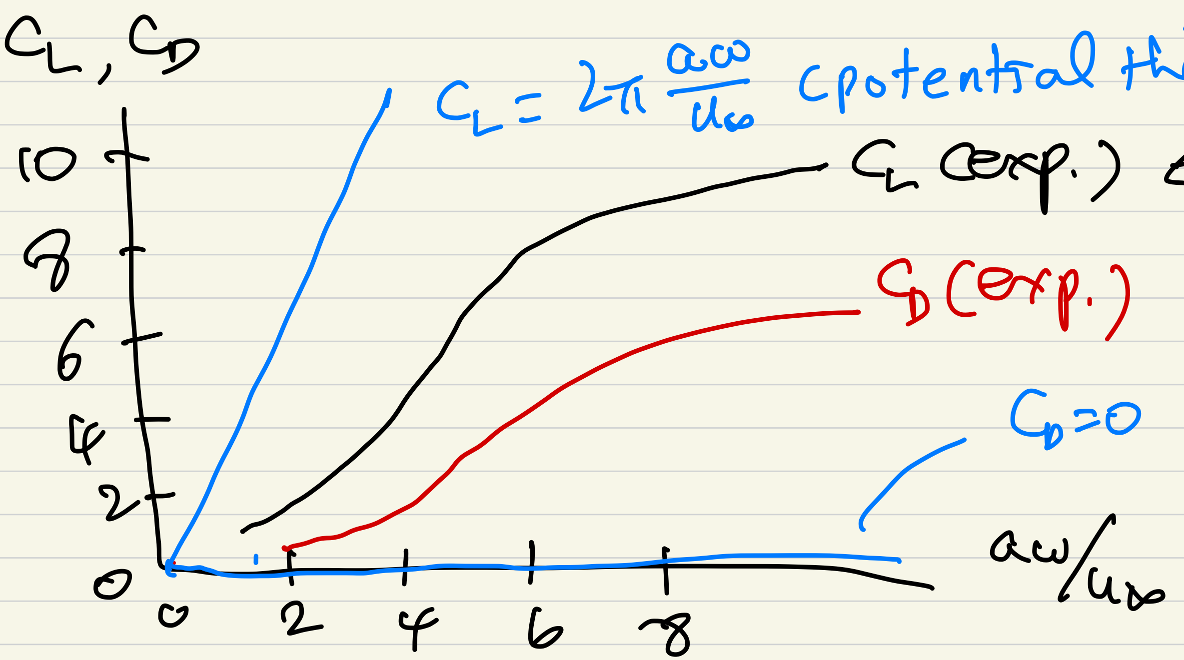
• Kutta-Joukowski lift theorem

$$L/b = -\rho u_\infty \Gamma \qquad L:\ \text{lift force}$$

$$\Gamma = \oint \underline{u} \cdot d\underline{l} = a\omega \cdot 2\pi a$$

$$L/b = \rho u_\infty \cdot a\omega \cdot 2\pi a$$

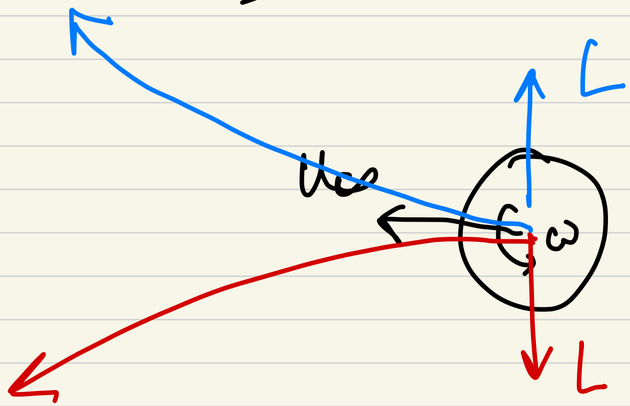
$$C_L = \frac{L}{\frac{1}{2}\rho u_\infty^2 \cdot (2ab)} = \frac{\cancel{\rho} \cancel{u_\infty} \cancel{a}\omega \cdot 2\pi a}{\cancel{\frac{1}{2}}\cancel{\rho} u_\infty^{\cancel{2}} \cancel{2a}} = \frac{2\pi a\omega}{u_\infty}$$



← This C_L is higher than that of typical airfoil w/ same chord length

L/D ↑

rotating ball has more drag than straight ball.

