- Model spectrum

$$E(k) = C\,\varepsilon^{2/3} k^{-5/3} f_L(kL)\, f_\eta(k\eta)$$

$$f_L(kL) = \cdots$$

$$f_\eta(k\eta) = \exp\left\{-\beta\left(\left[(k\eta)^4 + c_\eta^4\right]^{1/4} - c_\eta\right)\right\}$$ model spectrum

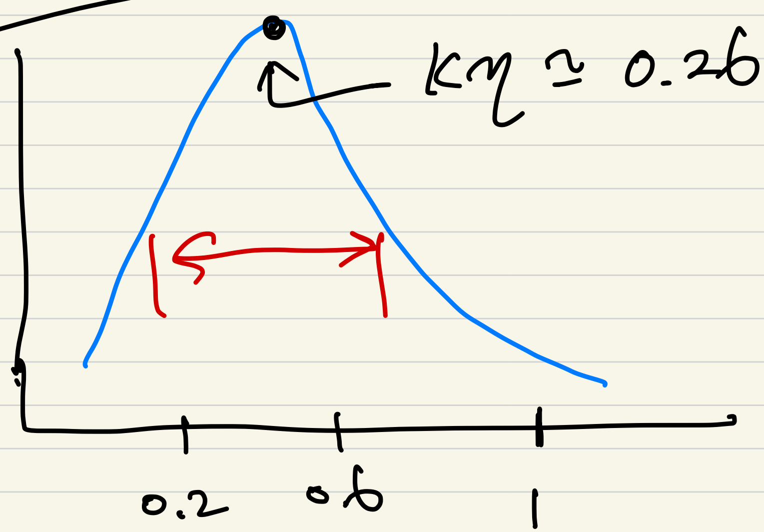
$$f_\eta(k\eta) = \exp\left(-\frac{3}{2} C (k\eta)^{4/3}\right)$$ Pao spectrum

not that good as compared to model spectrum.

