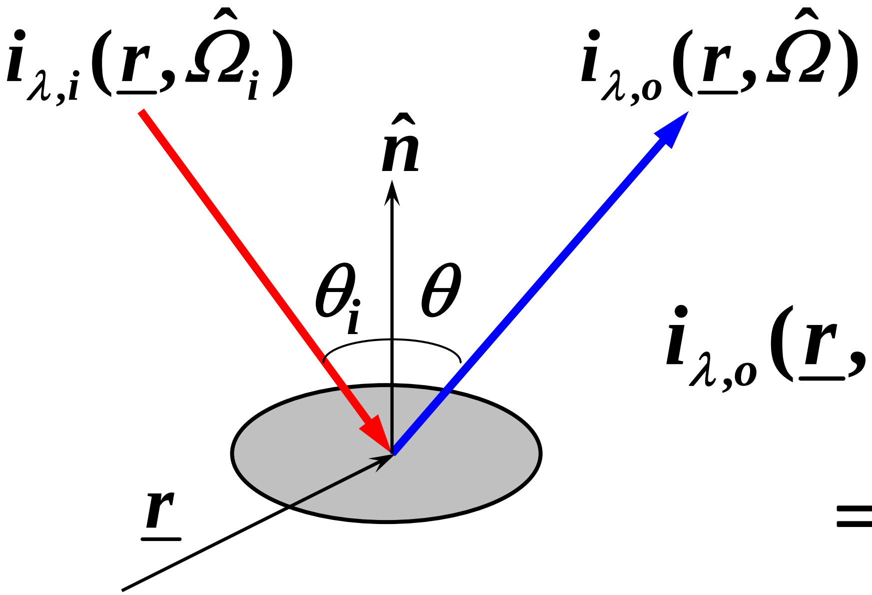
THE EXCHANGE OF THERMAL RADIATION BETWEEN NONDIFFUSE NONGRAY SURFACES

- General Formulation
- Exchange in an Enclosure
- Diffuse-Nongray Surfaces
- Directional-Gray Surfaces

General Formulation



$$i_{\lambda,o}(\underline{r},\hat{\Omega}) = i_{\lambda,e}(\underline{r},\hat{\Omega}) + i_{\lambda,r}(\underline{r},\hat{\Omega})$$

$$= \varepsilon'_{\lambda}(\underline{r},\hat{\Omega}) i_{\lambda b}(\underline{r}) + i_{\lambda,r}(\underline{r},\hat{\Omega})$$

$$di_{\lambda,r}(\underline{r},\hat{\Omega}_i,\hat{\Omega}) = \rho''_{\lambda}(\underline{r},\hat{\Omega}_i,\hat{\Omega}) i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i$$

$\rho''_{\lambda}(\underline{r},\hat{\Omega}_i,\hat{\Omega})$: **bidirectional spectral reflectivity**

$$i_{\lambda,r}(\underline{r},\hat{\Omega}) = \int_{\cap_i} \rho''_{\lambda}(\underline{r},\hat{\Omega}_i,\hat{\Omega}) i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i$$

$$i_{\lambda,o}(\underline{r},\hat{\Omega}) = \varepsilon'_\lambda(\underline{r},\hat{\Omega})i_{\lambda b}(\underline{r})$$

$$+\int_{\cap_i} \rho''_\lambda(\underline{r},\hat{\Omega}_i,\hat{\Omega})i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i$$

net radiative heat flux

$$q''(\underline{r}) = J(\underline{r}) - G(\underline{r}) = \int_0^\infty J_\lambda(\underline{r})d\lambda - \int_0^\infty G_\lambda(\underline{r})d\lambda$$

$$= \int_0^\infty \int_\cap i_{\lambda,o}(\underline{r},\hat{\Omega})\cos\theta d\omega d\lambda$$

$$-\int_0^\infty \int_{\cap_i} i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i d\lambda$$