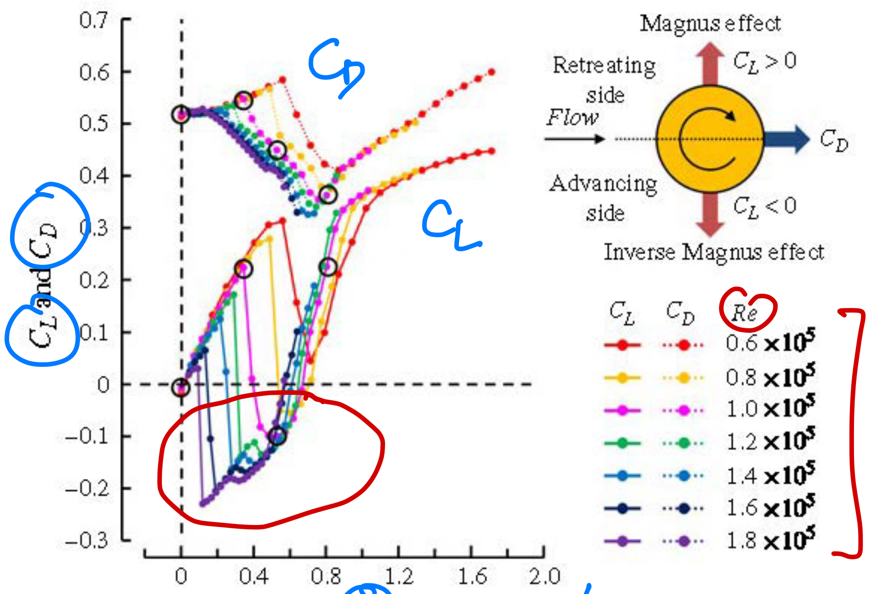


L : inverse Magnus effect

L : Magnus effect

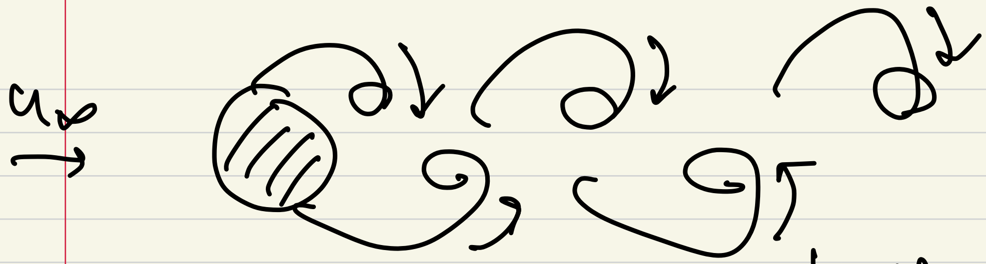
Inverse Magnus effect on a rotating sphere: when and why

Kim & Choi (JFM)

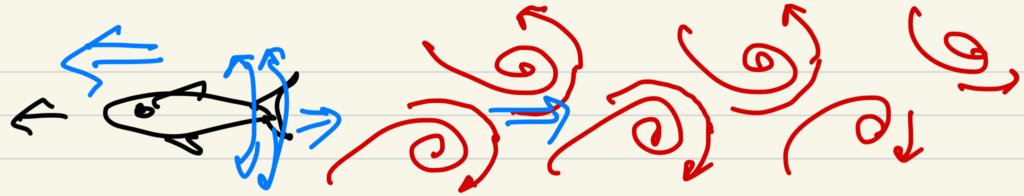


$\alpha = a\omega / u_\infty$

FIGURE 2. Variations of the lift and drag coefficients with the spin ratio at $Re = 0.6 \times 10^5$–1.8×10^5. The lift coefficient is defined to be positive when the lift force is exerted from the advancing to the retreating side (i.e. the Magnus effect occurs) and vice versa. It should be noted that the eight circles on the left of the figure denote the cases investigated in figure 4(a–d).