dissipation

$\dfrac{D(k)}{u_\eta^3}$

$D(k) = 2\nu k^2 E(k)$

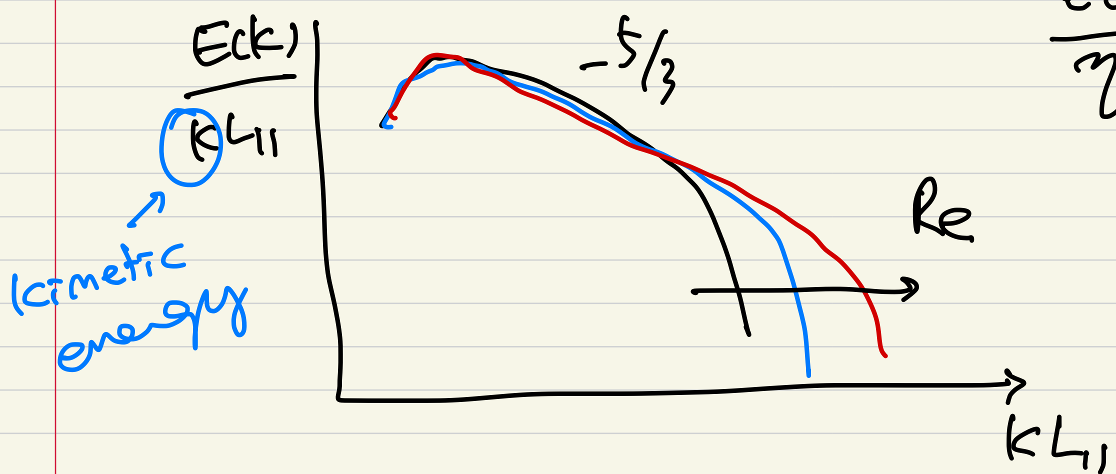


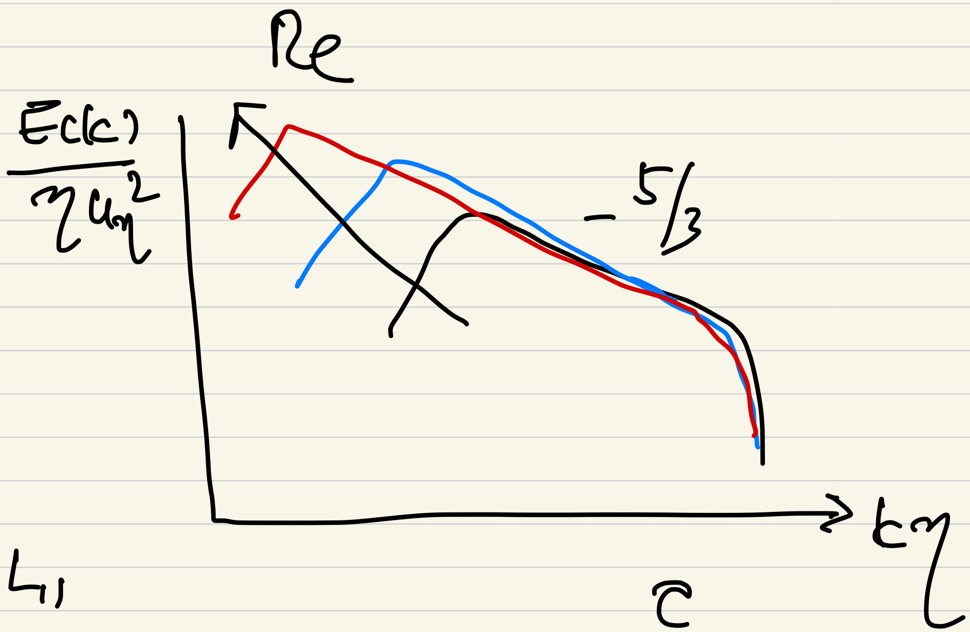
$k\eta$

$k\eta \simeq 0.26 \rightarrow \ell/\eta \simeq 24$

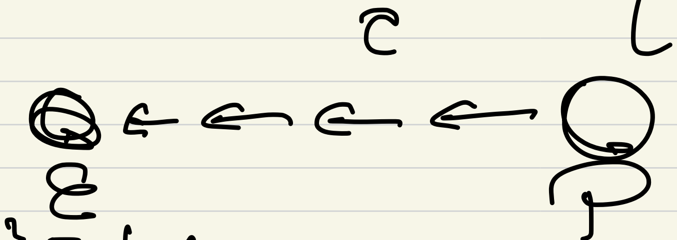
motions responsible for bulk of dissipation ($0.1 < k\eta < 0.75$; $8 < \ell/\eta < 60$) are considerably larger than the Kolmogorov scale.

