457.562 Special Issue on River Mechanics (Sediment Transport) .15 Morphodynamics and turbidity currents

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Equations governing the Morphodynamics of Rivers

- Morphodynamics
  - Rivers evolve over time in accordance with the interaction between the flow and sediment-transport fields over an ero dible bed and the changing morphology of the bed.
  - This co-evolution is termed as "morphodynamics"
- The relations governing the morphodynamics of turbidity currents

$$Uh = \int_0^h \bar{u}\, dz$$

WHERE U IS DEPTH-AVERAGED FLOW VELOCITY

h is the flow depth,

η is bed elevation

z is upward normal coordinate with its origina at the bed

$\bar{u}$ is local streamwise flow velocity averaged over turbulence at z

Equations governing the Morphodynamics of Rivers

- Simplified one dimensional St. Venant shallow-water equations.

$$\frac{\partial h}{\partial t}+\frac{\partial Uh}{\partial x}=0$$

$$\frac{\partial Uh}{\partial t}+\frac{\partial U^2 h}{\partial x}=-gh\frac{\partial h}{\partial x}+ghS-C_f U^2$$

- The river is assumed to carry a dilute suspension of sediment. Let the depth-flux averaged volume concentration C of suspended sediment be given by the relation

$$q_s=UCh=\int_0^h \overline{uc}\,dz$$

- An approximate form for depth-averaged conservation of suspended sediment is

$$\frac{\partial Ch}{\partial t}+\frac{\partial UCh}{\partial x}=v_s\left(E_s-\overline{c}_b\right)$$

Equations governing the Morphodynamics of Rivers

- For the bedform, the Exner equation of conservation of bed se diment

$$\left(1-\lambda_p\right)\frac{\partial \eta}{\partial t}=-\frac{\partial q_b}{\partial x}+v_s\left(\bar{c}_b-E_s\right)$$

- The set of the above equations is the basic equations for the one-dimensional morphodynamics.
- We need to solve (close), (1) volume bed load transport, (2) di mensionless rate of entrainment of bed sediment into suspens ion, (3) near-bed suspended-sediment concentration.

Equations governing the Morphodynamics of Rivers

- If the flow and suspension do not deviate too strongly from eq uilibrium (logarithmic and Rousean profiles)

$$\bar{c}_b = r_0 C$$

$$r_0 = \frac{\ln\left(11\frac{h}{k_c}\right)}{\int_{\zeta_b}^{1}\left[\frac{(1-\zeta)/\zeta}{(1-\zeta_b)/\zeta_b}\right]^{Z_R}\ln\left(11\frac{h}{k_c}\zeta\right)d\zeta}$$

$$Z_R = \frac{v_s}{\kappa u_*}, \qquad \zeta = \frac{z}{h}$$

- Where $\zeta \ll 1$ is a near-bed parameter. When $\zeta_b = b/h = 0.05$ which is the commonly used value for this parameter, the shape factor r_0 can be from Rousean $r_0 = 1 + 31.5\left(\frac{u_*}{v_s}\right)^{-1.46}$

Equations governing the Morphodynamics of Rivers

- The equations have forms

$$\frac{\partial Ch}{\partial t}+\frac{\partial UCh}{\partial x}=v_s\left(E_s-r_0C\right)$$

$$\left(1-\lambda_p\right)\frac{\partial \eta}{\partial t}=-\frac{\partial q_b}{\partial x}+v_s\left(r_0C-E_s\right)$$

- With some modifications

$$\left(1-\lambda_p\right)\frac{\partial \eta}{\partial t}+\frac{\partial Ch}{\partial t}=-\frac{\partial q_b}{\partial x}-\frac{\partial q_s}{\partial x}$$

- For the case of a dilute suspension, the amount of sediment s tored in suspension can be be neglected compared to the am ount of sediment stored in the bed, so that the above equation ,

$$\left(1-\lambda_p\right)\frac{\partial \eta}{\partial t}=-\frac{\partial q_b}{\partial x}-\frac{\partial q_s}{\partial x}=-\frac{\partial q_t}{\partial x}$$