$$i_{\lambda,o}(\underline{r},\hat{\Omega}) = \varepsilon'_\lambda(\underline{r},\hat{\Omega}) i_{\lambda b}(\underline{r})$$

$$+\int_{\cap_i} \rho''_\lambda(\underline{r},\hat{\Omega}_i,\hat{\Omega}) i_{\lambda,i}(\underline{r},\hat{\Omega}_i) \cos\theta_i d\omega_i$$

$$J_\lambda(\underline{r}) = \int_\cap i_{\lambda,o}(\underline{r},\hat{\Omega}) \cos\theta d\omega$$

$$= \int_\cap \varepsilon'_\lambda(\underline{r},\hat{\Omega}) i_{\lambda b}(\underline{r}) \cos\theta d\omega$$

$$+\int_\cap \left[\int_{\cap_i} \rho''_\lambda(\underline{r},\hat{\Omega}_i,\hat{\Omega}) i_{\lambda,i}(\underline{r},\hat{\Omega}_i) \cos\theta_i d\omega_i \right] \cos\theta d\omega$$

since $\rho'_\lambda(\underline{r},\hat{\Omega}_i) = \int_\cap \rho''_\lambda(\underline{r},\hat{\Omega}_i,\hat{\Omega}) \cos\theta d\omega$

$$\int_\cap \left[\int_{\cap_i} \rho''_\lambda(\underline{r},\hat{\Omega}_i,\hat{\Omega}) i_{\lambda,i}(\underline{r},\hat{\Omega}_i) \cos\theta_i d\omega_i \right] \cos\theta d\omega$$

$$= \int_{\cap_i} \left[\int_\cap \rho''_\lambda(\underline{r},\hat{\Omega}_i,\hat{\Omega}) \cos\theta d\omega \right] i_{\lambda,i}(\underline{r},\hat{\Omega}_i) \cos\theta_i d\omega_i$$

$$= \int_{\cap_i} \rho'_\lambda(\underline{r},\hat{\Omega}_i) i_{\lambda,i}(\underline{r},\hat{\Omega}_i) \cos\theta_i d\omega_i$$

$$J_\lambda(\underline{r}) = \int_\cap \varepsilon'_\lambda(\underline{r},\hat{\Omega}) i_{\lambda b}(\underline{r})\cos\theta d\omega$$

$$+\int_{\cap_i} \rho'_\lambda(\underline{r},\hat{\Omega}_i) i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i$$

$$\varepsilon_\lambda(\underline{r}) = \frac{\int_\cap \varepsilon'_\lambda(\underline{r},\hat{\Omega}) i_{\lambda b}(\underline{r})\cos\theta d\omega}{\int_\cap i_{\lambda b}(\underline{r})\cos\theta d\omega},$$

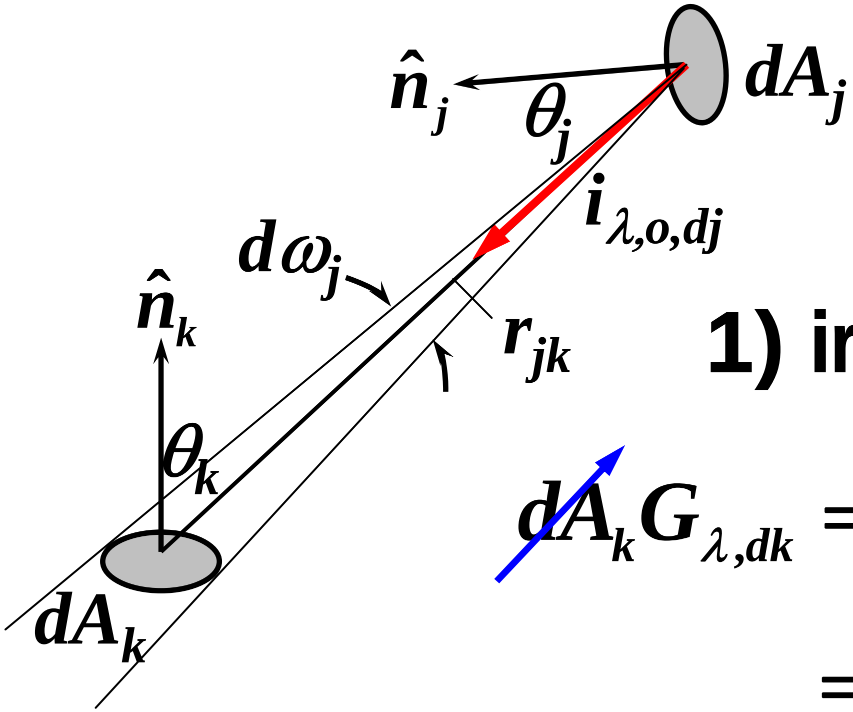
$$\rho_\lambda(\underline{r}) = \frac{\int_{\cap_i} \rho'_\lambda(\underline{r},\hat{\Omega}_i) i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i}{\int_{\cap_i} i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i}$$

