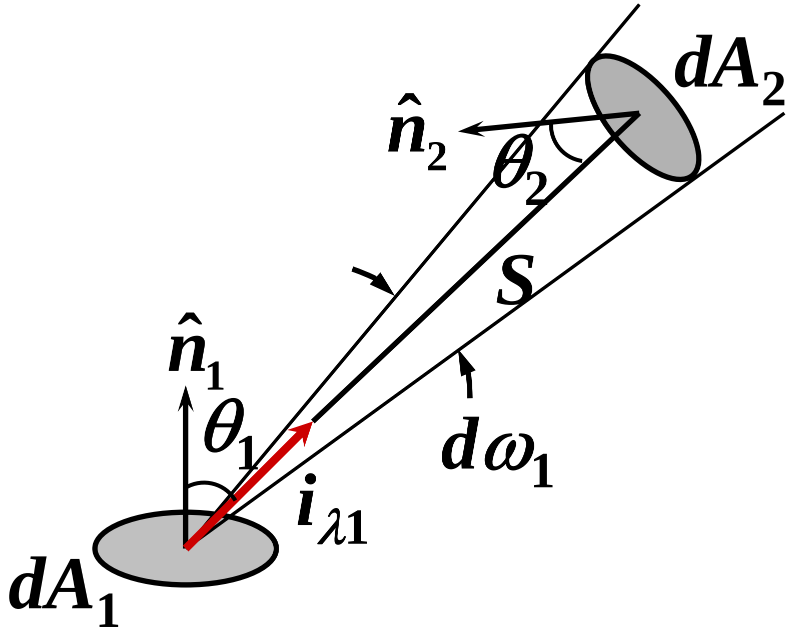
CONFIGURATION FACTORS FOR SURFACES TRANSFERRING UNIFORM DIFFUSE RADIATION

configuration factor, view factor,
angle factor, shape factor

- Between two differential area elements
- Between a differential element and a finite area
- Between two finite areas
- Methods of configuration factor evaluation

Between Two Differential Area Elements

differential configuration factor



$$dF_{d1-d2} = \frac{\text{energy intercepted by } dA_2}{\text{energy leaving } dA_1 \text{ hemispherically}}$$

$$= \frac{Q_{d1 \to d2}}{Q_{d1}}$$

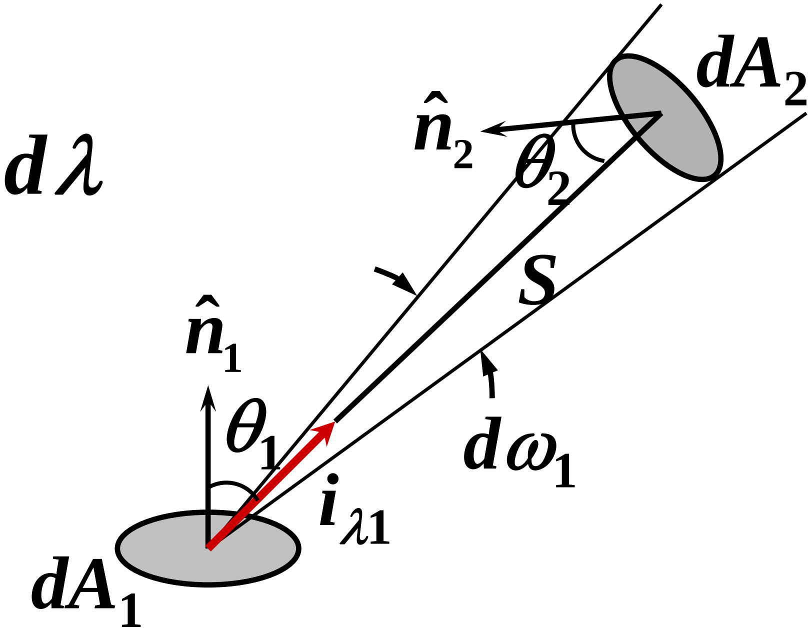
$$Q_{d1} = \int_{\cap}\int_0^{\infty} i_{\lambda 1} dA_1 \cos\theta_1 d\lambda d\omega_1$$

$$= \int_{\cap} i_1 dA_1 \cos\theta_1 d\omega_1$$

$$Q_{d1\rightarrow d2} = \int_0^{\infty} i_{\lambda 1} dA_1 \cos\theta_1 d\omega_1 d\lambda$$

$$= i_1 dA_1 \cos\theta_1 d\omega_1$$

$$d\omega_1 = \frac{dA_2 \cos\theta_2}{S^2}$$



$$dF_{d1-d2} = \frac{i_1 dA_1 \cos\theta_1 d\omega_1}{\int_{\cap} i_1 dA_1 \cos\theta_1 d\omega_1}$$

$$= \frac{i_1 dA_1 \cos\theta_1}{\int_{\cap} i_1 dA_1 \cos\theta_1 d\omega_1} \frac{dA_2 \cos\theta_2}{S^2}$$

$$dF_{d1-d2} = \frac{i_1 dA_1 \cos\theta_1}{\int_{\cap} i_1 dA_1 \cos\theta_1 d\omega_1} \frac{dA_2 \cos\theta_2}{S^2}$$

when i_1 is independent of θ, ϕ (diffuse radiation)

$$Q_{d1} = \int_{\cap} i_1 dA_1 \cos\theta_1 d\omega_1 = \pi i_1 dA_1$$

radiosity:

$$J = \int_{\cap}\int_0^{\infty} i_{\lambda,o} \cos\theta d\lambda d\omega = \int_{\cap} i_o \cos\theta d\omega, \; i_{\lambda,o} = i_{\lambda,e} + i_{\lambda,r}$$

for a diffuse surface, $J = \pi i_o$

$$Q_{d1} = \pi i_1 dA_1 = J_1 dA_1$$

$$Q_{d1\to d2} = i_1 dA_1 \cos\theta_1 d\omega_1 = J_1 dA_1 \frac{\cos\theta_1 d\omega_1}{\pi}$$

$$= J_1 dA_1 \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2 \equiv J_1 dA_1 dF_{d1-d2}$$

Thus, $$dF_{d1-d2} = \frac{Q_{d1\to d2}}{Q_{d1}} = \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2$$

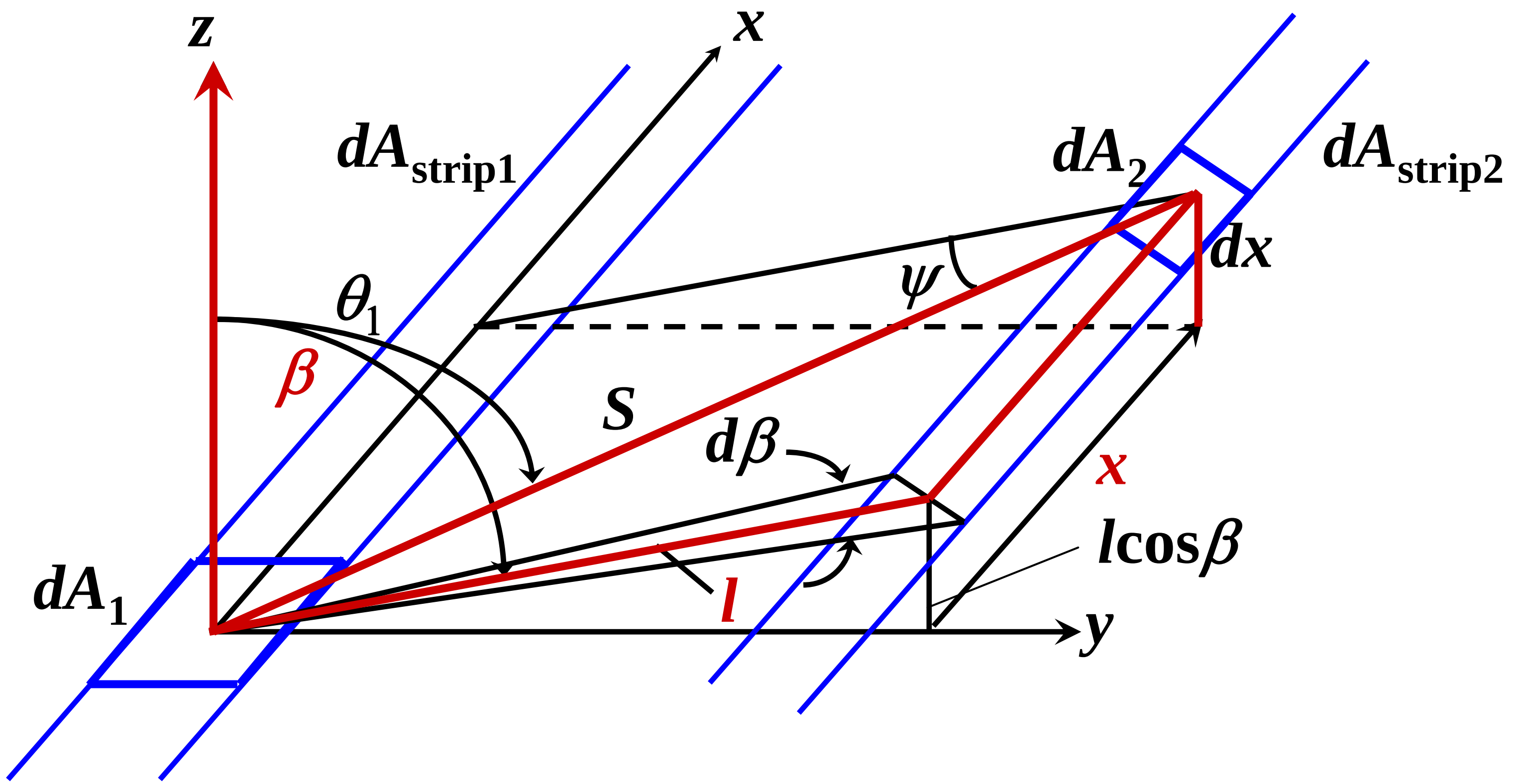
Similarly,

$$Q_{d2\to d1} = J_2 dA_2 \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_1$$

$$dF_{d2-d1} = \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_1$$

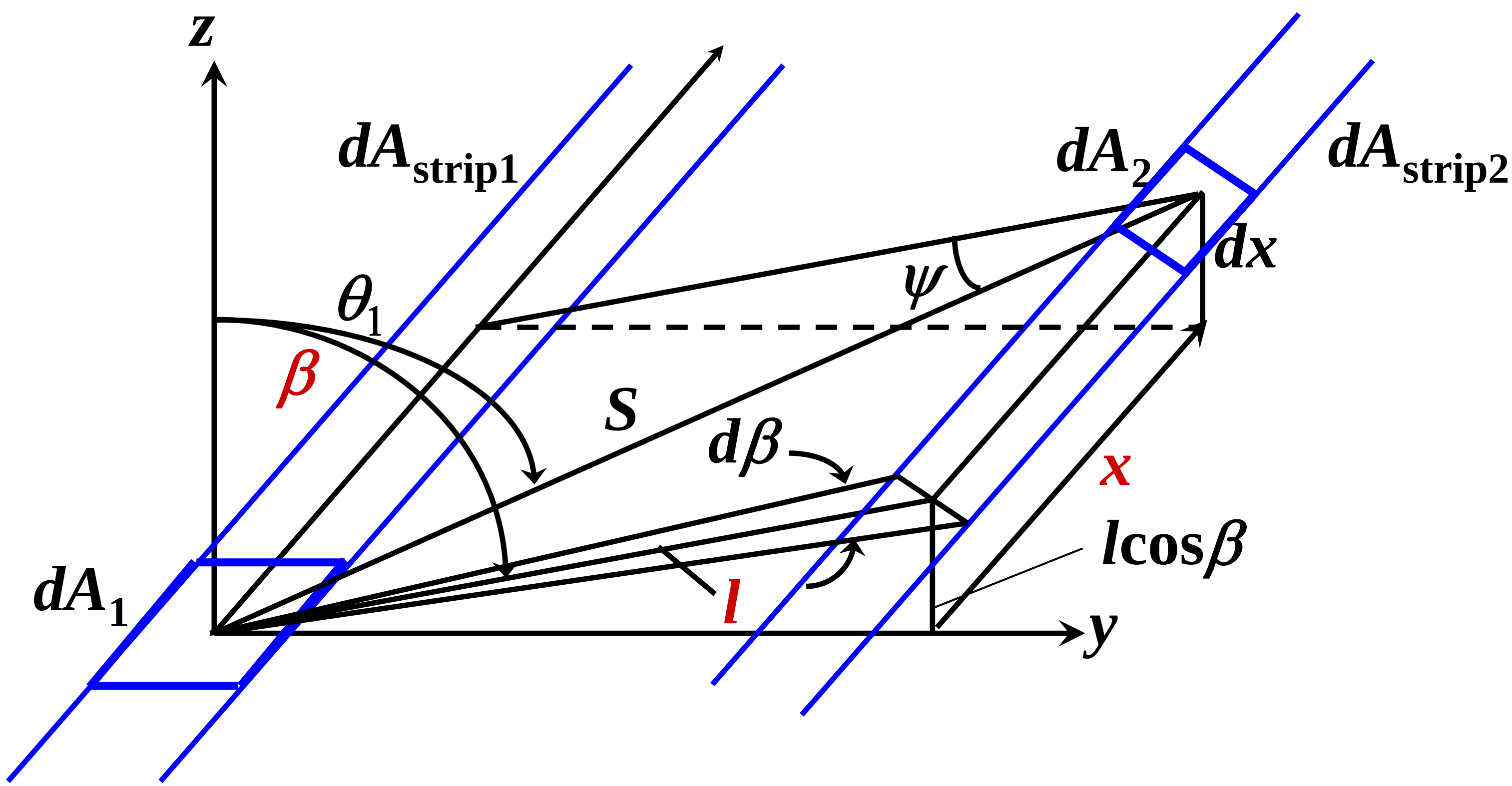
Reciprocity $$dA_1 dF_{d1-d2} = dA_2 dF_{d2-d1}$$

