ENGINEERING TREATMENT OF GAS RADIATION IN ENCLOSURES

- net-radiation method for enclosure filled with isothermal gas: spectral relations
- mean beam length for radiation from an entire gas volume to all or part of its boundary
- exchange of total radiation in an enclosure by application of mean beam length
- zonal method: non-isothermal gases

Net-Radiation Method for Enclosures Filled with Isothermal Gases

Equation of Transfer in an absorbing and emitting medium

$$\frac{di_\lambda(\underline{r},\hat{\Omega})}{ds} = -a_\lambda i_\lambda(\underline{r},\hat{\Omega}) + a_\lambda i_{\lambda b}(\underline{r})$$

in terms of optical thickness

$$\tau_\lambda(s) = \int_0^s a_\lambda(s')ds', \qquad \frac{d}{ds} = \frac{d}{d\tau_\lambda}\frac{d\tau_\lambda}{ds} = a_\lambda \frac{d}{d\tau_\lambda}$$

$$a_\lambda \frac{di_\lambda}{d\tau_\lambda} = -a_\lambda i_\lambda + a_\lambda i_{\lambda b} \rightarrow \frac{di_\lambda}{d\tau_\lambda} + i_\lambda = i_{\lambda b}$$

formal solution

in a given direction of propagation

$$\frac{di_\lambda}{d\tau_\lambda} + i_\lambda = i_{\lambda b}$$

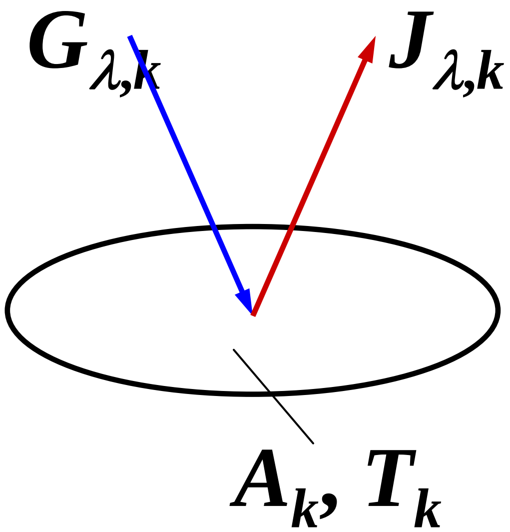
integrating factor $\exp(\tau_\lambda)$

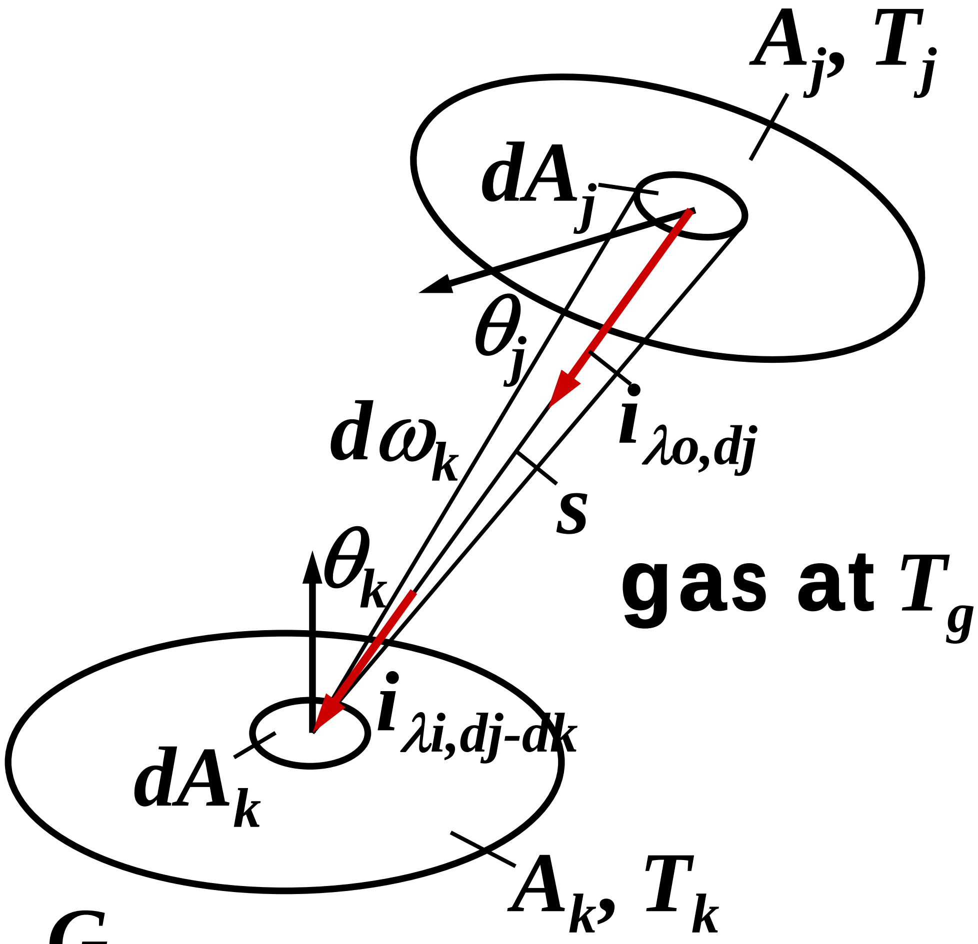
$$\frac{d}{d\tau_\lambda}\left(e^{\tau_\lambda} i_\lambda\right) = e^{\tau_\lambda} i_{\lambda b}, \;\; d\left(e^{\tau_\lambda} i_\lambda\right) = e^{\tau_\lambda} i_{\lambda b} d\tau_\lambda$$

$$\int_0^{\tau_\lambda} d\left[e^{\tau'_\lambda} i_\lambda(\tau'_\lambda)\right] = \int_0^{\tau_\lambda} e^{\tau'_\lambda} i_{\lambda b}(\tau'_\lambda) d\tau'_\lambda$$

$$i_\lambda(\tau_\lambda) = i_\lambda(0)\exp(-\tau_\lambda) + \int_0^{\tau_\lambda} i_{\lambda b}(\tau'_\lambda)\exp\left[-(\tau_\lambda - \tau'_\lambda)\right] d\tau'_\lambda$$

Spectral Geometric-Mean Transmission and Absorption Factors





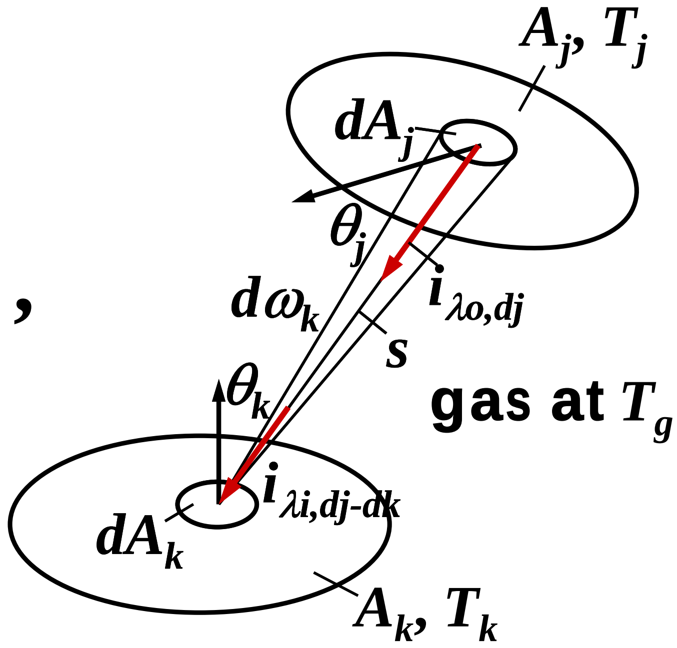
$$q''_{\lambda,k} = J_{\lambda,k} - G_{\lambda,k}$$

$$J_{\lambda,k} = \varepsilon_{\lambda,k} e_{\lambda b,k} + \rho_{\lambda,k} G_{\lambda,k}$$

without scattering

$$i_\lambda(\tau_\lambda) = i_\lambda(0)\exp(-\tau_\lambda) + \int_0^{\tau_\lambda} i_{\lambda b,g}(\tau'_\lambda)\exp[-(\tau_\lambda - \tau'_\lambda)]d\tau'_\lambda,$$

$$i_{\lambda i,dj-dk} = i_{\lambda o,dj}\exp(-\tau_\lambda) + \int_0^{\tau_\lambda} i_{\lambda b,g}(\tau'_\lambda)\exp[-(\tau_\lambda - \tau'_\lambda)]d\tau'_\lambda$$



for an isothermal gas $i_{\lambda b,g}$ = constant,
assume a_λ = constant, then $\tau_\lambda = a_\lambda s$

$$i_{\lambda i,dj-dk} = i_{\lambda o,dj}\exp(-a_\lambda s) + a_\lambda i_{\lambda b,g}\int_0^s \exp[-a_\lambda(s - s')]ds'$$

$$= i_{\lambda o,dj}\exp(-a_\lambda s) + i_{\lambda b,g}[1 - \exp(-a_\lambda s)]$$

Spectral transmittance : $\tau_\lambda(s) \equiv \exp(-a_\lambda s)$

Spectral absorptance : $\alpha_\lambda(s) \equiv 1 - \exp(-a_\lambda s)$

$$i_{\lambda i,dj-dk} = i_{\lambda o,dj} \exp(-a_\lambda s) + i_{\lambda b,g}\left[1 - \exp(-a_\lambda s)\right]$$

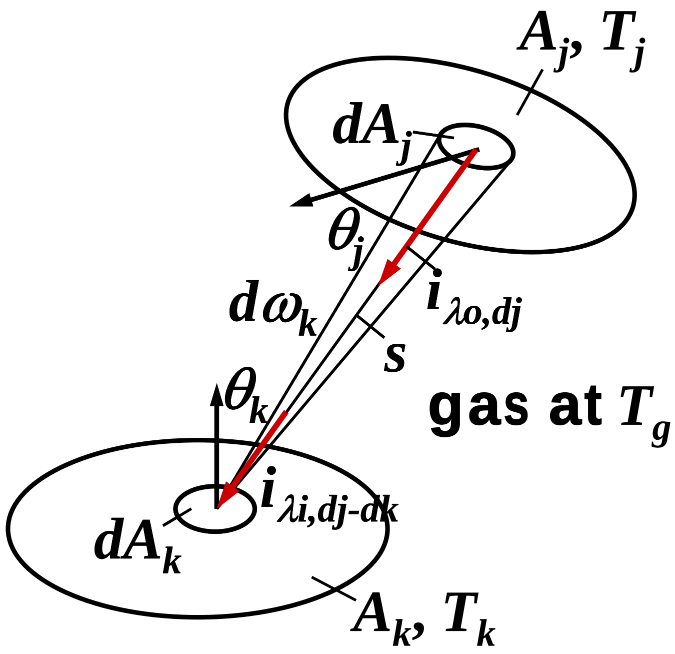
$$i_{\lambda i,dj-dk} = i_{\lambda o,dj}\tau_\lambda(s) + i_{\lambda b,g}\alpha_\lambda(s)$$

For diffuse surface

$$J_{\lambda,dj} = \int_{\cap} i_{\lambda o,dj} \cos\theta_j d\omega_j = \pi i_{\lambda o,dj}$$

$$i_{\lambda i,dj-dk} = \frac{J_{\lambda,dj}}{\pi}\tau_\lambda(s) + \frac{e_{\lambda b,g}}{\pi}\alpha_\lambda(s)$$

Irradiation



$$G_{\lambda,dk,dj-dk}dA_k$$

$$= i_{\lambda i,dj-dk}\cos\theta_k dA_k d\omega_k$$

$$= \left[\frac{J_{\lambda,dj}}{\pi}\tau_\lambda(s) + \frac{e_{\lambda b,g}}{\pi}\alpha_\lambda(s)\right]\frac{\cos\theta_k\cos\theta_j}{s^2}dA_j dA_k$$

or $G_{\lambda,dk,dj-dk}dA_k$

$$= \left[J_{\lambda,dj}\tau_\lambda(s) + e_{\lambda b,g}\alpha_\lambda(s)\right]\frac{\cos\theta_k\cos\theta_j}{\pi s^2}dA_j dA_k$$

$$G_{\lambda,k,j-k} A_k =$$

$$\int_{A_j}\int_{A_k}\left[J_{\lambda,dj}\tau_\lambda(s)+e_{\lambda b,g}\alpha_\lambda(s)\right]\frac{\cos\theta_k\cos\theta_j}{\pi s^2}dA_j dA_k$$

$$F_{j-k} = \frac{1}{A_j}\int_{A_j}\int_{A_k}\frac{\cos\theta_k\cos\theta_j}{\pi s^2}dA_k dA_j$$

Geometric-mean transmittance: $\bar{\tau}_{\lambda,j-k}$

$$F_{j-k}\bar{\tau}_{\lambda,j-k} \equiv \frac{1}{A_j}\int_{A_j}\int_{A_k}\frac{\tau_\lambda(s)\cos\theta_k\cos\theta_j}{\pi s^2}dA_k dA_j$$

Geometric-mean absorptance: $\bar{\alpha}_{\lambda,j-k}$

$$F_{j-k}\bar{\alpha}_{\lambda,j-k} \equiv \frac{1}{A_j}\int_{A_j}\int_{A_k}\frac{\alpha_\lambda(s)\cos\theta_k\cos\theta_j}{\pi s^2}dA_k dA_j$$

$$\bar{\alpha}_{\lambda,j-k} = 1 - \bar{\tau}_{\lambda,j-k}$$

Geometric transmission factor: $A_j F_{j-k} \bar{\tau}_{\lambda,j-k}$

Geometric absorption factor: $A_j F_{j-k} \bar{\alpha}_{\lambda,j-k}$

Irradiation in terms of geometric factors

$$G_{\lambda,k,j-k} A_k =$$

$$\int_{A_j} \int_{A_k} \left[J_{\lambda,dj} \tau_\lambda(s) + e_{\lambda b,g} \alpha_\lambda(s) \right] \frac{\cos\theta_k \cos\theta_j}{\pi s^2} dA_j dA_k$$

$$F_{j-k} = \frac{1}{A_j} \int_{A_j} \int_{A_k} \frac{\cos\theta_k \cos\theta_j}{\pi s^2} dA_k dA_j$$

$$G_{\lambda,k,j-k} A_k = \left(A_j F_{j-k} \bar{\tau}_{\lambda,j-k} \right) J_{\lambda,j} + \left(A_j F_{j-k} \bar{\alpha}_{\lambda,j-k} \right) e_{\lambda b,g}$$

for an enclosure with n surfaces

$$G_{\lambda,k} A_k = \sum_{j=1}^{n} \left[A_j F_{j-k} \bar{\tau}_{\lambda,j-k} J_{\lambda,j} + A_j F_{j-k} \bar{\alpha}_{\lambda,j-k} e_{\lambda b,g} \right]$$

$$G_{\lambda,k} = \sum_{j=1}^{n} \left[J_{\lambda,j} F_{k-j} \bar{\tau}_{\lambda,k-j} + e_{\lambda b,g} F_{k-j} \bar{\alpha}_{\lambda,k-j} \right]$$

$$q''_{\lambda,k} = \varepsilon_{\lambda,k} \left(e_{\lambda b,k} - G_{\lambda,k} \right) = \frac{\varepsilon_{\lambda,k}}{1-\varepsilon_{\lambda,k}} \left(e_{\lambda b,k} - J_{\lambda,k} \right)$$

or $$q''_{\lambda,k} = J_{\lambda,k} - \sum_{j=1}^{n} \left[J_{\lambda,j} F_{k-j} \bar{\tau}_{\lambda,k-j} + e_{\lambda b,g} F_{k-j} \bar{\alpha}_{\lambda,k-j} \right]$$

or $$\sum_{j=1}^{n} \left(\frac{\delta_{kj}}{\varepsilon_{\lambda,j}} - F_{k-j} \frac{1-\varepsilon_{\lambda,j}}{\varepsilon_{\lambda,j}} \bar{\tau}_{\lambda,k-j} \right) q''_{\lambda,j}$$

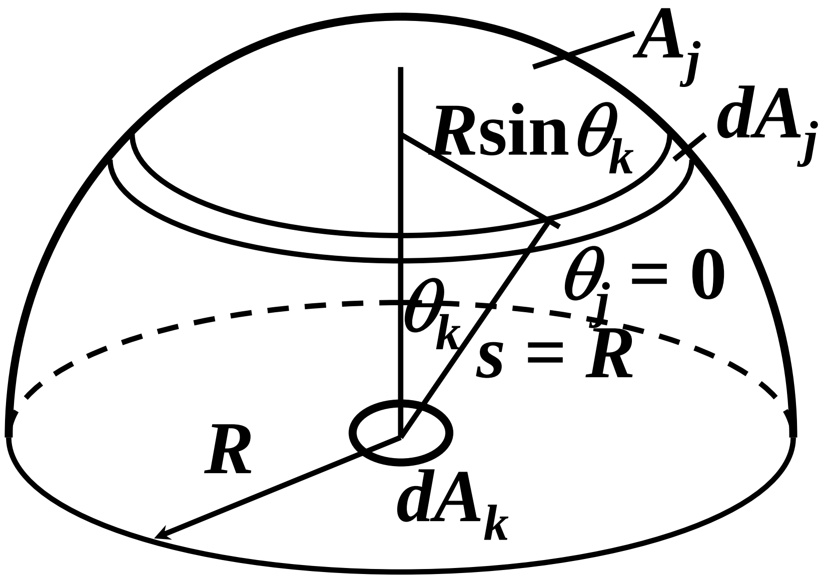
$$= \sum_{j=1}^{n} \left[(\delta_{kj} - F_{k-j} \bar{\tau}_{\lambda,k-j}) e_{\lambda b,j} - F_{k-j} \bar{\alpha}_{\lambda,k-j} e_{\lambda b,g} \right]$$

