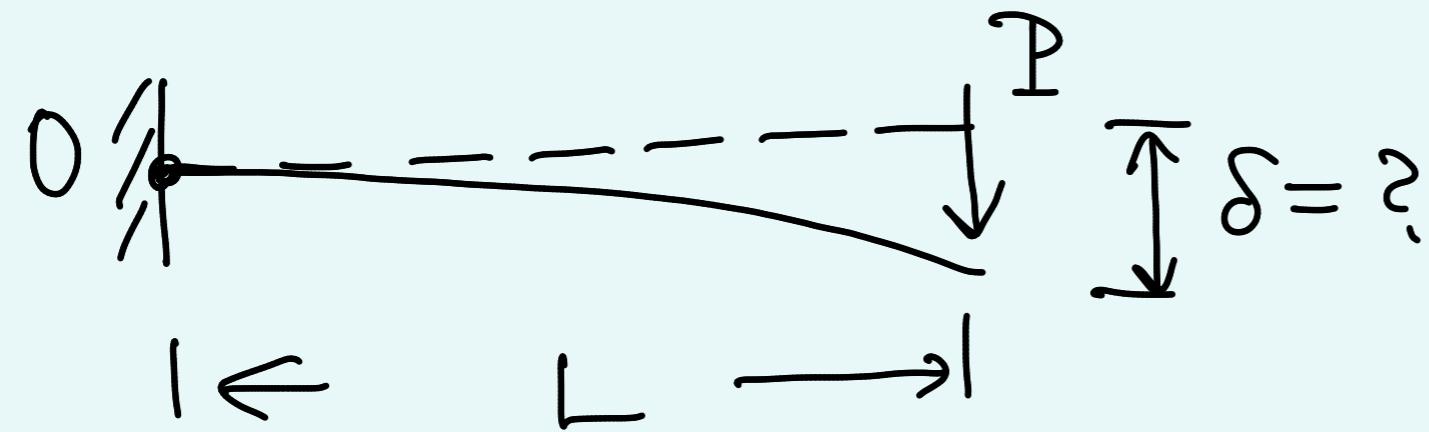


beam bending - scaling analysis

$$K \approx \frac{d^2y}{dx^2}$$



$$M_b \sim EI K''$$

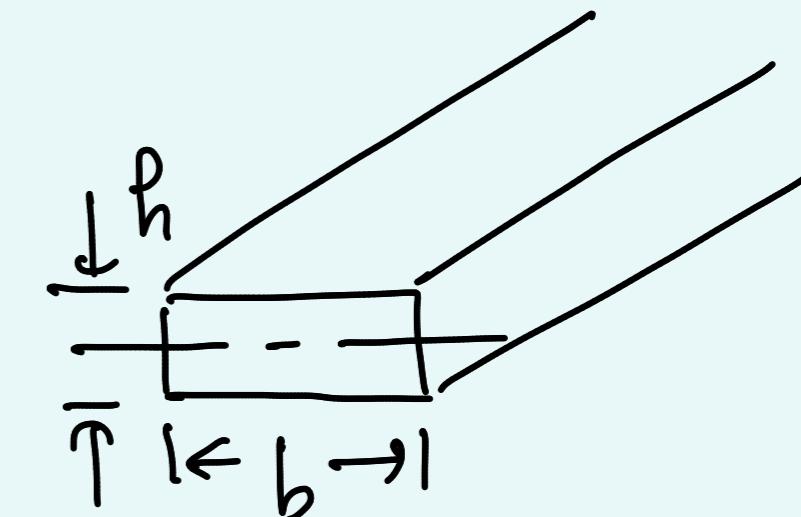
$$PL \sim EI \frac{\delta}{L^2}$$