Ex 6-1 Two elmental areas located on parallel strips



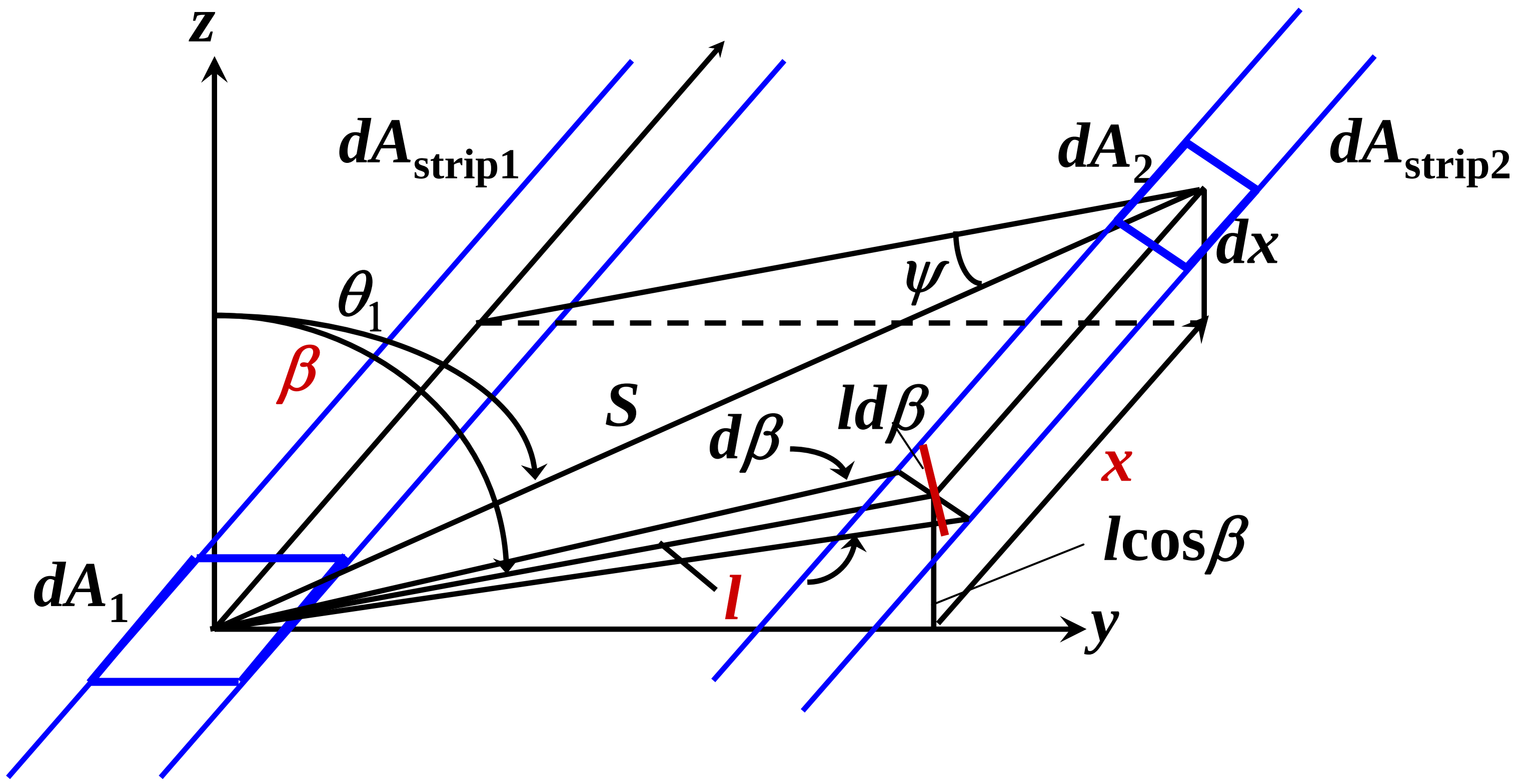
$$dF_{d1-d2} = \frac{\cos\theta_1 \cos\theta_2 dA_2}{\pi S^2} = \frac{\cos\theta_1}{\pi} d\omega_1,\ S^2 = l^2 + x^2$$

$$\cos\theta_1 = \frac{l\cos\beta}{S} = \frac{l\cos\beta}{\left(l^2 + x^2\right)^{1/2}}$$



$$d\omega_1 = \frac{\text{projected area of } dA_2}{S^2}$$

$$= \frac{(\text{projected width})(\text{projected length})}{S^2}$$



$$d\omega_1 = \frac{(l d\beta)(dx \cos\psi)}{S^2} = \frac{l^2 d\beta dx}{S^3} \left(\because \cos\psi = \frac{l}{S} \right)$$

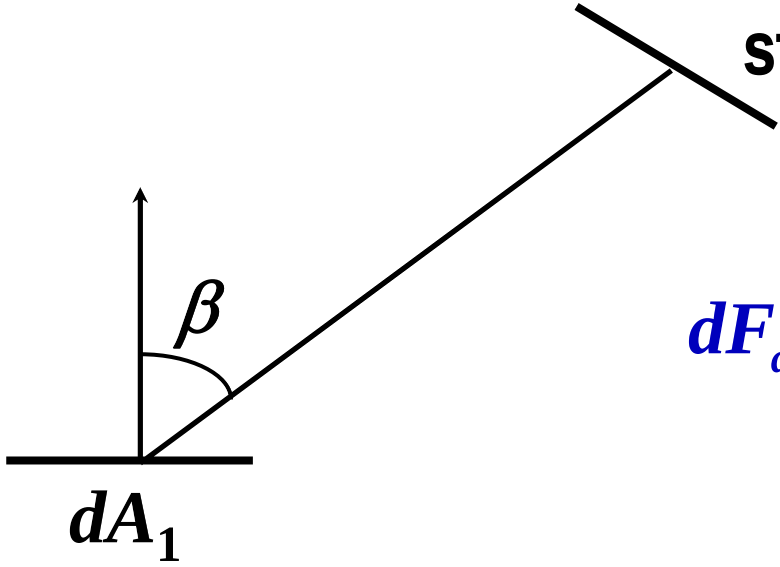
$$= \frac{l^2 d\beta dx}{\left(l^2 + x^2\right)^{3/2}}$$

$$dF_{d1-d2} = \frac{\cos\theta_1}{\pi} d\omega_1$$

$$= \frac{l\cos\beta}{\pi\left(l^2 + x^2\right)^{1/2}} \frac{l^2 d\beta dx}{\left(l^2 + x^2\right)^{3/2}}$$

$$= \frac{l^3 \cos\beta d\beta dx}{\pi\left(l^2 + x^2\right)^2}$$

Ex 6-2 dA_1 and long parallel strip2



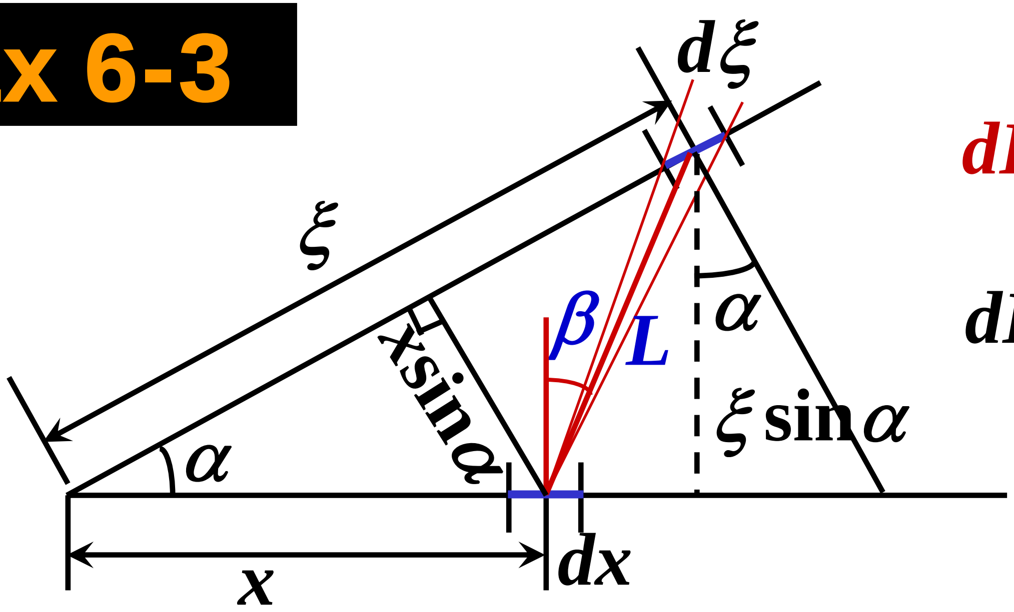
$$dF_{d1-d2} = \frac{\ell^3 \cos\beta d\beta dx}{\pi\left(\ell^2 + x^2\right)^2}$$

$$dF_{d1-\text{strip}2} = \frac{\ell^3 \cos\beta d\beta}{\pi}\int_{-\infty}^{\infty}\frac{dx}{\left(\ell^2 + x^2\right)^2}$$

$$= \frac{\cos\beta d\beta}{2} = \frac{1}{2}d(\sin\beta)$$

$$dF_{\text{strip1-strip2}} = \frac{1}{2}d(\sin\beta)$$

Ex 6-3



$$dF_{dx-d\xi} = ?$$

$$dF_{dx-d\xi} = \frac{1}{2} d(\sin\beta)$$

$$= \frac{1}{2} \cos\beta d\beta$$

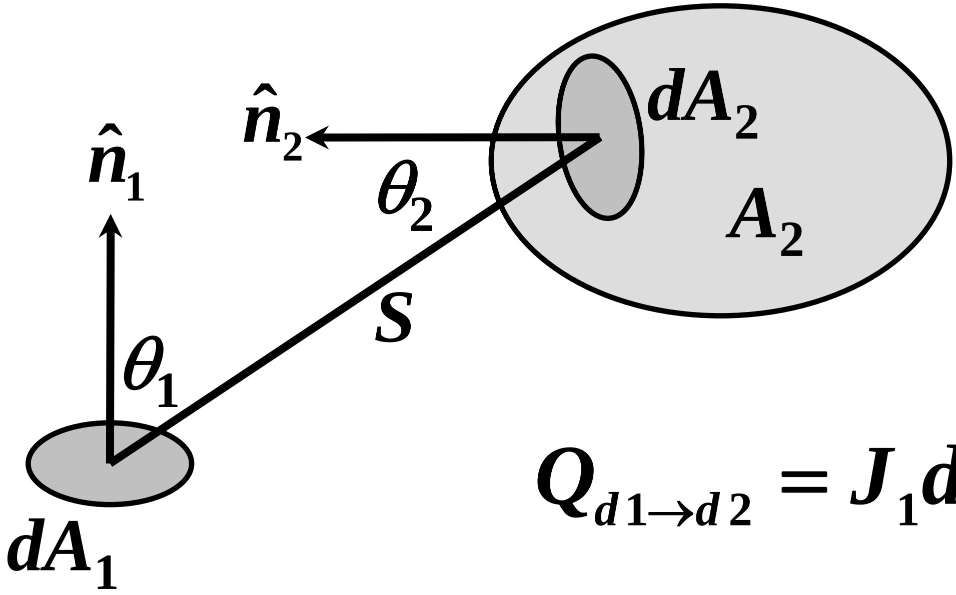
$$Ld\beta = d\xi \cos(\alpha+\beta)$$

$$d\beta = \frac{d\xi \cos(\alpha+\beta)}{L} = \frac{d\xi}{L} \frac{x\sin\alpha}{L}$$

$$L^2 = x^2 + \xi^2 - 2x\xi\cos\alpha, \quad \cos\beta = \frac{\xi\sin\alpha}{L}$$

$$dF_{dx-d\xi} = \frac{1}{2} \frac{x\xi\sin^2\alpha}{L^3} d\xi = \frac{1}{2} \frac{x\xi\sin^2\alpha}{\left(x^2+\xi^2-2x\xi\cos\alpha\right)^{3/2}} d\xi$$

Between a Differential Element and a Finite Area



$$Q_{d1\to d2} = J_1 dA_1 \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2$$

$$Q_{d1\to 2} = \int_{A_2} J_1 dA_1 \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2$$

$$= J_1 dA_1 \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2 = J_1 dA_1 \int_{A_2} dF_{d1-d2}$$

$$F_{d1-2} = \int_{A_2} dF_{d1-d2} = \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2$$

$$Q_{d2\to d1} = J_2 dA_2 \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_1$$

$$Q_{2\to d1} = \int_{A_2} Q_{d2\to d1} = \int_{A_2} J_2 dA_2 \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_1$$

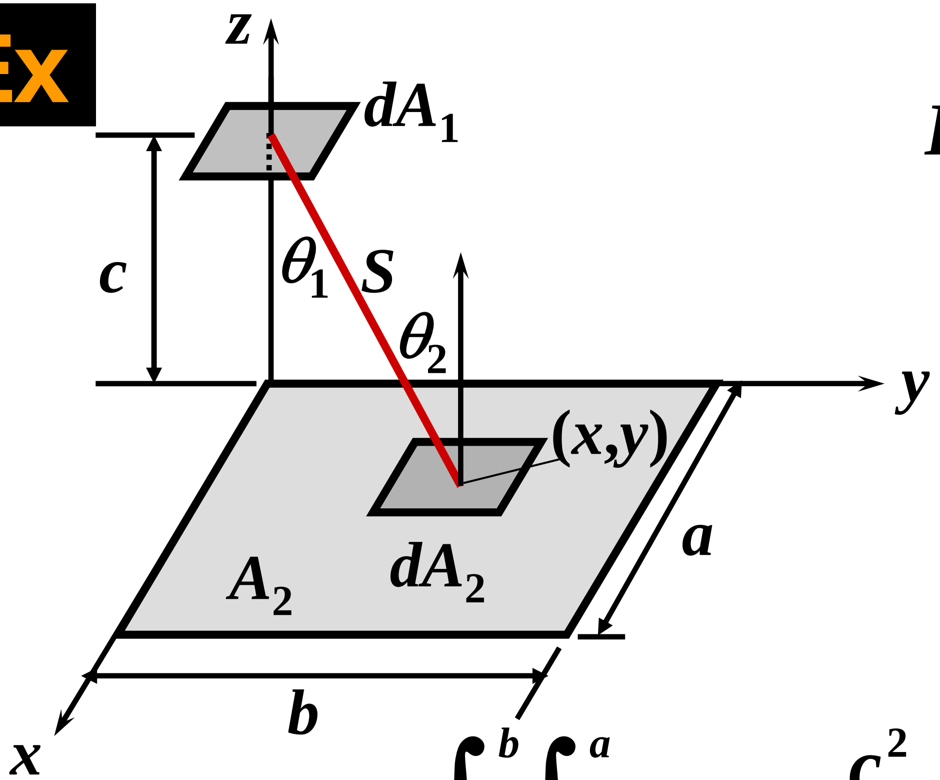
Assume J_2 is uniform over A_2, then

$$Q_{2\to d1} = J_2 A_2 \frac{dA_1}{A_2} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2 \equiv J_2 A_2 dF_{2-d1}$$

$$dF_{2-d1} = \frac{dA_1}{A_2} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2 = \frac{dA_1}{A_2} F_{d1-2}$$

Reciprocity $A_2 dF_{2-d1} = dA_1 F_{d1-2}$

Ex



$$F_{d1-2} = \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2$$

$$dA_2 = dxdy$$

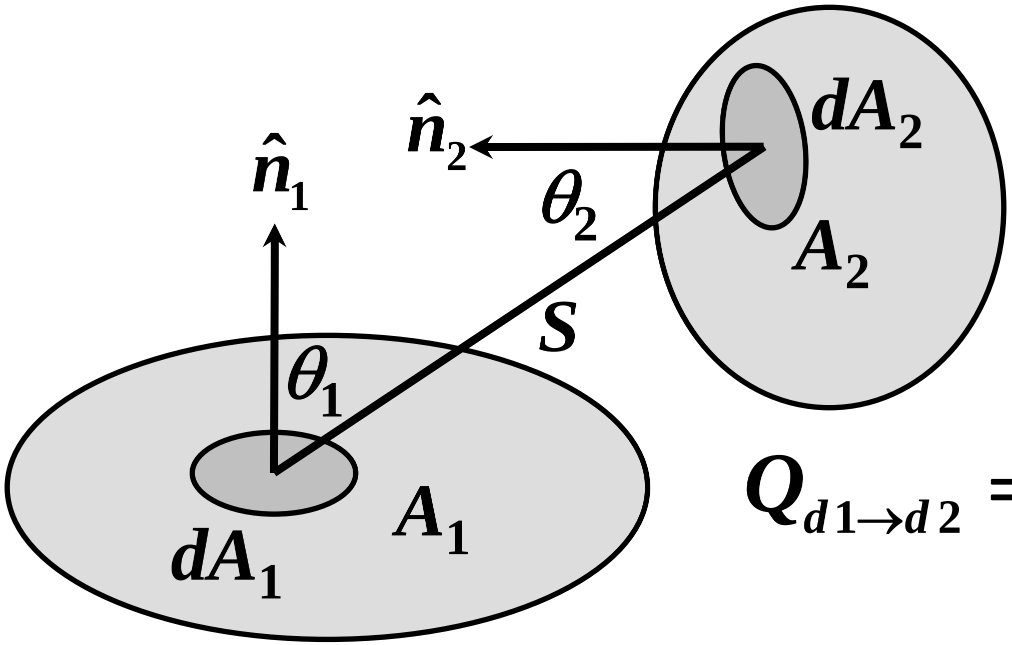
$$\cos\theta_1 = \cos\theta_2 = \frac{c}{S}$$

$$= \frac{c}{\sqrt{x^2 + y^2 + c^2}}$$

$$F_{d1-2} = \int_0^b \int_0^a \frac{c^2}{\pi\left(x^2 + y^2 + c^2\right)^2} dxdy$$

$$= \frac{1}{2\pi}\left[\frac{a}{\sqrt{a^2 + c^2}} \sin^{-1}\frac{b}{\sqrt{a^2 + b^2 + c^2}} + \frac{b}{\sqrt{b^2 + c^2}} \sin^{-1}\frac{a}{\sqrt{a^2 + b^2 + c^2}}\right]$$