Evaluation of spectral geometric-mean transmittance and absorption factors

$$A_j F_{j-k} \bar{\tau}_{\lambda,j-k} = \int_{A_j} \int_{A_k} \frac{\exp(-a_\lambda s)\cos\theta_k \cos\theta_j}{\pi s^2} dA_k dA_j,$$

$$A_j F_{j-k} \bar{\alpha}_{\lambda,j-k} = A_j F_{j-k}\left(1 - \bar{\tau}_{\lambda,j-k}\right)$$

Ex **hemisphere to differential area at the center of its base**

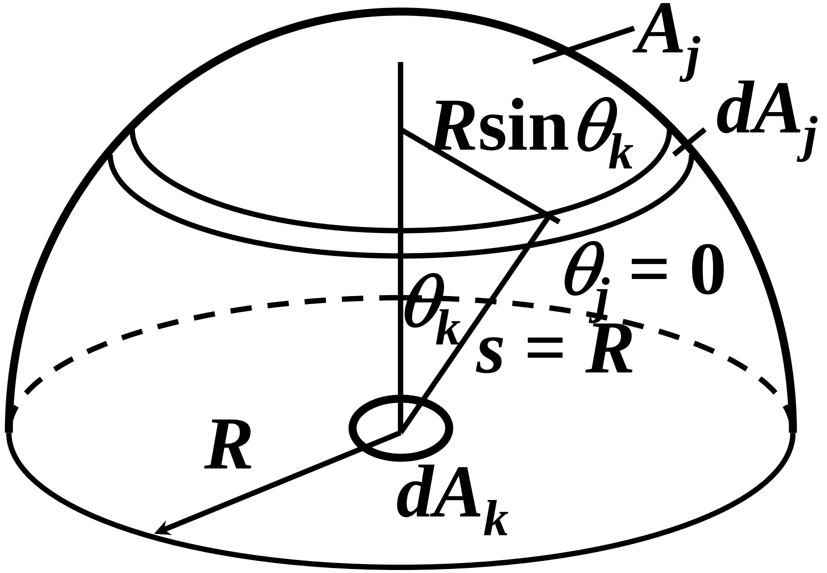


$$A_j dF_{j-dk} \bar{\tau}_{\lambda,j-dk}$$

$$= dA_k \int_{Aj} \frac{\exp(-a_\lambda R)\cos\theta_k}{\pi R^2} dA_j$$

$$dA_j = R d\theta_k \cdot 2\pi R \sin\theta_k$$

$$= 2\pi R^2 \sin\theta_k d\theta_k$$



$$A_j dF_{j-dk}\bar{\tau}_{\lambda,j-dk}$$
$$= dA_k \int_{Aj} \frac{\exp(-a_\lambda R)\cos\theta_k}{\pi R^2} dA_j$$

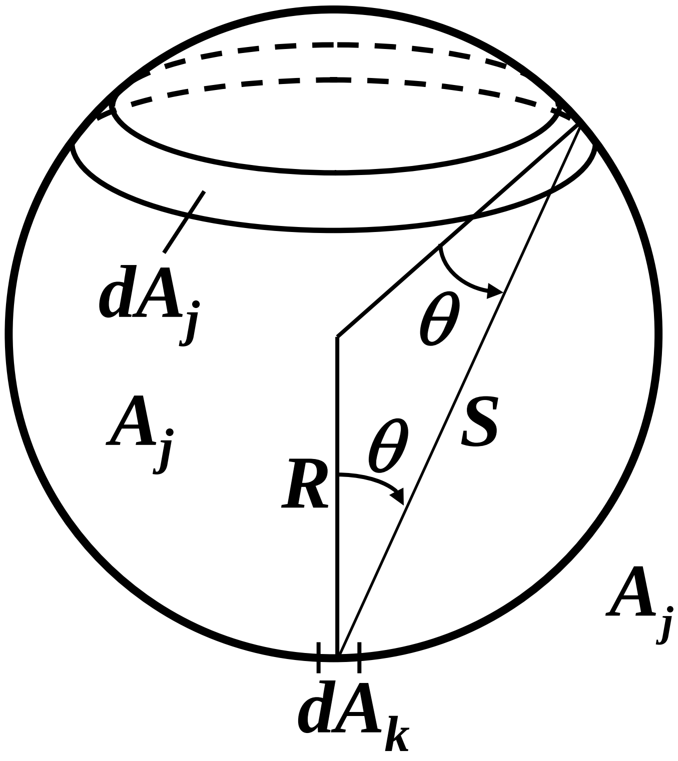
$$dA_j = 2\pi R^2 \sin\theta_k d\theta_k$$

$$A_j dF_{j-dk}\bar{\tau}_{\lambda,j-dk}$$
$$= dA_k \exp(-a_\lambda R)\cdot 2\int_0^{\pi/2} \cos\theta_k \sin\theta_k d\theta_k$$
$$= dA_k \exp(-a_\lambda R)$$

but $\quad A_j dF_{j-dk} = dA_k F_{dk-j} = dA_k$

Thus, $\quad \bar{\tau}_{\lambda,j-dk} = \exp(-a_\lambda R)$

Ex **entire sphere to any area on its surface or to its entire surface**



$$A_j dF_{j-dk}\bar{\tau}_{j-dk}$$

$$= dA_k \int_{A_j} \frac{\exp(-aS)\cos\theta_k \cos\theta_j}{\pi S^2} dA_j$$

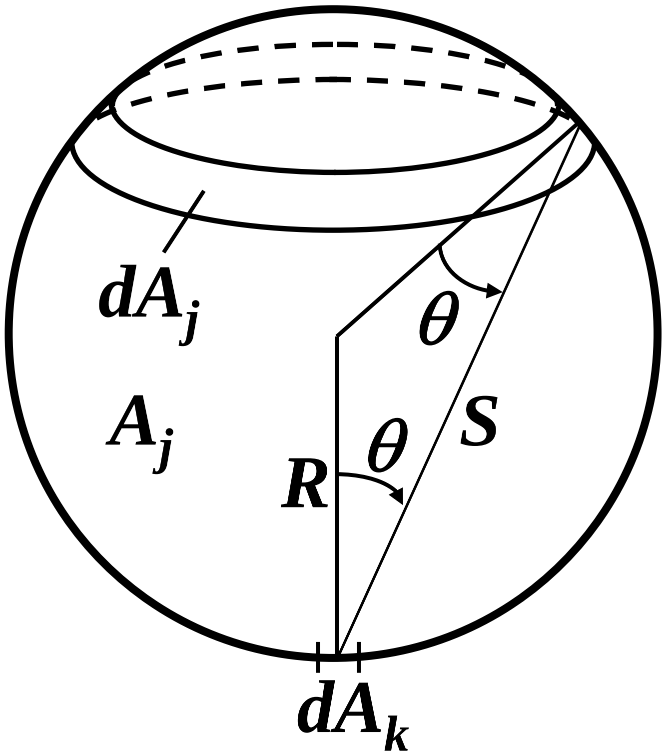
$$\theta_k = \theta_j = \theta,\ \ d\omega = \frac{dA_j \cos\theta}{S^2}$$

$$A_j dF_{j-dk}\bar{\tau}_{j-dk} = \frac{dA_k}{\pi}\int_{\cap} \exp(-aS)\cos\theta \, d\omega$$

$$= \frac{dA_k}{\pi} 2\pi \int_0^{\pi/2} \exp(-aS)\cos\theta \sin\theta \, d\theta$$

$$= 2dA_k \int_{\theta=0}^{\pi/2} \exp(-aS)\cos\theta \sin\theta \, d\theta$$

A_j: **area of entire sphere** $= 4\pi R^2$



$$A_j dF_{j-dk}\bar{\tau}_{j-dk}$$

$$= 2dA_k \int_{\theta=0}^{\pi/2} \exp(-aS)\cos\theta\sin\theta d\theta$$

$$S = 2R\cos\theta \rightarrow dS = -2R\sin\theta d\theta$$

and $\cos\theta = \dfrac{S}{2R}$

$$A_j dF_{j-dk}\bar{\tau}_{j-dk} = 2dA_k \int_0^{2R} \exp(-aS)\frac{S}{2R}\frac{dS}{2R}$$

$$= \frac{2dA_k}{(2R)^2}\int_0^{2R} S\exp(-aS)dS = \frac{2dA_k}{(2R)^2}\left[-\frac{\exp(-aS)}{a}\left(S+\frac{1}{a}\right)\right]_0^{2R}$$

$$A_j dF_{j-dk}\overline{\tau}_{j-dk} = \frac{2dA_k}{(2R)^2}\left[-\frac{\exp(-2aR)}{a}\left(2R+\frac{1}{a}\right)+\frac{1}{a^2}\right]$$

$$= \frac{2dA_k}{(2aR)^2}\left[1-(2aR+1)\exp(-2aR)\right]$$

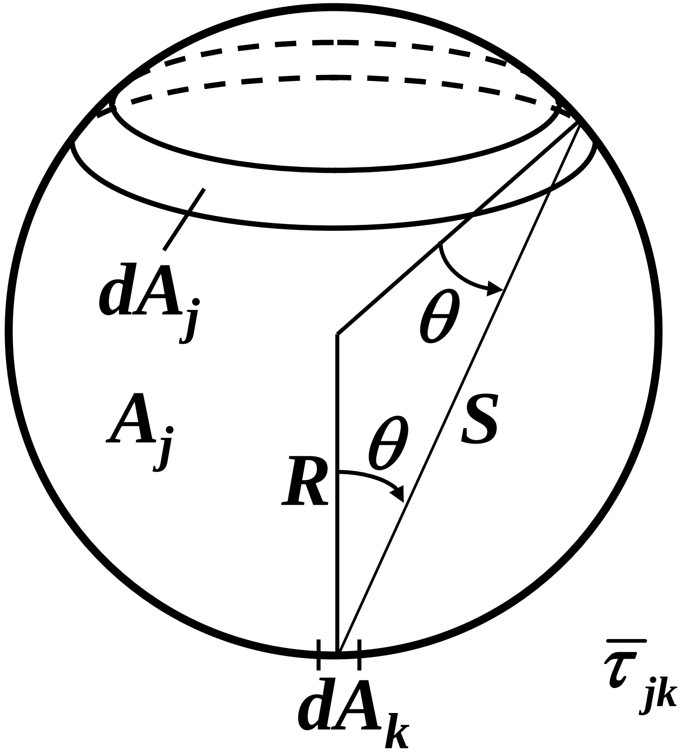
$$A_j dF_{j-dk}\overline{\tau}_{j-dk} = dA_k\int_{A_j}\frac{\exp(-aS)\cos\theta_k\cos\theta_j}{\pi S^2}dA_j$$

$$A_j F_{jk}\overline{\tau}_{jk} = \int_{A_k}\int_{A_j}\frac{\exp(-aS)\cos\theta_k\cos\theta_j}{\pi S^2}dA_j dA_k$$

$$= \int_{A_k}\frac{2dA_k}{(2aR)^2}\left[1-(2aR+1)\exp(-2aR)\right]$$

$$= \frac{2A_k}{(2aR)^2}\left[1-(2aR+1)\exp(-2aR)\right]$$

$$A_j F_{jk} \overline{\tau}_{jk} = \frac{2A_k}{(2aR)^2}\left[1-(2aR+1)\exp(-2aR)\right]$$

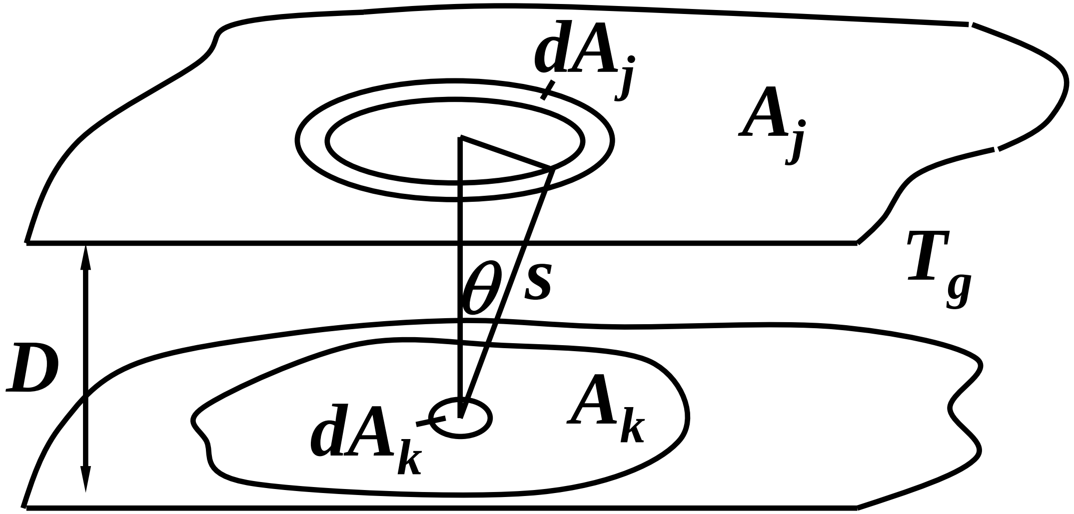


Recall inside sphere method

$$F_{jk} = \frac{A_k}{A_j}$$

$$\overline{\tau}_{jk} = \frac{2}{(2aR)^2}\left[1-(2aR+1)\exp(-2aR)\right]$$

Ex infinite plate to any area on parallel plate



$$\theta_j = \theta_k = \theta,$$

$$dA_j = 2\pi s \sin\theta \frac{s d\theta}{\cos\theta},$$

$$s\cos\theta = D$$

or $\quad s = \dfrac{D}{\cos\theta}$

$$A_j dF_{j-dk}\bar{\tau}_{\lambda,j-dk} = dA_k \int_{A_j} \frac{\exp(-a_\lambda s)\cos^2\theta}{\pi s^2} dA_j$$

$$= dA_k \int_0^{\pi/2} \exp(-a_\lambda s) 2\cos\theta \sin\theta d\theta$$

$$= 2dA_k \int_0^1 \exp\left(-\frac{a_\lambda D}{\mu}\right) \mu d\mu, \quad \mu = \cos\theta$$

exponential integral function

$$E_n(x) = \int_0^1 \mu^{n-2} e^{-\frac{x}{\mu}} d\mu$$

$$A_j dF_{j-dk} \bar{\tau}_{\lambda,j-dk} = 2dA_k \int_0^1 \exp\left(-\frac{a_\lambda D}{\mu}\right) \mu d\mu$$