$$\delta \sim \frac{PL^3}{EI}$$

exact $\delta = \frac{PL^3}{3EI}$

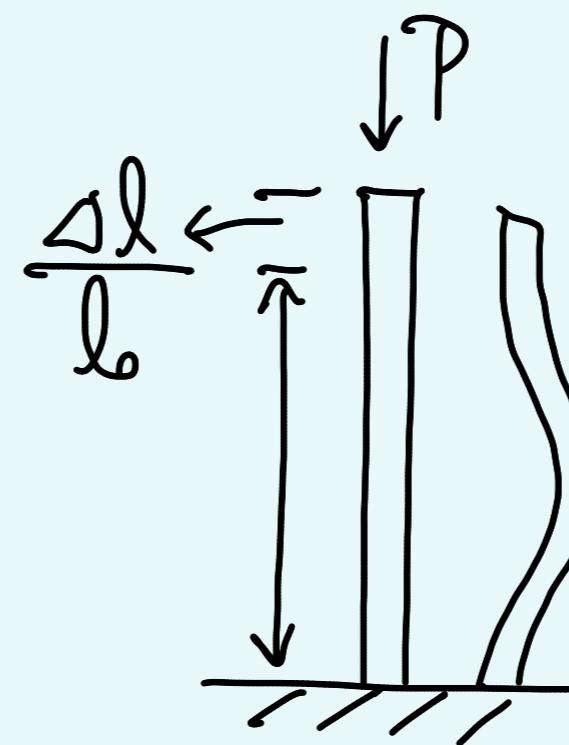
$$\delta \sim \frac{P}{k}$$

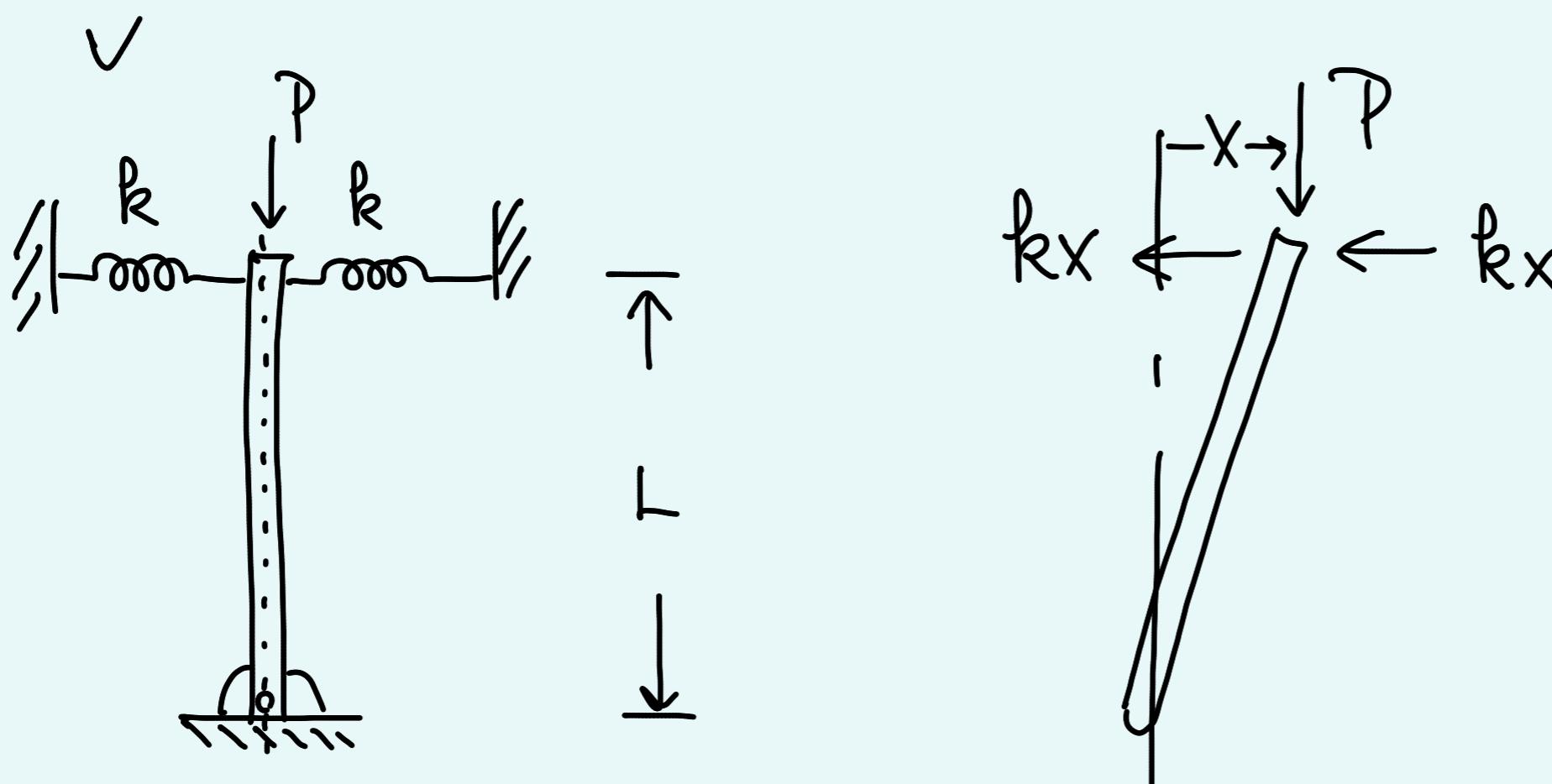
$$k \sim \frac{P}{\delta} \sim \frac{3EI}{L^3}$$