• Reynolds number effect



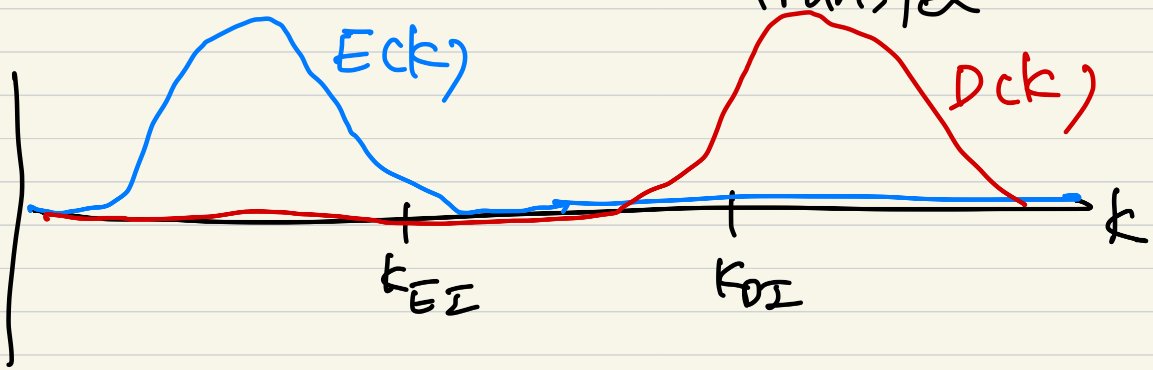


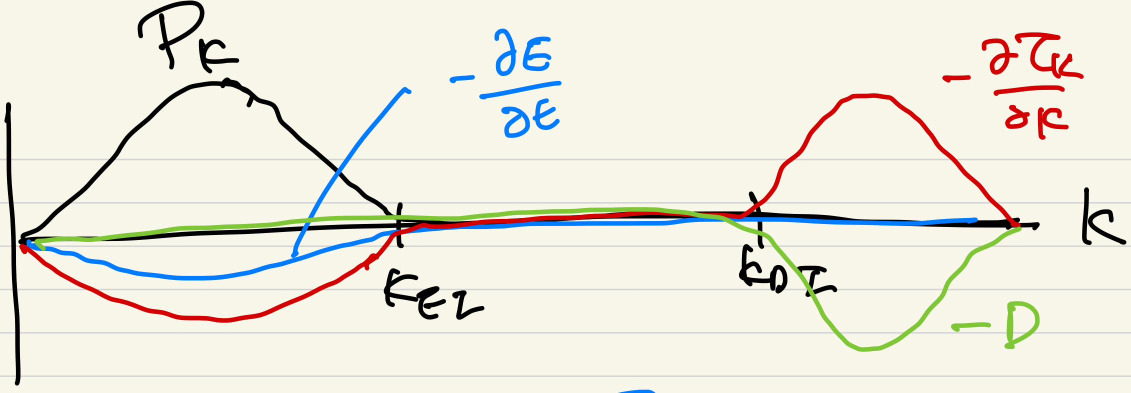
6.6 Spectral view of energy cascade

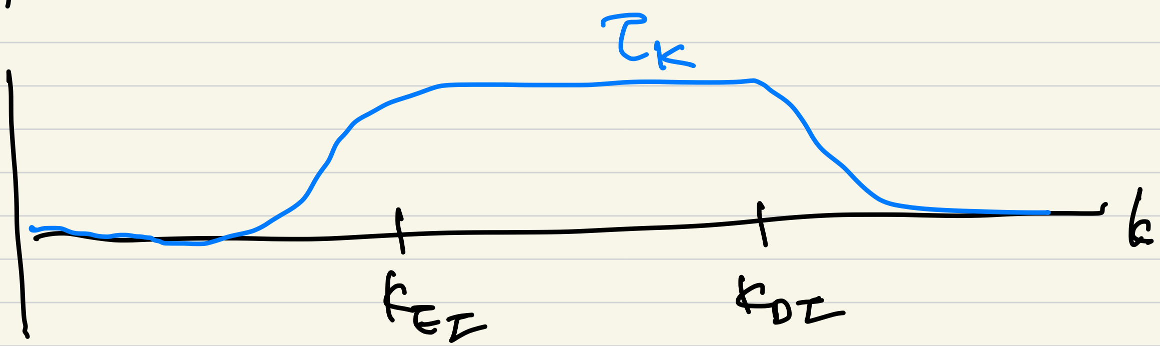


$$\frac{\partial}{\partial t}E(k,t) = P_k(k,t) - \frac{\partial}{\partial k}\mathcal{T}_k(k,t) - 2\nu k^2 E(k,t)$$

production   spectral transfer   dissipation D(k)