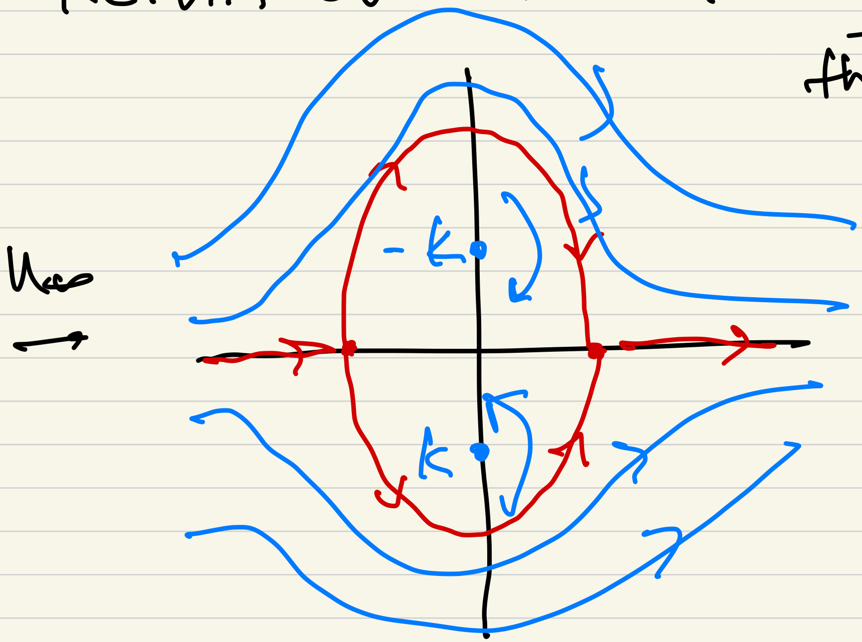


alternating Karman vortex shedding

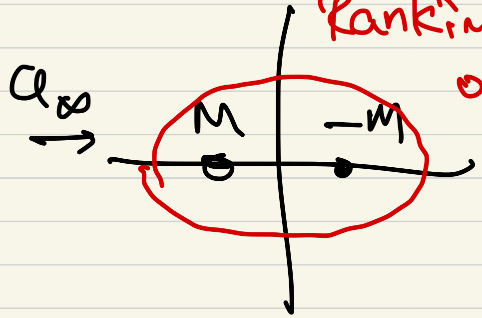


inverse Karman vortex shedding

- Kelvin oval : a family of body shapes taller than they are wide

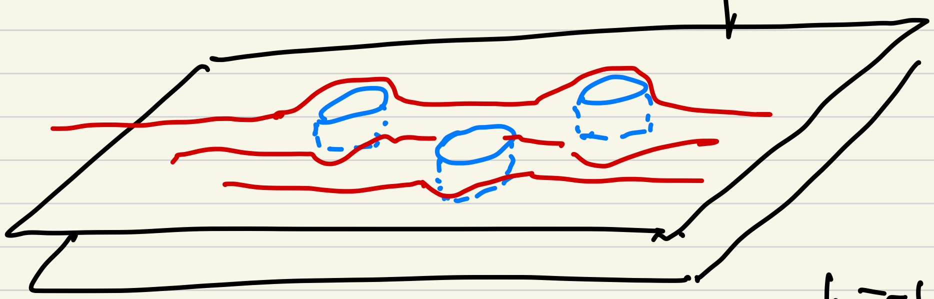


uniform stream + vortex pair



Rankine oval

- Potential-flow analogs



$h \ll O(1)$

↳ $Re \ll 1$

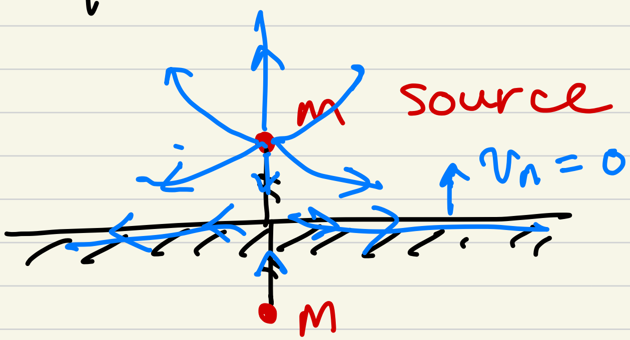
highly viscous flow → no flow separation