$$I = \frac{1}{12}bh^3$$

scaling for buckling





unstable if $P_x > 2kxL$

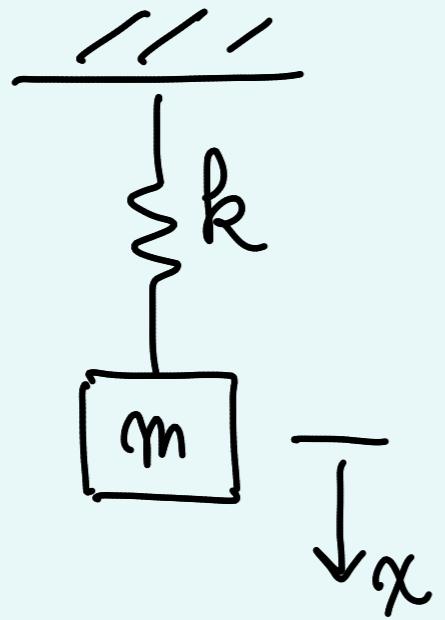
$$P > 2kL$$

critical load $P_c \sim kL$, $k \sim \frac{EI}{L^3}$

$$\therefore P_c \sim \frac{EI}{L^2}$$

• Vibration of a beam

• spring-mass



$$m\ddot{x} + kx = 0$$

$$m\omega^2 \sim k$$

$$\omega \sim \sqrt{\frac{k}{m}}$$

$$\ddot{x} = \frac{d^2x}{dt^2} \sim \frac{x}{T^2}$$

$$\left. \begin{aligned} \frac{1}{T} &= f = \frac{\omega}{2\pi} \\ x\omega^2 & \end{aligned} \right\}$$

exchange of kinetic and potential energy

$$mU^2 \sim kx^2 \quad . \quad U \sim \omega x$$

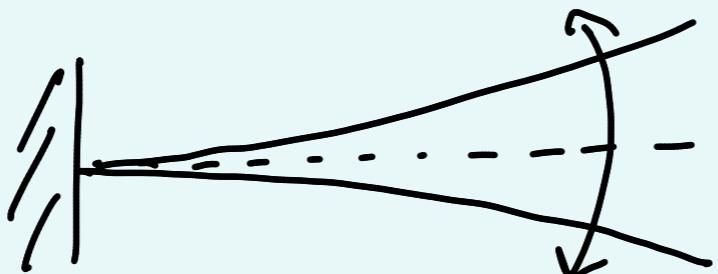
$$m\omega^2 x^2 \sim kx^2$$

$$\omega^2 \sim \frac{k}{m}$$

: natural frequency

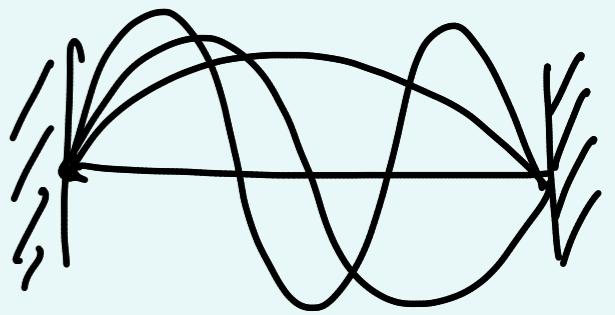
$$\omega^2 \sim \frac{k}{m} \sim \frac{EI/L^3}{\rho_s A L} \sim \frac{EI}{\rho_s A L^4}$$