Between Two Finite Areas



$$Q_{d1\to d2} = J_1 dA_1 \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2$$

J_1, J_2 is uniform over A_1, A_2, respectively

$$Q_{1\to 2} = \int_{A_1}\int_{A_2} J_1 dA_1 \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2$$

$$= J_1 A_1 \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2 dA_1 \equiv J_1 A_1 F_{12}$$

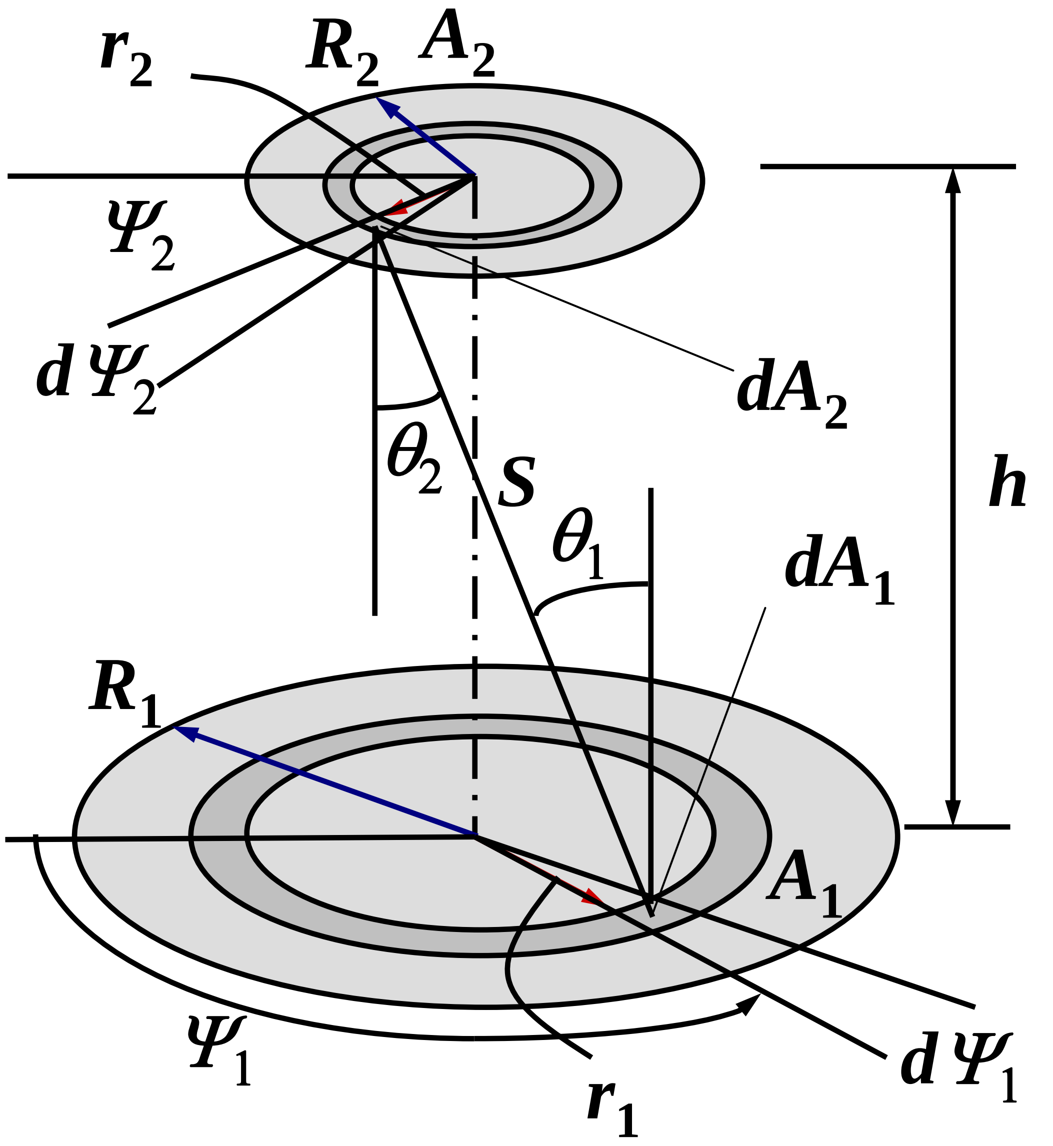
$$F_{12} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2 dA_1$$

Similarly

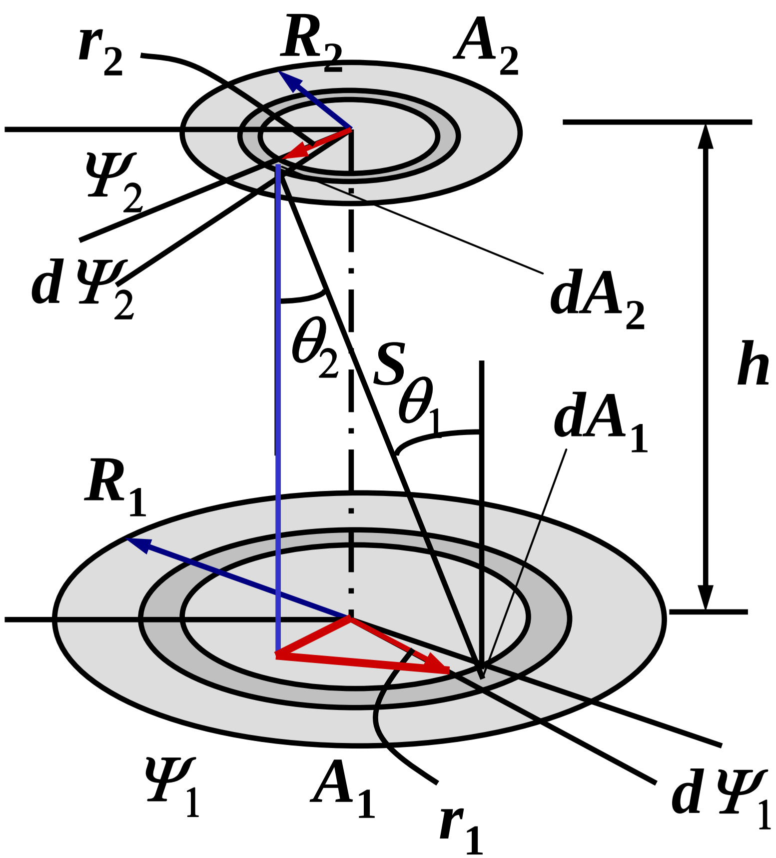
$$F_{21} = \frac{1}{A_2}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2 dA_1$$

Reciprocity $A_1 F_{12} = A_2 F_{21}$

Ex

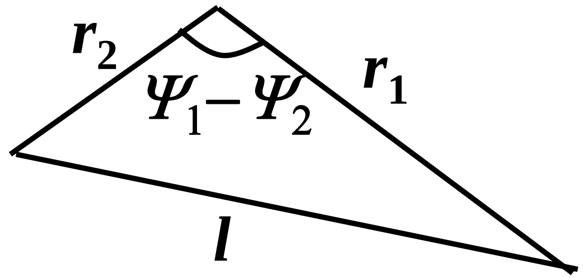


$$F_{12} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2 dA_1$$



$$\cos\theta_1 = \cos\theta_2 = \frac{h}{S}$$

$$dA_1 = r_1 d\psi_1 dr_1, \quad dA_2 = r_2 d\psi_2 dr_2$$



$$S^2 = h^2 + l^2$$

$$= h^2 + r_1^2 + r_2^2 - 2r_1 r_2 \cos(\psi_1 - \psi_2)$$

$$F_{12} = \frac{1}{\pi R_1^2}\int_0^{2\pi}\int_0^{R_1}\int_0^{2\pi}\int_0^{R_2}\frac{h^2 r_1 r_2}{\pi S^2} dr_2 d\psi_2 dr_1 d\psi_1$$

$$= \frac{1}{2}\left[\xi - \sqrt{\xi^2 - 4\left(\frac{\eta_2}{\eta_1}\right)^2}\right]$$

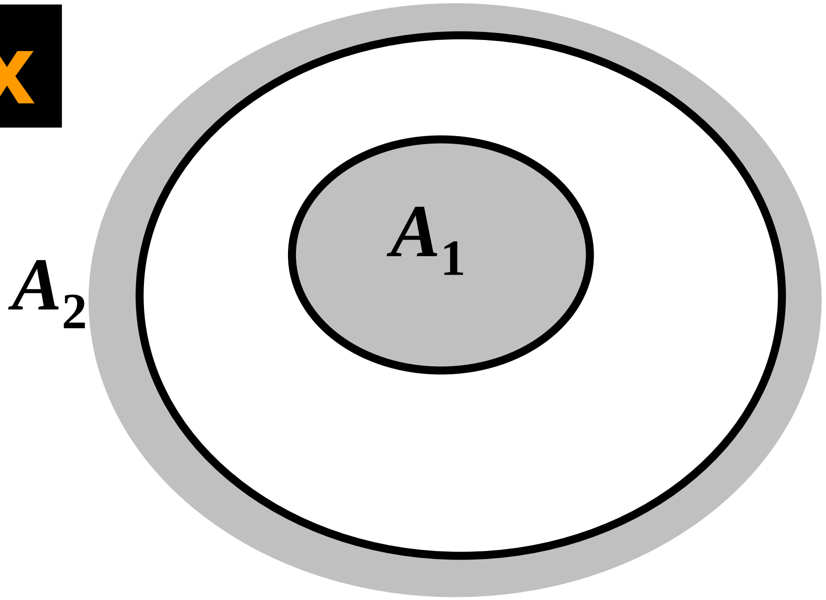
where

$$\xi = 1 + \frac{1+\eta_2^2}{\eta_1^2}, \quad \eta_1 = \frac{R_1}{h}, \quad \eta_2 = \frac{R_2}{h}$$

View factor relations

$$A_i F_{ij} = A_j F_{ji}, \quad \sum_{j=1}^{N} F_{ij} = 1$$

Ex



$$F_{11}, F_{12}, F_{21}, F_{22} = ?$$

$$F_{11} = 0$$

$$F_{12} = 1$$

$$A_2 F_{21} = A_1 F_{12} \rightarrow F_{21} = \frac{A_1}{A_2}$$

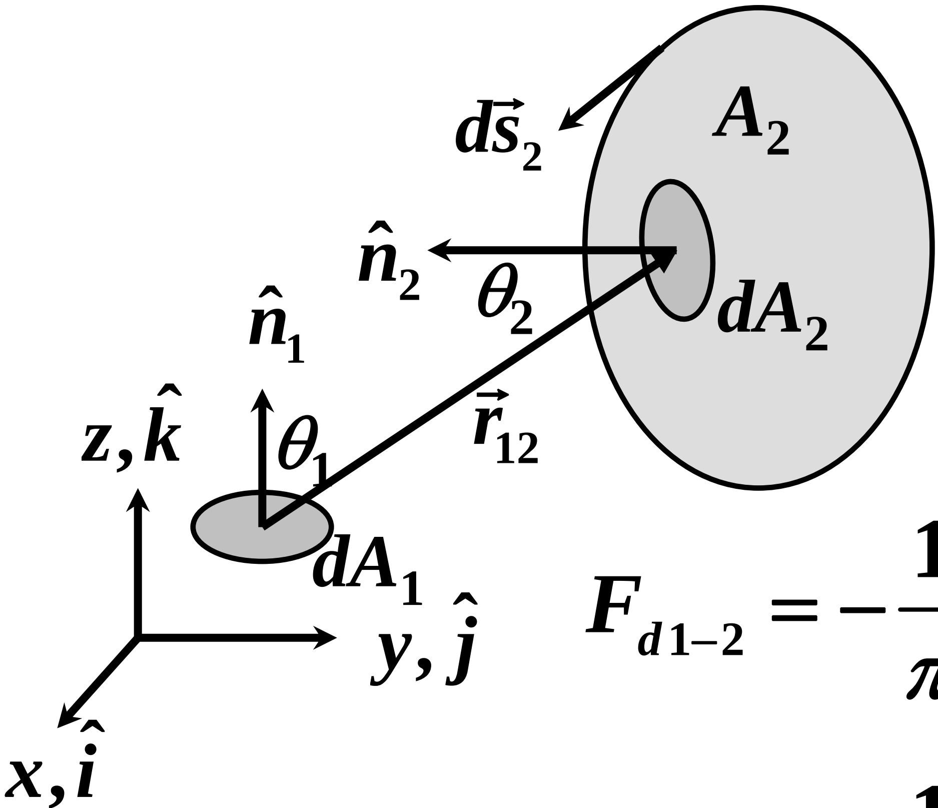
$$F_{21} + F_{22} = 1 \rightarrow F_{22} = 1 - F_{21} = 1 - \frac{A_1}{A_2}$$

Methods of Configuration Factor Evaluation

- **Direct integration**
  - Area integral
  - Contour integral
- **Flux algebra**
  - Cross-string method: 2D only
  - Decomposition of shapes
- **Sphere method**
  - Unit sphere method: only from a differential area
  - Inside sphere method

1) Contour integration

$$F_{d1-2} = \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_2$$



$$\cos\theta_1 = \frac{\hat{n}_1 \cdot \vec{r}_{12}}{r}, \quad |\vec{r}_{12}| = r$$

$$\cos\theta_2 = \frac{\hat{n}_2 \cdot \vec{r}_{21}}{r}$$

$$= -\frac{\hat{n}_2 \cdot \vec{r}_{12}}{r}$$

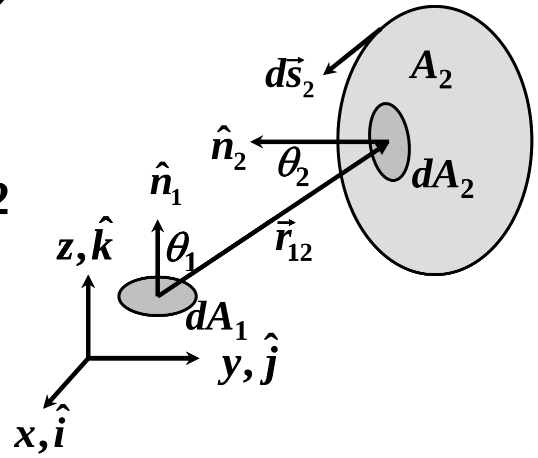
$$F_{d1-2} = -\frac{1}{\pi} \int_{A_2} \left(\frac{\hat{n}_1 \cdot \vec{r}_{12}}{r^2} \right) \left(\frac{\hat{n}_2 \cdot \vec{r}_{12}}{r^2} \right) dA_2$$

$$= -\frac{1}{\pi} \int_{A_2} \hat{n}_2 \cdot \left[\frac{\vec{r}_{12}}{r^2} \left(\frac{\hat{n}_1 \cdot \vec{r}_{12}}{r^2} \right) \right] dA_2$$

