_6,z_{j,pre}\right)+(1-c)\cdot h_{11}\left(P_{11},T_{11},v_{11},z_{j,pre}\right)$$
$$=h_{12}\left(P_{12},T_{12},v_{12},z_{j,pre}\right)\qquad P_6=P_{11},\ P_6=P_{12}$$

Mathematical Model for the Determination of Optimal Operating Condition of the DMR Cycle (2/2)

Composition of the refrigerant
Precooling: Ethane, Propane, n-Butane
Main cooling: Nitrogen, Methane, Ethane, Propane
Natural Gas: Methane(87.5%), Ethane(5.5%), Nitrogen(4.0%), Propane(2.1%), n-Butane(0.5%), i-Butane(0.3%), i-Pentane(0.1%)

1. Design variables (Operating conditions) [107]

: $P_i, T_i, v_i\left(i=1,...,26,28,29,30\right), T_{S,1}, T_{S,11}, T_{S,13}, v_{S,1}, v_{S,11}, v_{S,13}, w_1, w_2, w_3, c, M_{pre}, M_{main}, v\_f, z_{j,pre}\left(j=1,2,3\right), z_{k,main}\left(k=1,2,3,4\right)$

T: Temperature / *P*: Pressure / *v*: Specific volume / $z_{j,pre}$: mole fraction of the component *j* at the precooling part / w: work input to the compressor per mass / c: flow rate ratio between inlet and outlet 4 / m_{pre}: mass flow rate at the precooling refrigerant / *v_f*: Vapor fraction at stream 16
*Subscript 'NG': natural gas, Subscript 'main': main cooling refrigerant

2. Equality constraints[91]

2.2 Equality constraints of main cooling part [40]

10) Compressor 3: [6]

$$h_{26}\left(P_{26},T_{26},v_{26},z_{k,main}\right)+w_3=h\left(P_{13},T_{13},v_{13},z_{k,main}\right)$$
$$\eta=\frac{h_{S,13}\left(P_{13},T_{S,13},v_{S,13},z_{k,main}\right)-h_{26}\left(P_{26},T_{26},v_{26},z_{k,main}\right)}{h_{13}\left(P_{13},T_{13},v_{13},z_{k,main}\right)-h_{26}\left(P_{26},T_{26},v_{26},z_{k,main}\right)}$$
$$s_{26}\left(P_{26},T_{26},v_{26},z_{k,main}\right)=s_{13}\left(P_{13},T_{S,13},v_{S,13},z_{k,main}\right)$$
$$v_{26}=v_{26}(T_{26},P_{26},z_{k,main})$$
$$v_{S,13}=v_{13}(T_{S,13},P_{13},z_{k,main})$$
$$v_{13}=v_{13}(T_{13},P_{13},z_{k,main})$$

13) Heat exchanger 3: [10]

$$v\_f\cdot M_{main}\cdot h_{20}\left(P_{20},T_{20},v_{20},v\_f\cdot z_{k,main}\right)+(1-v\_f)\cdot M_{main}\cdot h_{17}\left(P_{17},T_{17},v_{17},(1-v\_f)z_{k,main}\right)$$
$$+M_{main}\cdot h_{25}\left(P_{25},T_{25},v_{25},z_{k,main}\right)+M_{NG}\cdot h_{29}\left(P_{29},T_{29},v_{29},z_{l,NG}\right)$$
$$=v\_f\cdot M_{main}\cdot h_{21}\left(P_{21},T_{21},v_{21},v\_f\cdot z_{k,main}\right)+(1-v\_f)\cdot M_{main}\cdot h_{18}\left(P_{18},T_{18},v_{18}\left(1-v\_f\right)z_{k,main}\right)$$
$$+M_{main}\cdot h_{26}\left(P_{26},T_{26},v_{26},z_{k,main}\right)+M_{NG}\cdot h_{30}\left(P_{30},T_{30},v_{30},z_{l,NG}\right)$$
$$P_{20}=P_{21},\ P_{17}=P_{18},P_{25}=P_{26},P_{29}=P_{30}\qquad T_{21}=T_{30},\quad T_{21}=T_{18}$$
$$v_{21}=v_{21}\left(T_{21},P_{21},v\_f\cdot z_{k,main}\right),\ v_{18}=v_{18}\left(T_{18},P_{18},(1-v\_f)z_{k,main}\right),\ \ v_{30}=v_{30}\left(T_{30},P_{30},z_{l,NG}\right)$$