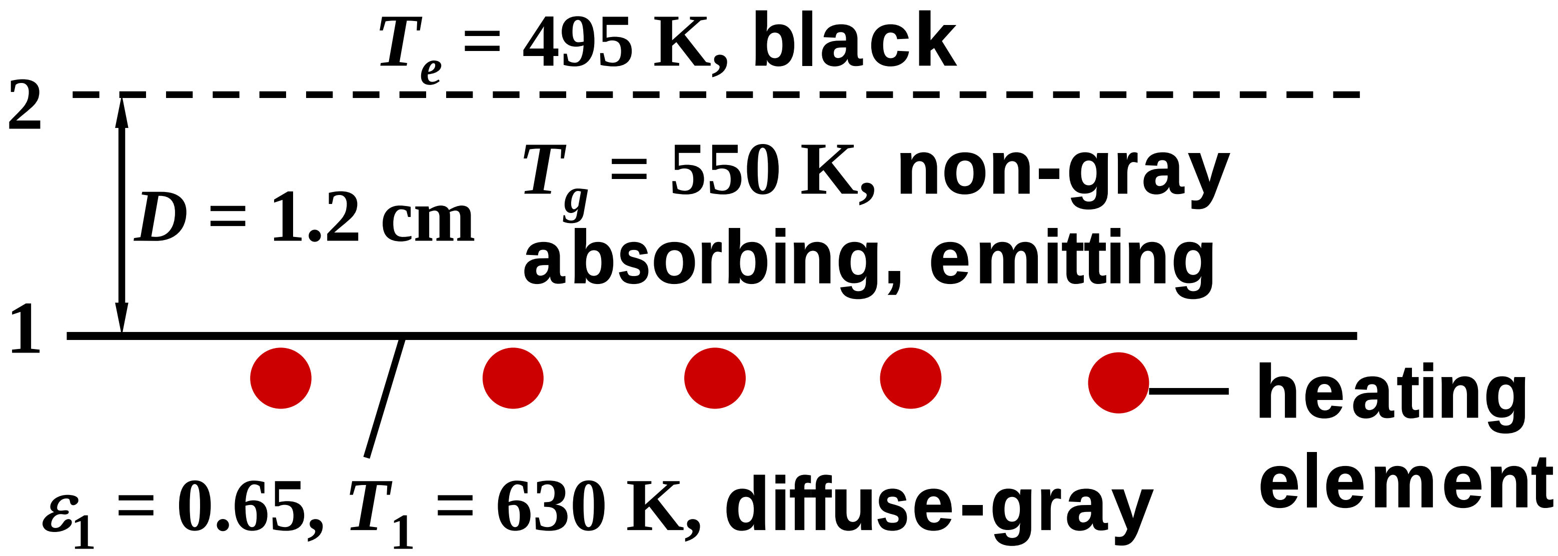
$$= 2dA_k E_3(a_\lambda D)$$

$$dA_k F_{dk-j} \bar{\tau}_{\lambda,j-dk} = 2dA_k E_3(a_\lambda D)$$

$$A_k F_{k-j} \bar{\tau}_{\lambda,j-k} = 2A_k E_3(a_\lambda D)$$

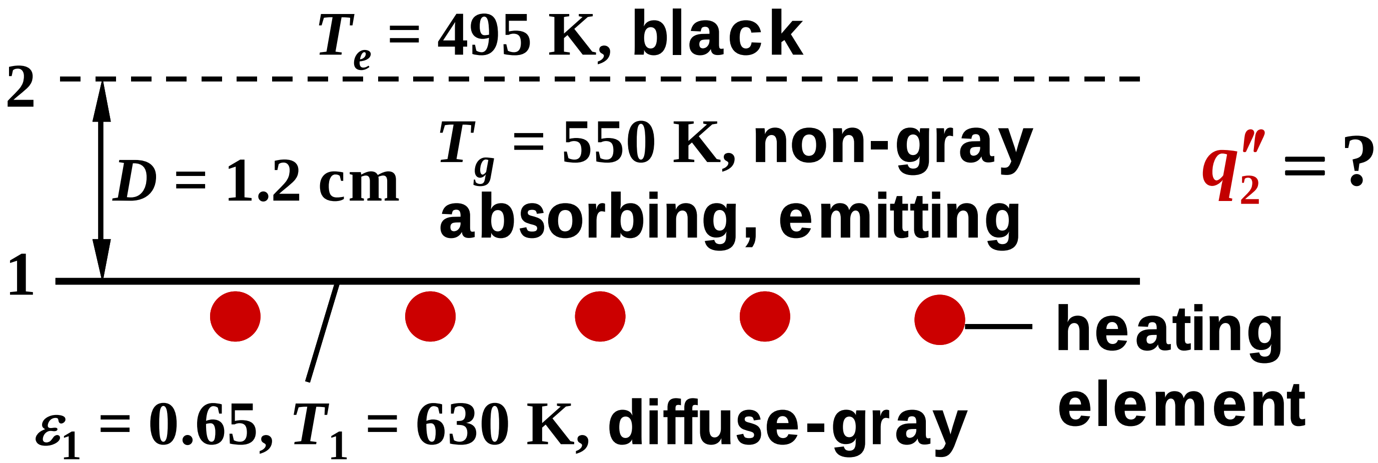
$$F_{k-j} = 1 \;\rightarrow\; \bar{\tau}_{\lambda,j-k} = 2E_3(a_\lambda D)$$

Ex 13-3



$$a_\lambda = \begin{cases} 0.16\ \text{cm}^{-1} & 0 \le \lambda \le 6.5\,\mu\text{m} \\ 0.65\ \text{cm}^{-1} & \lambda > 6.5\,\mu\text{m} \end{cases}$$

$q_2'' = ?$



$$q''_{\lambda,k} = J_{\lambda,k} - G_{\lambda,k}$$

$$G_{\lambda,k} = \sum_{j=1}^{n} \left[J_{\lambda,j} F_{k-j} \bar{\tau}_{\lambda,k-j} + e_{\lambda b,g} F_{k-j} \bar{\alpha}_{\lambda,k-j} \right]$$

$$q''_{\lambda,2} = J_{\lambda,2} - \left(J_{\lambda,1} F_{21} \bar{\tau}_{\lambda} + e_{\lambda b,g} F_{21} \bar{\alpha}_{\lambda} \right)$$

$$= e_{\lambda b,2} - \left(J_{\lambda,1} \bar{\tau}_{\lambda} + e_{\lambda b,g} \bar{\alpha}_{\lambda} \right)$$

$$J_{\lambda,1} = \varepsilon_{\lambda 1} e_{\lambda b1} + \left(1 - \varepsilon_{\lambda 1}\right) \left[e_{\lambda b,2} \bar{\tau}_{\lambda} + e_{\lambda b,g} \bar{\alpha}_{\lambda} \right]$$

$$q''_{\lambda,2} = e_{\lambda b,2} - \left\{ \varepsilon_{\lambda 1} e_{\lambda b1} \bar{\tau}_{\lambda} + \left(1 - \varepsilon_{\lambda 1}\right) \left(e_{\lambda b,2} \bar{\tau}_{\lambda}^{\,2} + e_{\lambda b,g} \bar{\alpha}_{\lambda} \bar{\tau}_{\lambda} \right) + e_{\lambda b,g} \bar{\alpha}_{\lambda} \right\}$$

$$\bar{\tau}_\lambda = 2E_3(a_\lambda D)$$

$$a_\lambda = \begin{cases} 0.16\ \text{cm}^{-1} & 0 \le \lambda \le 6.5\mu\text{m} \\ 0.65\ \text{cm}^{-1} & \lambda > 6.5\mu\text{m} \end{cases}$$

$$\bar{\tau}_\lambda = \begin{cases} 2E_3(0.16 \times 1.2) = 0.7142 \equiv \bar{\tau}_{\lambda 1} & \text{for } 0 \le \lambda \le 6.5\ \mu\text{m} \\ 2E_3(0.65 \times 1.2) = 0.2974 \equiv \bar{\tau}_{\lambda 2} & \text{for } \lambda > 6.5\ \mu\text{m} \end{cases}$$

$$q''_{\lambda,2} = e_{\lambda b,2} - \left\{ \varepsilon_{\lambda 1} e_{\lambda b1} \bar{\tau}_\lambda + (1 - \varepsilon_{\lambda 1})(e_{\lambda b,2} \bar{\tau}_\lambda{}^2 + e_{\lambda b,g} \bar{\alpha}_\lambda \bar{\tau}_\lambda) + e_{\lambda b,g} \bar{\alpha}_\lambda \right\}$$

$$q''_2 = \sigma T_2^4 - \left\{ \varepsilon_1 \sigma T_1^4 F_{0-\lambda T_1} \bar{\tau}_{\lambda 1} + \varepsilon_1 \sigma T_1^4 \bar{\tau}_{\lambda 2} F_{\lambda T_1 - \infty} \right.$$

$$+ (1 - \varepsilon_1) \left(\sigma T_2^4 F_{0-\lambda T_2} \bar{\tau}_{\lambda 1}{}^2 + \sigma T_2^4 F_{\lambda T_2 - \infty} \bar{\tau}_{\lambda 2}{}^2 \right.$$

$$\left. + \sigma T_g^4 F_{0-\lambda T_g} \bar{\tau}_{\lambda 1} \bar{\alpha}_{\lambda 1} + \sigma T_g^4 F_{\lambda T_g - \infty} \bar{\tau}_{\lambda 2} \bar{\alpha}_{\lambda 2} \right)$$

$$\left. + \sigma T_g^4 F_{o-\lambda T_g} \bar{\alpha}_{\lambda 1} + \sigma T_g^4 F_{\lambda T_g - \infty} \bar{\alpha}_{\lambda 2} \right\} = -2954\ \text{W/m}^2$$

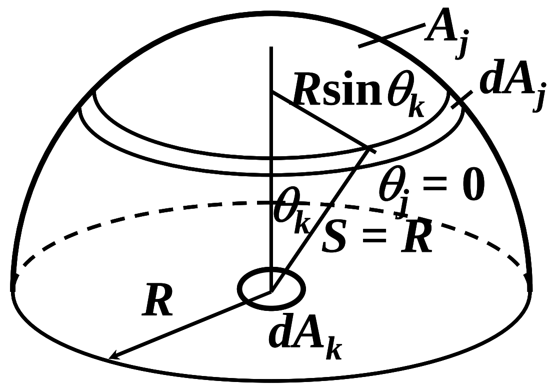
Mean Beam Length for Radiation from an Entire Gas Volume to All or Part of Its Boundary

Definition

$$G_{\lambda,k}A_k = \sum_{j=1}^{n}\left[A_j F_{j-k}\bar{\tau}_{\lambda,j-k}J_{\lambda,j} + A_j F_{j-k}\bar{\alpha}_{\lambda,j-k}e_{\lambda b,g}\right]$$

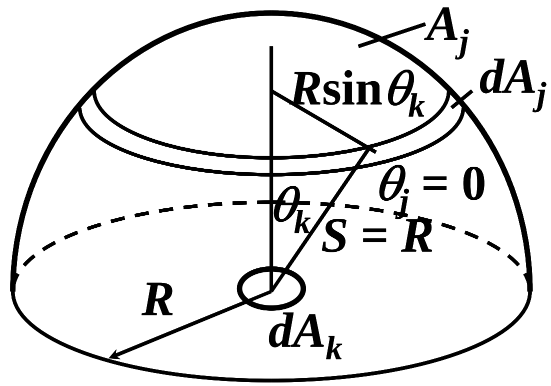
When $J_{\lambda,j}$ is negligible,

$$G_{\lambda,k}A_k = \sum_{j=1}^{n}A_j F_{j-k}\bar{\alpha}_{\lambda,j-k}e_{\lambda b,g}$$



for a hemisphere of gas radiating to an area element dA_k at the center of its base

$$\cancel{dA_k}G_{\lambda,dk} = A_j dF_{j-dk}\bar{\alpha}_{\lambda,j-dk}e_{\lambda b,g} = \cancel{dA_k}\overset{=1}{F_{dk-j}}\bar{\alpha}_{\lambda,j-dk}e_{\lambda b,g}$$



$$G_{\lambda,dk} = \bar{\alpha}_{\lambda,j-dk} e_{\lambda b,g}$$

$$\bar{\tau}_{\lambda,j-dk} = \exp(-a_\lambda R)$$

$$G_{\lambda,dk} = \left[1 - \exp(-a_\lambda R)\right] e_{\lambda b,g} \equiv \varepsilon_\lambda (a_\lambda R) e_{\lambda b,g}$$

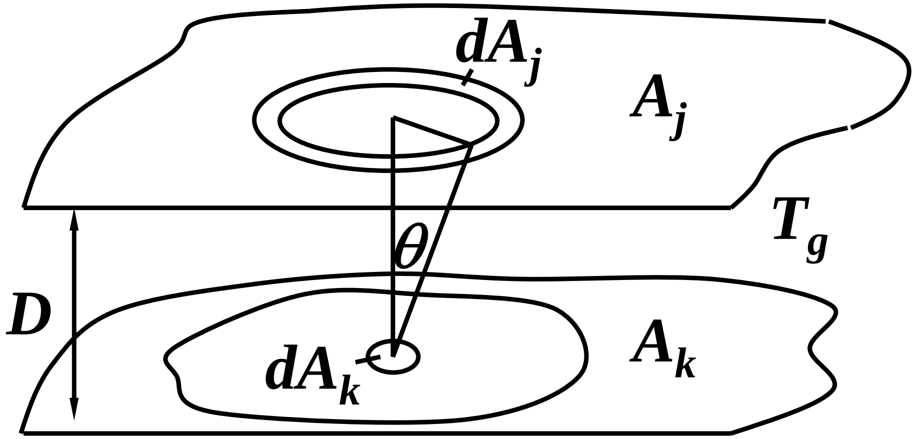
ε_λ: **spectral emittance of the gas**

mean beam length L_e for an arbitrary geometry of gas

$$G_{\lambda,k} = \varepsilon_\lambda (a_\lambda L_e) e_{\lambda b,g} = \left[1 - \exp(-a_\lambda L_e)\right] e_{\lambda b,g}$$

$$\varepsilon_\lambda (a_\lambda L_e) = 1 - \exp(-a_\lambda L_e)$$

Ex mean beam length for gas between parallel plates radiating to area on plate



$$\bar{\tau}_{\lambda,j-k} = 2E_3(a_\lambda D)$$

$$\varepsilon_\lambda(a_\lambda L_e) = 1 - \exp(-a_\lambda L_e) = 1 - 2E_3(a_\lambda D)$$

$$\exp(-a_\lambda L_e) = 2E_3(a_\lambda D)$$

$$L_e = -\frac{1}{a_\lambda}\ln\left[2E_3(a_\lambda D)\right]$$

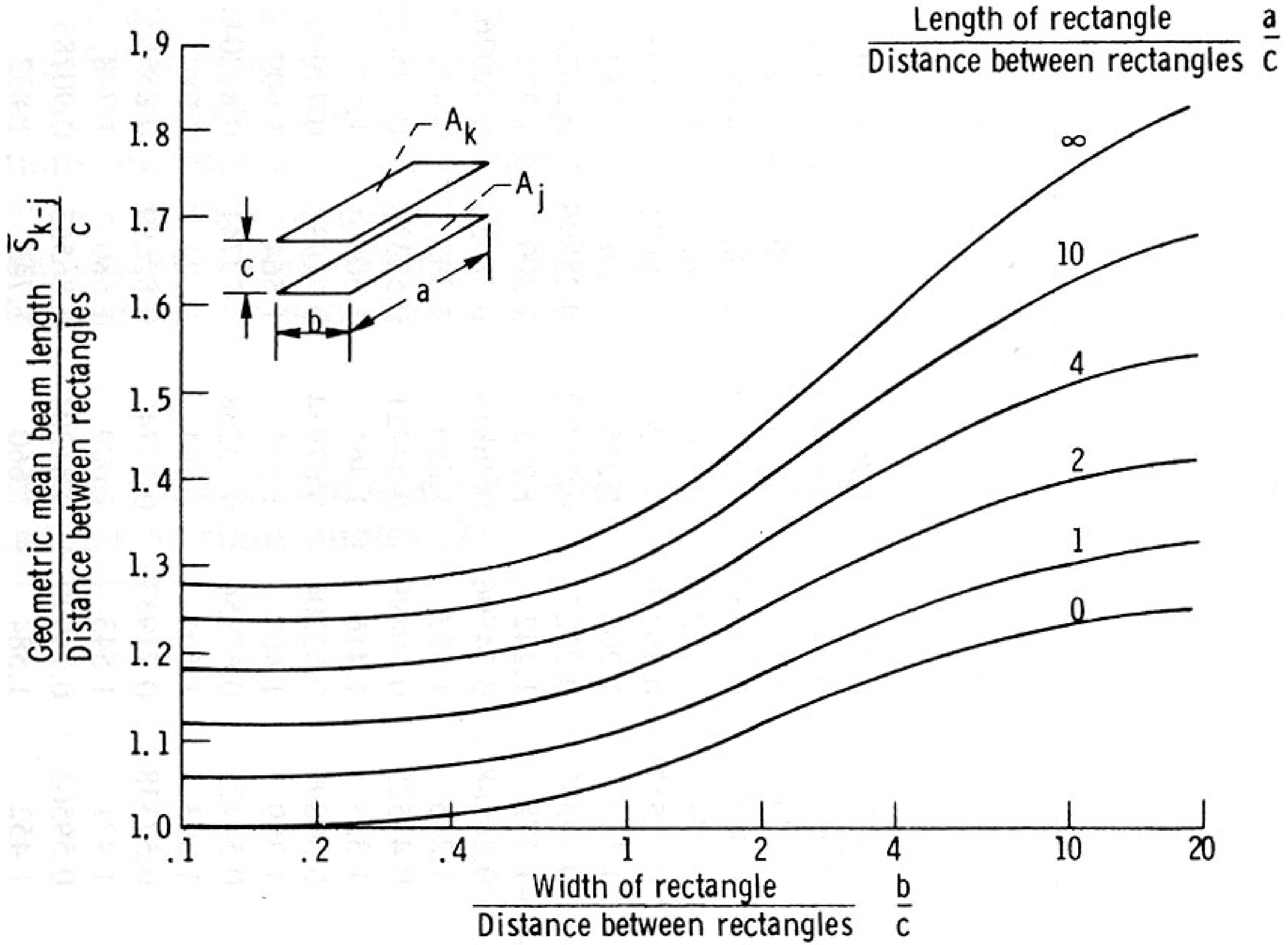
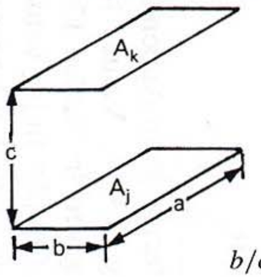


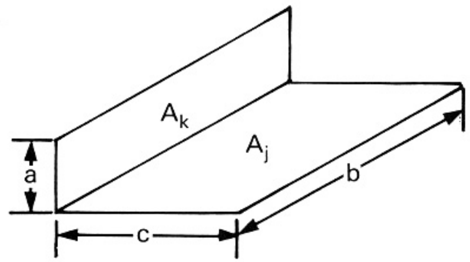
Figure 13-11 Geometric mean beam lengths for equal parallel rectangles [2].