• beam

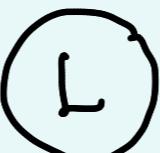


$$\tau = bh$$

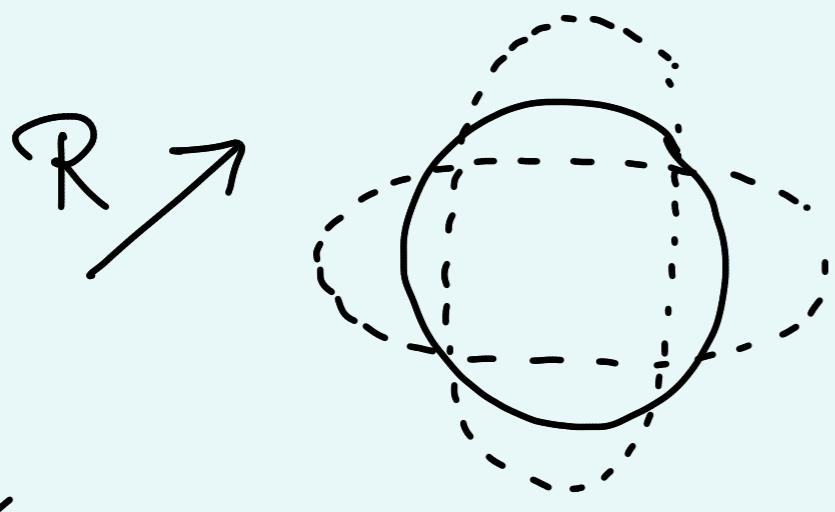
• Vibration of drops and bubbles



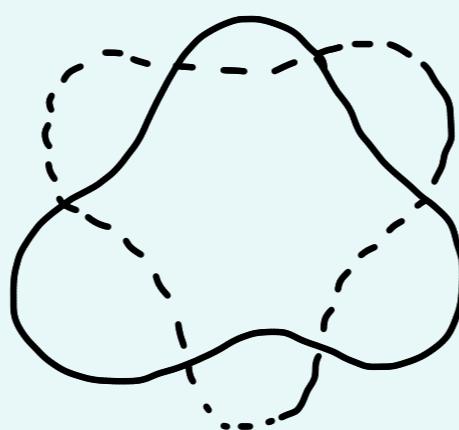
G



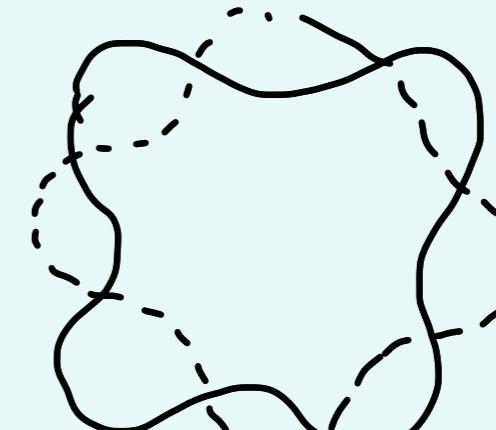
L



$m=2$

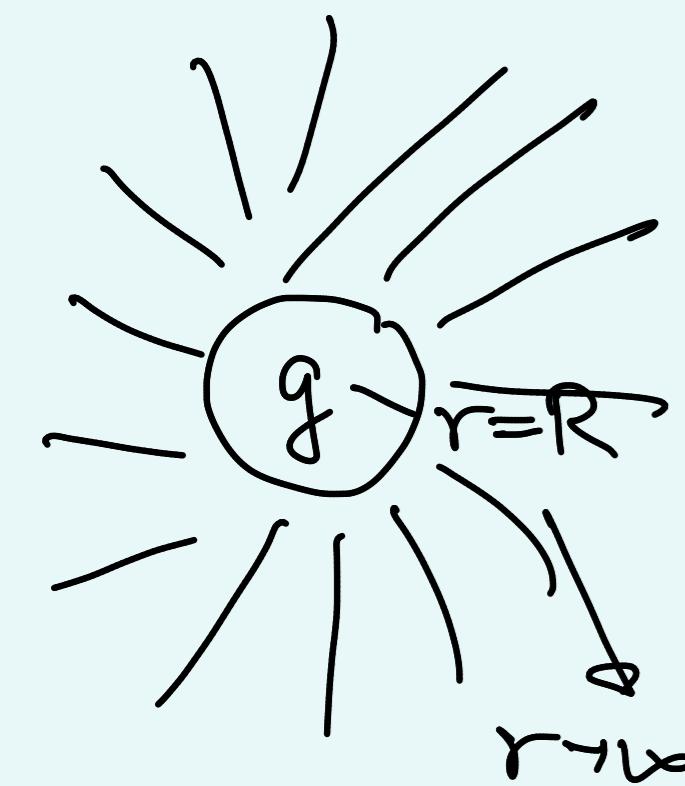


$m=3$



$m=4$

...



$$\rho R^3 U^2 \sim R^2 \gamma.$$

$$\omega^2 \sim \frac{\gamma}{\rho R^3}$$

$$U \sim R\omega$$

$$[\gamma] = [k] = \frac{N}{m}$$

$$\omega_{\text{drop}}^2 = m(m-1)(m+2) \frac{\gamma}{\rho R^3}$$

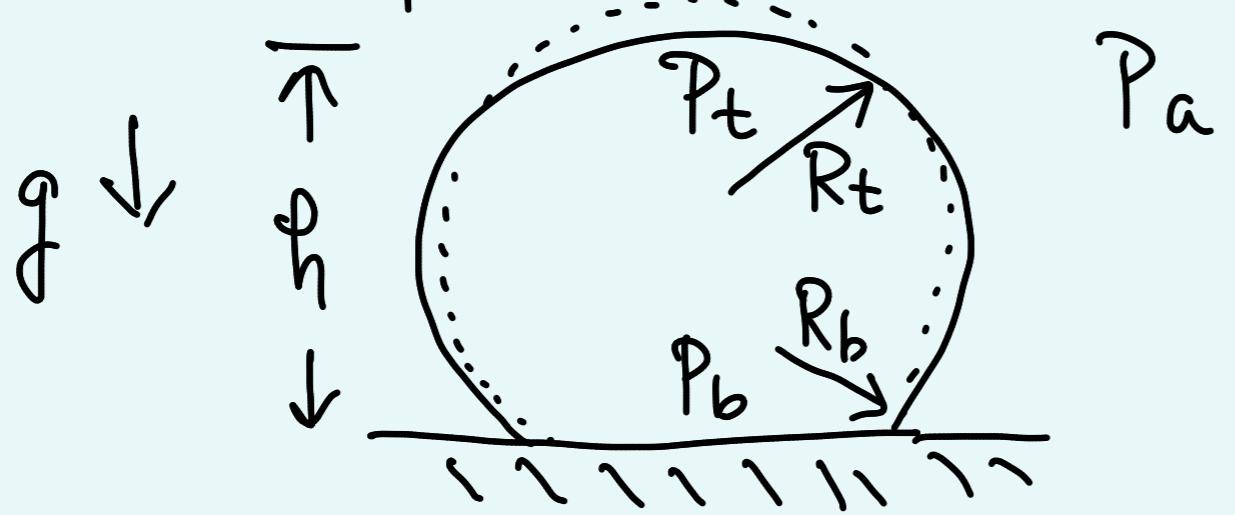
$$\omega_{\text{bubble}}^2 = (m+1)(m-1)(m+2) \frac{\gamma}{\rho R^3}$$