$$J_\lambda(\underline{r}) = \varepsilon_\lambda(\underline{r})\int_\cap i_{\lambda,b}(\underline{r})\cos\theta d\omega$$

$$+\rho_\lambda(\underline{r})\int_{\cap_i} i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i$$

$$= \varepsilon_\lambda(\underline{r}) e_{\lambda b}(\underline{r}) + \rho_\lambda(\underline{r}) G_\lambda(\underline{r})$$

Exchange in an Enclosure



$$q''_{\lambda,dk} = J_{\lambda,dk} - G_{\lambda,dk}$$

$$J_{\lambda,dk} = \varepsilon_{\lambda,dk} e_{\lambda b,dk} + \rho_{\lambda,dk} G_{\lambda,dk}$$

1) irradiation from dA_j to dA_k

$$dA_k G_{\lambda,dk} = i_{\lambda,o,dj} \cos\theta_j d\omega_j dA_j$$

$$= i_{\lambda,o,dj} \cos\theta_j \frac{\cos\theta_k dA_k}{r_{jk}^2} dA_j$$

$$= \pi i_{\lambda,o,dj} \frac{\cos\theta_j \cos\theta_k}{\pi r_{jk}^2} dA_j dA_k$$

$$= \pi i_{\lambda,o,dj} dA_k dF_{dk-dj}$$

$$\pi i_{\lambda,o,dj} \neq J_{\lambda,dj}$$

2) irradiation from A_j to dA_k

$$G_{\lambda,dk} = \int_{A_j} \pi i_{\lambda,o,dj} dF_{dk-dj}$$

3) irradiation from n surfaces

$$G_{\lambda,dk} = \sum_{j=1}^{n} \int_{A_j} \pi i_{\lambda,o,dj} dF_{dk-dj}$$

Summary

$$q''_{\lambda,dk} = J_{\lambda,dk} - G_{\lambda,dk}$$

$$J_{\lambda,dk} = \varepsilon_{\lambda,dk} e_{\lambda b,dk} + \rho_{\lambda,dk} \sum_{j=1}^{n} \int_{A_j} \pi i_{\lambda,o,dj} dF_{dk-dj}$$

Diffuse-Nongray Surfaces

For diffuse surfaces

$$J_{\lambda,dj} = \int_{\cap} i_{\lambda,o,dj} \cos\theta d\omega = \pi i_{\lambda,o,dj}$$

$$G_{\lambda,dk} = \sum_{j=1}^{n} \int_{A_j} \pi i_{\lambda,o,dj} dF_{dk-dj}$$

$$= \sum_{j=1}^{n} \int_{A_j} J_{\lambda,dj} dF_{dk-dj}$$

$$J_{\lambda,dk} = \varepsilon_{\lambda,dk} e_{\lambda b,dk} + \rho_{\lambda,dk} \sum_{j=1}^{n} \int_{A_j} \pi i_{\lambda,o,dj} dF_{dk-dj}$$