For energy-containing range,

$$\frac{dk}{dt} = P - \tau_{EI}$$

$$\tau_{EI} = \tau_k(k_{EI})$$

In the inertial subrange, spectral transfer only,

$$\tau_{EI} \simeq \tau_{DI}$$

In the dissipation range,

$$\tau_{DI} \simeq \varepsilon.$$

6.7 limitations

Kolmogorov hypotheses → oversimplification

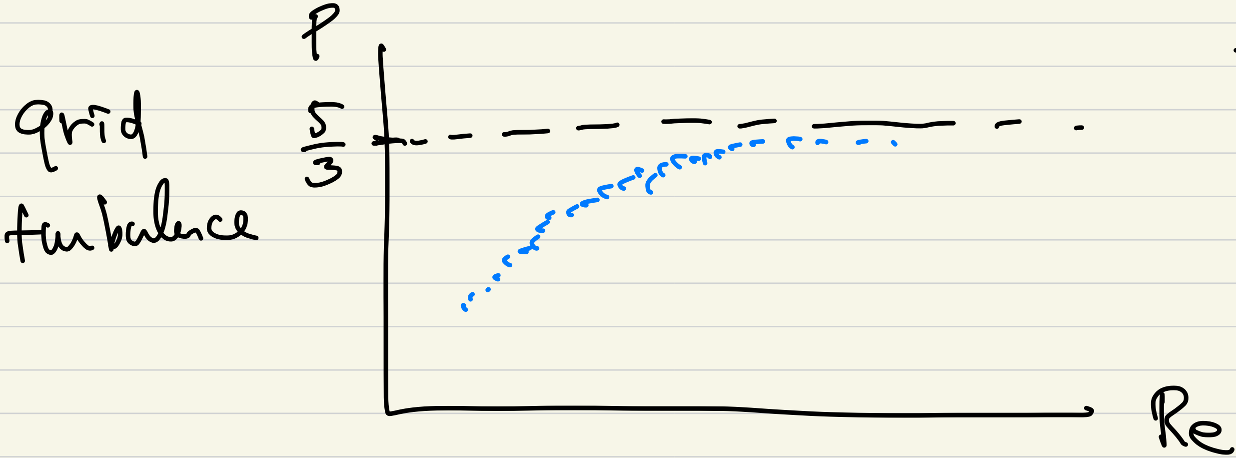
energy cascade consists of <u>one-way</u> transfer of energy from eddies of size l to those of somewhat smaller size and this energy transfer depends solely on motions of size l.

$$\eta \cdots \leftarrow \frac{1}{4}l \leftarrow \frac{1}{2}l \leftarrow l$$

energy backscatter

- In reality, Kolmogorov $-5/3$ spectrum is approached slowly as Reynolds number increases.

$$E(k) \sim k^{-p}$$



Ch. 7 Wall flows

- most turbulent flows are bounded by one or more solid surfaces

  internal flow : flow through pipes and ducts

  external flow : flow around an aircraft

- fully developed channel flow

  " " pipe "

  flat-plate boundary layer flow

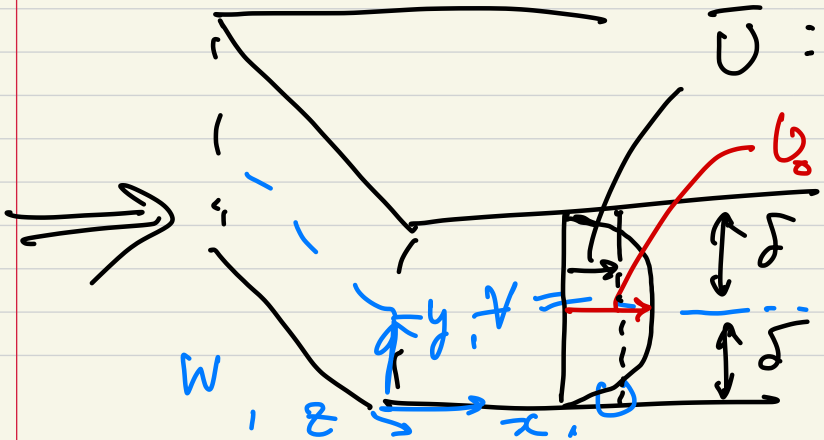
  (fully-developed Couette flow)