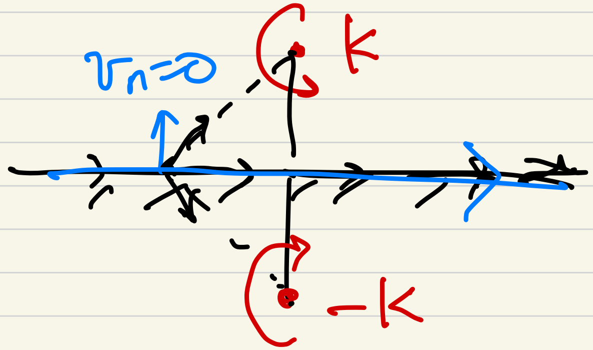
looks similar to potential flow

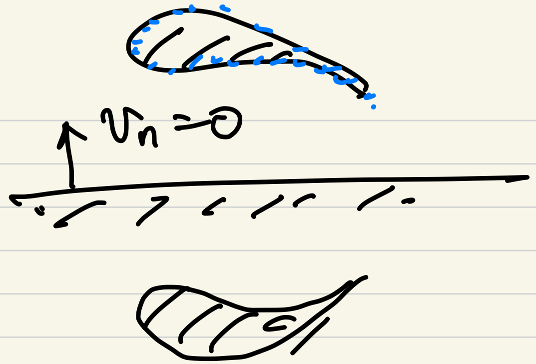
Hele-shaw flow

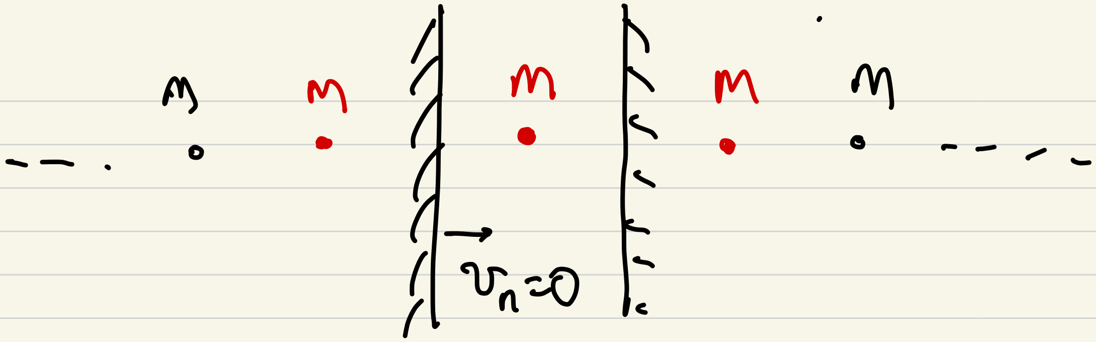
8.5 skip

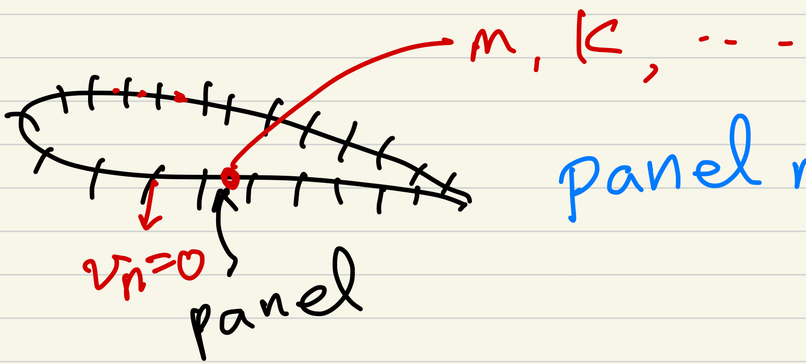
8.6 Images or Image method











panel method

8.7 Airfoil theory

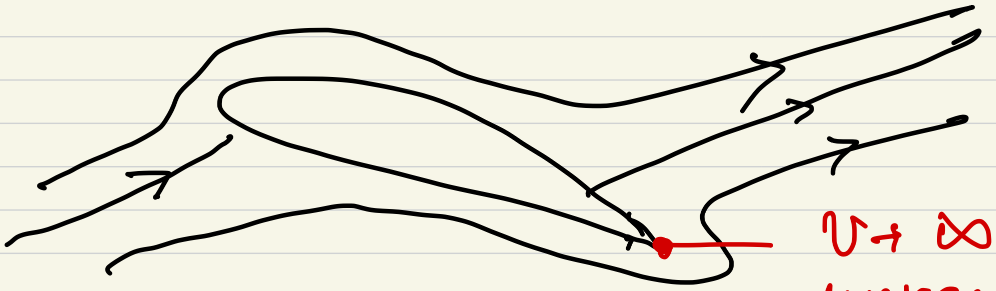
$$L/b = -\rho U_\infty \Gamma$$

How to determine Γ?

