#### **Engineering Economic Analysis**

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Chap. 23 FIRM SUPPLY

# Market Environments

- Firm's two important decisions
  - How much it should produce
  - What price it should set

#### Constraints

- Technological constraints
  Production function
- Economic constraints
  - Cost function
- Market constraints
  - Demand curve facing the firm
- Market environment
  - The way that firms respond to each other when they make their pricing and output decisions

- Market is *purely competitive* if each firm assumes that the market price is independent of its own level of output
  - An industry of many firms that produce an identical good, and that each firm is a small part of the market
  - Only worry about how much output it produces (supply)
  - Price taker
- Demand curve facing a competitive firm
  - Let  $p^e$  be the market price

$$D(p) = \begin{cases} 0 & \text{if } p > p^e \\ any \text{ amount if } p = p^e \\ \infty & \text{if } p < p^e \end{cases}$$

Demand curve facing a competitive firm



Demand curve facing a competitive firm



What does it mean to say that an individual firm is "small relative to the industry"?



 The individual firm's capacity to supply can cover only a small part of the total quantity demanded at the market price

• Profit-max problem

$$\max_{y} py - c(y)$$

• F.O.C.

 $p = c'(y^*)$  Marginal revenue = Marginal cost

- S.O.C.  $c''(y^*) \ge 0$
- Inverse supply function *p*(*y*): the price that must prevail in order for a firm to find it profitable to supply a given amount of output
  - **-** By F.O.C.

$$p(y) = c'(y)$$

- Supply function y(p): the profit-maximizing output at price of p
  - Whatever the level of market price *p*, the competitive firm will choose a level of output where p = c'(y(p))
  - In addition, by S.O.C., the firm will supply the output which satisfy  $c''(y(p)) \ge 0$
- Therefore, the supply curve of the competitive firm is the upward sloping part of its marginal cost curve!

Differentiating F.O.C.  $(p \equiv c'(y(p)))$  w.r.t. p

$$I = c'(y(p))y'(p)$$
  
By S.O.C.  $c''(y) > 0 \Rightarrow y'(p) > 0$ 

 Additionally, the competitive firm's economic profit level must not be negative since then the firm would exit the industry in the long-run. So,

$$\pi(y) = py - c(y) > 0$$
$$\Rightarrow p \ge \frac{c(y)}{y} = AC(y)$$

• LR Supply curve for a competitive firm

$$p(y) = \begin{cases} c'(y) & \text{if } p \ge AC(y) \\ 0 & o/w \end{cases}$$



- SR Supply curve considering SR cost curve
  - Let  $c(y) = c_v(y) + F$
  - Then  $\pi = p \cdot y(p) c_v(y(p)) F$
  - Producing a positive output (*y* >0) is profitable compared with the case of producing zero if

$$p \cdot y(p) - c_{v}(y(p)) - F \ge -F$$
$$p \ge \frac{c_{v}(y(p))}{y(p)} = AVC(y(p))$$

SR Supply curve for a competitive firm

$$p(y) = \begin{cases} c'(y) & \text{if } p \ge AVC(y) \\ 0 & o/w \end{cases}$$

# SR Supply curve of a competitive firm



- Shut-down is not the same as exit.
- Shutting-down means producing no output (but the firm is still in the industry and suffers its fixed cost because exit is impossible in the short-run).
- Exiting means leaving the industry, which the firm can do only in the long-run.

#### Producer's surplus

- Supply curve measures the amount that will be supplied at each price
- The area above the supply curve measures the surplus enjoyed by the suppliers of a good



Producer's surplus

$$PS(p) = \int_0^y [p - c'(x)] dx$$
$$= p \cdot y - \int_0^y c'(x) dx$$
$$= p \cdot y - c_v(y)$$

Profit and Producer's surplus

Profit =  $p \cdot y - c_v(y) - F$ Producer's surplus =  $p \cdot y - c_v(y)$ 

#### Measuring Producer's surplus

• measuring revenue minus variable cost.



Measuring Producer's surplus

• Measuring 
$$c_v(y) = \int_0^y MC(z) dz$$



- Measuring Producer's surplus
  - Measuring the box up until output z (area R) and then uses the area above the marginal cost curve (area T).



### Supply curve

• Example:

$$c(y) = y^2 + 1$$

- Price = Marginal cost p = 2y
- Supply curve  $S(p) = y = \frac{p}{2}$
- Max. profits for each price  $\pi(p) = py - c(y)$

$$= p\frac{p}{2} - \left(\frac{p}{2}\right)^2 - 1$$
$$= \frac{p^2}{4} - 1$$

# Supply curve

Example:



• Producer's surplus

$$A = \left(\frac{1}{2}\right) \left(\frac{p}{2}\right) p = \frac{p^2}{4}$$

• Producer's surplus = Profits + Fixed cost

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# Chap. 24 INDUSTRY SUPPLY

- Short-run industry supply curve
  - Since every firm in the industry is a price-taker, total quantity supplied at a given price is the sum of quantities supplied at that price by the individual firms.
  - In a short-run, the number of firms in the industry is, temporarily, fixed.
  - Let *n* be the number of firms, then the industry supply function is the sum of the individual firm supply function

$$S(p) = \sum_{i=1}^{n} S_i(p)$$



- Example: *n* firms with common cost  $c(y) = y^2 + 1$ 
  - Inverse supply function p(y) = MC(y) = 2y

- Note that  $MC(y) = 2y \ge AVC(y) = y$  for all  $y \ge 0$ 

- Individual supply function  $S_i(p) = \frac{p}{2}$
- Industry supply function  $S(p) = \sum_{i=1}^{n} S_i(p) = Y = \frac{p}{2}n$
- Inverse industry supply function  $p(Y) = \frac{2}{n}Y$