  } mean velocity vector is parallel or nearly " to the wall

7.1 Channel flow

$\bar{U}$: bulk velocity $= \frac{1}{2\delta}\int_0^{2\delta} \langle U \rangle \, dy$

U_0 : centerline vel. $= \langle U(y=\delta) \rangle$



centerline

fully developed flow : $\frac{\partial}{\partial x}(\cdot) = 0$ $\left(-\frac{dp}{dx} \neq 0\right)$
in the mean sense

$Re \equiv (2\delta)\bar{U}/\nu$: bulk Reynolds number

$Re_o \equiv U_o\delta/\nu$

laminar flow for $Re < 1350$
turbulent " " $Re > 1800$

statistically stationary – turbulence statistics doesn't change in time.

- continuity $\cancelto{0}{\frac{\partial\langle U\rangle}{\partial x}} + \frac{\partial\langle V\rangle}{\partial y} + \cancelto{0}{\frac{\partial\langle W\rangle}{\partial z}} = 0 \Rightarrow \langle V\rangle \equiv 0$

$\langle W\rangle = 0$

y-mtm eq. $\rightarrow$ $0 = -\frac{d}{dy}\langle v^2\rangle - \frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial y}$ $\cancelto{0}{+\nu\nabla^2\langle V\rangle}$

integrate $\rightarrow$ $\langle v^2\rangle + \frac{1}{\rho}\langle p\rangle = \frac{1}{\rho}p_w$ $\quad \because \langle v^2\rangle_w = 0$

$$\rightarrow \frac{\partial \langle P \rangle}{\partial x} = \frac{dP_w}{dx} \qquad \left(\because \frac{\partial}{\partial x}\langle v^2 \rangle = 0 \right)$$

x-mtm eq. $\rightarrow$
$$0 = -\frac{d}{dy}\langle uv \rangle - \frac{1}{\rho}\frac{\partial \langle P \rangle}{\partial x} + \nu \frac{d^2 \langle U \rangle}{dy^2}$$

$$\rightarrow \frac{d}{dy}\underbrace{\left(\mu \frac{d\langle U \rangle}{dy} - \rho \langle uv \rangle \right)}_{\text{ft. } y} = \underbrace{\frac{dP_w}{dx}}_{\text{ft. } x} \qquad \left(\langle U \rangle = \langle U \rangle(y) \right)$$

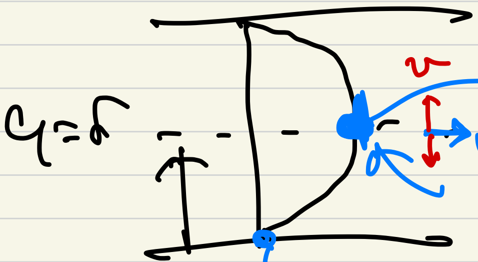
$\rightarrow \therefore$ const.

$$\boxed{\tau = \underbrace{\mu \frac{d\langle U \rangle}{dy}}_{\text{viscous}} - \underbrace{\rho \langle uv \rangle}_{\text{turbulent}}}$$: total shear stress

$$\rightarrow \frac{d\tau}{dy} = \frac{dP_w}{dx} = \text{const} \rightarrow \boxed{\tau(y) = \tau_w \left(1 - \frac{y}{\delta}\right)}$$

linear shear stress profile

$$\left(\because \tau(y=0) = \tau_w \qquad \left.\frac{d\langle U \rangle}{dy}\right|_{y=\delta} = \langle uv \rangle \Big|_{y=\delta} = 0 \right)$$