14) Expansion Valve 3: [2]

$$h_{18}\left(P_{18},T_{18},v_{18},(1-v\_f)z_{k,main}\right)=h_{19}\left(P_{19},T_{19},v_{19},(1-v\_f)z_{k,main}\right)$$
$$v_{19}=v_{19}\left(T_{19},P_{19}\right)$$

16) Expansion Valve 4: [2]

$$h_{22}\left(P_{22},T_{22},v_{22},v\_f\cdot z_{k,main}\right)=h_{23}\left(P_{23},T_{23},v_{23},v\_f\cdot z_{k,main}\right)$$
$$v_{23}=v_{23}\left(T_{23},P_{23},v\_f\cdot z_{k,main}\right)$$

11) Sea water cooler 2: [3]

The temperature of the outlet of the sea water cooler is usually given. T=305K

$$P_{13}=P_{14}\qquad v_{14}=v_{14}(T_{14},P_{14},z_{k,main})$$

15) Heat exchanger 4: [6]

$$v\_f\cdot M_{main}\cdot h_{21}\left(P_{21},T_{21},v_{21},v\_f\cdot z_{k,main}\right)+v\_f\cdot M_{main}\cdot h_{23}\left(P_{23},T_{23},v_{23},v\_f\cdot z_{k,main}\right)+M_{NG}\cdot h_{30}\left(P_{30},T_{30},v_{30},z_{l,NG}\right)$$
$$=v\_f\cdot M_{main}\cdot h_{22}\left(P_{22},T_{22},v_{22},v\_f\cdot z_{k,main}\right)+v\_f\cdot M_{main}\cdot h_{24}\left(P_{24},T_{24},v_{24},v\_f\cdot z_{k,main}\right)+M_{NG}\cdot h_{LNG}\left(P_{LNG},T_{LNG},v_{LNG},z_{l,NG}\right)$$
$$P_{21}=P_{22},\ P_{23}=P_{24},\ \ T_{22}=T_{31}\qquad v_{22}=v_{22}\left(T_{22},P_{22},v\_f\cdot z_{k,main}\right),\ v_{24}=v_{24}\left(T_{24},P_{24},v\_f\cdot z_{k,main}\right)$$

12) Phase separator: [7]

$$h_{16}\left(P_{16},T_{16},v_{16},z_{k,main}\right)=v\_f\cdot h_{20}\left(P_{20},T_{20},v_{20},v\_f\cdot z_{k,main}\right)+(1-v\_f)\cdot h_{17}\left(P_{17},T_{17},v_{17},(1-v\_f)z_{k,main}\right)$$
$$P_{16}=P_{17},\ \ P_{16}=P_{20}\qquad T_{17}=T_{20}\qquad T_{16}=T_{17}$$
$$v_{17}=v_{17}\left(T_{17},P_{17},(1-v\_f)z_{k,main}\right),\ v_{20}=v_{20}\left(T_{20},P_{20},v\_f\cdot z_{k,main}\right)$$

17) Common header 2: [4]

$$(1-v\_f)\cdot h_{19}\left(P_{19},T_{19},v_{19},(1-v\_f)z_{k,main}\right)$$
$$+v\_f\cdot h_{24}\left(P_{24},T_{24},v_{24},v\_f\cdot z_{k,main}\right)=h_{25}\left(P_{25},T_{25},v_{25},z_{k,main}\right)$$
$$P_{19}=P_{24},\ P_{19}=P_{25}\qquad v_{25}=v_{25}(T_{25},P_{25},z_{k,main})$$

Mathematical Model for the Determination of Optimal Operating Condition of the DMR Cycle

- Optimization Method

Modified optimization problem

- **Free variables** [16 = 107 - 91]
  $: P_1, P_5, P_9, P_{13}, P_{19}, T_3, T_8, T_{18}, c, M_{pre}, M_{main},$
  $z_{C2,pre}, z_{C3,pre}, z_{N2,main}, z_{C1,main}, z_{C2,main}$
- **Inequality constraints** [11]
- Objective function

$$\text{Minize } M_{pre} \cdot w_1 + M_{pre} \cdot w_2 + M_{main} \cdot w_3$$

➡ Use of sequential quadratic programming(SQP) method

Values of free variables ⇩ ⇧ Values of other design variables

Problem of nonlinear equations

- **Design variables** [91]
- **Equality constraints** [91]