- **Example:** Firm 1:  $c_1(y) = y^2$ , Firm 2:  $c_2(y) = 2y^2$ 
  - Each firm chooses a level of output where p=MC $p(y_1) = 2y_1, p(y_2) = 4y_2$
  - Individual supply function

$$y_1(p) = \frac{p}{2}, y_2(p) = \frac{p}{4}$$

• Industry supply function

$$Y(p) = \frac{p}{2} + \frac{p}{4} = \frac{3p}{4}$$

• Note that each firm that produces a positive amount of output must have the same MC

$$MC_1(y) = MC_2(y) = p = \frac{4}{3}y$$

In a short-run, neither entry nor exit can occur.

 $x_i(p)$ : individual *i*'s demand function, i = 1, ..., n $y_j(p)$ : supply function of firm j, j = 1, ..., m

• Equilibrium price *p*\*

 $\sum_{i} x_{i}\left(p^{*}\right) = \sum_{j} y_{j}\left(p^{*}\right)$ 



Firm 1 wishes to remain in the industry.

s Firm 2 wishes to exit from the industry. Firm 3 is indifferent.

In a short-run, neither entry nor exit can occur.

 $x_i(p)$ : individual *i*'s demand function, i = 1, ..., n $y_i(p)$ : supply function of firm j, j = 1, ..., m

• Equilibrium price *p*\*

 $\sum_{i} x_{i}\left(p^{*}\right) = \sum_{j} y_{j}\left(p^{*}\right)$ 

Consequently, in a short-run equilibrium, some firms may earn positive economics profits, others may suffer economic losses, and still others may earn zero economic profit

- Relationship between SR equilibrium price and the number of firms in a competitive market
  - Regard p as an implicit function of n, i.e., p(n)
  - Let *X*(*p*) be the arbitrary industry demand function and *y*(*p*) is a common supply function of individual n firms in a competitive market
  - Equilibrium condition: *X*(*p*)=*ny*(*p*)
  - Differentiating Eq. condition X(p(n))=ny(p(n)) w.r.t. *n*

$$X'(p)p'(n) = y(p) + n \cdot y'(p)p'(n)$$
  
$$\therefore p'(n) = \frac{y(p)}{X'(p) - n \cdot y'(p)}$$

As the number of firms increases, SR equil. price decreases

Since  $X'(p) < 0, y'(p) > 0 \implies p'(n) < 0$ 

### Entry & Long-Run Equilibrium

- In the long-run, every firm now in the industry is free to exit and firms now outside the industry are free to enter.
- The industry's long-run supply function must account for entry and exit as well as for the supply choices of firms that choose to be in the industry.
  - Positive economic profit induces entry.
  - Economic profit is positive when  $p^* > \min AC(y)$ .
  - Entry increases the number of firms, causing *p*\* to fall.
    (*dp*\*/*dn*<0)</li>
  - When does entry cease?

### Entry & Long-Run Equilibrium

• Suppose the industry initially contains only two firms.





- Then the market-clearing price is p<sub>2</sub>.
- Each firm produces y<sub>2</sub>\* units of output.



• Each firm makes a positive economic profit, inducing entry by another firm.



• Market supply shifts outwards.



- Market supply shifts outwards.
- Market price falls.



• Each firm produces less.



- Each firm produces less.
- Each firm's economic profit is reduced.



• Each firm's economic profit is still positive. Will another firm enter?



Market supply would shift outwards again.



- Market supply would shift outwards again.
- Market price would fall again.



• Each firm would produce less again.



- Each firm's economic profit would be negative.
- So the fourth firm would not enter.

- The long-run number of firms in the industry is the largest number for which the market price is at least as large as min AC(y).
- Example: cost function  $c(y) = y^2 + 1$ 
  - Break-even level of output is p=AC(y)
  - Since *p*=MC(*y*) in an equilibrium of competitive market, the break-even output is where *p*=AC(*y*)=MC(*y*)

$$2y = y + \frac{1}{y} \implies y = 1$$
  
Then  $p = MC(1) = 2$ 

• Thus firms will enter the industry as long as they will not drive the equilibrium price below 2

- Example: cost function  $c(y) = y^2 + 1$ 
  - Recall that market supply curve is  $Y(p) = \frac{p}{2}n$
  - If the market demand function is X(p) = a bp
  - Equilibrium price

$$p^* = \frac{a}{b+n/2}$$

• For the firms in the market to be profitable,

$$p^* = \frac{a}{b+n/2} \ge 2$$

• Thus, in the long-run the number of firms in the market is the largest integer *n* satisfying

 $n \le a - 2b$ 

• The competitive industry's long-run supply curve is horizontal at min AC(y).



 Now market demand is large enough to sustain only two firms in the industry.



• Suppose that market demand increases, then the market price rises, each firm produces more, and earns a higher profit.



• Will the 3rd firm enter? ▶ No! since negative profits.

• As market demand increases further, the market price rises further, the two incumbent firms each earn still higher economic profits -- until the 3rd firm becomes indifferent between entering and staying out.



• How much further can market demand increase before a fourth firm enters the industry?



• Continuing in this manner builds the industry's long-run supply curve, one section at-a-time from successive short-run industry supply curves.



Notice that the bottom of each segment of the supply curve is min AC(y).

• In the long-run, if there are a reasonable number of firms in the industry(or as firms become sufficiently small), the industry's long-run supply curve is horizontal at min AC(y).