Table 13-1 Geometric mean-beam-length ratios and configuration factors for parallel equal rectangles [2]



a/c		b/c: 0	0.1	0.2	0.4	0.6	1.0	2.0	4.0	6.0	10.0	20.0
0	$\bar{S}_{k-j}/c$	1.000	1.001	1.003	1.012	1.025	1.055	1.116	1.178	1.205	1.230	1.251
	F_{k-j}											
0.1	$\bar{S}_{k-j}/c$	1.001	1.002	1.004	1.013	1.026	1.056	1.117	1.179	1.207	1.233	1.254
	F_{k-j}		0.00316	0.00626	0.01207	0.01715	0.02492	0.03514	0.04210	0.04463	0.04671	0.04829
0.2	$\bar{S}_{k-j}/c$	1.003	1.004	1.006	1.015	1.028	1.058	1.120	1.182	1.210	1.235	1.256
	F_{k-j}		0.00626	0.01240	0.02391	0.03398	0.04941	0.06971	0.08353	0.08859	0.09271	0.09586
0.4	$\bar{S}_{k-j}/c$	1.012	1.013	1.015	1.024	1.037	1.067	1.129	1.192	1.220	1.245	1.267
	F_{k-j}		0.01207	0.02391	0.04614	0.06560	0.09554	0.13513	0.16219	0.17209	0.18021	0.18638
0.6	$\bar{S}_{k-j}/c$	1.025	1.026	1.028	1.037	1.050	1.080	1.143	1.206	1.235	1.261	1.282
	F_{k-j}		0.01715	0.03398	0.06560	0.09336	0.13627	0.19341	0.23271	0.24712	0.25896	0.26795
1.0	$\bar{S}_{k-j}/c$	1.055	1.056	1.058	1.067	1.080	1.110	1.175	1.242	1.272	1.300	1.324
	F_{k-j}		0.02492	0.04941	0.09554	0.13627	0.19982	0.28588	0.34596	0.36813	0.38638	0.40026
2.0	$\bar{S}_{k-j}/c$	1.116	1.117	1.120	1.129	1.143	1.175	1.246	1.323	1.359	1.393	1.421
	F_{k-j}		0.03514	0.06971	0.13513	0.19341	0.28588	0.41525	0.50899	0.54421	0.57338	0.59563
4.0	$\bar{S}_{k-j}/c$	1.178	1.179	1.182	1.192	1.206	1.242	1.323	1.416	1.461	1.505	1.543
	F_{k-j}		0.04210	0.08353	0.16219	0.23271	0.34596	0.50899	0.63204	0.67954	0.71933	0.74990
6.0	$\bar{S}_{k-j}/c$	1.205	1.207	1.210	1.220	1.235	1.272	1.359	1.461	1.513	1.564	1.609
	F_{k-j}		0.04463	0.08859	0.17209	0.24712	0.36813	0.54421	0.67954	0.73258	0.77741	0.81204
10.0	$\bar{S}_{k-j}/c$	1.230	1.233	1.235	1.245	1.261	1.300	1.393	1.505	1.564	1.624	1.680
	F_{k-j}		0.04671	0.09271	0.18021	0.25896	0.38638	0.57338	0.71933	0.77741	0.82699	0.86563
20.0	$\bar{S}_{k-j}/c$	1.251	1.254	1.256	1.267	1.282	1.324	1.421	1.543	1.609	1.680	1.748
	F_{k-j}		0.04829	0.09586	0.18638	0.26795	0.40026	0.59563	0.74990	0.81204	0.86563	0.90785
∞	$\bar{S}_{k-j}/c$	1.272	1.274	1.277	1.289	1.306	1.349	1.452	1.584	1.660	1.745	1.832
	F_{k-j}		0.04988	0.09902	0.19258	0.27698	0.41421	0.61803	0.78078	0.84713	0.90499	0.95125

Table 13-2 Configuration factors and mean beam-length functions for rectangles at right angles [2]



a/b		c/b 0.05	0.10	0.20	0.4	0.6	1.0
0.02	A_kF_{k-j}/b^2	0.007982	0.008875	0.009323	0.009545	0.009589	0.009628
	$A_kF_{k-j}\bar{S}_{k-j}/abc$	0.17840	0.12903	0.08298	0.04995	0.03587	0.02291
0.05	A_kF_{k-j}/b^2	0.014269	0.018601	0.02117	0.02243	0.02279	0.02304
	$A_kF_{k-j}\bar{S}_{k-j}/abc$	0.21146	0.18756	0.13834	0.08953	0.06627	0.04372
0.10	A_kF_{k-j}/b^2		0.02819	0.03622	0.04086	0.04229	0.04325
	$A_kF_{k-j}\bar{S}_{k-j}/abc$		0.20379	0.17742	0.12737	0.09795	0.06659
0.20	A_kF_{k-j}/b^2			0.05421	0.06859	0.07377	0.07744
	$A_kF_{k-j}\bar{S}_{k-j}/abc$			0.18854	0.15900	0.13028	0.09337
0.40	A_kF_{k-j}/b^2				0.10013	0.11524	0.12770
	$A_kF_{k-j}\bar{S}_{k-j}/abc$				0.16255	0.14686	0.11517
0.60	A_kF_{k-j}/b^2					0.13888	0.16138
	$A_kF_{k-j}\bar{S}_{k-j}/abc$					0.14164	0.11940
1.0	A_kF_{k-j}/b^2						0.20004
	$A_kF_{k-j}\bar{S}_{k-j}/abc$						0.11121
2.0	A_kF_{k-j}/b^2						
	$A_kF_{k-j}\bar{S}_{k-j}/abc$						
4.0	A_kF_{k-j}/b^2						
	$A_kF_{k-j}\bar{S}_{k-j}/abc$						
6.0	A_kF_{k-j}/b^2						
	$A_kF_{k-j}\bar{S}_{k-j}/abc$						
10.0	A_kF_{k-j}/b^2						
	$A_kF_{k-j}\bar{S}_{k-j}/abc$						
20.0	A_kF_{k-j}/b^2						
	$A_kF_{k-j}\bar{S}_{k-j}/abc$						

(Table continues on next page)

Radiation from entire gas volume to its entire boundary in limit when gas is optically thin

transmittance: $\tau_\lambda = \exp(-a_\lambda s)$

$$\lim_{a_\lambda s \to 0} \tau_\lambda = \lim_{a_\lambda s \to 0} \left\{ 1 - a_\lambda s + \frac{(a_\lambda s)^2}{2!} - \cdots \right\} = 1$$

emitted energy per unit volume

$$\int_{\omega=4\pi} a_\lambda i_{\lambda b,g} d\omega = 4\pi a_\lambda i_{\lambda b,g} = 4 a_\lambda e_{\lambda b,g}$$

for entire radiating volume for uniform-temperature gas: $4 a_\lambda e_{\lambda b,g} V$

average spectral flux received at the boundary:

$$4a_\lambda e_{\lambda b,g}\frac{V}{A}$$

when L_e is small, let $L_e = L_{e,o}$

$$G_\lambda = \left[1-\exp(-a_\lambda L_e)\right]e_{\lambda b,g}$$

$$=\left\{1-\left[1-a_\lambda L_{e,o}+\frac{(a_\lambda L_{e,o})^2}{2!}-\cdots\right]\right\}e_{\lambda b,g}$$

$$=a_\lambda L_{e,o} e_{\lambda b,g}$$

Thus, $L_{e,o}=\dfrac{4V}{A}$

$$L_{e,o} = \frac{4V}{A}$$

a sphere of diameter D :

$$L_{e,o} = \frac{4\pi D^3/6}{\pi D^2} = \frac{2}{3}D$$

infinitely long cylinder of diameter D:

$$L_{e,o} = \frac{4 \cdot \pi D^2/4}{\pi D} = D$$

between two infinite parallel plates:

$$L_{e,o} = \frac{4D}{2} = 2D$$

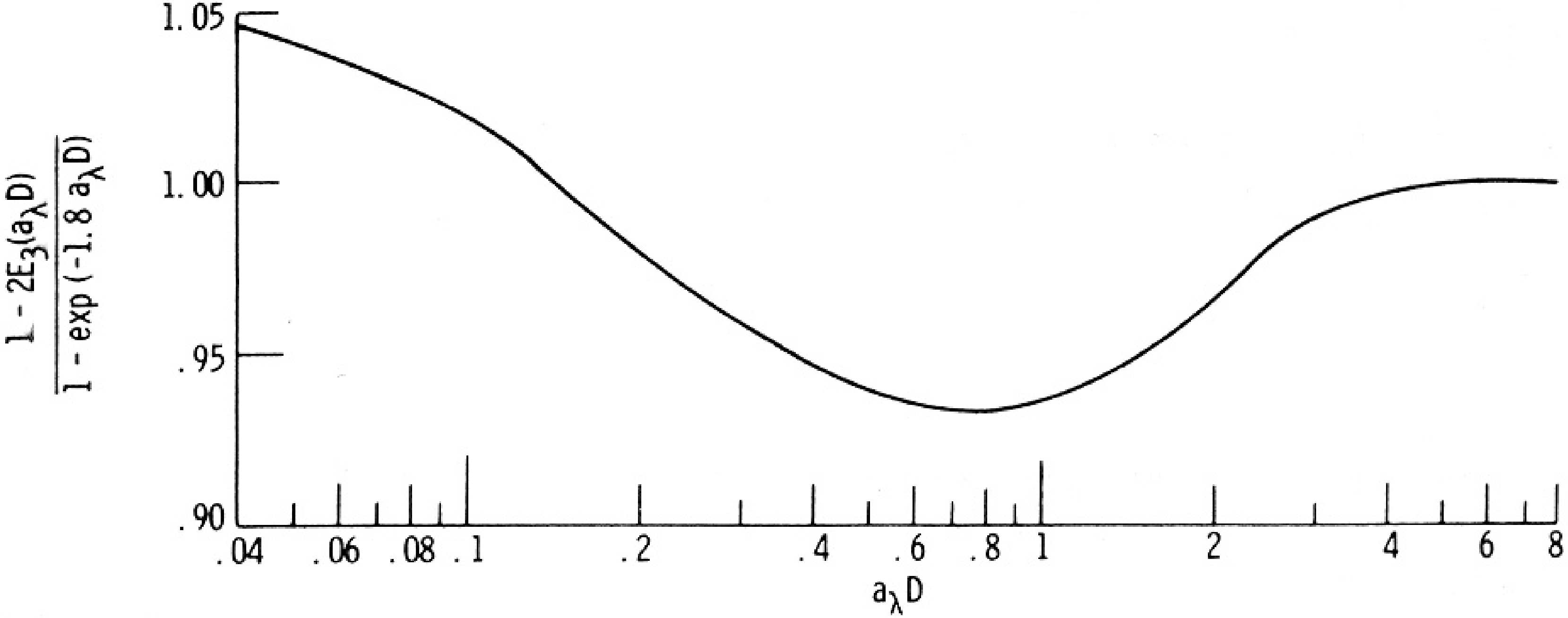


Figure 13-12 Ratio of emission by gas layer to that calculated using a mean beam length $L_e = 1.8D$.

Table 13-4 Mean beam lengths for radiation from entire gas volume

Geometry of radiating system	Characterizing dimension	Mean beam length for optical thickness $a_\lambda L_e \to 0$, $L_{e,0}$	Mean beam length corrected for finite optical thickness,[a] L_e	$C = L_e/L_{e,0}$
Hemisphere radiating to element at center of base	Radius R	R	R	1
Sphere radiating to its surface	Diameter D	$\frac{2}{3}D$	$0.65D$	0.97
Circular cylinder of infinite height radiating to concave bounding surface	Diameter D	D	$0.95D$	0.95
Circular cylinder of semiinfinite height radiating to:				
Element at center of base	Diameter D	D	$0.90D$	0.90
Entire base	Diameter D	$0.81D$	$0.65D$	0.80
Circular cylinder of height equal to diameter radiating to:				
Element at center of base	Diameter D	$0.77D$	$0.71D$	0.92
Entire surface	Diameter D	$\frac{2}{3}D$	$0.60D$	0.90

Circular cylinder of height equal to one-half the diameter radiating to:				
Plane end	Diameter D	$0.48D$	$0.43D$	0.90
Concave surface	Diameter D	$0.52D$	$0.46D$	0.88
Entire surface	Diameter D	$0.50D$	$0.45D$	0.90
Cylinder of infinite height and semicircular cross section radiating to element at center of plane rectangular face	Radius R		$1.26R$	
Infinite slab of gas radiating to:				
Element on one face	Slab thickness D	$2D$	$1.8D$	0.90
Both bounding planes	Slab thickness D	$2D$	$1.8D$	0.90
Cube radiating to a face	Edge X	$\frac{2}{3}X$	$0.6X$	0.90
Rectangular parallelepipeds $1 \times 1 \times 4$ radiating to:				
1×4 face	Shortest edge X	$0.90X$	$0.82X$	0.91

Table 13-4 Mean beam lengths for radiation from entire gas volume (*Continued*)

1 × 1 face		$0.86X$	$0.71X$	0.83
all faces		$0.89X$	$0.81X$	0.91
1 × 2 × 6 radiating to:				
2 × 6 face		$1.18X$		
1 × 6 face		$1.24X$		
1 × 2 face		$1.18X$		
all faces		$1.20X$		
Gas between infinitely long parallel concentric cylinders	Radius of outer cylinder R and of inner cylinder r	$2(R - r)$	See [4]	
Gas volume in the space between the outside of the tubes in an infinite tube bundle and radiating to a single tube:				
Equilateral triangular array:	Tube diameter D, and spacing between tube centers, S			
$S = 2D$		$3.4(S - D)$	$3.0(S - D)$	0.88
$S = 3D$		$4.45(S - D)$	$3.8(S - D)$	0.85
Square array:				
$S = 2D$		$4.1(S - D)$	$3.5(S - D)$	0.85

[a]Corrections are those suggested by Hottel et al. [3, 8] or Eckert [9]. Corrections were chosen to provide maximum L_e where these references disagree.

Exchange of Total Radiation in an Enclosure by Application of Mean Beam Length

Total radiation from entire gas volume to all or part of boundary

total heat flux from the gas that is incident on a surface

$$G_\lambda = \left[1 - \exp(-a_\lambda L_e)\right] e_{\lambda b,g}$$

$$G = \int_0^\infty \left[1 - \exp(-a_\lambda L_e)\right] e_{\lambda b,g} \, d\lambda$$

gas total emittance

$$G = \int_0^\infty \left[1 - \exp(-a_\lambda L_e)\right] e_{\lambda b,g} \, d\lambda \equiv \varepsilon_g \sigma T_g^4$$

$$\varepsilon_g = \frac{\int_0^\infty \left[1 - \exp(-a_\lambda L_e)\right] e_{\lambda b,g} \, d\lambda}{\sigma T_g^4}$$

radiation to area A_k from the gas volume

$$q_k = q_k'' A_k = A_k \varepsilon_g \sigma T_g^4$$

Hottel's charts for gas total emittance

$$\varepsilon_g = C_{\mathrm{CO_2}}\varepsilon_{\mathrm{CO_2}} + C_{\mathrm{H_2O}}\varepsilon_{\mathrm{H_2O}} - \Delta\varepsilon$$

for a mixture

$$\varepsilon_g = \frac{1}{\sigma T_g^4}\int_0^\infty \left[1 - e^{-(a_{\lambda 1}+a_{\lambda 2})L_e}\right] e_{\lambda b,g}\, d\lambda$$

$$= \frac{1}{\sigma T_g^4}\int_0^\infty \Big[1 - e^{-a_{\lambda 1}L_e} + 1 - e^{-a_{\lambda 2}L_e} - \left(1 - e^{-a_{\lambda 1}L_e}\right)\left(1 - e^{-a_{\lambda 2}L_e}\right)\Big] e_{\lambda b,g}\, d\lambda$$

$$= \varepsilon_1 + \varepsilon_2 - \frac{1}{\sigma T_g^4}\int_0^\infty \left(1 - e^{-a_{\lambda 1}L_e}\right)\left(1 - e^{-a_{\lambda 2}L_e}\right) e_{\lambda b,g}\, d\lambda$$