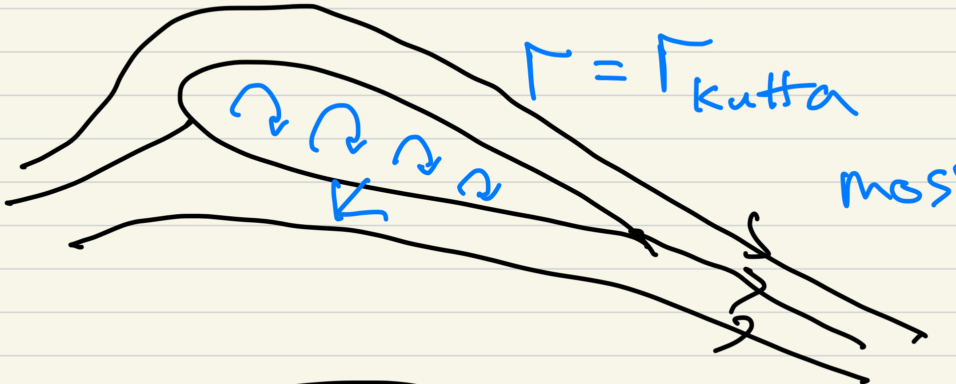
airfoil shape

angle of attack α

Kutta condition

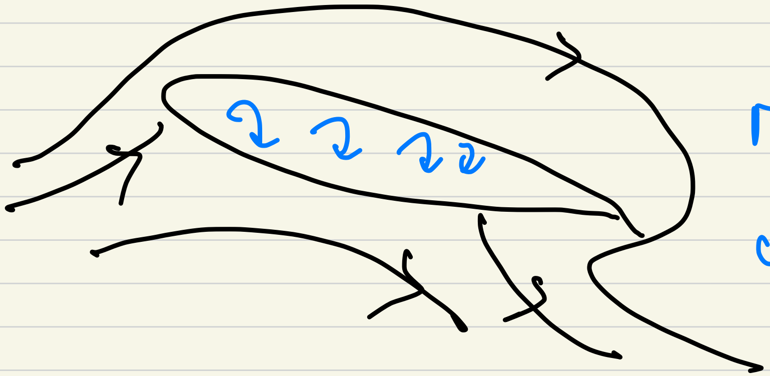


$v \to \infty$
unrealistic



$\Gamma = \Gamma_{kutta}$

most realistic

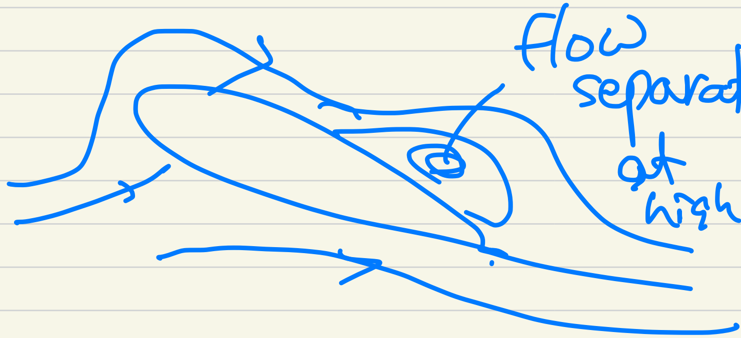


$\Gamma > \Gamma_{kutta}$

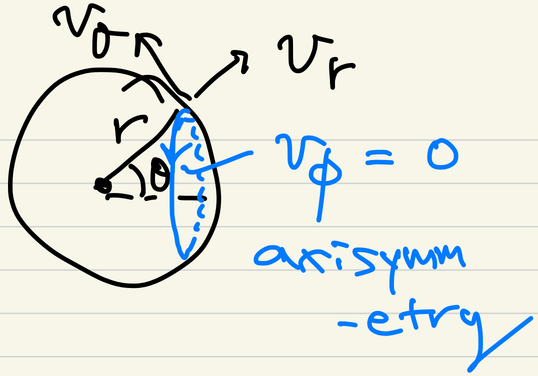
unrealistic

Kutta condition: velocity difference vanishes at the trailing edge.

real flow



8.8 Axisymmetric potential flow



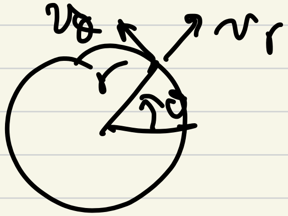
$\nabla \cdot \underline{V} = 0 \; : \; \frac{\partial}{\partial r}(r^2 v_r \sin\theta) + \frac{\partial}{\partial \theta}(r v_\theta \sin\theta) = 0$

$\rightarrow v_r = -\frac{1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta} \qquad , \qquad v_\theta = \frac{1}{r\sin\theta}\frac{\partial \psi}{\partial r}$

$\qquad = \frac{\partial \phi}{\partial r} \qquad \qquad \qquad \qquad = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$

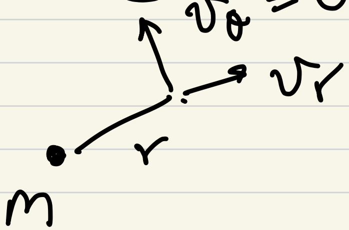
- uniform stream

$U_\infty \longrightarrow$



$$\left.\begin{aligned} v_r &= U_\infty \cos\theta \\ v_\theta &= -U_\infty \sin\theta \end{aligned}\right\} \rightarrow \begin{aligned} \psi &= -\frac{1}{2} U_\infty r^2 \sin\theta \\ \phi &= U_\infty r \cos\theta \end{aligned}$$