➡ Use of Newton-Raphson method

1. Design variables (operating conditions) [107]

$: P_i, T_i, v_i\,(i=1, \ldots, 26, 28, 29, 30),$
$T_{S,1}, T_{S,11}, T_{S,13}, v_{S,1}, v_{S,11}, v_{S,13}, w_1, w_2, w_3, c,$
$M_{pre}, M_{main}, v\_f, z_{j,pre}\,(j=1,2,3), z_{k,main}\,(k=1,2,3,4)$

2. Equality constraints [91]
   1) Composition of the refrigerant [2]
   2) Precooling part [49]
   3) Main cooling part [40]

3. Inequality constraints [11]
   1) Minimum temperature approach in heat exchanger [8]
   2) Inlet condition of the compressor [3]

4. Objective function: Minimize the compressors power

$$\text{Minize } M_{pre} \cdot w_1 + M_{pre} \cdot w_2 + M_{main} \cdot w_3$$

T: Temperature / P: Pressure / v: Specific volume / $z_{j,pre}$: mole fraction of the component j at the precooling part / w: work input to the compressor per mass / c: flow rate ratio between inlet and outlet 4 / M_{pre}: mass flow rate at the precooling part
*Subscript 'NG': natural gas, Subscript 'main': main cooling part

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Mathematical Model for the Determination of Optimal Operating Condition of the DMR Cycle

- Comparison between Optimization Result and Existing Study

P[bar], T[K], v[m^3/mol], w[J/mol], m[mol/s], W[kW]

Result obtained by this study:

P1	19.17	P11	7.59	P21	48.57	Ts1	349.62
T1	355.42	T11	312.51	T21	143.54	Ts11	305.49
v1	0.001274	v11	0.003078	v21	0.000045	Ts13	388.66
P2	19.20	P12	7.59	P22	48.57	vs1	0.0014673
T2	310.00	T12	308.94	T22	113.00	vs11	0.0032314
v2	0.000122	v12	0.003044	v22	0.000040	vs13	0.0006245
P3	19.20	P13	48.57	P23	3.18	w1[J/mol]	2738.87
T3	273.33	T13	413.82	T23	106.95	w2	1099.84
v3	0.000087	v13	0.000663	v23	0.000304	w3	8233.20
P4	19.20	P14	48.57	P24	3.18	c	0.6000
T4	273.33	T14	305.00	T24	140.47	mpre[mol/s]	0.8982
v4	0.000087	v14	0.000389	v24	0.002884	mmain	0.9776
P5	7.59	P15	48.57	P25	3.18	zpre_Ethane	0.2481
T5	269.94	T15	273.33	T25	140.36	zpre_Propane	0.6410
v5	0.000158	v15	0.000248	v25	0.001089	zpre_n-Butane	0.1110
P6	7.59	P16	48.57	P26	3.18	zmain_Nitrogen	0.0710
T6	306.51	T16	240.00	T26	236.43	zmain_Methane	0.4153
v6	0.003020	v16	0.000141	v26	0.006349	zmain_Ethane	0.2887
P7	19.20	P17	48.57	P28	65.00	**zmain_Propaane**	**0.225054**
T7	273.23	T17	240.00	T28	273.33	**Objective function(W)**	**11.497**
v7	0.000087	v17	0.000071	v28	0.000286		
P8	19.20	P18	48.57	P29	65.00		
T8	240.00	T18	143.54	T29	240.00		
v8	0.000079	v18	0.000052	v29	0.000210		
P9	2.80	P19	**3.18**	P30	65.00		
T9	236.90	T19	139.21	T30	143.54		
v9	0.000257	v19	0.000368	v30	0.000043		
P10	2.80	P20	48.57				
T10	267.67	T20	240.00				
v10	0.007509	v20	0.000312				