But $-\frac{\vec{r}_{12}}{r^2}\left(\frac{\hat{n}_1\cdot\vec{r}_{12}}{r^2}\right)=\frac{1}{2}\nabla\times\left(\frac{\vec{r}_{12}\times\hat{n}_1}{r^2}\right)$

$$F_{d1-2}=\frac{1}{2\pi}\int_{A_2}\hat{n}_2\cdot\left[\nabla\times\left(\frac{\vec{r}_{12}\times\hat{n}_1}{r^2}\right)\right]dA_2$$

$$=\frac{1}{2\pi}\oint_{c_2}\left(\frac{\vec{r}_{12}\times\hat{n}_1}{r^2}\right)\cdot d\vec{s}_2$$



in a cartesian coordinate system (x, y, z)

$$d\vec{s}_2=dx_2\hat{i}+dy_2\hat{j}+dz_2\hat{k}$$

$$r^2=\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2$$

$$\vec{r}_{12}=\left(x_2-x_1\right)\hat{i}+\left(y_2-y_1\right)\hat{j}+\left(z_2-z_1\right)\hat{k},$$

$$\hat{n}_1=\ell_1\hat{i}+m_1\hat{j}+n_1\hat{k}$$

$$F_{d1-2} = \frac{\ell_1}{2\pi} \oint_{c_2} \frac{(z_2 - z_1)dy_2 - (y_2 - y_1)dz_2}{r^2}$$

$$+ \frac{m_1}{2\pi} \oint_{c_2} \frac{(x_2 - x_1)dz_2 - (z_2 - z_1)dx_2}{r^2}$$

$$+ \frac{n_1}{2\pi} \oint_{c_2} \frac{(y_2 - y_1)dx_2 - (x_2 - x_1)dy_2}{r^2}$$

$$A_1 F_{12} = \int_{A_1} F_{d1-2} dA_1$$

$$= \frac{1}{2\pi} \oint_{c_2} \left[\int_{A_1} \frac{(y_2 - y_1)n_1 - (z_2 - z_1)m_1}{r^2} dA_1 \right] dx_2$$

$$+ \frac{1}{2\pi} \oint_{c_2} \left[\int_{A_1} \frac{(z_2 - z_1)\ell_1 - (x_2 - x_1)n_1}{r^2} dA_1 \right] dy_2$$

$$+ \frac{1}{2\pi} \oint_{c_2} \left[\int_{A_1} \frac{(x_2 - x_1)m_1 - (y_2 - y_1)\ell_1}{r^2} dA_1 \right] dz_2$$

$$\int_{A_1} \frac{(y_2 - y_1)n_1 - (z_2 - z_1)m_1}{r^2} dA_1$$

$$= \int_{A_1} \hat{n}_1 \cdot \left(\nabla \times \vec{V}_1\right) dA_1 = \oint_{c_1} \vec{V}_1 \cdot d\vec{s}_1 = \oint_{c_1} \ln r \, dx_1$$

where $\vec{V}_1 = \hat{i} \ln r$, $\nabla = \hat{i} \frac{\partial}{\partial x_1} + \hat{j} \frac{\partial}{\partial y_1} + \hat{k} \frac{\partial}{\partial z_1}$

Similarly

$$\int_{A_1} \frac{(z_2 - z_1)\ell_1 - (x_2 - x_1)n_1}{r^2} dA_1 = \oint_{c_1} \vec{V}_2 \cdot d\vec{s} = \oint_{c_1} \ln r \, dy_1$$

$$\int_{A_1} \frac{(x_2 - x_1)m_1 - (y_2 - y_1)\ell_1}{r^2} dA_1 = \oint_{c_1} \vec{V}_3 \cdot d\vec{s} = \oint_{c_1} \ln r \, dz_1$$

where $\vec{V}_2 = \hat{j} \ln r$, $\vec{V}_3 = \hat{k} \ln r$

$$A_1F_{12} = \frac{1}{2\pi}\oint_{c_2}\left(\oint_{c_1}\ln r\, dx_1\right)dx_2$$

$$+\frac{1}{2\pi}\oint_{c_2}\left(\oint_{c_1}\ln r\, dy_1\right)dy_2$$

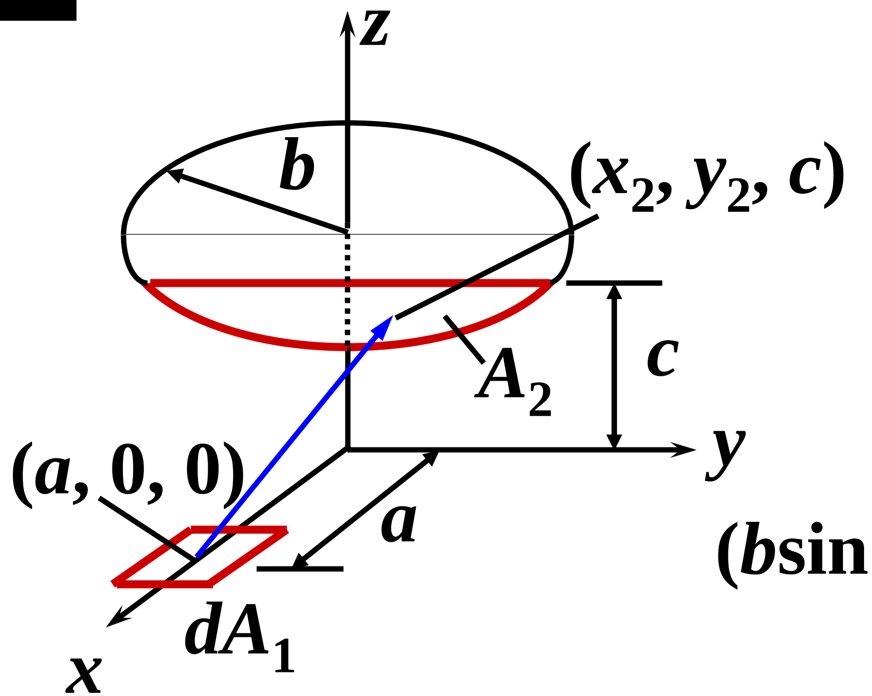
$$+\frac{1}{2\pi}\oint_{c_2}\left(\oint_{c_1}\ln r\, dz_1\right)dz_2$$

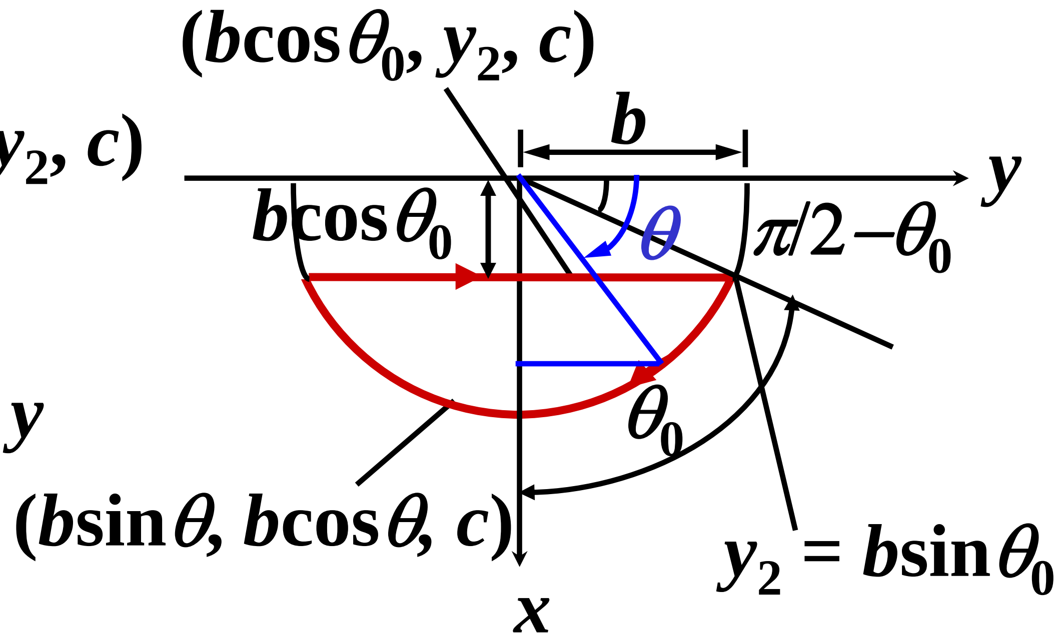
$$= \frac{1}{2\pi}\oint_{c_1}\oint_{c_2}\left(\ln r\, dx_2 dx_1 + \ln r\, dy_2 dy_1 + \ln r\, dz_2 dz_1\right)$$

where

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Ex





$$F_{d1-2} = \frac{1}{2\pi}\oint_{c_2}\left(\frac{\vec{r}_{12}\times\hat{n}_1}{r^2}\right)\cdot d\vec{s}_2$$

$$\vec{r}_{12} = \left(x_2 - a\right)\hat{i} + y_2\hat{j} + c\hat{k}, \quad \hat{n}_1 = \hat{k}$$

$$d\vec{s}_2 = dx_2\hat{i} + dy_2\hat{j}\ , \quad r^2 = (x_2 - a)^2 + y_2^2 + c^2$$

$$F_{d1-2} = \frac{1}{2\pi}\oint_{c_2}\left(\frac{\vec{r}_{12}\times\hat{n}_1}{r^2}\right)\cdot d\vec{s}_2$$

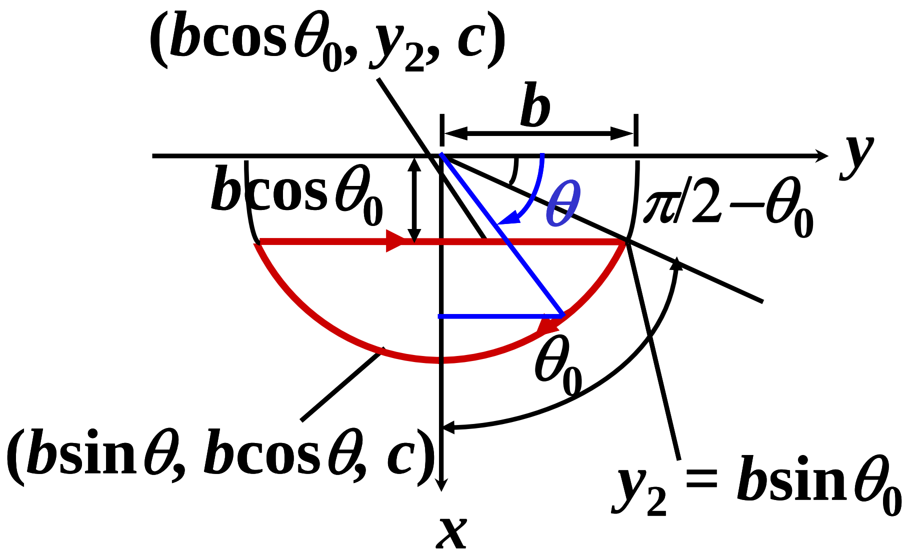
$$\left(\vec{r}_{12}\times\hat{n}_1\right)\cdot d\vec{s}_2 = \left(y_2\hat{i} - (x_2 - a)\hat{j}\right)\cdot\left(dx_2\hat{i} + dy_2\hat{j}\right)$$

$$= y_2 dx_2 - (x_2 - a)dy_2$$

$$F_{d1-2} = \frac{1}{2\pi}\oint_{c_2}\frac{y_2 dx_2 - (x_2 - a)dy_2}{(x_2 - a)^2 + y_2^2 + c^2}$$

integral along the arc

$$F_{d1-2} = \frac{1}{2\pi}\oint_{c_2} \frac{y_2 dx_2 - (x_2 - a)dy_2}{(x_2-a)^2 + y_2^2 + c^2}$$



$$x_2 = b\sin\theta,\quad y_2 = b\cos\theta$$

$$dx_2 = b\cos\theta d\theta,$$

$$dy_2 = -b\sin\theta d\theta$$

$$y_2 dx_2 = b^2\cos^2\theta d\theta$$

$$(x_2 - a)dy_2$$

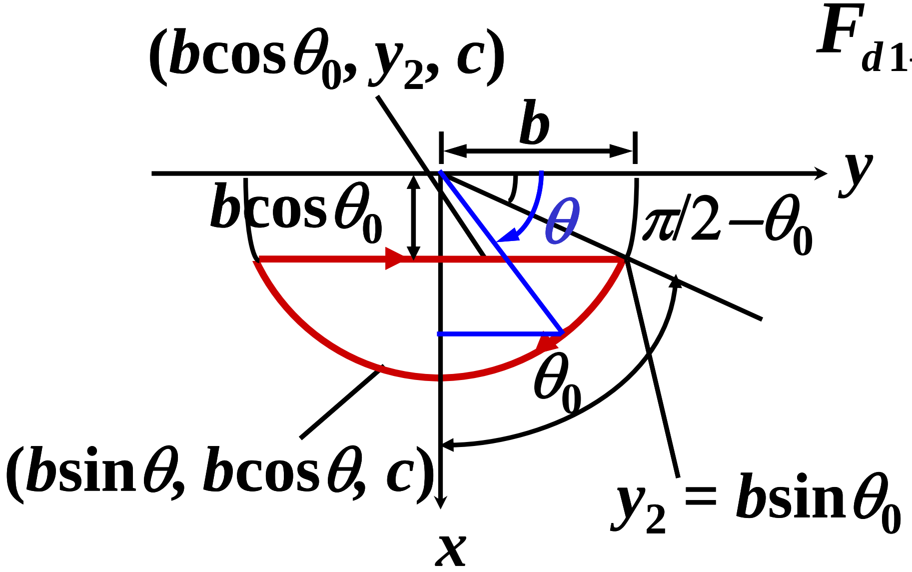
$$= (b\sin\theta - a)(-b\sin\theta)d\theta$$

$$= -(b^2\sin^2\theta + ab\sin\theta)d\theta$$

$$\int_{\cup} = 2\int_{\frac{\pi}{2}-\theta_0}^{\frac{\pi}{2}} \frac{b^2 - ab\sin\theta}{a^2+b^2+c^2-2ab\sin\theta}d\theta$$

$$\overset{\theta'=\frac{\pi}{2}-\theta}{=} 2\int_{\theta_0}^{0} \frac{ab\cos\theta' - b^2}{a^2+b^2+c^2-2ab\cos\theta'}d\theta'$$

integral along the straight line



$$F_{d1-2} = \frac{1}{2\pi}\oint_{c_2} \frac{y_2 dx_2 - (x_2 - a)dy_2}{(x_2-a)^2 + y_2^2 + c^2}$$

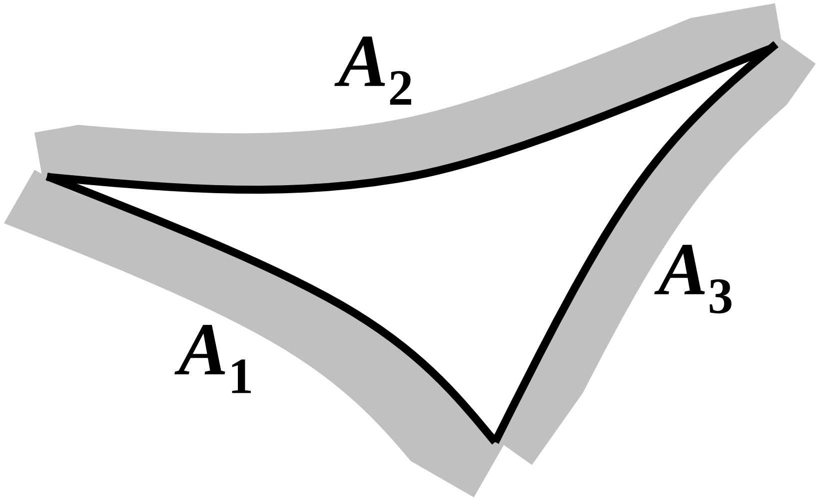
$$x_2 = b\cos\theta_0,\ dx_2 = 0,$$

$$x_2 - a = b\cos\theta_0 - a$$

$$\int_{\rightarrow} = 2\int_0^{b\sin\theta_0} \frac{-b\cos\theta_0 + a}{(b\cos\theta_0 - a)^2 + y_2^2 + c^2} dy_2$$

$$= -2(b\cos\theta_0 - a)\int_0^{b\sin\theta_0} \frac{dy_2}{(b\cos\theta_0 - a)^2 + y_2^2 + c^2}$$