- Point source

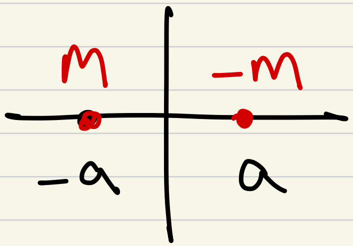


$Q = v_r \cdot 4\pi r^2$

$(m = \frac{Q}{4\pi}$: source strength$)$

$$\left.\begin{aligned} \rightarrow v_r &= \frac{Q}{4\pi r^2} = \frac{m}{r^2} \\ v_\theta &= 0 \end{aligned}\right] \qquad \begin{aligned} \psi &= m\cos\theta \\ \phi &= -\frac{m}{r} \end{aligned}$$

- Point doublet

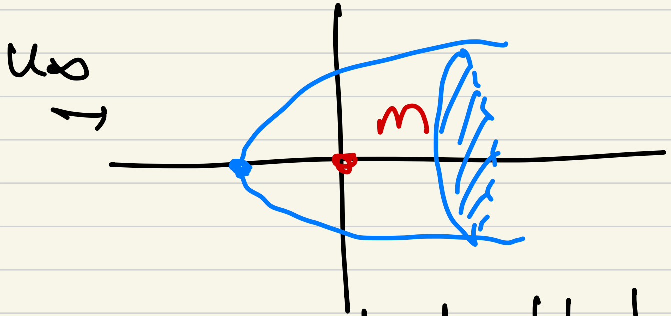


$$\left.\begin{array}{l} a \to 0 \\ 2am = \lambda \end{array}\right\} \quad \psi = \frac{\lambda \sin^2\theta}{r}, \qquad \phi = \frac{\lambda \cos\theta}{r^2}$$

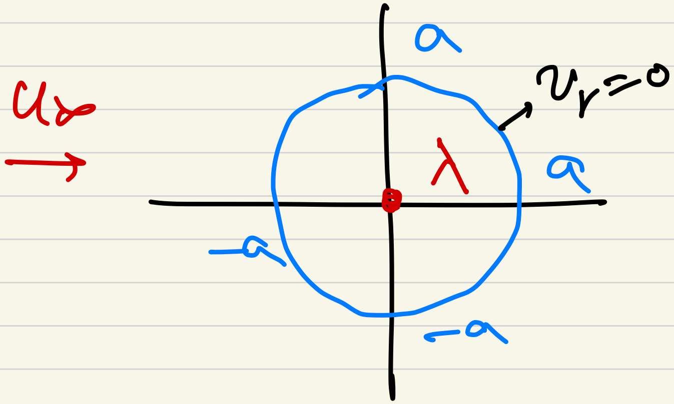
- Rankine half body of revolution

uniform stream

+ point source



- Sphere : unif. stream + point doublet



$$\psi = -\frac{1}{2}U_\infty r^2 \sin^2\theta + \frac{\lambda}{r}\sin^2\theta$$

$$v_r = -\frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta} = \cdots = \frac{2}{r^2}\underline{\left(\frac{1}{2}U_\infty r^2 - \frac{\lambda}{r}\right)}\cos\theta$$

@ $r = a$, $v_r = 0$ $\to$ $\frac{1}{2}U_\infty a^2 - \frac{\lambda}{a} = 0$ $\to$ $a = \left(\frac{2\lambda}{U_\infty}\right)^{\frac{1}{3}}$