$\Delta\varepsilon$ **: spectral overlap**

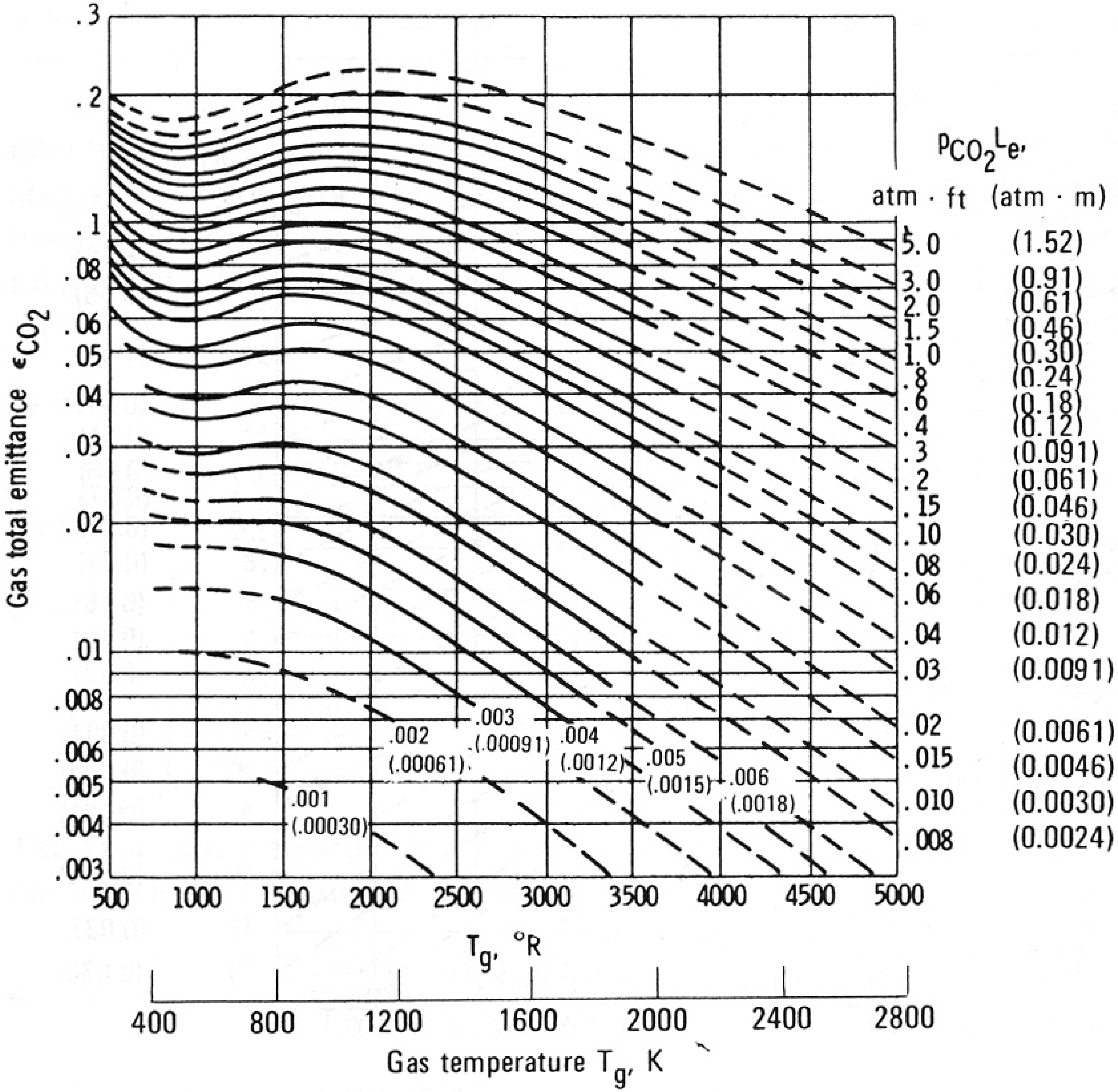


Figure 13-13 Total emittance of carbon dioxide in a mixture having a total pressure P of 1 atm [8].

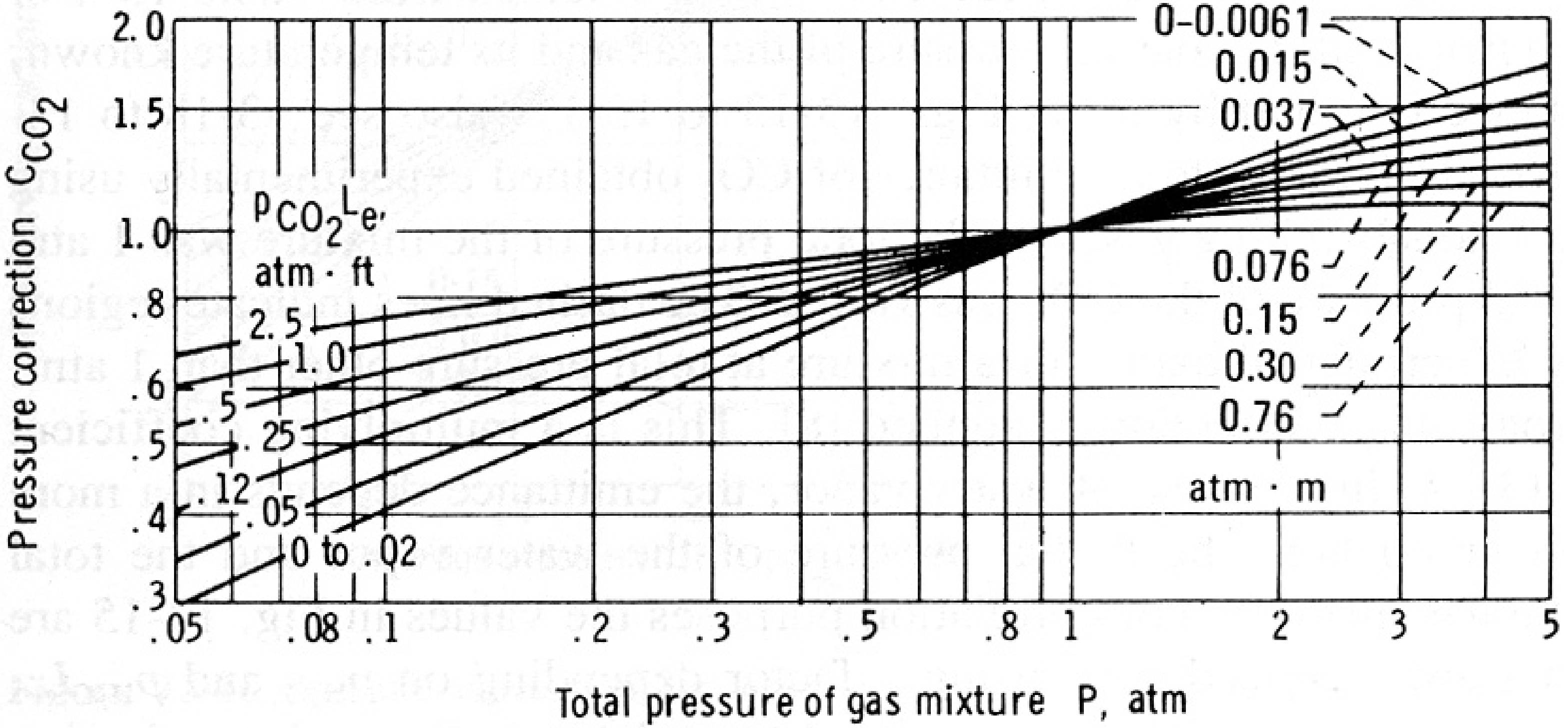


Figure 13-14 Pressure correction for CO_2 total emittance for values of P other than 1 atm [8].

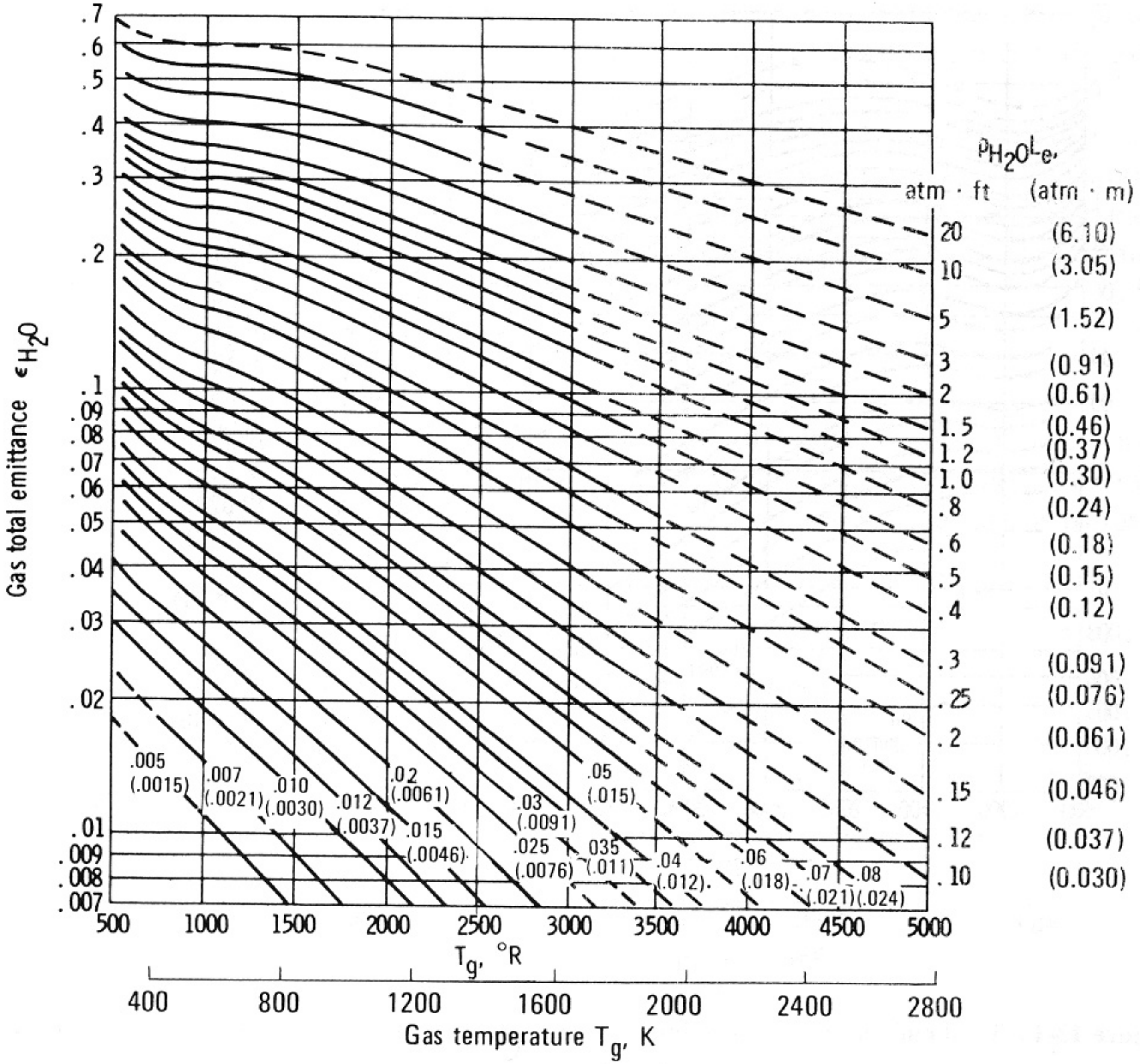


Figure 13-15 Total emittance of water vapor in limit of zero partial pressure in a mixture having a total pressure P of 1 atm [8].

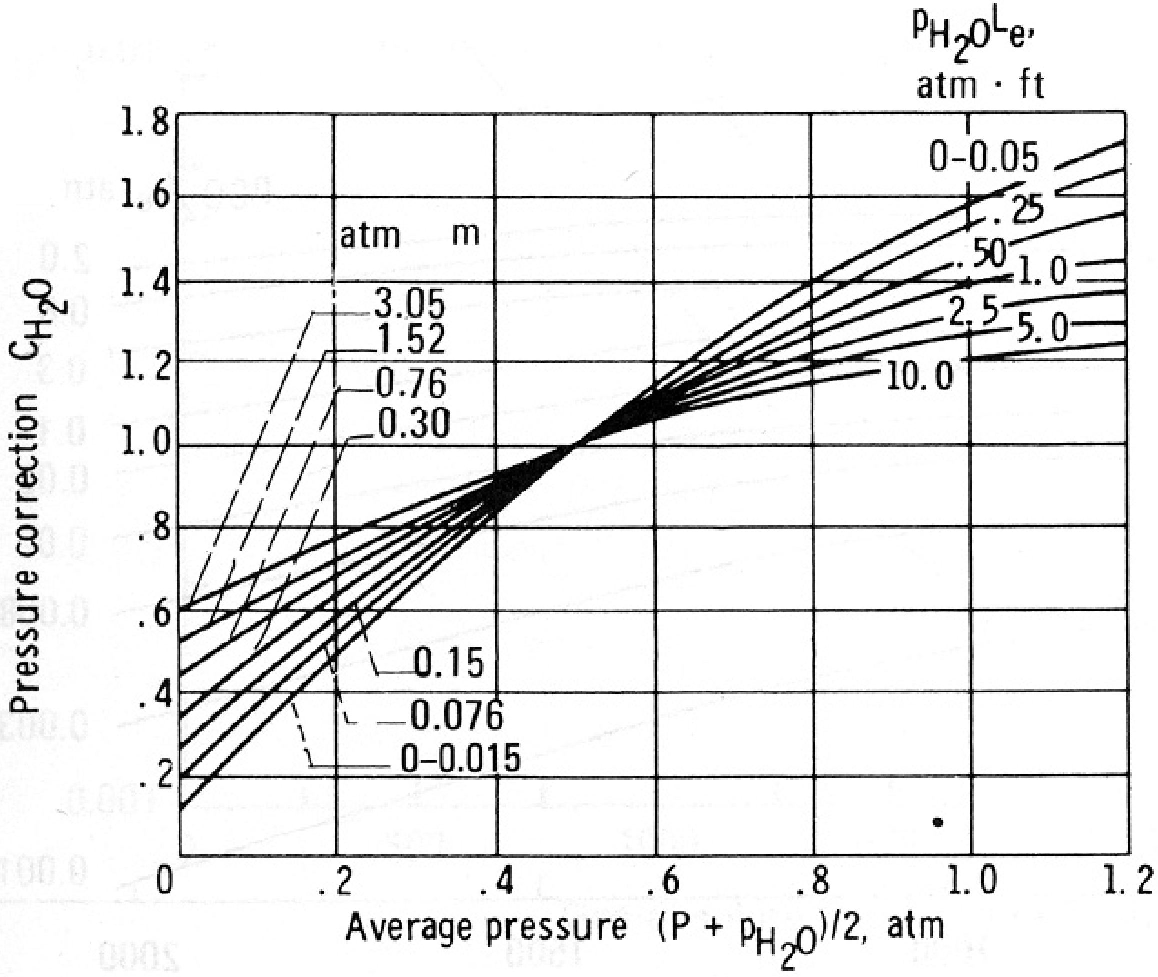
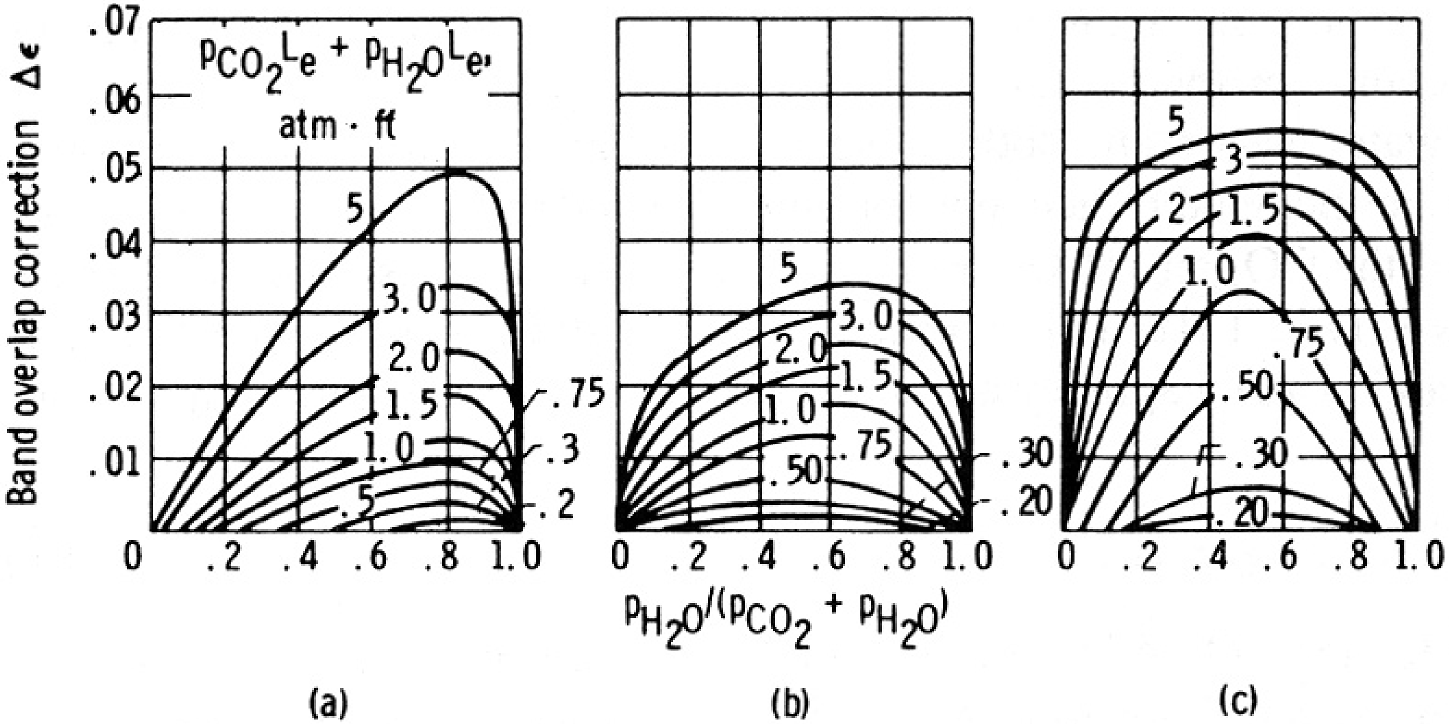


Figure 13-16 Pressure correction for water vapor total emittance for values of partial pressure p_{H_2O} and total pressure P other than 0 and 1 atm, respectively [8].



atm · ft	atm · m
5	1.52
3	0.91
2	0.61
1.5	0.46
1.0	0.30
0.75	0.23
0.50	0.15
0.30	0.091
0.20	0.061

Figure 13-17 Correction on total emittance for band overlap when both CO_2 and water vapor are present [8]. (*a*) Gas temperature T_g = 400 K (720°R); (*b*) gas temperature T_g = 810 K (1460°R); (*c*) gas temperature, $T_g \geq$ 1200 K (2160°R).