Result obtained by Venkatarathnam[1]: **[%]: Difference**

		[%}			[%			[%			[%
P1	19.20	0.2%	P11	7.60	0.2%	P21	48.60	0.1%	Ts1	349.59	0.0%
T1	360.25	1.3%	T11	313.59	0.3%	T21	144.70	0.8%	Ts11	305.75	0.1%
v1	0.001291	1.3%	v11	0.003089	0.3%	v21	0.000045	0.8%	Ts13	389.24	0.1%
P2	19.20	0.0%	P12	7.60	0.2%	P22	48.60	0.1%	vs1	0.001467	0.0%
T2	310.00	0.0%	T12	313.79	1.5%	T22	113.00	0.0%	vs11	0.003234	0.1%
v2	0.000122	0.0%	v12	0.003092	1.5%	v22	0.000040	0.0%	vs13	0.000625	0.1%
P3	19.20	0.0%	P13	48.60	0.1%	P23	3.00	6.0%	w1[J/mol]	2767.71	1.0%
T3	273.10	0.1%	T13	418.13	1.0%	T23	106.89	0.1%	w2	1103.87	0.4%
v3	0.000087	0.1%	v13	0.000669	1.0%	v23	0.000304	0.1%	w3	8441.32	2.5%
P4	19.20	0.0%	P14	48.60	0.1%	P24	3.00	6.0%	c	0.5980	0.3%
T4	273.10	0.1%	T14	305.00	0.0%	T24	141.79	0.9%	mpre[mol/s]	0.913	1.6%
v4	0.000087	0.1%	v14	0.000389	0.0%	v24	0.002911	0.9%	mmain	1	2.2%
P5	7.60	0.2%	P15	48.60	0.1%	P25	3.00	6.0%	z1_Ethane	0.2481	0.0%
T5	269.73	0.1%	T15	273.10	0.1%	T25	140.26	0.1%	z2_Propane	0.6416	0.1%
v5	0.000159	0.1%	v15	0.000248	0.1%	v25	0.001090	0.1%	z3_n-Butane	0.1103	0.6%
P6	7.60	0.2%	P16	48.60	0.1%	P26	3.00	5.9%	z1_Nitrogen	0.070	1.4%
T6	313.93	2.4%	T16	240.00	0.0%	T26	236.95	0.2%	z2_Methane	0.4180	0.6%
v6	0.003093	2.4%	v16	0.000141	0.0%	v26	0.006363	0.2%	z3_Ethane	0.2990	3.5%
P7	19.20	0.0%	P17	48.60	0.1%	P28	65.00	0.0%	**z4_Propaane**	**0.2130**	**5.7%**
T7	273.00	0.1%	T17	240.00	0.0%	T28	273.10	0.1%	**Objective function(W)**	**11.976**	**4.0%**
v7	0.000087	0.1%	v17	0.000071	0.0%	v28	0.000286	0.1%			
P8	19.20	0.0%	P18	48.60	0.1%	P29	65.00	0.0%			
T8	240.00	0.0%	T18	144.70	0.8%	T29	240.00	0.0%			
v8	0.000079	0.0%	v18	0.000053	0.8%	v29	0.000210	0.0%			
P9	2.80	0.1%	P19	**3.00**	**6.0%**	P30	65.00	0.0%			
T9	236.50	0.2%	T19	139.14	0.1%	T30	144.70	0.8%			
v9	0.000258	0.2%	v19	0.000368	0.1%	v30	0.000044	0.8%			
P10	2.80	0.1%	P20	48.60	0.1%						
T10	268.67	0.4%	T20	240.00	0.0%						
v10	0.007537	0.4%	v20	0.000312	0.0%						

➡ The result of the optimal operating condition of the DMR cycle obtained by this study **saves 4.0% of the total required power consumption** compared with the past relevant research.

* Reference: 1) Venkatarathnam, G., 2008, Cryogenic Mixed Refrigerant Processes, Springer, New York

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Mathematical Model for the Determination of Optimal Operating Condition of the DMR Cycle

- Validation of the Optimization Result

1. Design variables (Operating Conditions) [107]

2. Equality constraints of Precooling part [91]

P[bar], T[K], v[m³/mol], w[J/mol], m[mol/s], W[kW]

➡ **To verify the mathematical model of** this research **for the DMR Cycle, 16 design variables are assumed** and the values of the other design variables are compared between the result obtained by the mathematical model and HYSYS.