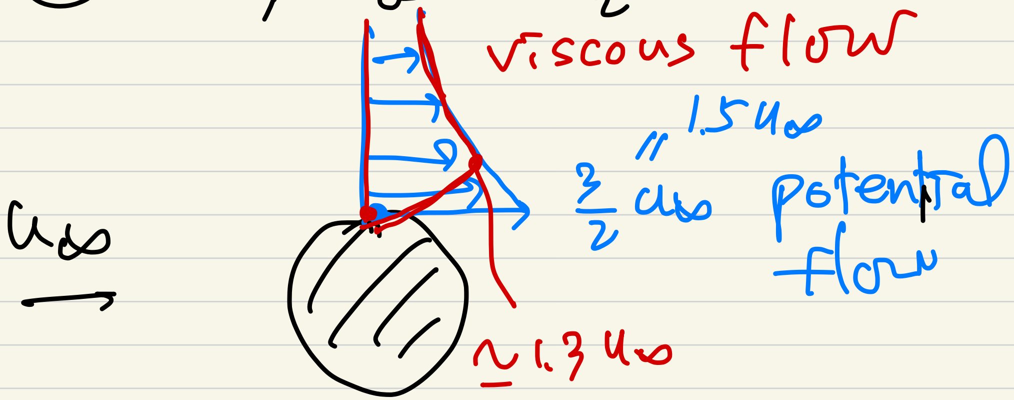
$\lambda = \frac{1}{2}U_\infty a^3$

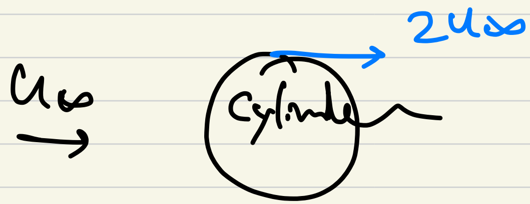
$$v_\theta = \frac{1}{r\sin\theta}\frac{\partial \psi}{\partial r} = -\left(u_\infty + \frac{\lambda}{r^3}\right)\sin\theta$$

$$= - -$$

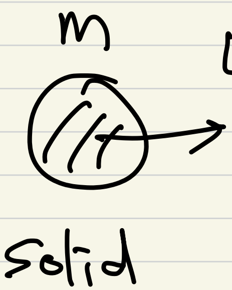
$$= -\frac{1}{2}u_\infty \sin\theta\left(2+\frac{a^3}{r^3}\right)$$

@ $r=a$, $v_\theta = -\frac{3}{2}u_\infty \sin\theta$





- Concept of hydrodynamic mass m_h (potential flow)



$U(t)$: pushes fluid too → need force

$$\boxed{\sum \underline{F} = (m+m_h)\frac{d\underline{U}}{dt}}$$

you feel like having heavier object.

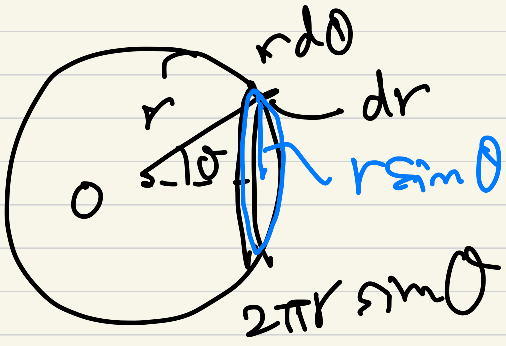
m_h: depend on the shape, direction

→ hydrodynamic mass, added mass, virtual mass

$$KE_{fluid} = \int \underset{\substack{\uparrow \\ \text{flow field}}}{\tfrac{1}{2} V^2}\, dm \equiv \frac{1}{2} m_h U^2$$

$V =$ flow over sphere - uniform stream

$$v_r = -\frac{Ua^3\cos\theta}{r^3}, \quad v_\theta = -\frac{Ua^3\sin\theta}{2r^3}$$



$$dm = \rho(2\pi r\sin\theta)\, r d\theta\, dr$$

$$KE_{fluid} = \int_{r=a}^{\infty}\int_0^{\pi} \frac{1}{2} V^2 \cdot \rho\, 2\pi r\sin\theta\, r d\theta\, dr$$

$$= \cdots = \frac{1}{3}\rho\pi a^3 U^2 \equiv \frac{1}{2} m_h U^2$$

$$\therefore \quad m_h(\text{sphere}) = \frac{2}{3}\rho\pi a^3$$

if $\rho_s = \rho\ (= \rho_f)$, $m_h =$ half of the sphere mass.

For cylinder, $m_h(\text{cylinder}) = \rho\pi a^2 L$

if $\rho_s = \rho$, $m_h = m$ (mass of cylinder)