2) Cross-string method : 2-D only



$$A_1F_{12} = A_2F_{21}$$

$$A_1F_{13} = A_3F_{31}$$

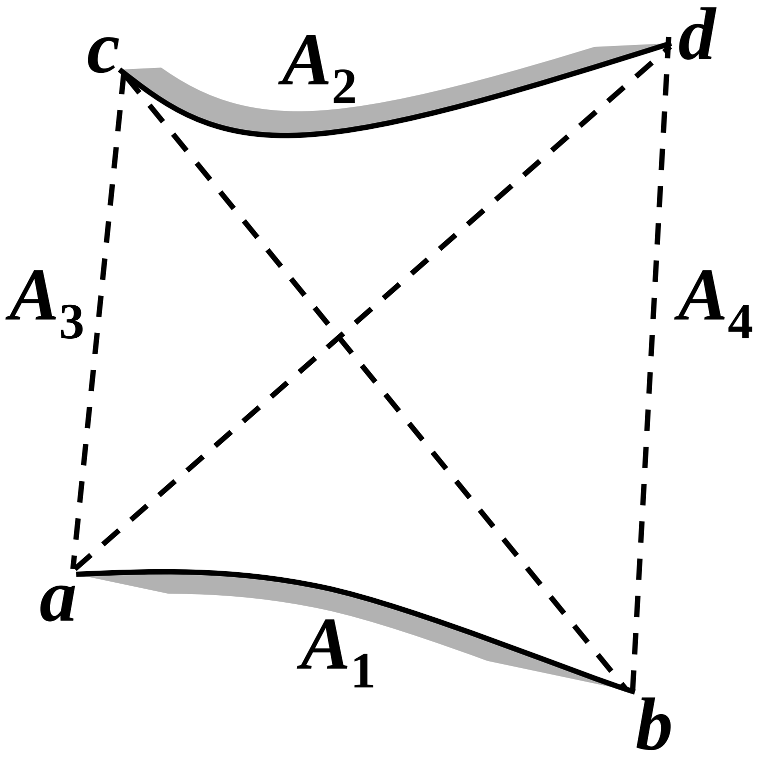
$$A_2F_{23} = A_3F_{32}$$

$$F_{11} = F_{22} = F_{33} = 0$$

$$F_{12} + F_{13} = 1, \quad F_{21} + F_{23} = 1, \quad F_{31} + F_{32} = 1$$

6 unknowns and 6 equations

$$F_{12} = \frac{A_1 + A_2 - A_3}{2A_1}, \qquad F_{23} = \frac{A_2 + A_3 - A_1}{2A_2}$$



$$F_{12} = 1 - F_{13} - F_{14}$$

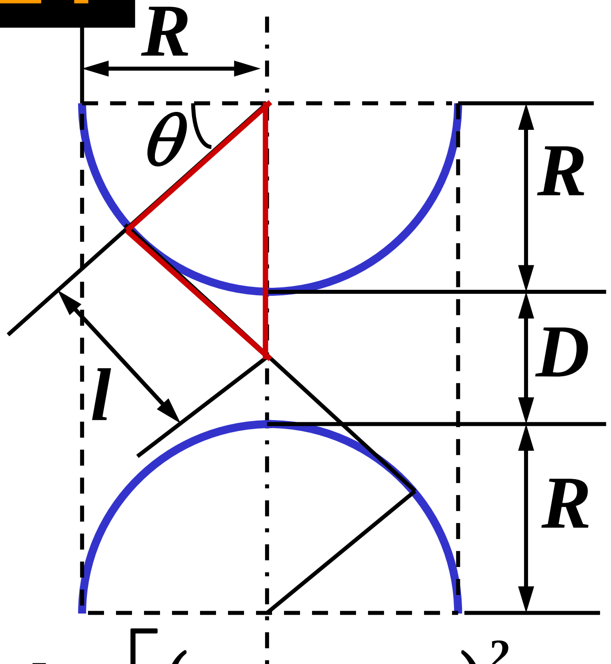
$$F_{13} = \frac{ab + ac - bc}{2ab}$$

$$F_{14} = \frac{ab + bd - ad}{2ab}$$

$$F_{12} = 1 - \frac{ab + ac - bc + ab + bd - ad}{2ab}$$

$$= \frac{(bc + ad) - (ac + bd)}{2ab}$$

Ex 6-14



$$F_{12} = \frac{2\left[R\theta + l\right] - 2\left[R + D/2\right]}{\pi R}$$

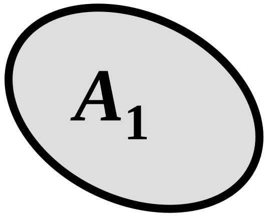
$$\sin\theta = \frac{R}{D/2 + R}$$

$$l = \left[\left(D/2 + R\right)^2 - R^2\right]^{1/2} = \left(D^2/4 + DR\right)^{1/2}$$

$$F_{12} = \frac{2\left(\frac{D^2}{4} + DR\right)^{1/2} + 2R\sin^{-1}\left(\frac{R}{D/2 + R}\right) - (D + 2R)}{\pi R}$$

$$F_{12} = \frac{2}{\pi}\left[\sqrt{X^2 - 1} + \sin^{-1}\frac{1}{X} - X\right] \quad \text{where} \quad X = 1 + \frac{D}{2R}$$

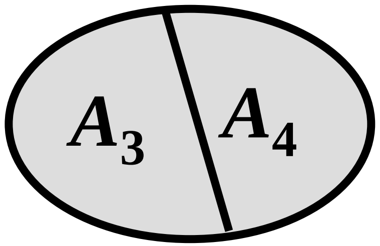
3) Decomposition of shapes



$$F_{12} = \text{known}$$

$$F_{14} = \text{known}$$

$$F_{13} = ?$$



$$A_2 = A_3 + A_4$$

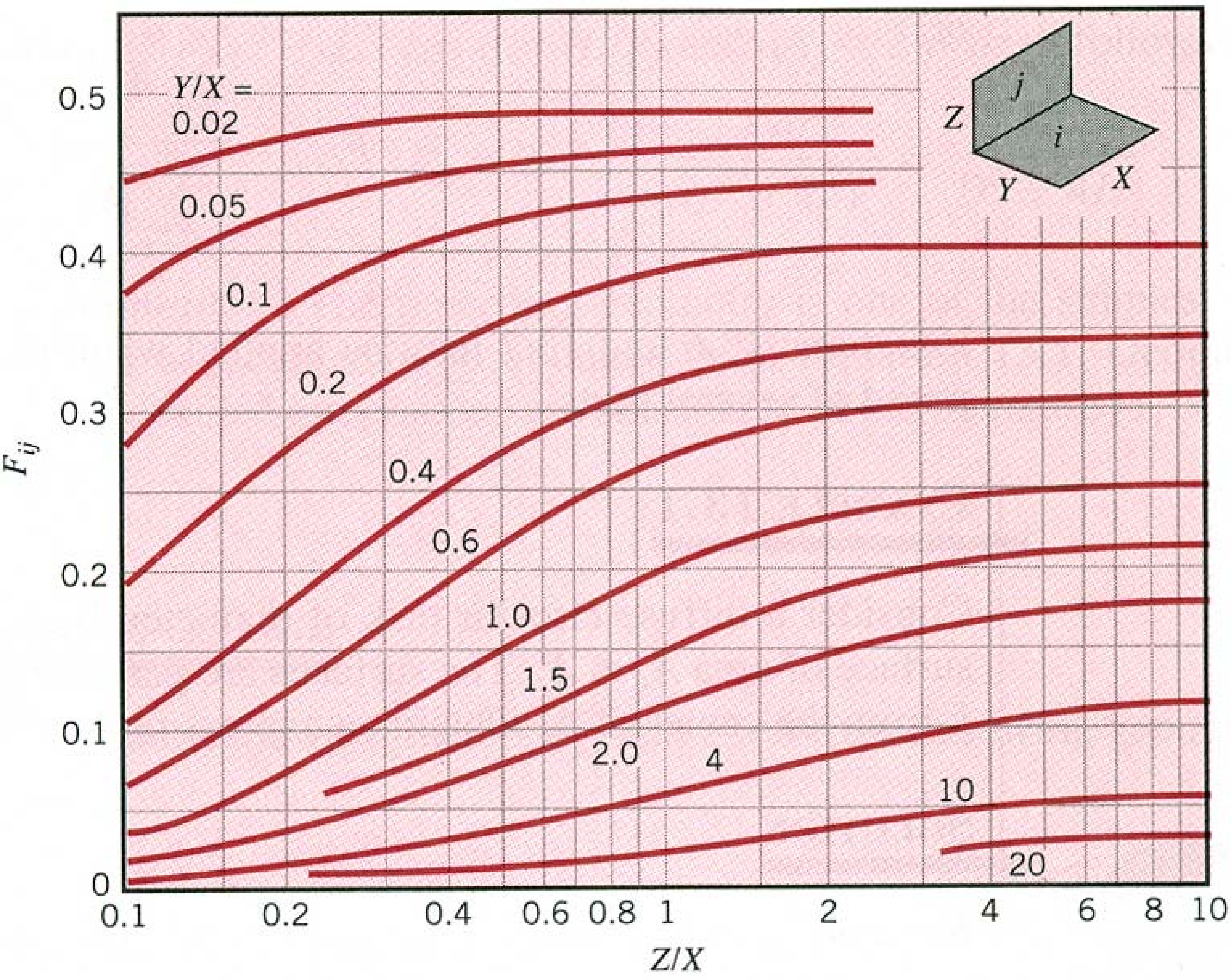
$$F_{12} = F_{13} + F_{14}$$

$$F_{13} = F_{12} - F_{14}$$

Remark: $F_{(3+4)-1} \neq F_{31} + F_{41}$

$$F_{(3+4)-1} = \frac{A_1}{A_{(3+4)}} F_{1-(3+4)} = \frac{A_1}{A_2}\left[F_{13} + F_{14}\right]$$

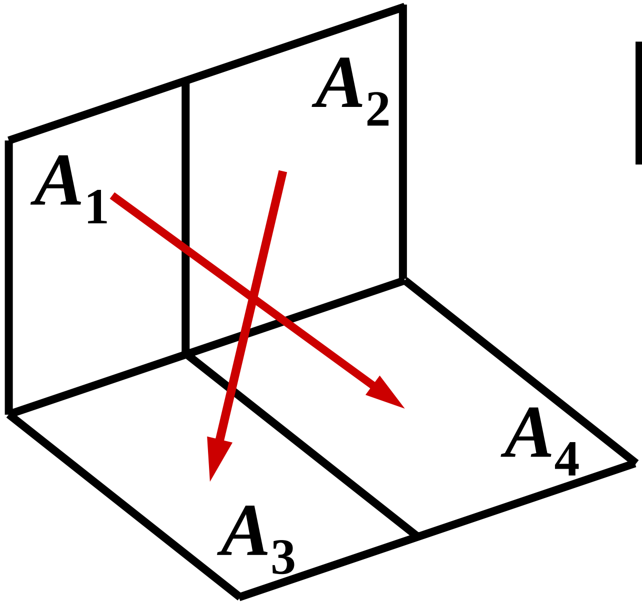
$$= \frac{A_1}{A_2}\left[\frac{A_3}{A_1} F_{31} + \frac{A_4}{A_1} F_{41}\right] = \frac{A_3}{A_2} F_{31} + \frac{A_4}{A_2} F_{41}$$



Y/X =
0.02
0.05
0.1
0.2
0.4
0.6
1.0
1.5
2.0
4
10
20
0.5
0.4
0.3
0.2
0.1
0
F_ij
0.1
0.2
0.4
0.6
0.8
1
2
4
6
8
10
Z/X
Z
j
i
Y
X

Cross reciprocity

$$A_1F_{14} = A_2F_{23}$$



Ex $F_{14} = ?$

$$F_{(1+2)-(3+4)} = F_{(1+2)-3} + F_{(1+2)-4}$$

$$= \frac{A_3}{A_{(1+2)}}F_{3-(1+2)} + \frac{A_4}{A_{(1+2)}}F_{4-(1+2)}$$

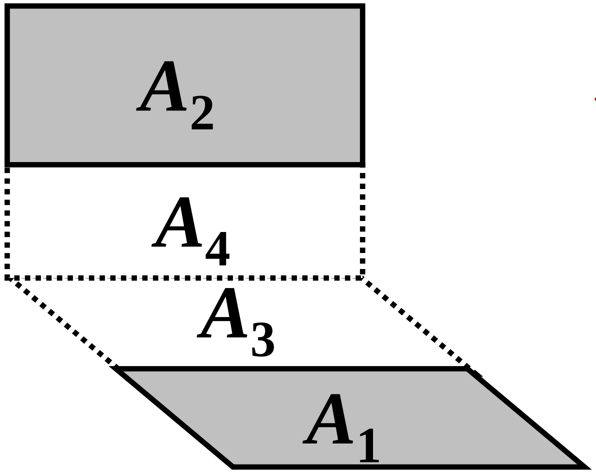
$$= \frac{A_3}{A_{(1+2)}}\left(F_{31} + F_{32}\right) + \frac{A_4}{A_{(1+2)}}\left(F_{41} + F_{42}\right)$$

$$A_3F_{32} = A_4F_{41} = A_1F_{14} \rightarrow F_{32} = \frac{A_1}{A_3}F_{14},\ F_{41} = \frac{A_1}{A_4}F_{14}$$

$$F_{(1+2)-(3+4)} = \frac{A_3}{A_{(1+2)}}\left(F_{31} + \frac{A_1}{A_3}F_{14}\right) + \frac{A_4}{A_{(1+2)}}\left(\frac{A_1}{A_4}F_{14} + F_{42}\right)$$

$$F_{14} = \frac{1}{2A_1}\left(A_{(1+2)}F_{(1+2)-(3+4)} - A_3F_{31} - A_4F_{42}\right)$$

Ex



$F_{12} = ?$

known configuration factors

$F_{(1+3)-(2+4)}$, $F_{(1+3)-4}$, $F_{3-(2+4)}$, F_{34}

$$F_{(1+3)-(2+4)} = F_{(1+3)-2} + F_{(1+3)-4} = \frac{A_2}{A_{13}} F_{2-(1+3)} + F_{(1+3)-4}$$

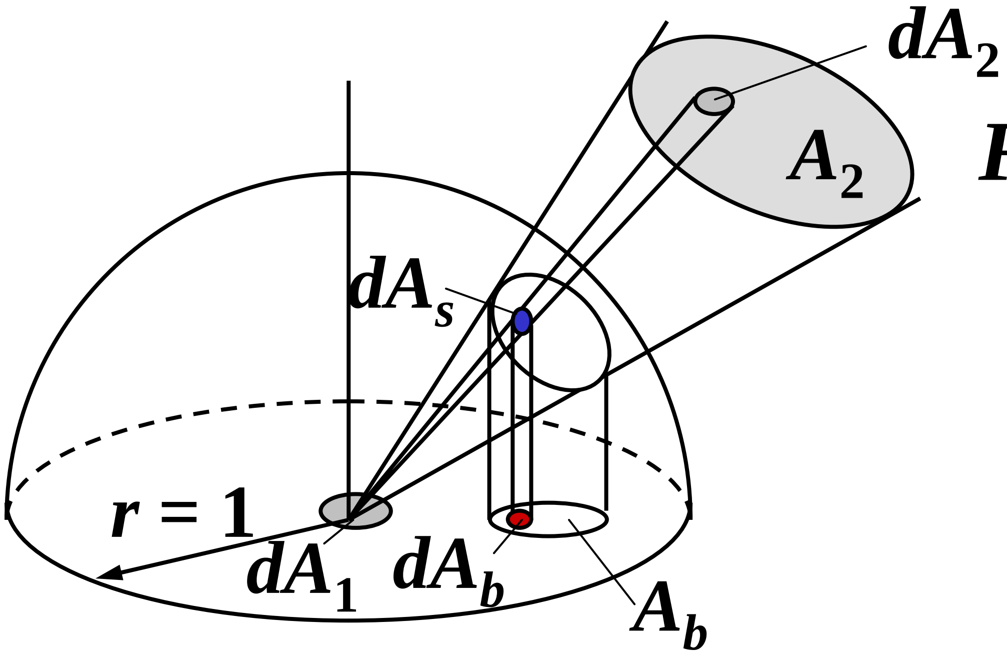
$$= \frac{A_2}{A_{13}}\left(F_{21} + F_{23}\right) + F_{(1+3)-4}, \quad F_{21} = \frac{A_1}{A_2} F_{12}$$

$$F_{3-(2+4)} = F_{32} + F_{34} = \frac{A_2}{A_3} F_{23} + F_{34} \rightarrow F_{23} = \frac{A_3}{A_2}\left(F_{3-(2+4)} - F_{34}\right)$$

$$F_{(1+3)-(2+4)} = \frac{A_2}{A_{13}}\left[\frac{A_1}{A_2} F_{12} + \frac{A_3}{A_2}\left(F_{3-(2+4)} - F_{34}\right)\right] + F_{(1+3)-4}$$

$$F_{12} = \frac{A_{13}F_{(1+3)-(2+4)} + A_3F_{34} - A_3F_{3-(2+4)} - A_{13}F_{(1+3)-4}}{A_1}$$