H. Lamb

"Hydrodynamics"

* gravitational sagging

- drop



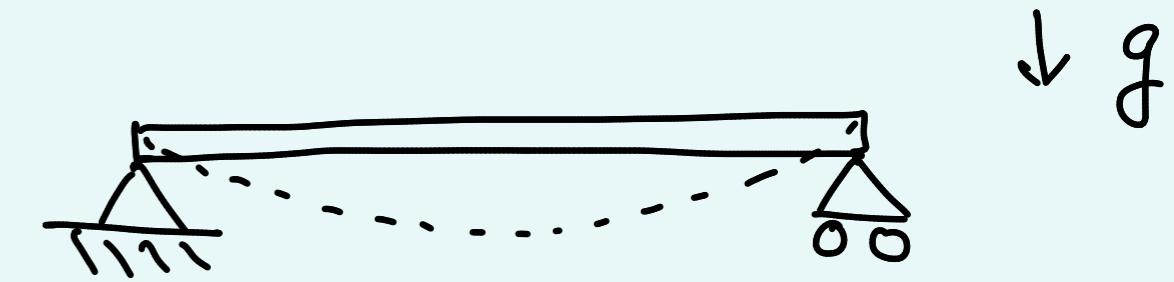
$$P_b = P_t + \rho g h$$

$$P_t - P_a \sim \frac{\gamma}{R_t}$$

$$P_b - P_a \sim \frac{\gamma}{R_b}$$

$$P_b - P_t \sim \gamma \left(\frac{1}{R_b} - \frac{1}{R_t} \right) \sim \rho g h$$