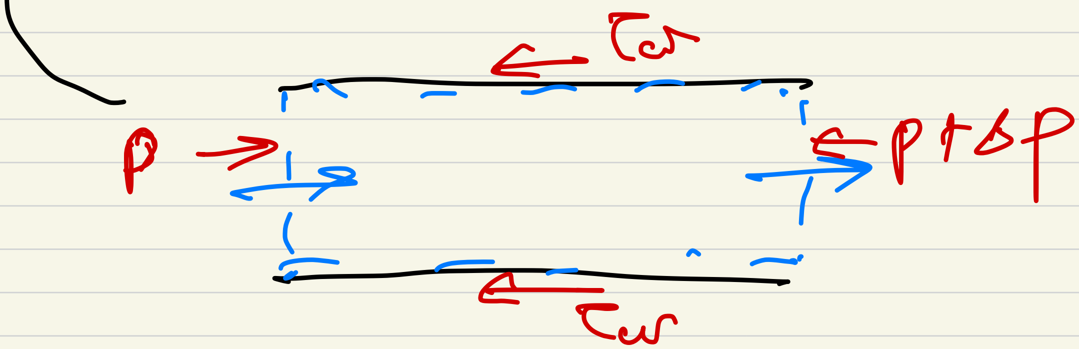
$$\frac{d\langle U \rangle}{dy} = 0$$

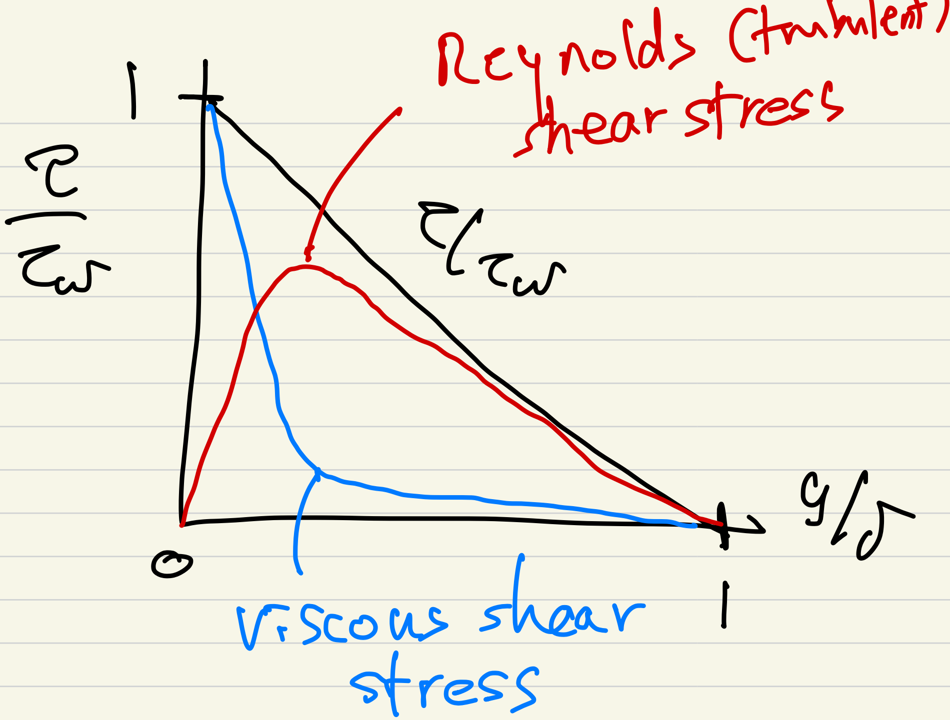
$$\langle uv \rangle = 0$$

$$\tau_w = \mu \left.\frac{d\langle U \rangle}{dy}\right|_w \ (y=0)$$

$$\boxed{-\frac{dP_w}{dx} = \boxed{-\frac{d\tau}{dy} = \frac{\tau_w}{\delta}}} \neq f(x)$$