Representation of total emittance in an analytical form

Weighted sum of gray gases

Gas is assumed to behave like a mixture of gray gases and a transparent (nonemitting) medium to account for the windows between the absorption bands

$$\varepsilon_g = a_1\left(1 - e^{-k_1 P L_e}\right) + a_2\left(1 - e^{-k_2 P L_e}\right) + \cdots$$

$$= \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} a_i e^{-k_i P L_e}$$

when path length is long $\quad \varepsilon_g \rightarrow \sum_{i=1}^{n} a_i < 1$

$$a_i = a_i(T_g)$$

Let $a_i = b_{1,i} + b_{2,i}\tau + b_{3,i}\tau^2$

$$A_S \equiv \sum_{i=1}^{n} a_i = c_1 + c_2\tau + c_3\tau^2$$

$$\tau = T(\mathrm{K})/1000$$

for CO_2
$$\left.\begin{aligned} &T : 300 \sim 1800\ \mathrm{K} \\ &PL_e : 0.01 \sim 10\ \mathrm{atm \cdot m} \\ &P = 1\ \mathrm{atm} \end{aligned}\right\} \rightarrow \text{Table 13-5}$$

for H_2O
$$\left.\begin{aligned} &T : 300 \sim 700\ K \\ &PL_e : 0.01 \sim 2\ \mathrm{atm \cdot m} \\ &P = 1\ \mathrm{atm} \end{aligned}\right\} \rightarrow \text{Table 13-6}$$

Table 13-5 Sum of gray gases coefficients for CO_2 [22]

	Coefficients of A_s		
	c_1	c_2	c_3
	2.7769×10^{-1}	3.869×10^{-2}	1.4249×10^{-5}

	Coefficients of a_i and k_i $(\text{atm-m})^{-1}$			
i	$b_{1,i}$	$b_{2,i}$	$b_{3,i}$	k_i
1	0.1074	−0.10705	0.072727	0.03647
2	0.027237	0.10127	−0.043773	0.3633
3	0.058438	−0.001208	0.0006558	3.10
4	0.019078	0.037609	−0.015424	14.96
5	0.056993	−0.025412	0.0026167	103.61
6	0.0028014	0.038826	−0.020198	780.7

Exchange between entire gas volume and emitting boundary

$$\frac{q_g}{A} = \frac{q_w}{A} = \sigma\left(\varepsilon_g T_g^4 - \alpha_g(T_w)T_w^4\right)$$ **: black wall**

$$\alpha_g = \alpha_{\mathrm{CO_2}} + \alpha_{\mathrm{H_2O}} - \Delta\alpha$$

$$\alpha_{\mathrm{CO_2}} = C_{\mathrm{CO_2}}\varepsilon^+_{\mathrm{CO_2}}\left(\frac{T_g}{T_w}\right)^{0.5}, \quad \alpha_{\mathrm{H_2O}} = C_{\mathrm{H_2O}}\varepsilon^+_{\mathrm{H_2O}}\left(\frac{T_g}{T_w}\right)^{0.5}$$

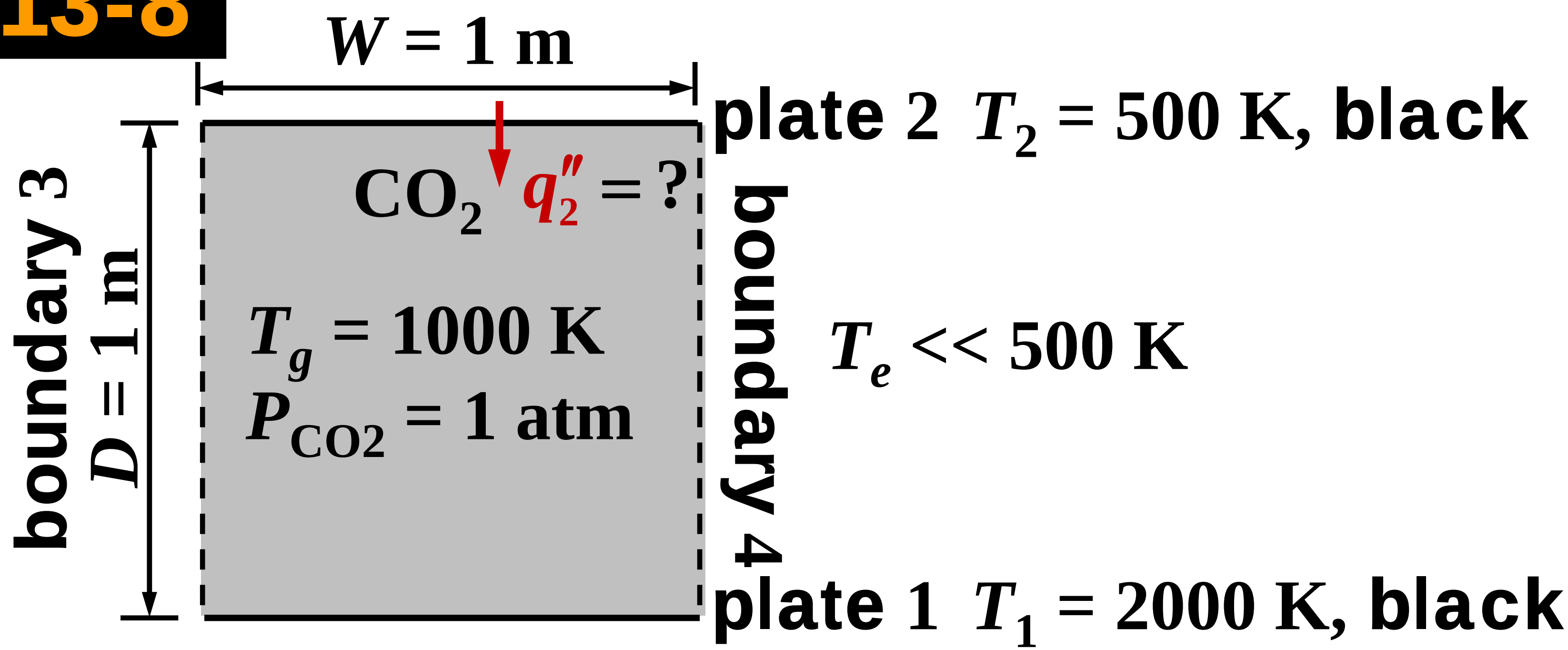
$$\Delta\alpha = (\Delta\varepsilon)_{\text{at } T_w}$$

$\varepsilon^+_{\mathrm{CO_2}}, \varepsilon^+_{\mathrm{H_2O}}$ **: evaluated at** T_w**, and** $L_e' = L_e\left(T_w / T_g\right)$

high pressure and temperature

$$L_e' = L_e\left(T_w / T_g\right)^{3/2}$$

Ex 13-8



$$q''_{\lambda,k} = J_{\lambda,k} - \sum_{j=1}^{n} \left[J_{\lambda,j} F_{k-j} \bar{\tau}_{\lambda,k-j} + e_{\lambda b,g} F_{k-j} \bar{\alpha}_{\lambda,k-j} \right]$$

$$q''_{\lambda,2} = J_{\lambda,2} - \left(J_{\lambda,1} F_{21} \bar{\tau}_{\lambda,21} + e_{\lambda b,g} F_{21} \bar{\alpha}_{\lambda,21} \right)$$

$$- \left(\cancel{J_{\lambda,3}} F_{23} \bar{\tau}_{\lambda,23} + e_{\lambda b,g} F_{23} \bar{\alpha}_{\lambda,23} \right)$$

$$- \left(\cancel{J_{\lambda,4}} F_{24} \bar{\tau}_{\lambda,24} + e_{\lambda b,g} F_{24} \bar{\alpha}_{\lambda,24} \right)$$

$$q''_{\lambda,2} = e_{\lambda b,2} - \left(e_{\lambda b,1} F_{21} \overline{\tau}_{\lambda,21} + e_{\lambda b,g} F_{21} \overline{\alpha}_{\lambda,21}\right)$$

$$- \left(e_{\lambda b,g} F_{23} \overline{\alpha}_{\lambda,23} + e_{\lambda b,g} F_{24} \overline{\alpha}_{\lambda,24}\right)$$

$$= e_{\lambda b,2} - e_{\lambda b,1} F_{21} \overline{\tau}_{\lambda,21} - e_{\lambda b,g} \left(F_{21} \overline{\tau}_{\lambda,21} + F_{23} \overline{\tau}_{\lambda,23} + F_{24} \overline{\tau}_{\lambda,24}\right)$$

$$q''_2 = \int_0^\infty q''_{\lambda,2} d\lambda$$

$$= \sigma T_2^4 - F_{21} \int_0^\infty \overline{\tau}_{\lambda,21} e_{\lambda b,1} d\lambda$$

$$- \int_0^\infty \left(F_{21} \overline{\alpha}_{\lambda,21} + F_{23} \overline{\alpha}_{\lambda,23} + F_{24} \overline{\alpha}_{\lambda,24}\right) e_{\lambda b,g} d\lambda$$

Let total transmittance and absorption factors

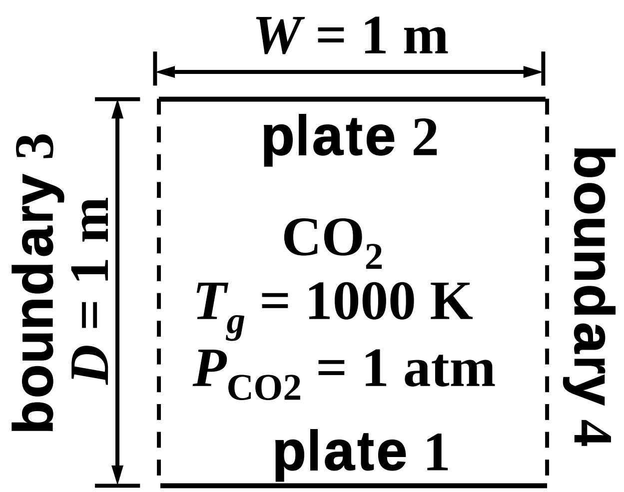
$$\bar{\tau}_{21} = \frac{\int_0^\infty \bar{\tau}_{\lambda,21} e_{\lambda b,1} d\lambda}{\sigma T_1^4}, \quad \bar{\alpha}_{21} = \frac{\int_0^\infty \bar{\alpha}_{\lambda,21} e_{\lambda b,g} d\lambda}{\sigma T_g^4}$$

$$q_2'' = \sigma T_2^4 - F_{21}\bar{\tau}_{21}\sigma T_1^4 - \left(F_{21}\bar{\alpha}_{21} + F_{23}\bar{\alpha}_{23} + F_{24}\bar{\alpha}_{24}\right)\sigma T_g^4$$

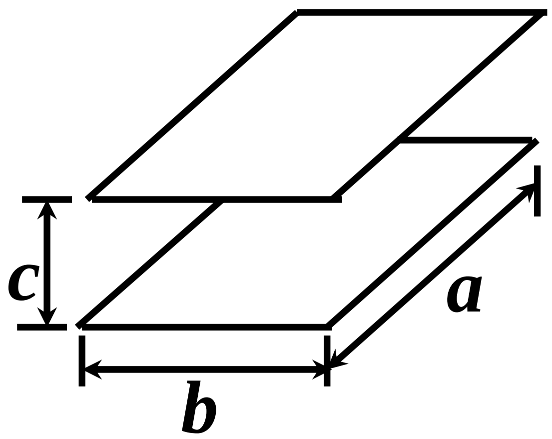
$$= \sigma T_2^4 - F_{21}\bar{\tau}_{21}\sigma T_1^4 - \left(F_{21}\bar{\alpha}_{21} + 2F_{23}\bar{\alpha}_{23}\right)\sigma T_g^4$$

$$F_{21} = \sqrt{2} - 1 = 0.414$$

$$F_{23} = \frac{1 + 1 - \sqrt{2}}{2} = 0.293$$



$\bar{\alpha}_{21}$:



$$\frac{a}{c} \to \infty, \ \frac{b}{c} = 1$$

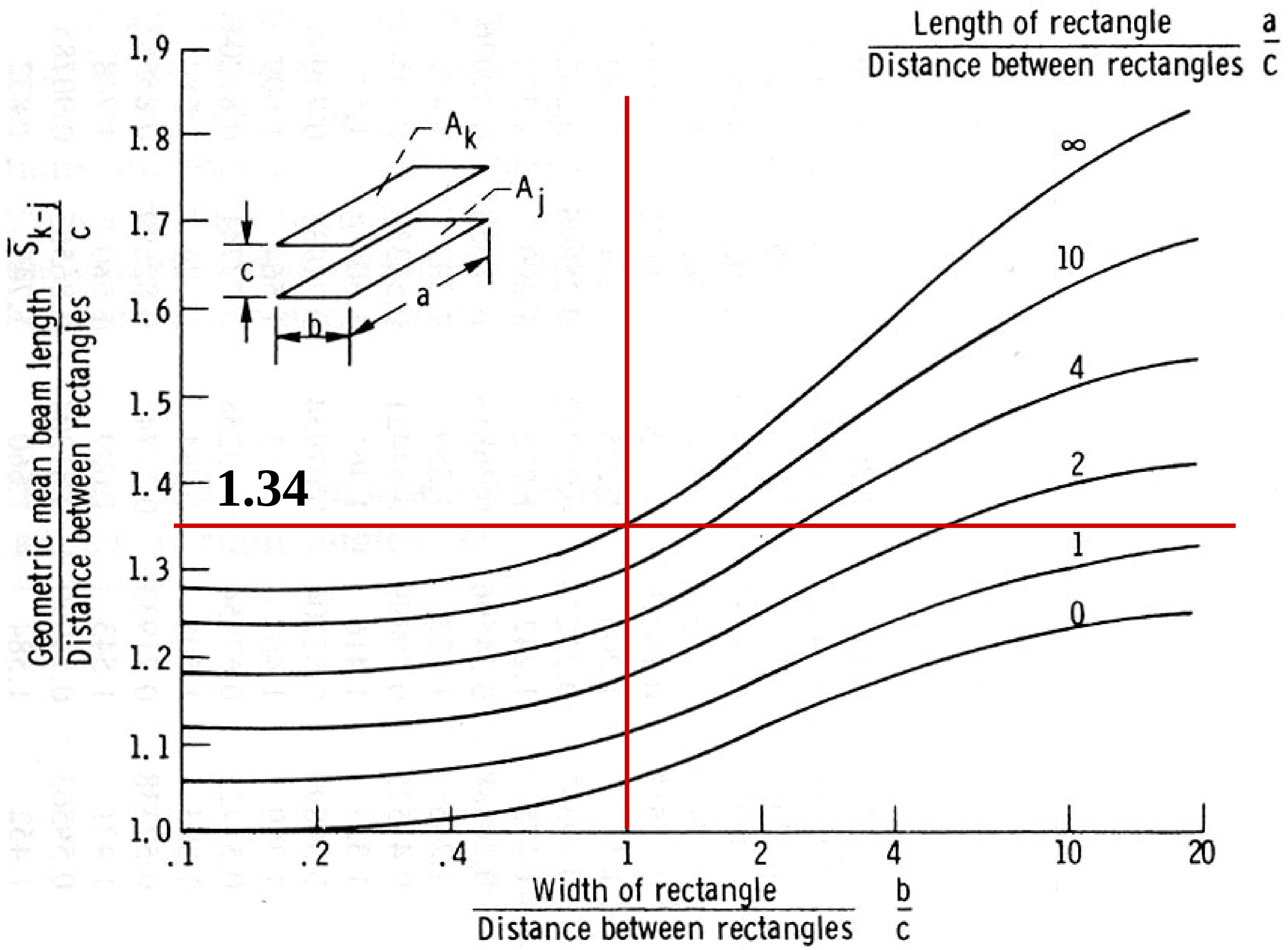


Figure 13-11 Geometric mean beam lengths for equal parallel rectangles [2].