- beam



$$l_c^2 \sim \frac{\gamma}{\rho g}$$

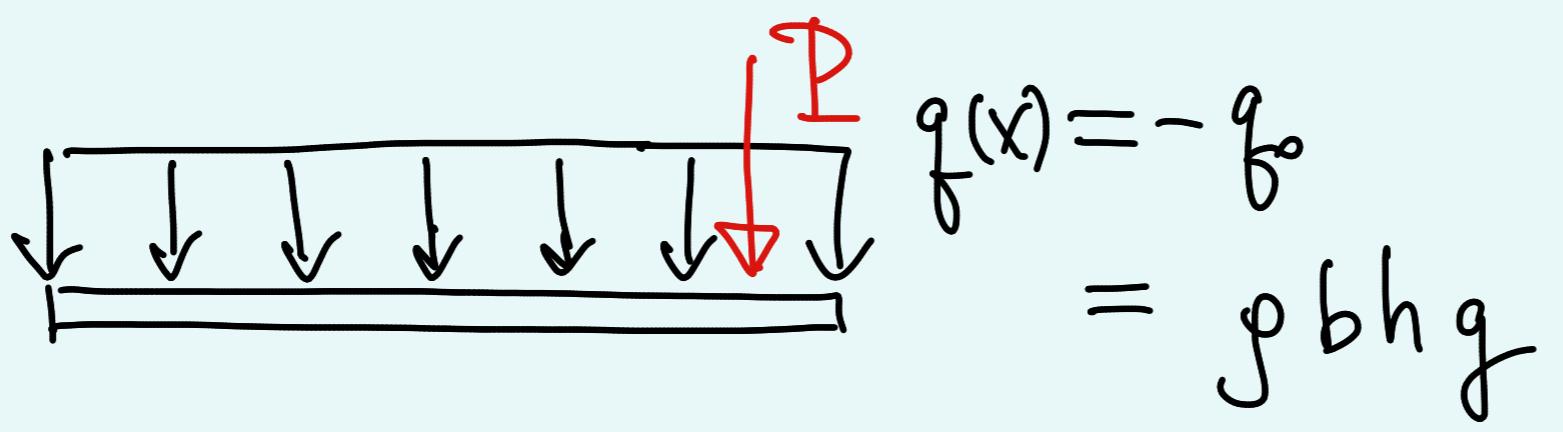
for water $\gamma = 0.072 \text{ N/m}$

$$\rho g \approx 10^4$$

$$l_c = \sqrt{\frac{\gamma}{\rho g}} = 2.7 \text{ mm}$$

- gravitational effects negligible when

$$\rho g R \ll \frac{\gamma}{R}$$



$$q(x) = -q_0 \\ = \rho b h g$$

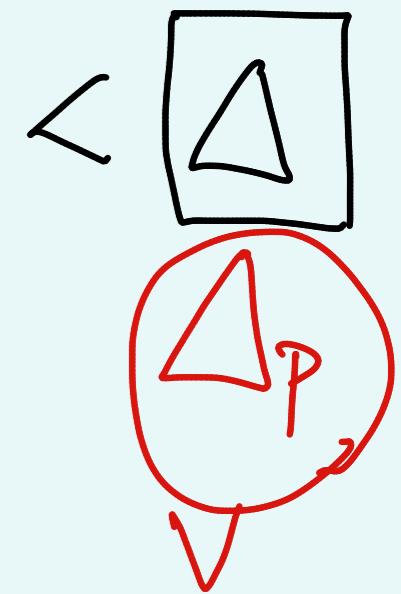
$$EIy''' = f$$

$$\delta \sim \frac{\rho_s A L^4 g}{EI} \sim \frac{\rho_s b h L^4 g}{E b h^3}$$

$$\sim \frac{\rho g L^3}{E h^2}$$

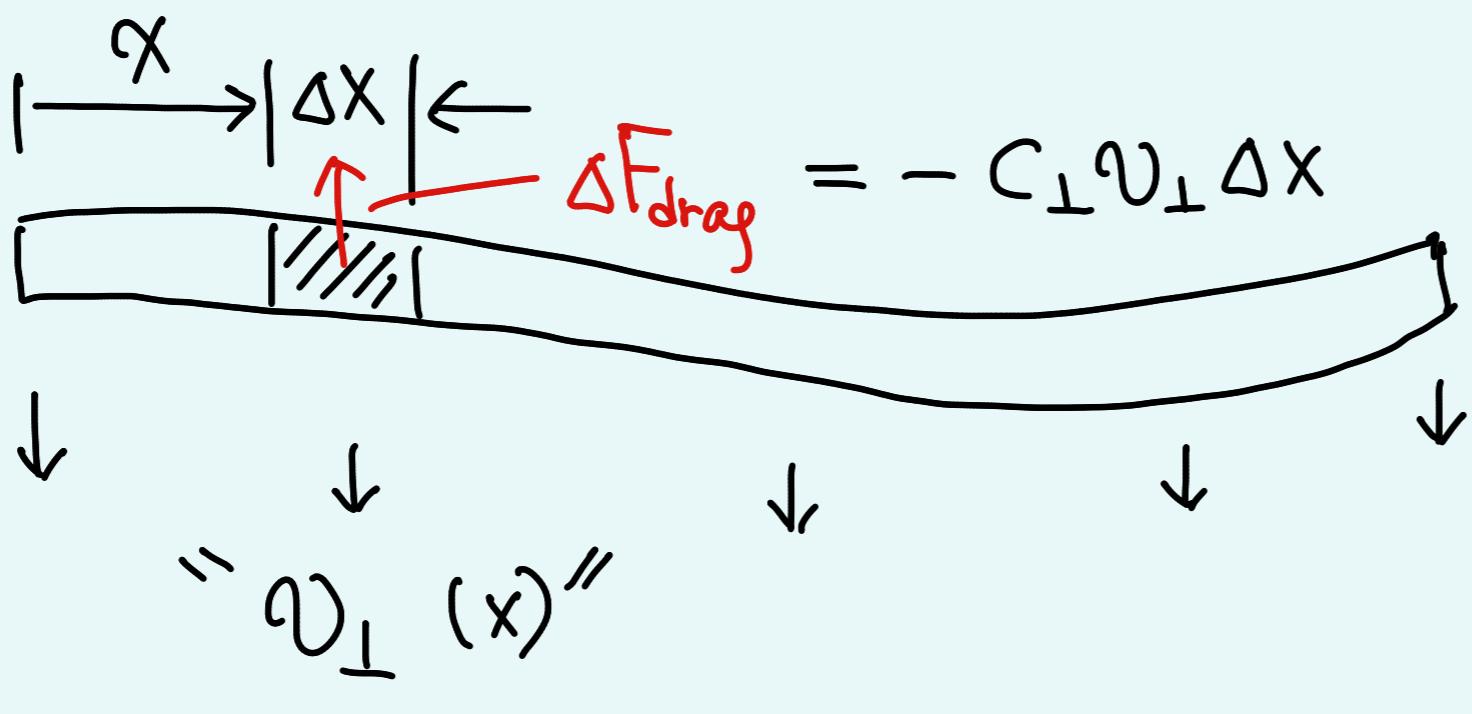
" " "