4) Unit sphere method
(only from a differential area)



$$F_{d1-2} = \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2$$

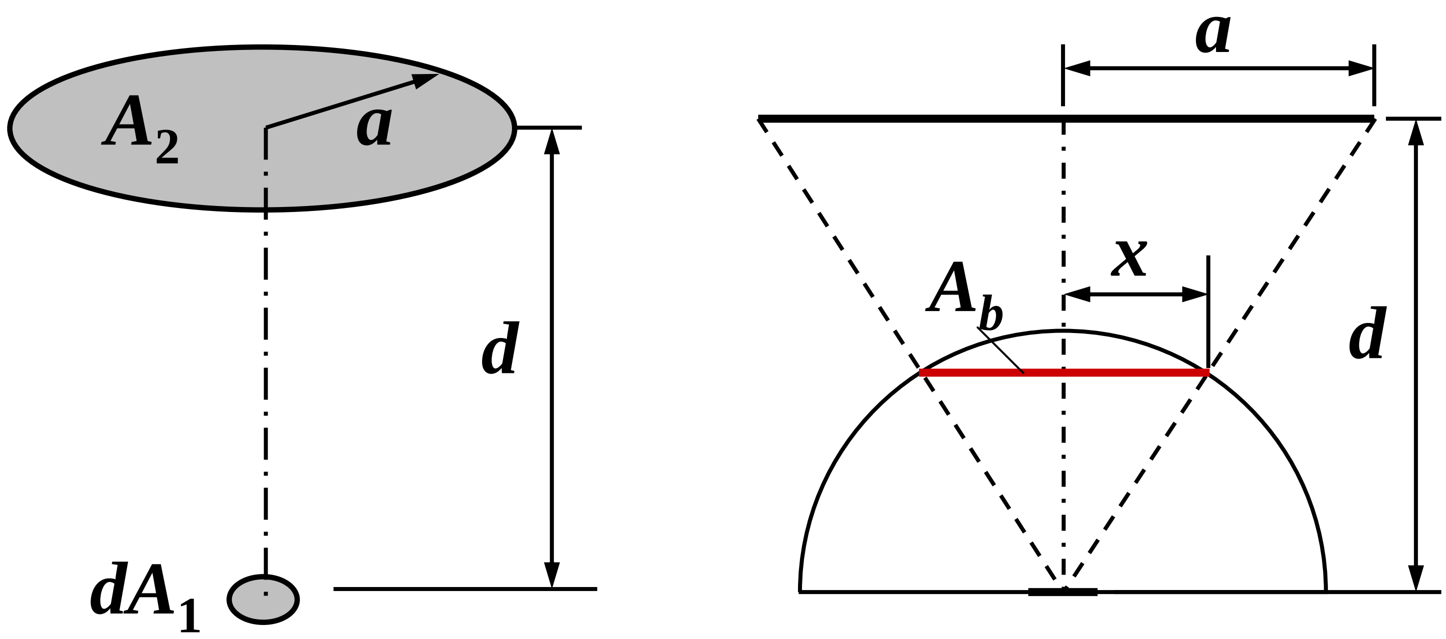
$$= \frac{1}{\pi} \int_{A_2} \cos\theta_1 d\omega_1$$

$$d\omega_1 = \frac{dA_2 \cos\theta_2}{S^2}$$

$$d\omega_1 = \frac{dA_2 \cos\theta_2}{S^2} = \frac{dA_s}{r^2} = dA_s \quad (r = 1)$$

$$F_{d1-2} = \frac{1}{\pi} \int_{A_s} \cos\theta_1 dA_s = \frac{1}{\pi} \int_{A_b} dA_b = \frac{A_b}{\pi}$$

Ex

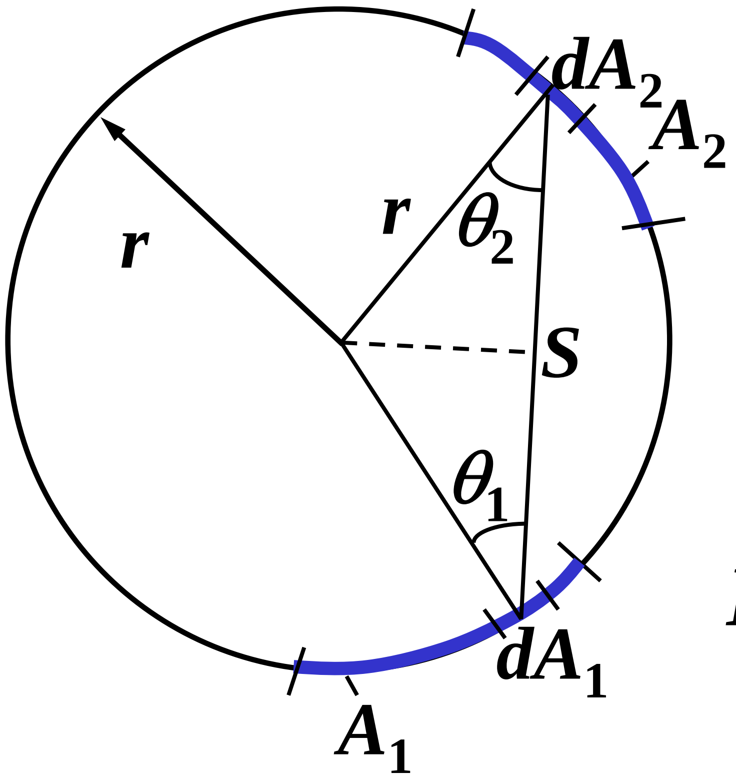


$$F_{d1-2} = \frac{A_b}{\pi}$$

$$\sqrt{a^2 + d^2} : a = 1 : x \rightarrow x = \frac{a}{\sqrt{a^2 + d^2}}$$

$$F_{d1-2} = \frac{1}{\pi} \cdot \pi \left(\frac{a}{\sqrt{a^2 + d^2}} \right)^2 = \frac{a^2}{a^2 + d^2}$$

5) Inside sphere method



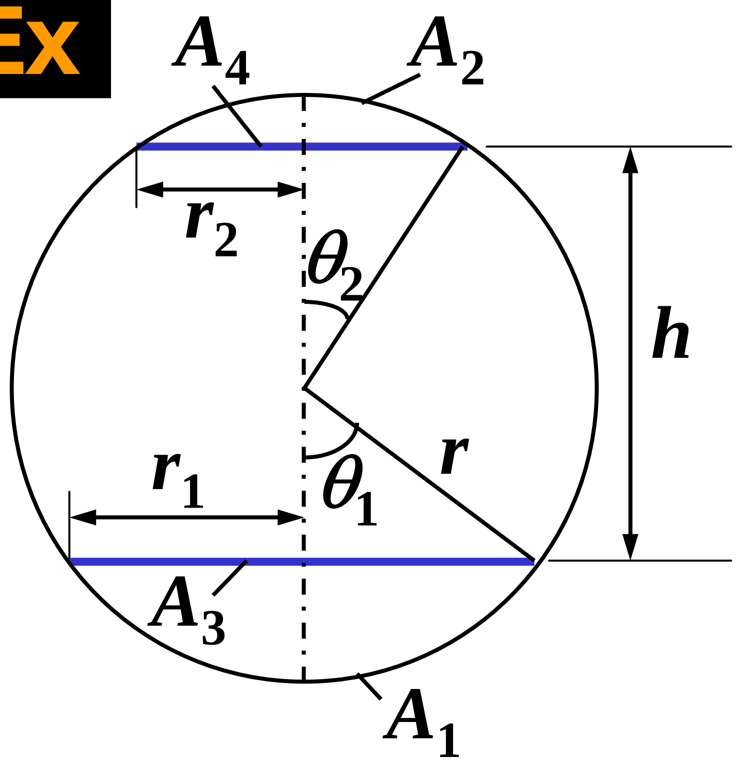
$$F_{d1-2} = \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi S^2} dA_2$$

$$\cos\theta_1 = \cos\theta_2 = \frac{S}{2r}$$

$$F_{d1-2} = \int_{A_2} \frac{dA_2}{4\pi r^2} = \frac{A_2}{4\pi r^2} = \frac{A_2}{A_s}$$

$$F_{12} = \frac{1}{A_1}\int_{A_1} F_{d1-2} dA_1 = \frac{1}{A_1}\int_{A_1} \frac{A_2}{A_s} dA_1 = \frac{A_2}{A_s} = \frac{A_2}{4\pi r^2}$$

Ex



$$F_{12} = \frac{A_2}{A_s}$$

$$d\omega = \frac{dA_2}{r^2}, \quad d\omega = \sin\theta d\theta d\phi$$

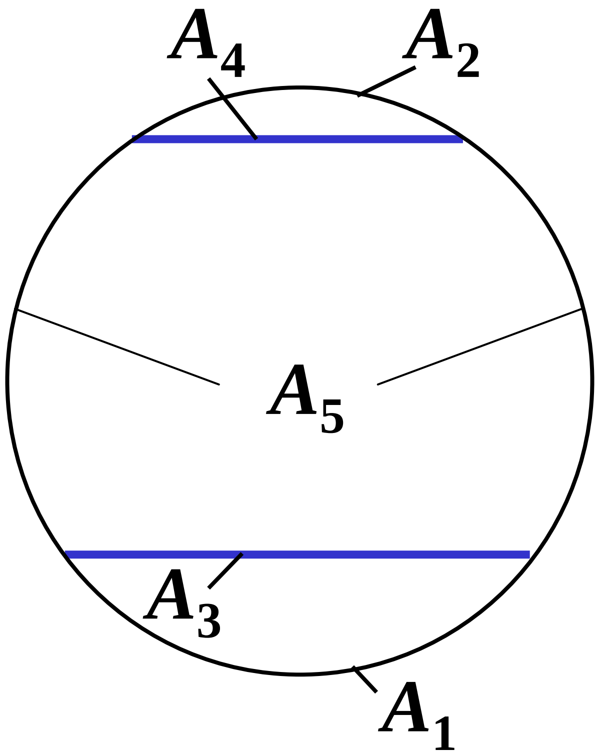
$$A_2 = \int_0^{2\pi}\int_0^{\theta_2} r^2 \sin\theta d\theta d\phi$$

$$= 2\pi r^2\left(1-\cos\theta_2\right)$$

$$A_s = 4\pi r^2, \qquad F_{12} = \frac{1-\cos\theta_2}{2}$$

$$A_1F_{12} = A_3F_{34} \;\rightarrow F_{34} = \frac{A_1}{A_3}F_{12}$$

$$A_1 = 2\pi r^2\left(1-\cos\theta_1\right), \; A_3 = \pi\left(r\sin\theta_1\right)^2 = \pi r^2\left(1-\cos^2\theta_1\right)$$



$$F_{11} + F_{12} + F_{15} = 1$$

$$F_{11} + F_{14} + F_{15} = 1$$

Thus, $F_{12} = F_{14}$

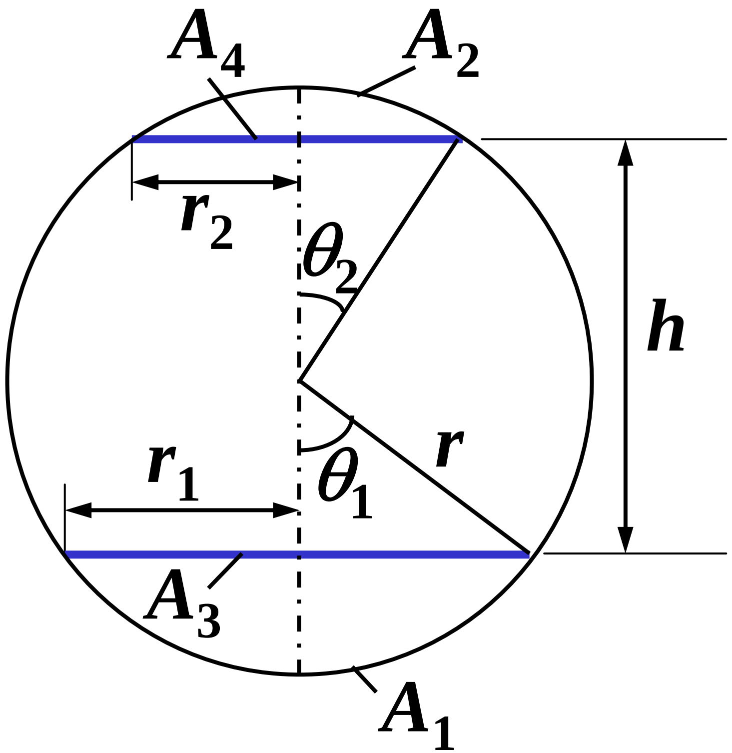
$$F_{41} + F_{45} = 1$$

$$F_{43} + F_{45} = 1$$

Thus, $F_{41} = F_{43}$

$$F_{12} = F_{14} = \frac{A_4}{A_1} F_{41} = \frac{A_4}{A_1} F_{43} = \frac{A_4}{A_1}\frac{A_3}{A_4} F_{34} = \frac{A_3}{A_1} F_{34}$$

$$A_1 F_{12} = A_3 F_{34}$$



$$F_{34} = \frac{A_1}{A_3} F_{12}$$

$$F_{12} = \frac{1 - \cos\theta_2}{2}$$

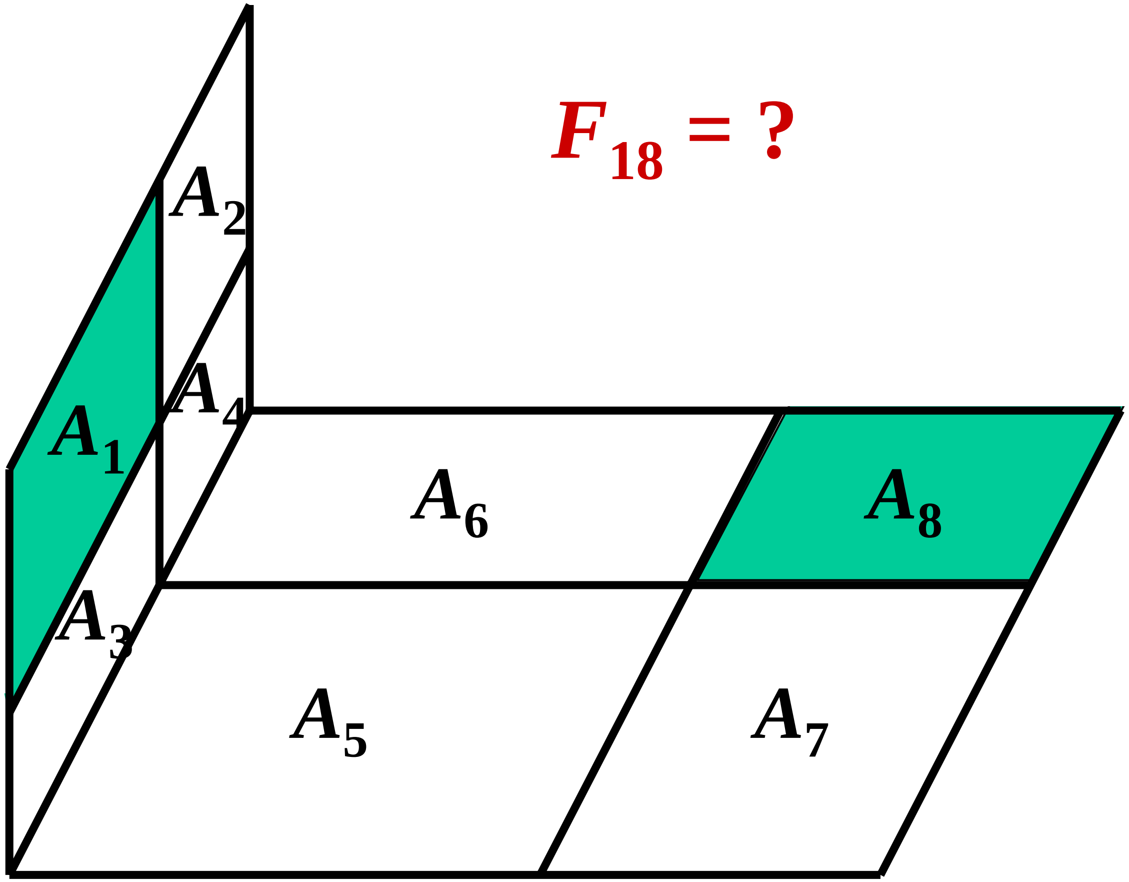
$$A_1 = 2\pi r^2 \left(1 - \cos\theta_1\right)$$

$$A_3 = \pi r^2 \left(1 - \cos^2\theta_1\right)$$

$$F_{34} = \frac{2\pi r^2 \left(1 - \cos\theta_1\right)}{\pi r^2 \left(1 - \cos^2\theta_1\right)} \frac{1 - \cos\theta_2}{2} = \frac{1 - \cos\theta_2}{1 + \cos\theta_1}$$

$$\begin{bmatrix} r\left(\cos\theta_1 + \cos\theta_2\right) = h \\ r\sin\theta_2 = r_2 \\ r\sin\theta_1 = r_1 \end{bmatrix}$$