[%]: Difference

P1	**19.64**	P11	8.19	P21	48.92	Ts1	346.97
T1	352.31	T11	313.47	T21	140.36	Ts11	306.24
v1	0.001208	v11	0.002846	v21	0.000043	Ts13	395.94
P2	19.64	P12	8.19	P22	48.92	vs1	0.001467
T2	310.00	T12	307.89	T22	113.00	vs11	0.003234
v2	0.000092	v12	0.002774	v22	0.000039	vs13	0.000625
P3	19.64	P13	**48.92**	P23	**2.79**	w1[J/mol]	2505.86
T3	275.01	T13	422.24	T23	105.80	w2	1187.98
v3	0.000081	v13	0.000669	v23	0.000360	w3	8746.96
P4	19.64	P14	48.92	P24	2.79	c	**0.584643**
T4	275.01	T14	305.00	T24	137.74	Mpre[mol/s]	**0.932866**
v4	0.000081	v14	0.000379	v24	0.003016	Mmain	**0.957021**
P5	**8.19**	P15	48.92	P25	2.79	zpre_Ethane	**0.253895**
T5	272.01	T15	275.01	T25	137.41	zpre_Propane	**0.63883**
v5	0.000142	v15	0.000242	v25	0.001016	zpre_n-Butane	0.107275
P6	8.19	P16	48.92	P26	2.79	zmain_Nitrogen	**0.069317**
T6	303.97	T16	239.64	T26	237.65	zmain_Methane	**0.405874**
v6	0.002722	v16	0.000131	v26	0.006835	zmain_Ethane	**0.2964**
P7	19.64	P17	48.92	P28	65.00	zmain_Propaane	0.228409
T7	275.01	T17	239.64	T28	**275.01**		
v7	0.000081	v17	0.000068	v28	0.000290		
P8	19.64	P18	48.92	P29	65.00		
T8	239.64	T18	140.36	T29	**239.64**		
v8	0.000074	v18	0.000050	v29	0.000209		
P9	**2.86**	P19	2.79	P30	65.00		
T9	236.58	T19	136.08	T30	**140.36**		
v9	0.000232	v19	0.000344	v30	0.000042		
P10	2.86	P20	48.92				
T10	265.92	T20	239.64				
v10	0.007301	v20	0.000311				

Mathematical model of this study for the DMR Cycle

		[%]			[%			[%			[%
P1	**19.64**	0.00%	P11	8.19	0.00%	P21	48.92	0.00%	Ts1	347.36	0.11%
T1	352.71	0.11%	T11	313.95	0.15%	T21	140.36	0.00%	Ts11	306.71	0.15%
v1	0.001209	0.01%	v11	0.002847	0.02%	v21	0.000043	0.00%	Ts13	394.75	0.30%
P2	19.64	0.00%	P12	8.19	0.00%	P22	48.92	0.00%	vs1	0.001467	0.03%
T2	310.00	0.00%	T12	308.27	0.12%	T22	113.00	0.00%	vs11	0.003234	0.02%
v2	0.000092	0.00%	v12	0.002774	0.01%	v22	0.000039	0.00%	vs13	0.000625	0.03%
P3	19.64	0.00%	P13	**48.92**	0.00%	P23	**2.79**	0.00%	w1[J/mol]	2514.02	0.32%
T3	275.01	0.00%	T13	420.94	0.31%	T23	105.74	0.06%	w2	1191.01	0.25%
v3	0.000081	0.00%	v13	0.000669	0.03%	v23	0.000360	0.01%	w3	8705.69	0.47%
P4	19.64	0.00%	P14	48.92	0.00%	P24	2.79	0.00%	c	**0.584643**	0.00%
T4	275.01	0.00%	T14	305.00	0.00%	T24	137.87	0.09%	Mpre[mol/s]	**0.932866**	0.00%
v4	0.000081	0.00%	v14	0.000379	0.00%	v24	0.003016	0.01%	Mmain	**0.957021**	0.00%
P5	**8.19**	0.00%	P15	48.92	0.00%	P25	2.79	0.00%	zpre_Ethane	**0.253895**	0.00%
T5	272.00	0.01%	T15	275.01	0.00%	T25	137.26	0.11%	zpre_Propane	**0.63883**	0.00%
v5	0.000142	0.00%	v15	0.000242	0.00%	v25	0.001016	0.01%	zpre_n-Butane	0.107275	0.00%
P6	8.19	0.00%	P16	48.92	0.00%	P26	2.79	0.00%	zmain_Nitrogen	**0.069317**	0.00%
T6	304.20	0.08%	T16	239.64	0.00%	T26	236.62	0.44%	zmain_Methane	**0.405874**	0.00%
v6	0.002722	0.01%	v16	0.000131	0.00%	v26	0.006838	0.04%	zmain_Ethane	**0.2964**	0.00%
P7	19.64	0.00%	P17	48.92	0.00%	P28	65.00	0.00%	zmain_Propaane	0.228409	0.00%
T7	275.01	0.00%	T17	239.64	0.00%	T28	**275.01**	0.00%			
v7	0.000081	0.00%	v17	0.000068	0.00%	v28	0.000290	0.00%			
P8	19.64	0.00%	P18	48.92	0.00%	P29	65.00	0.00%			
T8	239.64	0.00%	T18	140.36	0.00%	T29	**239.64**	0.00%			
v8	0.000074	0.00%	v18	0.000050	0.00%	v29	0.000209	0.00%			
P9	**2.86**	0.00%	P19	2.79	0.00%	P30	65.00	0.00%			
T9	236.56	0.01%	T19	135.97	0.08%	T30	**140.36**	0.00%			
v9	0.000232	0.00%	v19	0.000344	0.01%	v30	0.000042	0.00%			
P10	2.86	0.00%	P20	48.92	0.00%						
T10	266.38	0.17%	T20	239.64	0.00%						
v10	0.007303	0.02%	v20	0.000311	0.00%						