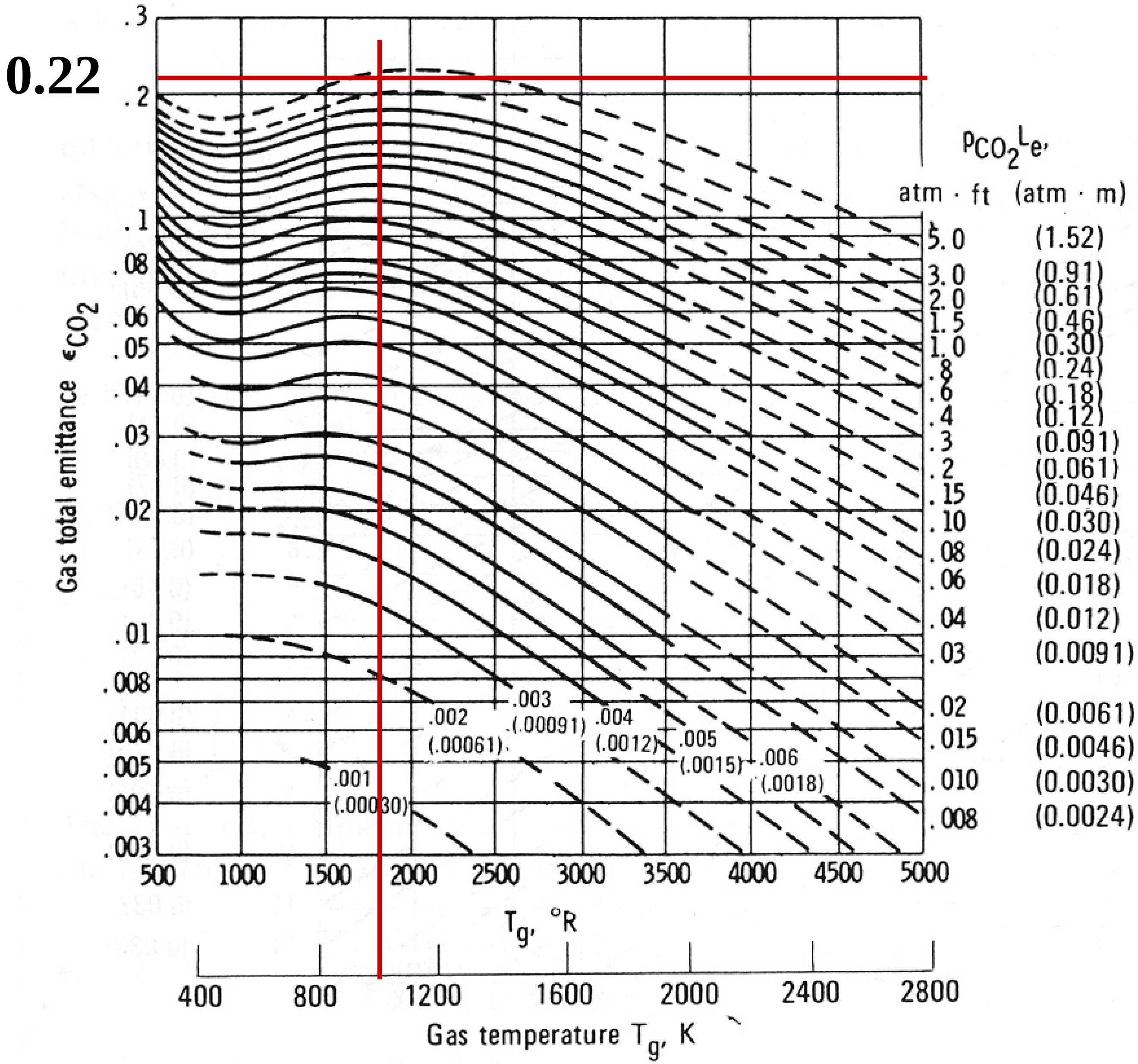


Figure 13-13 Total emittance of carbon dioxide in a mixture having a total pressure P of 1 atm [8].

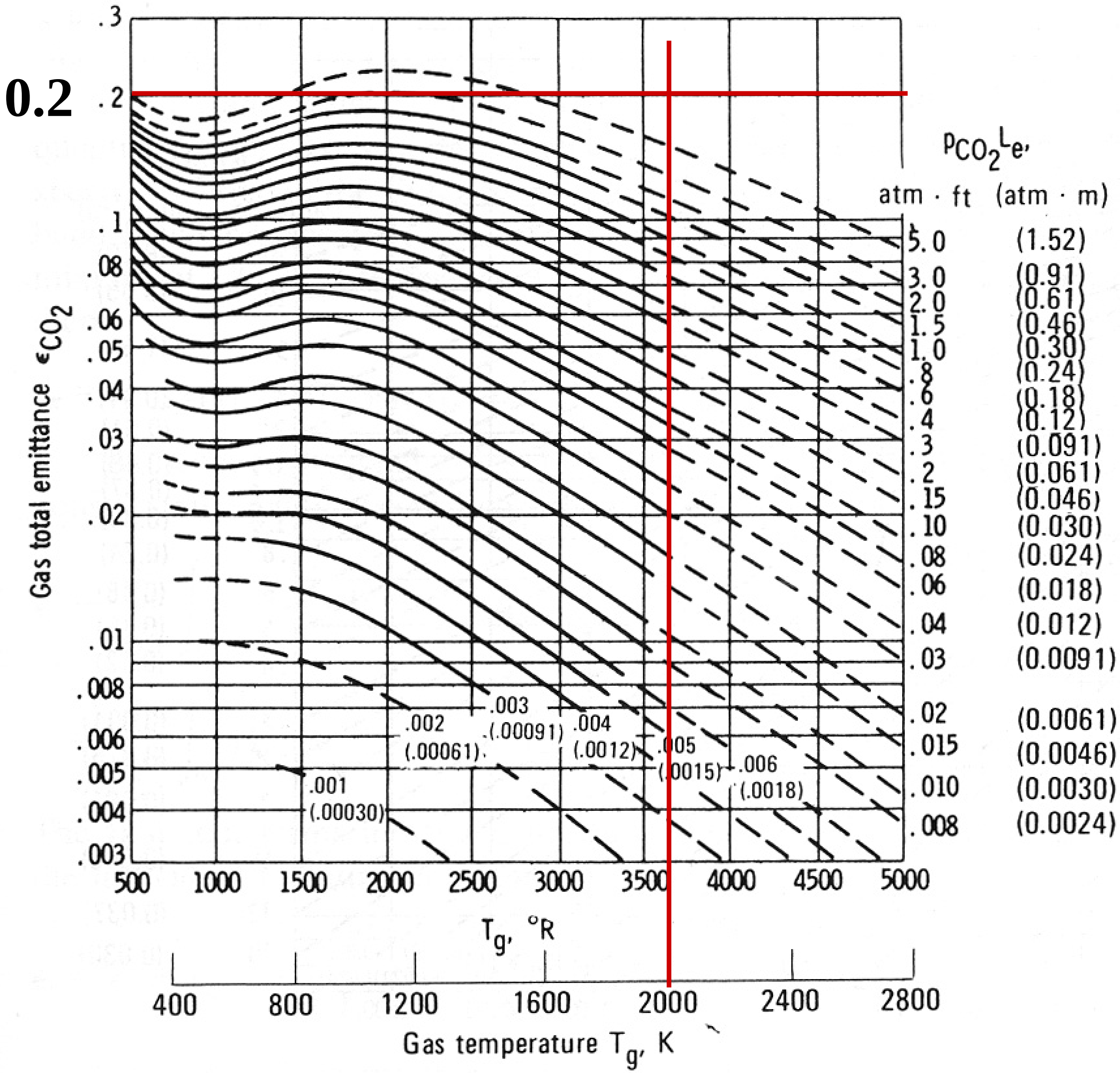


Figure 13-13 Total emittance of carbon dioxide in a mixture having a total pressure *P* of 1 atm [8].

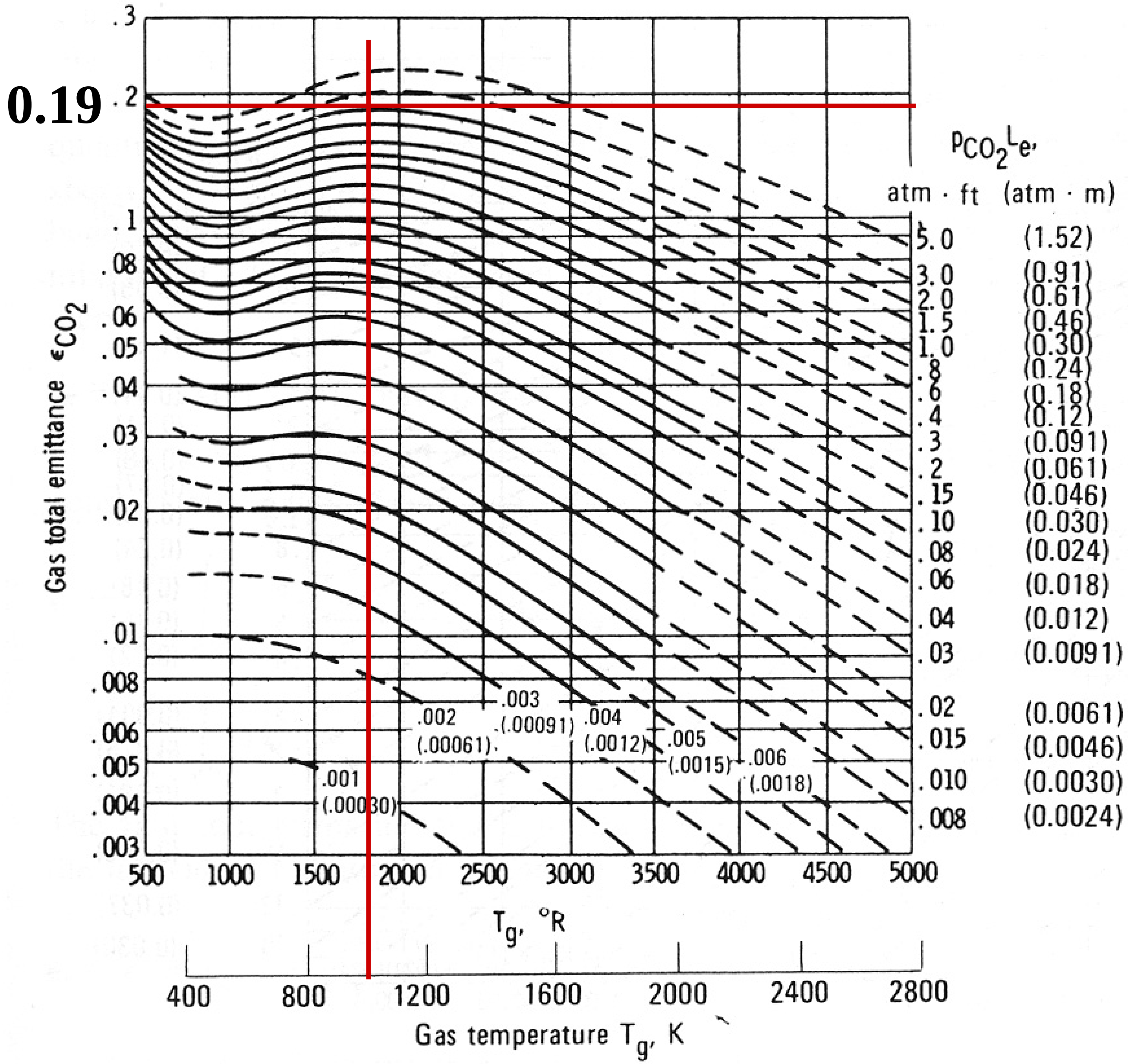
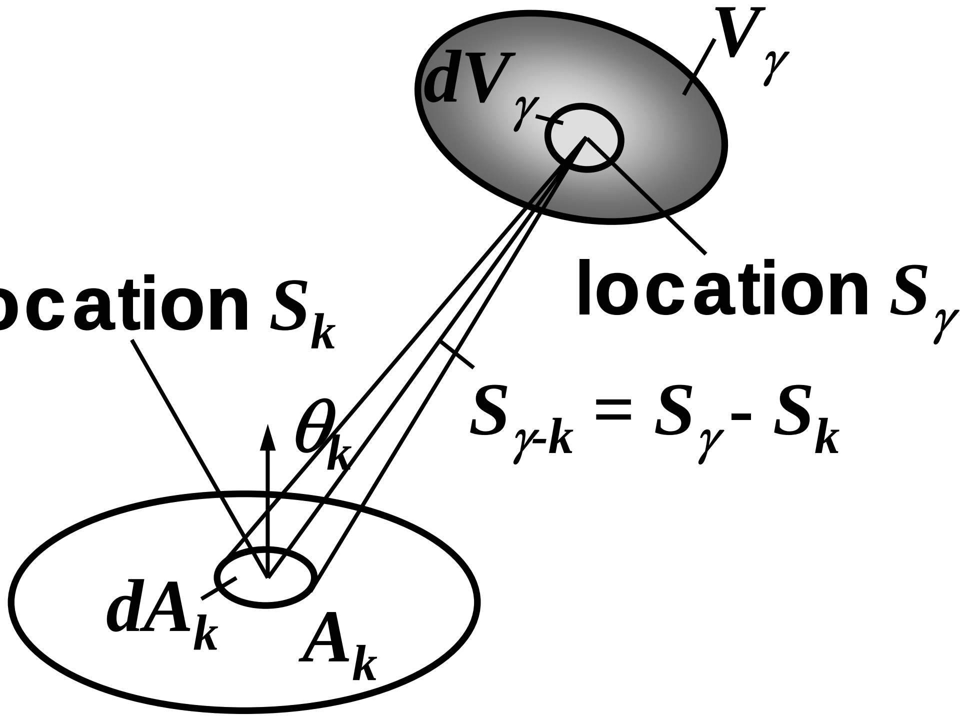


Figure 13-13 Total emittance of carbon dioxide in a mixture having a total pressure P of 1 atm [8].

Zonal Method: Nonisothermal Gases

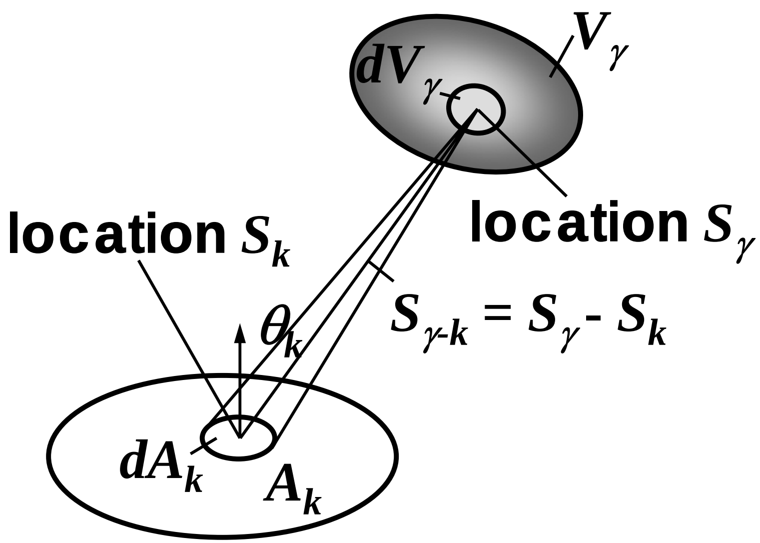
Gas-surface direct exchange area



spectral emissive power per unit volume :

$$4\pi a_\lambda i_{\lambda b,g}$$

per unit solid angle around dV_γ **:** $a_\lambda i_{\lambda b,g}$



solid angle: $\dfrac{dA_k \cos\theta_k}{S_{\gamma-k}^2}$

the fraction transmitted through $S_{\gamma\text{-}k}$

$$\exp\left[-\int_{S_\gamma}^{S_k} a_\lambda(S')dS'\right]$$

spectral energy arriving at A_k from gas volume V_γ :

$$G_{\lambda,\gamma-k}A_k = \int_{V_\gamma}\int_{A_k} \frac{a_\lambda i_{\lambda b,g}\cos\theta_k}{S_{\gamma-k}^2}\exp\left[-\int_{S_\gamma}^{S_k} a_\lambda(S')dS'\right]dA_k\,dV_\gamma$$

assume a_λ uniform, and conditions are uniform over each V_γ :

$$G_{\lambda,\gamma-k}A_k = \int_{V_\gamma}\int_{A_k} \frac{a_\lambda i_{\lambda b,g}\cos\theta_k}{S_{\gamma-k}^2}\exp\left[-\int_{S_\gamma}^{S_k} a_\lambda(S')dS'\right]dA_k dV_\gamma$$

$$G_{\lambda,\gamma-k}A_k = a_\lambda i_{\lambda b,g}\int_{V_\gamma}\int_{A_k} \frac{\cos\theta_k}{S_{\gamma-k}^2}\tau_{\lambda,\gamma-k}dA_k dV_\gamma$$

for gray gas,

$$G_{\gamma-k}A_k = a\frac{\sigma T_\gamma^4}{\pi}\int_{V_\gamma}\int_{A_k} \frac{\cos\theta_k}{S_{\gamma-k}^2}\tau_{\gamma-k}dA_k dV_\gamma$$

gas-surface direct exchange area:

$$\overline{g_\gamma s_k} \equiv \frac{a}{\pi}\int_{V_\gamma}\int_{A_k} \frac{\cos\theta_k}{S_{\gamma-k}^2}\tau_{\gamma-k}dA_k dV_\gamma$$

then, $G_{\gamma-k}A_k = \overline{g_\gamma s_k}\sigma T_\gamma^4$

for Γ **finite regions,**

$$G_k = \frac{1}{A_k}\sum_{\gamma=1}^{\Gamma}\overline{g_\gamma s_k}\sigma T_\gamma^4$$