Equations governing the Morphodynamics of Rivers

- The previous st Vennant and exner's equations are simplified version. The more general forms are

$$\frac{\partial Uh}{\partial t}+\frac{\partial \alpha_1 U^2 h}{\partial x}=-gh\frac{\partial h}{\partial x}+ghS-C_f U^2$$

$$\alpha_1=\int_0^1\left(\frac{\overline{u}}{U}\right)^2 d\zeta$$

$$\frac{\partial \alpha_2 Ch}{\partial t}+\frac{\partial UCh}{\partial x}=v_s\left(E_s-r_0 C\right)$$

$$\alpha_2=\int_0^1\frac{\overline{c}}{C}d\zeta=\frac{\int_0^1\overline{u}\,d\zeta\int_0^1\overline{c}\,d\zeta}{\int_0^1\overline{uc}\,d\zeta}$$

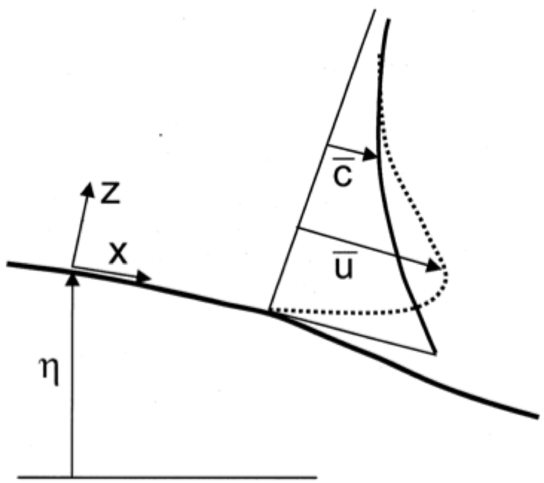
α_1 and α_2 are dimensionless shape factors governing the velocity and concentration profiels. (almost unity)

Equations governing Turbidity currents

- The turbidity current flows because it is laden with sediment, which renders it heavier that the ambient water above.

$$\rho_t = \rho\left(1-\overline{c}\right) + \rho_s \overline{c} = \rho\left(1+R\overline{c}\right)$$

$$R = \frac{\rho_s}{\rho} - 1$$



- One dimensional layer-averaged equations governing the flow of turbidity currents can be derived in analogy to the St. Venant equations of shallow water flow in rivers.

$$Uh = \int_0^\infty \overline{u}dz \qquad U^2h = \int_0^\infty \overline{u}^2dz \qquad UCh = \int_0^\infty \overline{uc}dz$$

- Main difference between equations of river and turbidity curr ent in boundary condition (∞). It means that there is no “laye r thickness”

Equations governing Turbidity currents

- The one-dimensional equations contain a number of dimensionless shaper factors analogous to alpha's in the previous cases, all of which take the value unity for the special case of the velocity and concentration profiles

$$\frac{\bar{u}}{U} = \frac{\bar{c}}{C} = \begin{cases} 1 & : \quad 0 < z \leq h \\ 0 & : \quad z > z \end{cases}$$

- The one-dimensional layer-averaged equation of conservation of water mass takes the form

$$\frac{\partial h}{\partial t} + \frac{\partial Uh}{\partial x} = e_w U$$

$e_w =$ Coefficient of entrainment of ambient water from above into the turbidity current.

Equations governing Turbidity currents

- The one-dimensional layer-averaged equation of conservation of streamwise flow momentum is

$$\frac{\partial Uh}{\partial t}+\frac{\partial U^2 h}{\partial x}=-\frac{1}{2}g\frac{\partial Ch^2}{\partial x}+gRChS-C_f U^2$$

- This equation is same to the form as for a river.
- Important difference
  - A river that carries no suspended sediment continues to flow, because the pull of gravity acts directly on the water. This effect is *ghS*.
  - In the case of a turbidity current, the corresponding downslope impelling term is *gRChS*. If C drops to zero, then no flow.

Equations governing Turbidity currents

- Parker et al (1987) suggested the entrainment coefficient as

$$e_w = \frac{0.075}{\left(1 + 718 \times Ri^{0.24}\right)^{0.5}} \qquad Ri = \frac{RgCh}{U^2}$$

- The bulk Richardson number is related to the densimetric Froude number Fr_d as

$$Fr_d = \frac{U}{gRCh} = Ri^{-1/2}$$

- Fr > 1, supercritical, to entrain ambient water from above.
- Fr < 1, subcritical flow, if really small Fr then no entrainment.