$$= \varepsilon_{\lambda,dk} e_{\lambda b,dk} + \rho_{\lambda,dk} \sum_{j=1}^{n} \int_{A_j} J_{\lambda,dj} dF_{dk-dj}$$

$$q''_{\lambda,dk} = J_{\lambda,dk} - G_{\lambda,dk}$$

$$= \varepsilon_{\lambda,dk} e_{\lambda b,dk} + \rho_{\lambda,dk} G_{\lambda,dk} - G_{\lambda,dk}$$

$$= \varepsilon_{\lambda,dk} e_{\lambda b,dk} - \varepsilon_{\lambda,dk} \sum_{j=1}^{n} \int_{A_j} J_{\lambda,dj} dF_{dk-dj}$$

$$J_{\lambda,dk} = \varepsilon_{\lambda,dk} e_{\lambda b,dk} + \left(1 - \varepsilon_{\lambda,dk}\right) \sum_{j=1}^{n} \int_{A_j} J_{\lambda,dj} dF_{dk-dj}$$

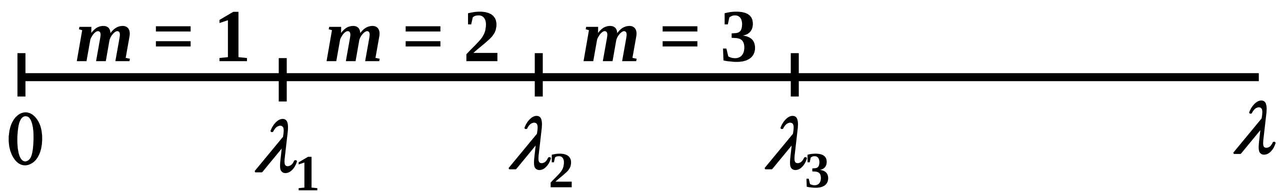
Total quantities

$$q''_{dk} = \int_0^\infty \varepsilon_{\lambda,dk} e_{\lambda b,dk} d\lambda - \int_0^\infty \left(\varepsilon_{\lambda,dk} \sum_{j=1}^{n} \int_{A_j} J_{\lambda,dj} dF_{dk-dj} \right) d\lambda$$

$$J_{dk} = \int_0^\infty \varepsilon_{\lambda,dk} e_{\lambda b,dk} d\lambda$$

$$+ \int_0^\infty \left[\left(1 - \varepsilon_{\lambda,dk}\right) \sum_{j=1}^{n} \int_{A_j} J_{\lambda,dj} dF_{dk-dj} \right] d\lambda$$

Band approximation



$$q''_{dk} = \int_0^{\infty} \varepsilon_{\lambda,dk} e_{\lambda b,dk} d\lambda - \int_0^{\infty} \left(\varepsilon_{\lambda,dk} \sum_{j=1}^{n} \int_{A_j} J_{\lambda,dj} dF_{dk-dj} \right) d\lambda$$

$$q''_{dk,m} = \varepsilon_{dk,m} e_{b,dk,m} - \varepsilon_{dk,m} \sum_{j=1}^{n} \int_{A_j} J_{dj,m} dF_{dk-dj}$$

$$J_{dk,m} = \varepsilon_{dk,m} e_{b,dk,m} + \rho_{dk,m} \sum_{j=1}^{n} \int_{A_j} J_{dj,m} dF_{dk-dj}$$

$$J_{dk} = \sum_{m} J_{dk,m}, \quad q''_{dk} = \sum_{m} q''_{dk,m}$$

Simplified zone analysis

$$A_k G_{\lambda,k} = \int_{A_k} G_{\lambda,dk} dA_k = \int_{A_k} \left(\sum_{j=1}^{n} \int_{A_j} J_{\lambda,dj} dF_{dk-dj} \right) dA_k$$

$$= \sum_{j=1}^{n} \int_{A_k} \int_{A_j} J_{\lambda,dj} dF_{dk-dj} dA_k$$

$$= \sum_{j=1}^{n} J_{\lambda,j} \int_{A_k} \int_{A_j} dF_{dk-dj} dA_k$$

$$F_{kj} = \frac{1}{A_k} \int_{A_k} \int_{A_j} dF_{dk-dj} dA_j dA_k$$

$$= \sum_{j=1}^{n} J_{\lambda,j} A_k F_{kj} = A_k \sum_{j=1}^{n} J_{\lambda,j} F_{kj}$$