$\cancel{\sqrt{}}$



- Elastohydrodynamics



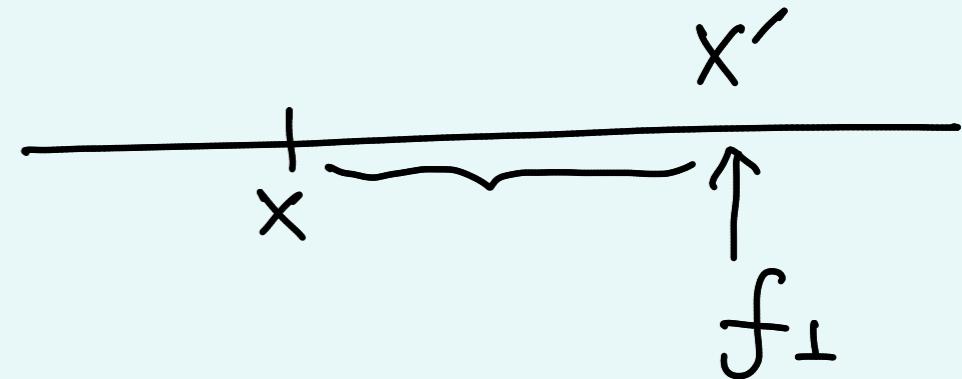


drag force per unit length

$$f_{\perp}(x) = -C_{\perp}v_{\perp}(x) = -C_{\perp}\frac{\partial y}{\partial t}(x)$$

bending moment at pt x due to drag force on
the RHS of the pt.

$$M(x) = \int_x^L f_{\perp}(x') (x' - x) dx'$$



$$\frac{\partial M}{\partial x^2} = f_{\perp}(x) = -C_{\perp}\frac{\partial y}{\partial t}$$

$$EI \frac{\partial^2 y}{\partial x^2} = M$$

$$EI \frac{\partial^4 y}{\partial x^4} = \frac{\partial^2 M}{\partial x^2} = -C_{\perp}\frac{\partial^2 y}{\partial t^2}$$

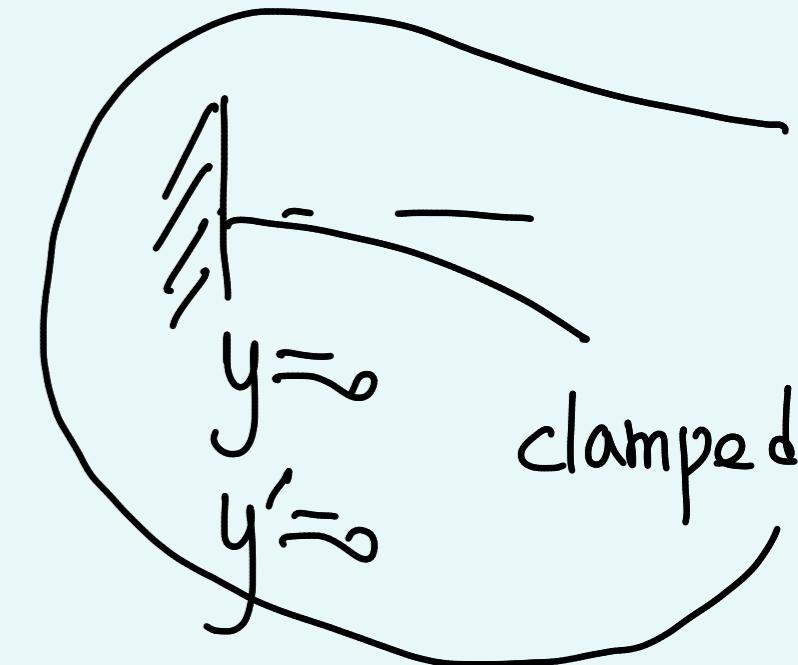
$$\frac{\partial^4 y}{\partial x^4} = - \frac{c_L}{EI} \frac{\partial y}{\partial t}$$