Problem 6-18



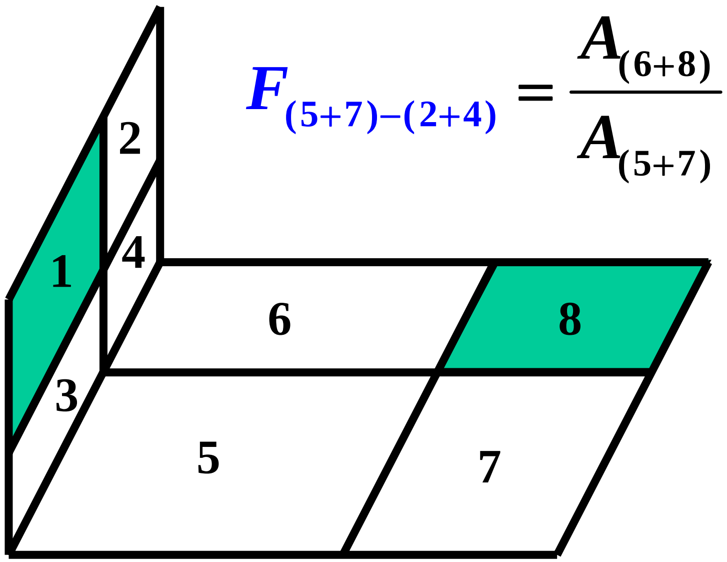
$$F_{18} = ?$$

$$F_{(1+2+3+4)-(5+6+7+8)}$$

$$= F_{(1+2+3+4)-(5+7)} + F_{(1+2+3+4)-(6+8)}$$

$$= \frac{A_{(5+7)}}{A_{(1+2+3+4)}} F_{(5+7)-(1+2+3+4)}$$

$$+ \frac{A_{(6+8)}}{A_{(1+2+3+4)}} F_{(6+8)-(1+2+3+4)}$$

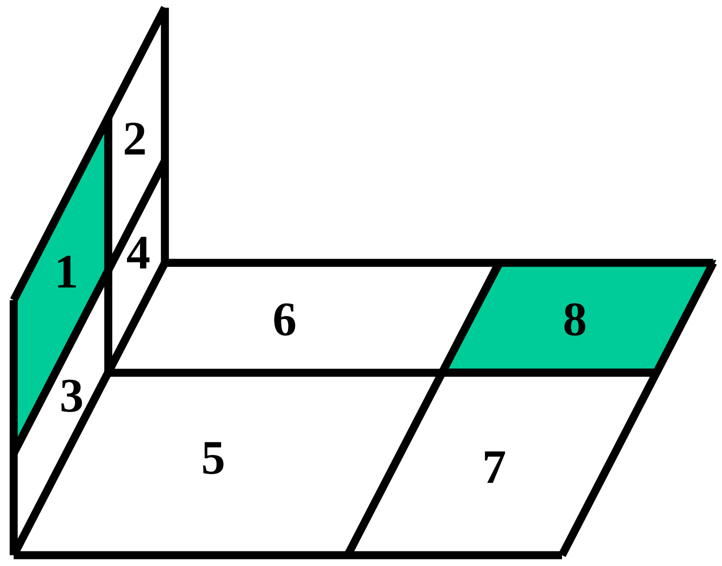


$$F_{(5+7)-(2+4)} = \frac{A_{(6+8)}}{A_{(5+7)}} F_{(6+8)-(1+3)}$$

$$= \frac{A_{(5+7)}}{A_{(1+2+3+4)}} \left(F_{(5+7)-(1+3)} + F_{(5+7)-(2+4)} \right) + \frac{A_{(6+8)}}{A_{(1+2+3+4)}} \left(F_{(6+8)-(2+4)} + F_{(6+8)-(1+3)} \right)$$

$$= \frac{A_{(5+7)}}{A_{(1+2+3+4)}} \left(F_{(5+7)-(1+3)} + \frac{A_{(6+8)}}{A_{(5+7)}} F_{(6+8)-(1+3)} \right)$$

$$+ \frac{A_{(6+8)}}{A_{(1+2+3+4)}} \left(F_{(6+8)-(2+4)} + F_{(6+8)-(1+3)} \right)$$

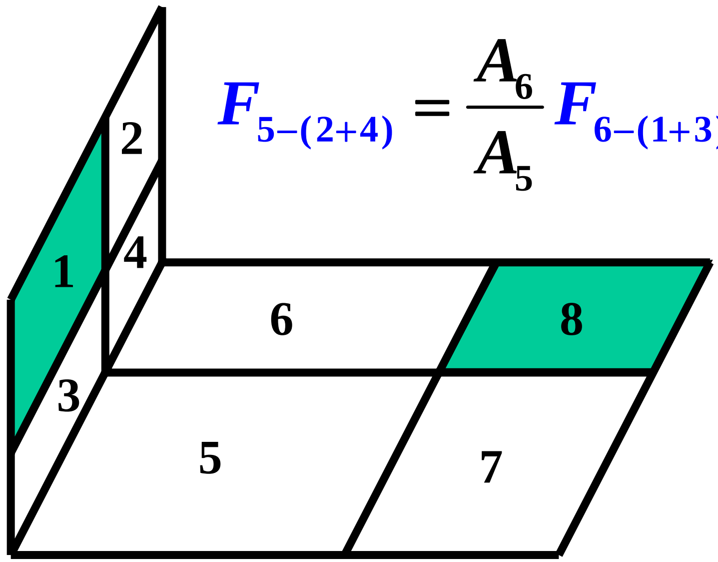


$$\frac{2A_{(6+8)}}{A_{(1+2+3+4)}} F_{(6+8)-(1+3)}$$

$$= F_{(1+2+3+4)-(5+6+7+8)} - \frac{A_{(5+7)}}{A_{(1+2+3+4)}} F_{(5+7)-(1+3)} - \frac{A_{(6+8)}}{A_{(1+2+3+4)}} F_{(6+8)-(2+4)}$$

$$F_{(6+8)-(1+3)}$$

$$= \frac{A_{(1+2+3+4)}}{2A_{(6+8)}} F_{(1+2+3+4)-(5+6+7+8)} - \frac{A_{(5+7)}}{2A_{(6+8)}} F_{(5+7)-(1+3)} - \frac{1}{2} F_{(6+8)-(2+4)}$$



$$F_{5-(2+4)} = \frac{A_6}{A_5} F_{6-(1+3)}$$

$$F_{(1+2+3+4)-(5+6)} = F_{(1+2+3+4)-5} + F_{(1+2+3+4)-6}$$

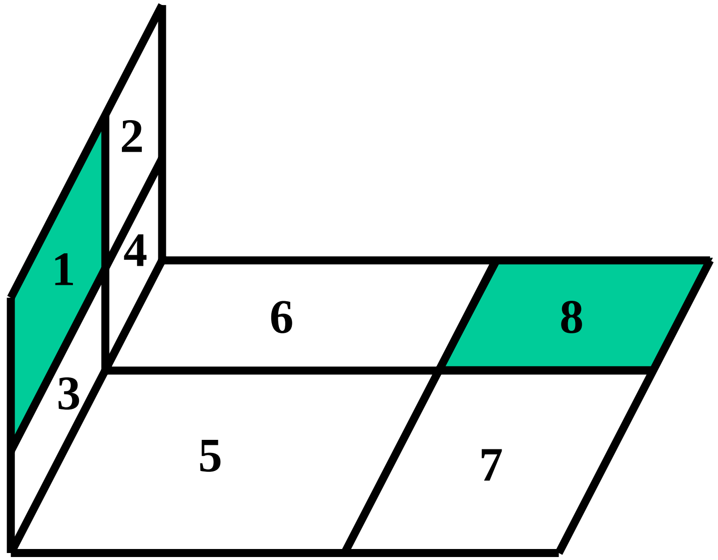
$$= \frac{A_5}{A_{(1+2+3+4)}} F_{5-(1+2+3+4)} + \frac{A_6}{A_{(1+2+3+4)}} F_{6-(1+2+3+4)}$$

$$= \frac{A_5}{A_{(1+2+3+4)}} \left(F_{5-(1+3)} + F_{5-(2+4)} \right) + \frac{A_6}{A_{(1+2+3+4)}} \left(F_{6-(2+4)} + F_{6-(1+3)} \right)$$

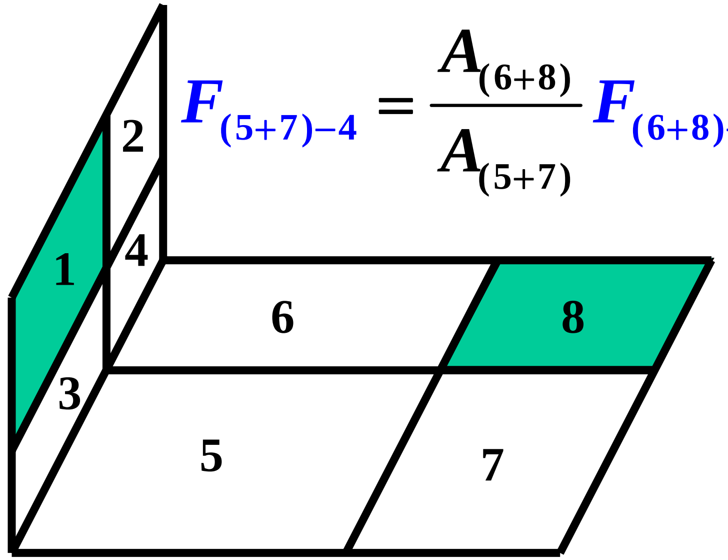
$$= \frac{A_5}{A_{(1+2+3+4)}} \left(F_{5-(1+3)} + \frac{A_6}{A_5} F_{6-(1+3)} \right) + \frac{A_6}{A_{(1+2+3+4)}} \left(F_{6-(2+4)} + F_{6-(1+3)} \right)$$

$$\frac{2A_6}{A_{(1+2+3+4)}} F_{6-(1+3)} = F_{(1+2+3+4)-(5+6)}$$

$$-\frac{A_5}{A_{(1+2+3+4)}} F_{5-(1+3)} - \frac{A_6}{A_{(1+2+3+4)}} F_{6-(2+4)}$$



$$F_{6-(1+3)} = \frac{A_{(1+2+3+4)}}{2A_6} F_{(1+2+3+4)-(5+6)} - \frac{A_5}{2A_6} F_{5-(1+3)} - \frac{1}{2} F_{6-(2+4)}$$



$$F_{(5+7)-4} = \frac{A_{(6+8)}}{A_{(5+7)}} F_{(6+8)-3}$$

$$F_{(3+4)-(5+6+7+8)} = F_{(3+4)-(5+7)} + F_{(3+4)-(6+8)}$$

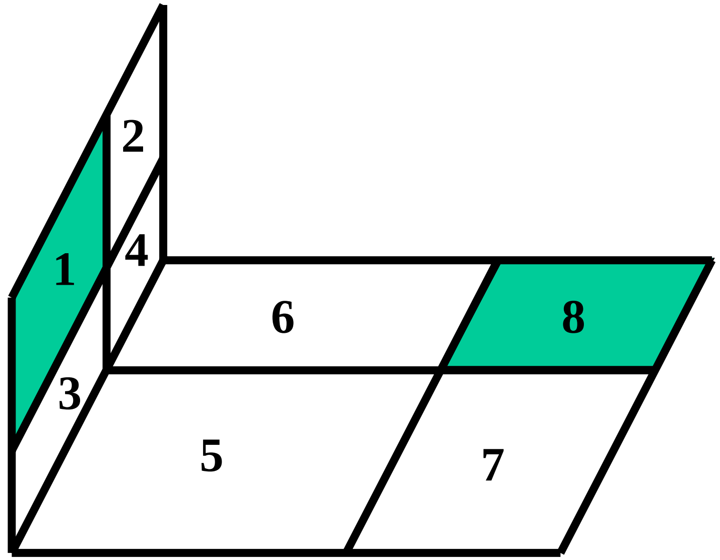
$$= \frac{A_{(5+7)}}{A_{(3+4)}} F_{(5+7)-(3+4)} + \frac{A_{(6+8)}}{A_{(3+4)}} F_{(6+8)-(3+4)}$$

$$= \frac{A_{(5+7)}}{A_{(3+4)}} \left(F_{(5+7)-3} + F_{(5+7)-4} \right) + \frac{A_{(6+8)}}{A_{(3+4)}} \left(F_{(6+8)-4} + F_{(6+8)-3} \right)$$

$$= \frac{A_{(5+7)}}{A_{(3+4)}} \left(F_{(5+7)-3} + \frac{A_{(6+8)}}{A_{(5+7)}} F_{(6+8)-3} \right) + \frac{A_{(6+8)}}{A_{(3+4)}} \left(F_{(6+8)-4} + F_{(6+8)-3} \right)$$

$$\frac{2A_{(6+8)}}{A_{(3+4)}} F_{(6+8)-3} = F_{(3+4)-(5+6+7+8)}$$

$$-\frac{A_{(5+7)}}{A_{(3+4)}} F_{(5+7)-3} - \frac{A_{(6+8)}}{A_{(3+4)}} F_{(6+8)-4}$$



$$F_{(6+8)-3} = \frac{A_{(3+4)}}{2A_{(6+8)}} F_{(3+4)-(5+6+7+8)} - \frac{A_{(5+7)}}{2A_{(6+8)}} F_{(5+7)-3} - \frac{1}{2} F_{(6+8)-4}$$

$$F_{(3+4)-(5+6)} = F_{(3+4)-5} + F_{(3+4)-6}$$

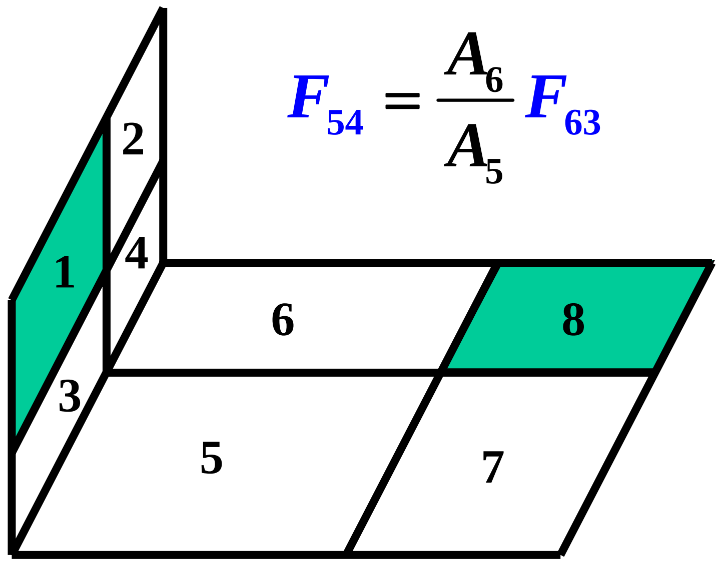
$$= \frac{A_5}{A_{(3+4)}} F_{5-(3+4)} + \frac{A_6}{A_{(3+4)}} F_{6-(3+4)}$$

$$= \frac{A_5}{A_{(3+4)}} \left(F_{53} + F_{54} \right) + \frac{A_6}{A_{(3+4)}} \left(F_{64} + F_{63} \right)$$

$$= \frac{A_5}{A_{(3+4)}} \left(F_{53} + \frac{A_6}{A_5} F_{63} \right) + \frac{A_6}{A_{(3+4)}} \left(F_{64} + F_{63} \right)$$

$$\frac{2A_6}{A_{(3+4)}} F_{63} = F_{(3+4)-(5+6)} - \frac{A_5}{A_{(3+4)}} F_{53} - \frac{A_6}{A_{(3+4)}} F_{64}$$

$$F_{63} = \frac{A_{(3+4)}}{2A_6} F_{(3+4)-(5+6)} - \frac{A_5}{2A_6} F_{53} - \frac{1}{2} F_{64}$$