Thus, $$G_{\lambda,k} = \sum_{j=1}^{n} J_{\lambda,j} F_{kj}$$

$$q''_{\lambda,k} = \varepsilon_{\lambda,k} e_{\lambda b,k} - \varepsilon_{\lambda,k} \sum_{j=1}^{n} J_{\lambda,j} F_{kj}$$

$$J_{\lambda,k} = \varepsilon_{\lambda,k} e_{\lambda b,k} + \left(1 - \varepsilon_{\lambda,k}\right) \sum_{j=1}^{n} J_{\lambda,j} F_{kj}$$

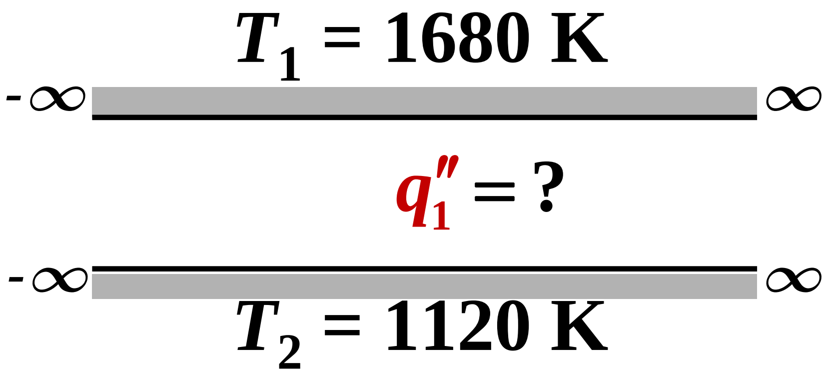
Band approximation

$$q''_{k,m} = \varepsilon_{k,m} e_{bk,m} - \varepsilon_{k,m} \sum_{j=1}^{n} J_{j,m} F_{kj}$$

$$J_{k,m} = \varepsilon_{k,m} e_{kb,m} + \left(1 - \varepsilon_{k,m}\right) \sum_{j=1}^{n} J_{j,m} F_{kj}$$

$$J_{k} = \sum_{m} J_{k,m}, \quad q''_{k} = \sum_{m} q''_{k,m}$$

Ex 8-2 Two infinite parallel plates



$$\varepsilon_{\lambda 1} = \begin{cases} 0.4 & 0 \le \lambda \le 3\mu\text{m} \\ 0.8 & 3\mu\text{m} < \lambda \end{cases}$$

$$\varepsilon_{\lambda 2} = \begin{cases} 0.7 & 0 \le \lambda \le 5\mu\text{m} \\ 0.3 & 5\mu\text{m} < \lambda \end{cases}$$

$$q''_{\lambda 1} = J_{\lambda 1} - G_{\lambda 1}, \quad J_{\lambda,k} = \varepsilon_{\lambda,k} e_{\lambda b,k} + \left(1 - \varepsilon_{\lambda,k}\right) \sum_{j=1}^{n} J_{\lambda,j} F_{kj}$$

$$J_{\lambda 1} = \varepsilon_{\lambda 1} e_{\lambda b1} + \left(1 - \varepsilon_{\lambda 1}\right) G_{\lambda 1}, \quad G_{\lambda 1} = J_{\lambda 2} F_{12} = J_{\lambda 2}$$

$$J_{\lambda 2} = \varepsilon_{\lambda 2} e_{\lambda b2} + \left(1 - \varepsilon_{\lambda 2}\right) J_{\lambda 1}$$