: hydrodynamic beam equation

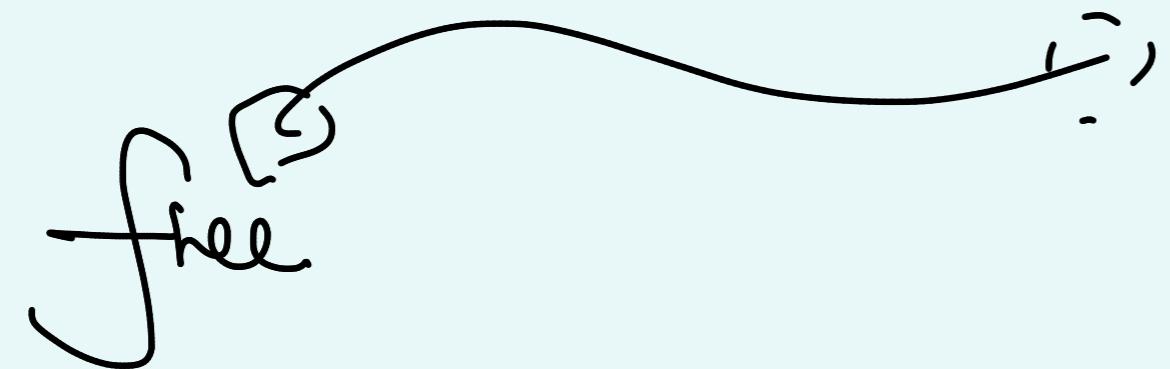
$$\uparrow \qquad \qquad \uparrow$$

$$y = y_0 \text{ at } t=0 \\ \bar{y}(x)$$

$$f_L = - c_L \frac{\partial y}{\partial t}$$



B.C. for an unconstrained filament



$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^3 y}{\partial x^3} = 0 \quad \text{at } x=0 \text{ &} \\ x=L$$

$$M_b = 0, V \approx 0.$$

by separation of variables

$$y_n(x,t) = e^{-t/\tau_n} \left[\sinh \alpha_n \cos \frac{2\alpha_n}{L} \left(x - \frac{L}{2} \right) - \sin \alpha_n \cosh \frac{2\alpha_n}{L} \left(x - \frac{L}{2} \right) \right] \quad n. \text{ odd}$$

$$y_m(x,t) = e^{-t/\tau_m} \left[\cosh \alpha_m \sinh \frac{\omega_m}{L} \left(x - \frac{L}{2} \right) + \cos \alpha_m \sinh \frac{\omega_m}{L} \left(x - \frac{L}{2} \right) \right]. \quad m - \text{even}$$

where $\tau_n = \frac{C_1}{EI} \left(\frac{L}{2\alpha_n} \right)^4$, relaxation time

$$\tanh \alpha_n = (-1)^n \tanh \alpha_n. \quad n=1, 2, 3, \dots$$

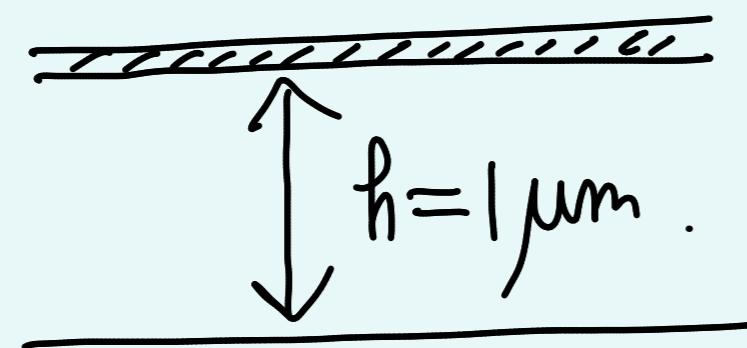
approximate values for α_n

$$\alpha_n \approx (n + \frac{1}{2}) \frac{\pi}{2}, \quad n = 1, 2, 3, \dots \quad (\alpha_1 \approx 2.365)$$

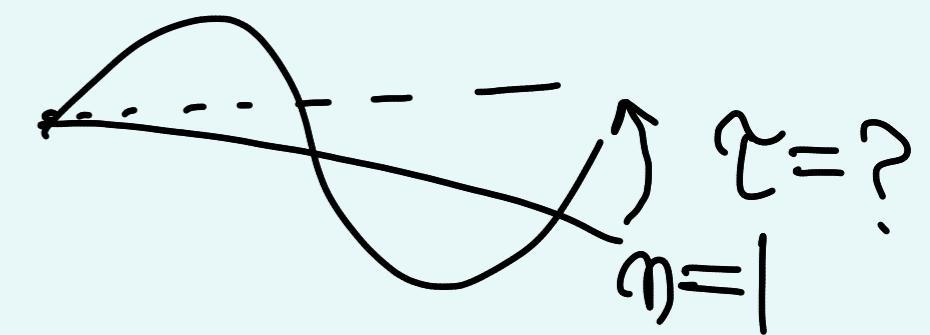
e.g. Relaxation of microtubules and actin filaments

microtubules

$$L = 50 \mu\text{m}, \quad \text{radius} = 15 \text{ nm}, \quad EI = 30 \times 10^{-24} \text{ N} \cdot \text{m}^2$$



$$\mu = 1 \text{ mPa} \cdot \text{s}$$



relaxation time for $m=1$

$$C_L = \frac{4\pi\mu}{\ln(2h/r)} = 2.6 \times 10^{-3}$$

$$\textcircled{C_m} = \frac{C_L}{EI} \left(\frac{L}{2\alpha_h} \right)^4 \quad (m=1)$$

$$= 1.1 \text{ s}$$