HYSYS

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Comparison between the Cycle by Optimal Synthesis and DMR Cycle

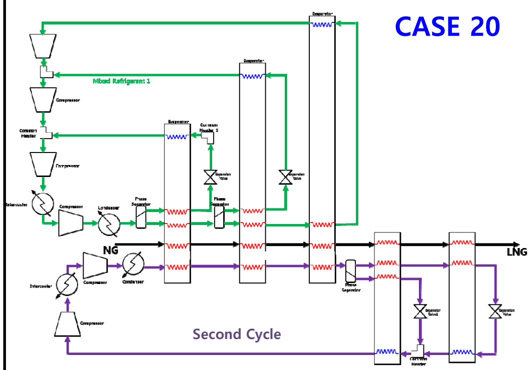
- After the optimal operating conditions for the two cycles are achieved, the power required by the compressors for the two cycles are compared.

• **Common condition**

1) Given: NG T=26.85°C, P=65 bar,
LNG T=-160.15°C, P=65 bar
$\dot{m}_{NG} = 49.21\ kg/h\ (= 0.0004\ MMTA)$

2) Refrigerant:
- Mixed refrigerant 1 is composed of Ethane(C_2H_6), Propane(C_3H_8), n-Butane(C_4H_{10}) for precooling
- Mixed refrigerant 2 is composed of Nitrogen(N_2), Methane(C_1H_4), Ethane(C_2H_6), Propane(C_3H_8) for main cooling

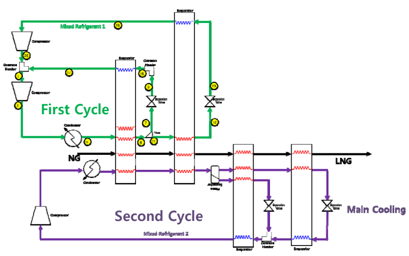
CASE 20



[Result of the Synthesis of the Liquefaction Cycle]

- Precooling 3 stage compression refrigeration
- Main cooling 2 stage compassion, 2 stage refrigeration

CASE 17



[Configuration of the Dual Mixed Refrigerant Cycle[1]]

- Precooling 2 stage compression refrigeration
- Main cooling 1 stage compression, 2 stage refrigeration

Total required power consumption of the compressors and pumps: **11.359[kW]**

Total required power consumption of the compressors[2]: **11.497[kW]**

➡ The result of the synthesis of the liquefaction cycle **saves 1.2% of the total required power consumption**.

* Reference: 1) Venkatarathnam, G., Cryogenic Mixed Refrigerant Processes, Springer, New York, 2008.
2) K. Y. Lee, J. H. Cha, J. C. Lee, M. I. Roh, and J. H. Hwang, Determination of the Optimal Operating Condition of Dual Mixed Refrigerant Cycle at the Pre-FEED stage of LNG FPSO Topside Liquefaction Process, ISOPE, June 2011.

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