$$q''_{\lambda 1} = J_{\lambda 1} - G_{\lambda 1} = \frac{e_{\lambda b1} - e_{\lambda b2}}{\dfrac{1}{\varepsilon_{\lambda 1}} + \dfrac{1}{\varepsilon_{\lambda 2}} - 1}$$

$$q_1'' = \int_0^\infty \frac{e_{\lambda b1} - e_{\lambda b2}}{\frac{1}{\varepsilon_{\lambda 1}} + \frac{1}{\varepsilon_{\lambda 2}} - 1} d\lambda$$

$$\varepsilon_{\lambda 1} = \begin{cases} 0.4 & 0 \le \lambda \le 3\mu\text{m} \\ 0.8 & 3\mu\text{m} < \lambda \end{cases}$$

$$\varepsilon_{\lambda 2} = \begin{cases} 0.7 & 0 \le \lambda \le 5\mu\text{m} \\ 0.3 & 5\mu\text{m} < \lambda \end{cases}$$

$$= \int_0^3 \frac{e_{\lambda b1} - e_{\lambda b2}}{\frac{1}{0.4} + \frac{1}{0.7} - 1} d\lambda$$

$$+ \int_3^5 \frac{e_{\lambda b1} - e_{\lambda b2}}{\frac{1}{0.8} + \frac{1}{0.7} - 1} d\lambda + \int_5^\infty \frac{e_{\lambda b1} - e_{\lambda b2}}{\frac{1}{0.8} + \frac{1}{0.3} - 1} d\lambda$$

$$= 0.341\left[\sigma T_1^4\int_0^3 \frac{e_{\lambda b1}}{\sigma T_1^4}d\lambda - \sigma T_2^4\int_0^3 \frac{e_{\lambda b2}}{\sigma T_2^4}d\lambda\right]$$

$$+0.596\left[\sigma T_1^4\int_3^5 \frac{e_{\lambda b1}}{\sigma T_1^4}d\lambda - \sigma T_2^4\int_3^5 \frac{e_{\lambda b2}}{\sigma T_2^4}d\lambda\right]$$

$$+0.299\left[\sigma T_1^4\int_5^\infty \frac{e_{\lambda b1}}{\sigma T_1^4}d\lambda - \sigma T_2^4\int_5^\infty \frac{e_{\lambda b2}}{\sigma T_2^4}d\lambda\right]$$

$$= 0.341\left[\sigma T_1^4 F_{0-3T_1} - \sigma T_2^4 F_{0-3T_2}\right]$$

$$+0.596\left[\sigma T_1^4 F_{3T_1-5T_1} - \sigma T_2^4 F_{3T_2-5T_2}\right]$$

$$+0.279\left[\sigma T_1^4 F_{5T_1-\infty} - \sigma T_2^4 F_{5T_2-\infty}\right] = 140,500\ \mathrm{W/m^2}$$

Remark: using average property

$$\varepsilon_{\lambda 1} = \begin{cases} 0.4 & 0 \le \lambda \le 3\mu\text{m} \\ 0.8 & 3\mu\text{m} < \lambda \end{cases}$$

$$\varepsilon_{\lambda 2} = \begin{cases} 0.7 & 0 \le \lambda \le 5\mu\text{m} \\ 0.3 & 5\mu\text{m} < \lambda \end{cases}$$

$$\varepsilon_1 = \frac{\int_0^\infty \varepsilon_{\lambda 1} e_{\lambda b1} d\lambda}{\sigma T_1^4} = 0.4F_{0-3T_1} + 0.8F_{3T_1-\infty}$$