↑ mean pressure gradient





$$c_f \equiv \tau_w \Big/ \tfrac{1}{2}\rho U_0^2 \quad \text{skin-friction coefficient.}$$

$$C_f \equiv \tau_w \Big/ \tfrac{1}{2}\rho \bar{U}^2$$

In turbulent flow,

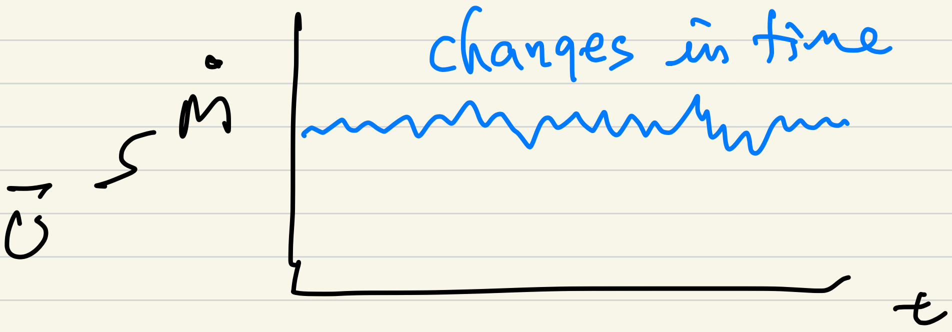
if flow is defined by ρ, ν, δ and $-\frac{dP_w}{dx}$,

U_0 (or $\bar{U}$) are not known a priori,

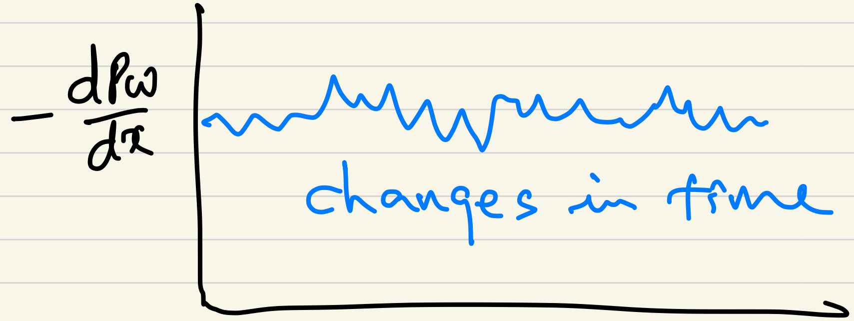
if $\bar{U}$ is imposed, $-dP_w/dx$ is not known a priori.

For laminar flow, all these quantities are determined.

Given $-\frac{dP_w}{dx}$ (const)



Given $\dot{m}$



- $\tau = \underbrace{\mu \frac{d\langle U\rangle}{dy}}_{\text{viscous shear stress}} \underbrace{- \rho\langle uv\rangle}_{\text{or Reynolds (turbulent) shear stress}}$

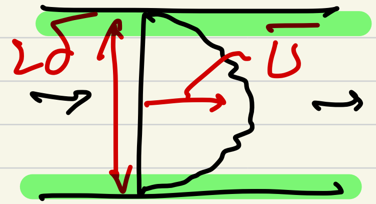
$$\tau_w = \mu \frac{d\langle U\rangle}{dy}\Big|_{y=0}$$

Close to the wall, ν and τ_w are important parameters.

viscous scales : velocity and length scales in the near-wall region

friction velocity (wall-shear ")

$$\boxed{u_\tau \equiv \sqrt{\frac{\tau_w}{\rho}}}$$



viscous length scale $\boxed{\delta_v \equiv \frac{\nu}{u_\tau}} = \nu\sqrt{\frac{\rho}{\tau_w}}$

$$\frac{u_\tau \delta_v}{\nu} = 1$$

$$\boxed{Re_\tau \equiv \frac{u_\tau \delta}{\nu} = \frac{\delta}{\delta_v}}$$ Friction Reynolds number

$Re_\tau = 180$ @ $Re = 5600$ Kim, Moin & Moser (JFM, 1987)

395 13750

5200 ← 10^{10} grid pts. Lee & Moser

$$\boxed{y^+ \equiv \frac{y u_\tau}{\nu}} = \frac{y}{\delta_v}$$ wall units