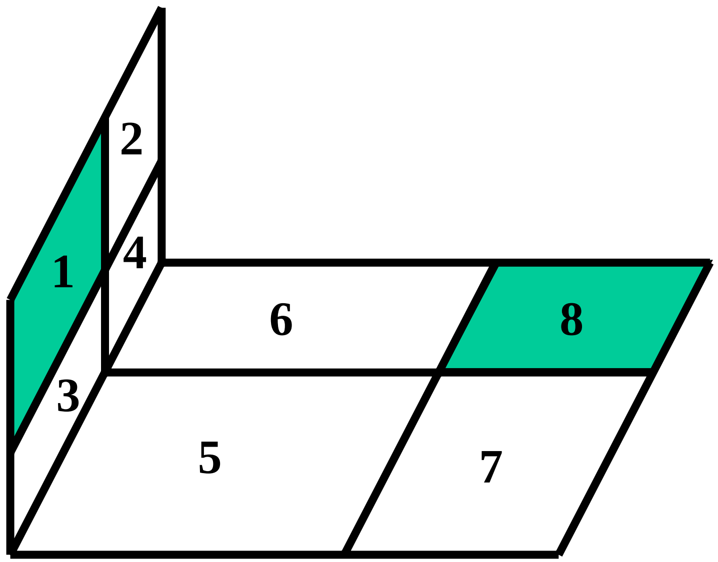


$$F_{54} = \frac{A_6}{A_5} F_{63}$$

$$F_{(6+8)-(1+3)}$$

$$= \frac{A_{(1+2+3+4)}}{2A_{(6+8)}} F_{(1+2+3+4)-(5+6+7+8)}$$

$$- \frac{A_{(5+7)}}{2A_{(6+8)}} F_{(5+7)-(1+3)} - \frac{1}{2} F_{(6+8)-(2+4)}$$

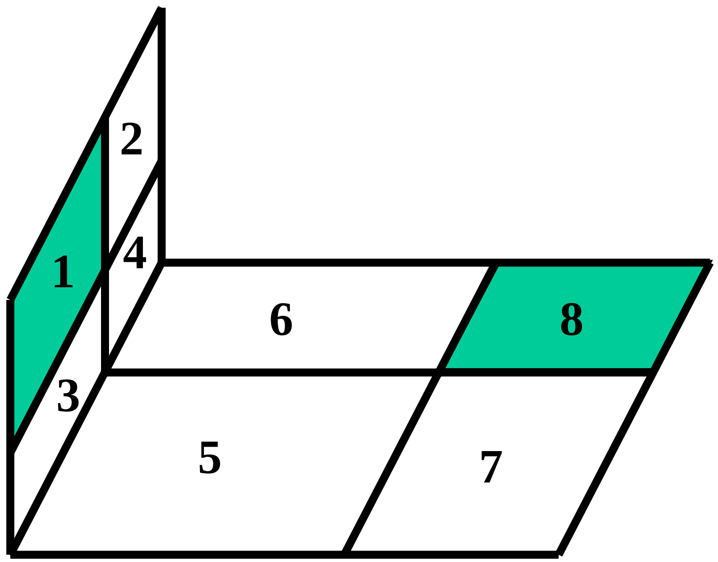


$$F_{6-(1+3)} = \frac{A_{(1+2+3+4)}}{2A_6} F_{(1+2+3+4)-(5+6)} - \frac{A_5}{2A_6} F_{5-(1+3)} - \frac{1}{2} F_{6-(2+4)}$$

$$F_{(6+8)-3} = \frac{A_{(3+4)}}{2A_{(6+8)}} F_{(3+4)-(5+6+7+8)} - \frac{A_{(5+7)}}{2A_{(6+8)}} F_{(5+7)-3} - \frac{1}{2} F_{(6+8)-4}$$

$$F_{63} = \frac{A_{(3+4)}}{2A_6} F_{(3+4)-(5+6)} - \frac{A_5}{2A_6} F_{53} - \frac{1}{2} F_{64}$$

$$F_{(6+8)-(1+3)} = F_{(6+8)-1} + F_{(6+8)-3}$$



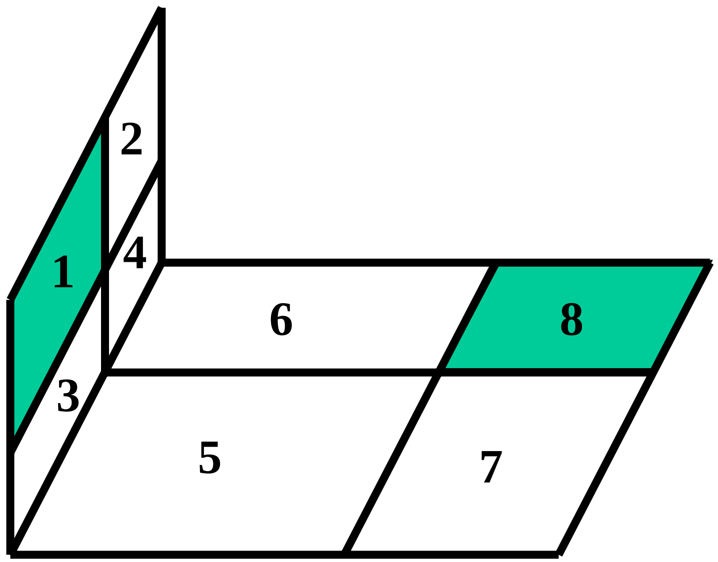
$$F_{(6+8)-1} = \frac{A_1}{A_{(6+8)}} F_{1-(6+8)}$$

$$= \frac{A_1}{A_{(6+8)}} \left(F_{16} - F_{18} \right) = F_{(6+8)-(1+3)} - F_{(6+8)-3}$$

$$F_{6-(1+3)} = \frac{A_{(1+2+3+4)}}{2A_6} F_{(1+2+3+4)-(5+6)} - \frac{A_5}{2A_6} F_{5-(1+3)} - \frac{1}{2} F_{6-(2+4)}$$

$$F_{61} + F_{63} = \frac{A_{(1+2+3+4)}}{2A_6} F_{(1+2+3+4)-(5+6)} - \frac{A_5}{2A_6} F_{5-(1+3)} - \frac{1}{2} F_{6-(2+4)}$$

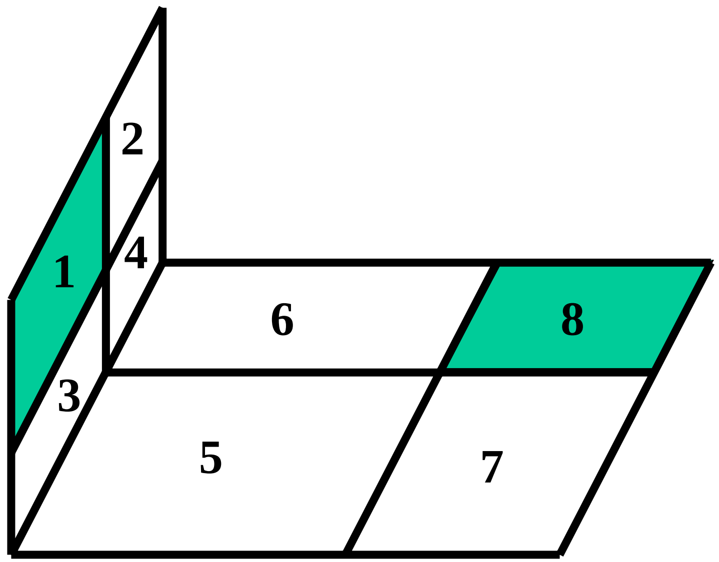
$$F_{61} = \frac{A_{(1+2+3+4)}}{2A_6} F_{(1+2+3+4)-(5+6)} - \frac{A_5}{2A_6} F_{5-(1+3)} - \frac{1}{2} F_{6-(2+4)} - F_{63}$$



$$F_{16} = \frac{A_6}{A_1} F_{61}$$

$$= \frac{A_6}{A_1}\left[\frac{A_{(1+2+3+4)}}{2A_6} F_{(1+2+3+4)-(5+6)} - \frac{A_5}{2A_6} F_{5-(1+3)} - \frac{1}{2} F_{6-(2+4)} - F_{63} \right]$$

$$= \frac{A_{(1+2+3+4)}}{2A_1} F_{(1+2+3+4)-(5+6)} - \frac{A_5}{2A_1} F_{5-(1+3)} - \frac{A_6}{2A_1} F_{6-(2+4)} - \frac{A_6}{A_1} F_{63}$$



$$F_{16} =$$

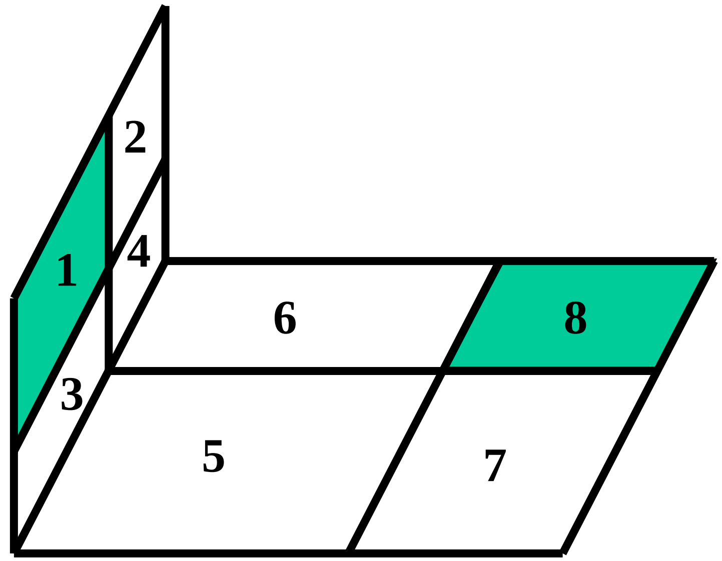
$$\frac{A_{(1+2+3+4)}}{2A_1} F_{(1+2+3+4)-(5+6)} - \frac{A_5}{2A_1} F_{5-(1+3)} - \frac{A_6}{2A_1} F_{6-(2+4)}$$

$$- \frac{A_6}{A_1} \left(\frac{A_{(3+4)}}{2A_6} F_{(3+4)-(5+6)} - \frac{A_5}{2A_6} F_{53} - \frac{1}{2} F_{64} \right)$$

$$= \frac{A_{(1+2+3+4)}}{2A_1} F_{(1+2+3+4)-(5+6)} - \frac{A_5}{2A_1} F_{5-(1+3)} - \frac{A_6}{2A_1} F_{6-(2+4)}$$

$$- \frac{A_{(3+4)}}{2A_1} F_{(3+4)-(5+6)} + \frac{A_5}{2A_1} F_{53} + \frac{A_6}{2A_1} F_{64}$$

$$\frac{A_1}{A_{(6+8)}}\left(F_{16}-F_{18}\right)=F_{(6+8)-(1+3)}-F_{(6+8)-3}$$



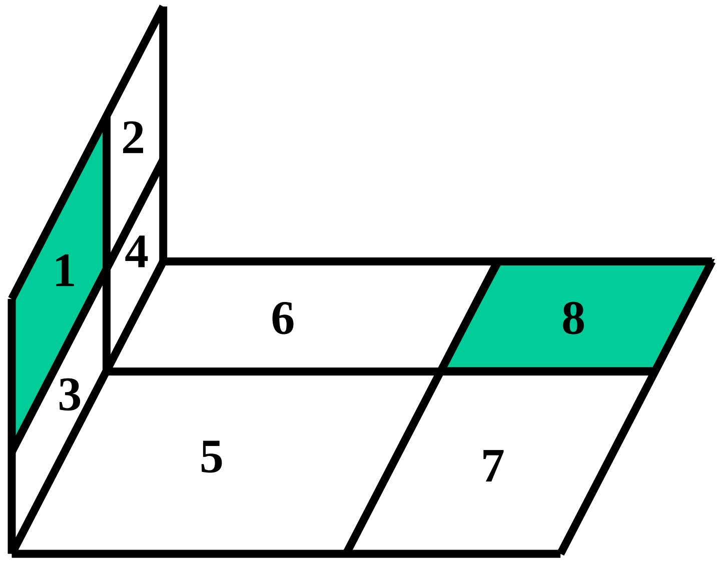
$$F_{18}=F_{16}-\frac{A_{(6+8)}}{A_1}\left(F_{(6+8)-(1+3)}-F_{(6+8)-3}\right)$$

$$=F_{16}-\frac{A_{(6+8)}}{A_1}F_{(6+8)-(1+3)}+\frac{A_{(6+8)}}{A_1}F_{(6+8)-3}$$

$$\frac{A_{(6+8)}}{A_1}F_{(6+8)-(1+3)}$$

$$=\frac{A_{(6+8)}}{A_1}\left(\frac{A_{(1+2+3+4)}}{2A_{(6+8)}}F_{(1+2+3+4)-(5+6+7+8)}-\frac{A_{(5+7)}}{2A_{(6+8)}}F_{(5+7)-(1+3)}-\frac{1}{2}F_{(6+8)-(2+4)}\right)$$

$$=\frac{A_{(1+2+3+4)}}{2A_1}F_{(1+2+3+4)-(5+6+7+8)}-\frac{A_{(5+7)}}{2A_1}F_{(5+7)-(1+3)}-\frac{A_{(6+8)}}{2A_1}F_{(6+8)-(2+4)}$$



$$\frac{A_{(6+8)}}{A_1} F_{(6+8)-3}$$

$$= \frac{A_{(6+8)}}{A_1}\left(\frac{A_{(3+4)}}{2A_{(6+8)}} F_{(3+4)-(5+6+7+8)} - \frac{A_{(5+7)}}{2A_{(6+8)}} F_{(5+7)-3} - \frac{1}{2} F_{(6+8)-4} \right)$$

$$= \frac{A_{(3+4)}}{2A_1} F_{(3+4)-(5+6+7+8)} - \frac{A_{(5+7)}}{2A_1} F_{(5+7)-3} - \frac{A_{(6+8)}}{2A_1} F_{(6+8)-4}$$

$$F_{18} = \frac{A_{(1+2+3+4)}}{2A_1} F_{(1+2+3+4)-(5+6)}$$

$$- \frac{A_5}{2A_1} F_{5-(1+3)} - \frac{A_6}{2A_1} F_{6-(2+4)}$$

$$- \frac{A_{(3+4)}}{2A_1} F_{(3+4)-(5+6)} + \frac{A_5}{2A_1} F_{53} + \frac{A_6}{2A_1} F_{64}$$

$$- \frac{A_{(1+2+3+4)}}{2A_1} F_{(1+2+3+4)-(5+6+7+8)} + \frac{A_{(5+7)}}{2A_1} F_{(5+7)-(1+3)} + \frac{A_{(6+8)}}{2A_1} F_{(6+8)-(2+4)}$$

$$+ \frac{A_{(3+4)}}{2A_1} F_{(3+4)-(5+6+7+8)} - \frac{A_{(5+7)}}{2A_1} F_{(5+7)-3} - \frac{A_{(6+8)}}{2A_1} F_{(6+8)-4}$$