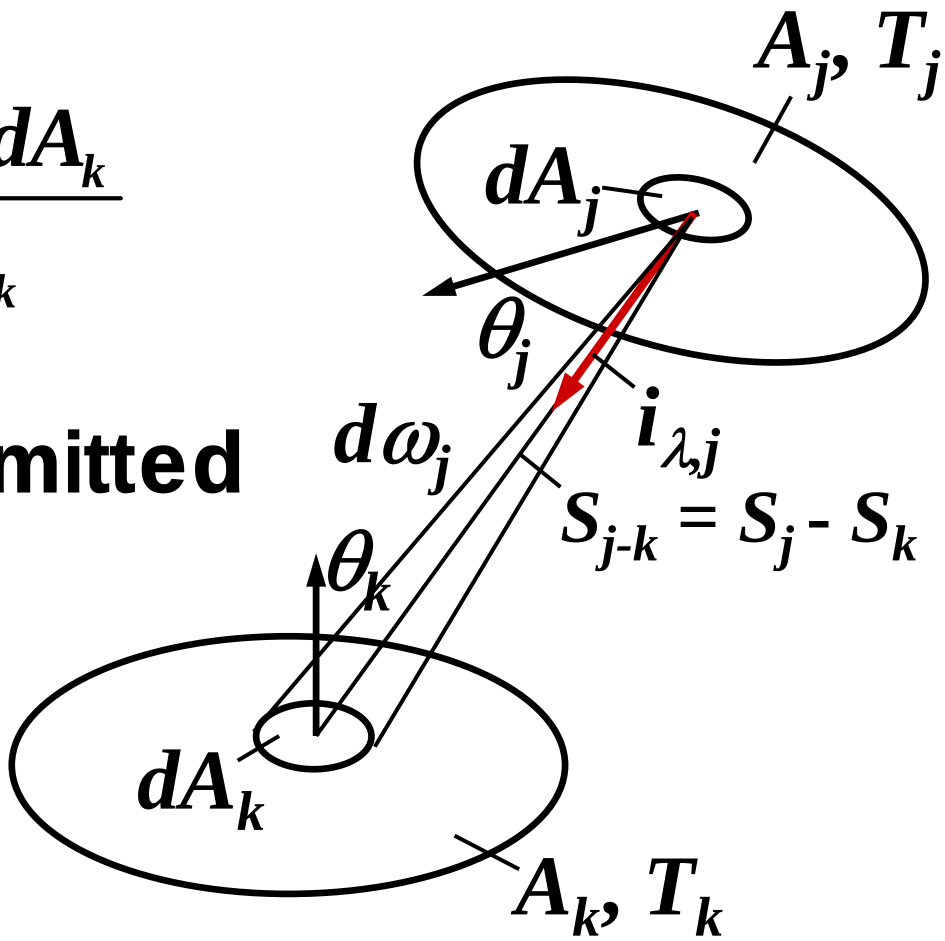
Surface-surface direct exchange area

energy leaving dA_j in θ_j direction

$$i_{\lambda,j}\cos\theta_j dA_j$$

solid angle: $\dfrac{\cos\theta_k dA_k}{S_{j-k}^2}$

the fraction transmitted through S_{j-k}: $\tau_{\lambda,j\text{-}k}$



$$G_{\lambda,j-k}A_k = \int_{A_k}\int_{A_j} \frac{i_{\lambda,j}\cos\theta_j \cos\theta_k}{S_{j-k}^2} \tau_{\lambda,j-k}\, dA_j\, dA_k$$

assume diffuse surface, and uniform radiosity

$$G_{\lambda,j-k}A_k = J_{\lambda,j}\int_{A_k}\int_{A_j} \frac{\cos\theta_j \cos\theta_k}{\pi S_{j-k}^2} \tau_{\lambda,j-k}\, dA_j\, dA_k$$

for gray surface,

$$G_{j-k}A_k = J_j\int_{A_k}\int_{A_j} \frac{\cos\theta_j \cos\theta_k}{\pi S_{j-k}^2} \tau_{j-k}\, dA_j\, dA_k$$

surface-surface direct exchange area:

$$\overline{s_j s_k} \equiv \int_{A_k}\int_{A_j} \frac{\cos\theta_j \cos\theta_k}{\pi S_{j-k}^2} \tau_{j-k}\, dA_j\, dA_k$$

then, $G_k A_k = \overline{s_j s_k} J_j$

for N surfaces, $G_k = \frac{1}{A_k} \sum_{j=1}^{N} \overline{s_j s_k} J_j$

all irradiation:

$$G_k = \frac{1}{A_k} \left(\sum_{j=1}^{N} \overline{s_j s_k} J_j + \sum_{\gamma=1}^{\Gamma} \overline{g_\gamma s_k} \sigma T_\gamma^4 \right)$$

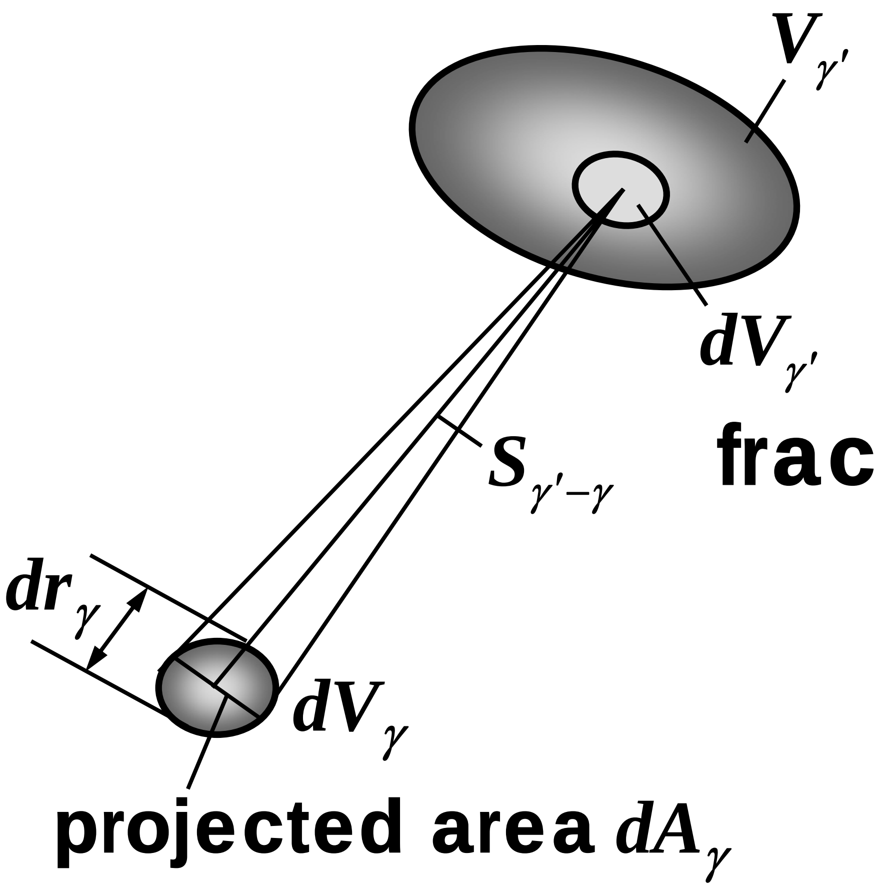
and $q_k'' = J_k - G_k, \; J_k = \varepsilon_k \sigma T_k^4 + (1 - \varepsilon_k) G_k$

when T_g is given for all gas volume, T_k and q_k'' can be determined.

In the case when T_g is unknown

Energy balance on gas volume should be considered.

Gas-gas direct exchange area

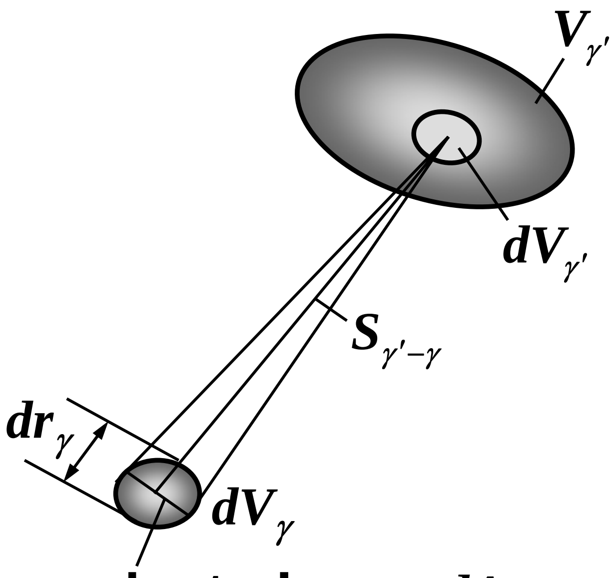


energy emitted by $dV_{\gamma'}$

$$4a\sigma T_{\gamma'}^4 dV_{\gamma'}$$

fraction crosses dA_γ into dV_γ

$$\frac{dA_\gamma}{4\pi S_{\gamma'-\gamma}^2}\tau_{\gamma'-\gamma}$$



fraction absorbed: adr_γ

energy absorbed by dV_γ $(dV_{\gamma'} \to dV_\gamma)$:

$$4a\sigma T_{\gamma'}^4 dV_{\gamma'} \cdot \frac{dA_\gamma}{4\pi S_{\gamma'-\gamma}^2}\tau_{\gamma'-\gamma} \cdot adr_\gamma$$

$$= \frac{a\sigma T_{\gamma'}^4 dV_{\gamma'}}{\pi}\tau_{\gamma'-\gamma}\frac{adV_\gamma}{S_{\gamma'-\gamma}^2}$$

total energy absorbed by V_γ $(V_{\gamma'} \to V_\gamma)$:

$$\int_{V_\gamma}\int_{V_{\gamma'}}\frac{a^2}{\pi}\sigma T_{\gamma'}^4 \tau_{\gamma'-\gamma}\frac{dV_{\gamma'}dV_\gamma}{S_{\gamma'-\gamma}^2}$$

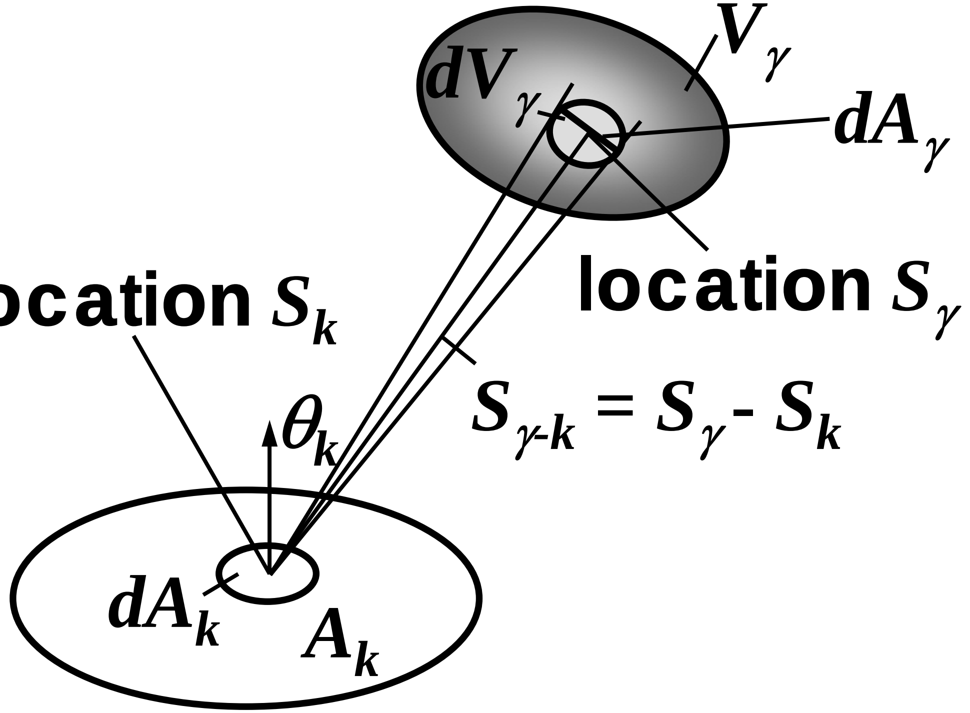
gas-gas direct exchange area:

$$\overline{g_{\gamma'} g_\gamma} \equiv \frac{a^2}{\pi} \int_{V_\gamma} \int_{V_{\gamma'}} \tau_{\gamma'-\gamma} \frac{dV_{\gamma'} dV_\gamma}{S^2_{\gamma'-\gamma}}$$

total energy absorbed:

$$\int_{V_\gamma} \int_{V_{\gamma'}} \frac{a^2}{\pi} \sigma T^4_{\gamma'} \tau_{\gamma'-\gamma} \frac{dV_{\gamma'} dV_\gamma}{S^2_{\gamma'-\gamma}} = \overline{g_{\gamma'} g_\gamma} \sigma T^4_{\gamma'}$$

Surface-gas direct exchange area



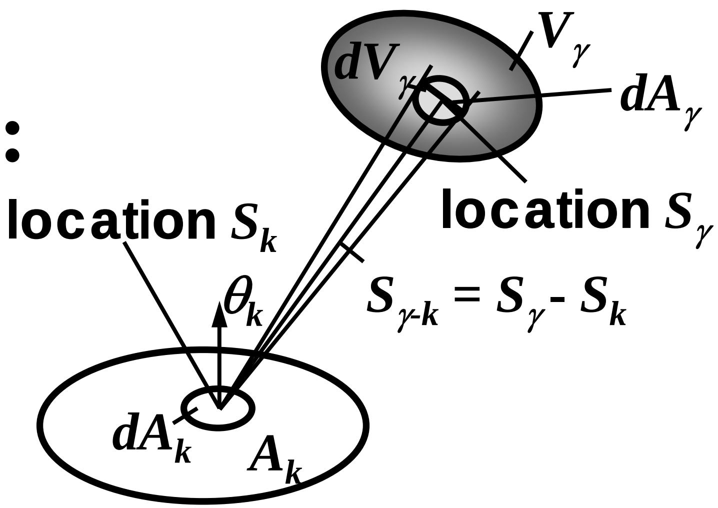
energy leaving dA_k per unit solid angle:

$$\frac{J_k}{\pi} dA_k \cos\theta_k$$

fraction crosses dA_k into dA_γ: $\dfrac{dA_\gamma}{S_{k-\gamma}^2}\tau_{k-\gamma}$

fraction absorbed: adr_γ

energy absorbed $dA_k \rightarrow dV_\gamma$ **:**



$$\frac{J_k}{\pi} dA_k \cos\theta_k \cdot \frac{dA_\gamma}{S_{k-\gamma}^2} \tau_{k-\gamma} \cdot adr_\gamma$$

$$= \frac{J_k \cos\theta_k}{\pi} dA_k \tau_{k-\gamma} \frac{adV_\gamma}{S_{k-\gamma}^2}$$

total energy absorbed $A_k \rightarrow V_\gamma$ **:**

$$\int_{V_\gamma} \int_{A_k} \frac{J_k \cos\theta_k}{\pi} dA_k \tau_{k-\gamma} \frac{adV_\gamma}{S_{k-\gamma}^2}$$

surface-gas direct exchange area

$$\overline{s_k g_\gamma} = \frac{a}{\pi} \int_{V_\gamma} \int_{A_k} \frac{\cos\theta_k}{S_{k-\gamma}^2} \tau_{k-\gamma} dA_k dV_\gamma$$

reciprocity: $\overline{s_k g_\gamma} = \overline{g_\gamma s_k}$

total energy absorbed

$$\int_{V_\gamma} \int_{A_k} \frac{J_k \cos\theta_k}{\pi} dA_k \tau_{k-\gamma} \frac{a dV_\gamma}{S_{k-\gamma}^2} = J_k \overline{g_\gamma s_k}$$

energy balance:

$$4a\sigma T_\gamma^4 V_\gamma = \sum_{\gamma'=1}^{\Gamma} \sigma T_{\gamma'}^4 \overline{g_{\gamma'} g_\gamma} + \sum_{k=1}^{N} J_k \overline{g_\gamma s_k}, \quad \gamma = 1, 2, 3, \cdots \Gamma$$