$$= 0.4F_{0-5040} + 0.8\left(1 - F_{0-5040}\right) = 0.545$$

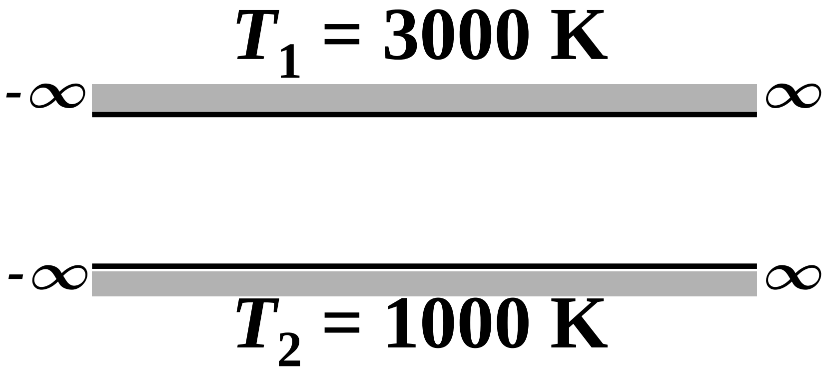
$$\varepsilon_2 = 0.7F_{0-5600} + 0.3\left(1 - F_{0-5600}\right) = 0.580$$

$$q_1'' = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{5.67\times 10^{-8}\left[(1680)^4 - (1120)^4\right]}{\frac{1}{0.545} + \frac{1}{0.580} - 1}$$

$$= 141{,}637 \text{ W/m}^2$$ **0.8% error**

$$140{,}500 \text{ W/m}^2$$

Ex



$$\varepsilon_{\lambda 1} = \begin{cases} 0.8 & 0 \le \lambda \le 2\ \mu\text{m} \\ 0.2 & \lambda > 2\ \mu\text{m} \end{cases}$$

$$\varepsilon_{\lambda 2} = \begin{cases} 0.2 & 0 \le \lambda \le 4\ \mu\text{m} \\ 0.8 & \lambda > 4\ \mu\text{m} \end{cases}$$

$$q_1'' = \int_0^2 \frac{e_{\lambda b1} - e_{\lambda b2}}{\dfrac{1}{0.8} + \dfrac{1}{0.2} - 1} d\lambda + \int_2^4 \frac{e_{\lambda b1} - e_{\lambda b2}}{\dfrac{1}{0.2} + \dfrac{1}{0.2} - 1} d\lambda$$

$$+ \int_4^\infty \frac{e_{\lambda b1} - e_{\lambda b2}}{\dfrac{1}{0.2} + \dfrac{1}{0.8} - 1} d\lambda$$

$$= 0.190\left[\sigma T_1^4 F_{0-6000} - \sigma T_2^4 F_{0-2000}\right]$$

$$+0.111\left[\sigma T_1^4 F_{6000-12000} - \sigma T_2^4 F_{2000-4000}\right]$$

$$+0.190\left[\sigma T_1^4 F_{12000-\infty} - \sigma T_2^4 F_{4000-\infty}\right]$$

$$= 788{,}374 \text{ W/m}^2$$

using average property

$$\varepsilon_1 = 0.8F_{0-6000} + 0.2F_{6000-\infty} = 0.643$$

$$\varepsilon_2 = 0.2F_{0-4000} + 0.8F_{4000-\infty} = 0.511$$

$$q_1'' = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{0.643} + \frac{1}{0.511} - 1} = \frac{5.67\left[30^4 - 10^4\right]}{\frac{1}{0.643} + \frac{1}{0.511} - 1}$$

$$= 1{,}805{,}328 \text{ W/m}^2$$ **129% error**

Directional-Gray Surfaces

General formulation

$$J_\lambda(\underline{r}) = \int_\cap i_{\lambda,o}(\underline{r},\hat{\Omega})\cos\theta d\omega$$

$$i_{\lambda,o}(\underline{r},\hat{\Omega}) = \varepsilon'_\lambda(\underline{r},\hat{\Omega})i_{\lambda b}(\underline{r}) + \int_{\cap_i} \rho''_\lambda(\underline{r},\hat{\Omega}_i,\hat{\Omega})i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i$$

$$J(\underline{r}) = \int_\cap \int_0^\infty i_{\lambda,o}(\underline{r},\hat{\Omega})\cos\theta d\lambda d\omega$$

$$= \int_\cap \cos\theta \Big[\int_0^\infty \varepsilon'_\lambda(\underline{r},\hat{\Omega})i_{\lambda b}(\underline{r})d\lambda + \int_0^\infty \int_{\cap_i} \rho''_\lambda(\underline{r},\hat{\Omega}_i,\hat{\Omega})i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i d\lambda \Big] d\omega$$

$$\int_{\cap} \cos\theta \int_0^{\infty} \varepsilon'_\lambda(\underline{r},\hat{\Omega}) i_{\lambda b}(\underline{r})\, d\lambda\, d\omega =$$

$$\sigma T^4(\underline{r}) \frac{1}{\pi} \int_{\cap} \varepsilon'(\underline{r},\hat{\Omega}) \cos\theta\, d\omega = \varepsilon(\underline{r}) \sigma T^4(\underline{r})$$

$$\varepsilon'(\underline{r},\hat{\Omega}) = \frac{\pi \int_0^{\infty} \varepsilon'_\lambda(\underline{r},\hat{\Omega}) i_{\lambda b}(\underline{r})\, d\lambda}{\sigma T^4(\underline{r})}$$

$$\int_0^{\infty} \varepsilon'_\lambda(\underline{r},\hat{\Omega}) i_{\lambda b}(\underline{r})\, d\lambda = \frac{\varepsilon'(\underline{r},\hat{\Omega})}{\pi} \sigma T^4(\underline{r})$$

$$\int_{\cap}\cos\theta\left[\int_0^{\infty}\int_{\cap_i}\rho''_{\lambda}(\underline{r},\hat{\Omega}_i,\hat{\Omega})i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i d\lambda\right]d\omega$$

$$=\int_0^{\infty}\int_{\cap_i}\rho'_{\lambda}(\underline{r},\hat{\Omega}_i)i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i d\lambda$$

$$=\int_0^{\infty}\rho_{\lambda}(\underline{r})\int_{\cap_i}i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i d\lambda$$

$$=\int_0^{\infty}\rho_{\lambda}(\underline{r})G_{\lambda}(\underline{r})d\lambda=\rho(\underline{r})\int_0^{\infty}G_{\lambda}(\underline{r})d\lambda$$

$$=\left[1-\varepsilon(\underline{r})\right]G(\underline{r})$$

$$\rho'_{\lambda}(\underline{r},\hat{\Omega}_i)=\int_{\cap}\rho''_{\lambda}(\underline{r},\hat{\Omega}_i,\hat{\Omega})\cos\theta d\omega$$

$$\rho_{\lambda}(\underline{r})=\frac{\int_{\cap_i}\rho'_{\lambda}(\underline{r},\hat{\Omega}_i)i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i}{\int_{\cap_i}i_{\lambda,i}(\underline{r},\hat{\Omega}_i)\cos\theta_i d\omega_i}$$

Finally,

$$J(\underline{r}) = \varepsilon(\underline{r})\sigma T^4(\underline{r}) + \left[1 - \varepsilon(\underline{r})\right]G(\underline{r})$$

For an enclosure

$$G_{dk} = \int_0^\infty \left[\sum_{j=1}^{n} \int_{A_j} \pi i_{\lambda,o,dj} dF_{dk-dj} \right] d\lambda = \sum_{j=1}^{n} \int_{A_j} \pi i_{o,dj} dF_{dk-dj}$$

$$= \sum_{j=1}^{n} \int_{A_j} \pi i_{o,dj} \frac{\cos\theta_k \cos\theta_j}{\pi r_{ij}^2} dA_j$$

$$J_{dk} = \varepsilon_{dk}\sigma T_{dk}^4 + \left(1 - \varepsilon_{dk}\right)G_{dk}$$

$$q''_{dk} = \varepsilon_{dk}\left[\sigma T_{dk}^4 - \sum_{j=1}^{n} \int_{A_j} \pi i_{o,dj} dF_{dk-dj} \right]$$