Engineering Economic Analysis

2019 SPRING

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Chap. 25

MONOPOLY

Introduction

- A monopolized market has a single seller.
- The monopolist's demand curve is the (downward sloping) market demand curve.
- So the monopolist can decide the market price by adjusting its output level (Price setter).
- What causes monopolies?
  - a legal fiat; Tobacco and Ginseng in Korea
  - a patent; a new drug
  - sole ownership of a resource; a toll Airport highway
  - formation of a cartel; OPEC
  - large economies of scale; local utility companies.

Maximizing Profits

- Profit maximization

$$\max_{p,y} p \cdot y - c(y)$$
$$s.t. \quad D(p) = y$$

$$\Rightarrow \max_{p} p \cdot D(p) - c(D(p))$$

  - Let $p(y)$ be the inverse demand

$$\max_{y} p(y) \cdot y - c(y)$$

  - F.O.C.

$$p(y) + p'(y) \cdot y = c'(y)$$

Marginal revenue = Marginal cost

  - S.O.C.

$$2p'(y) + p''(y)y - c''(y) \leq 0$$

Maximizing Profits

- Price elasticity & Monopolistic optimal
  - F.O.C. can be rearranged as

$$p(y)\left[1+\frac{y}{p}\frac{dp}{dy}\right]=c'(y)$$

  - Price elasticity

$$\varepsilon(y)=\frac{y/dy}{p/dp}=\frac{p}{y}\frac{dy}{dp}$$

  - Thus, F.O.C. becomes

$$p(y)\left[1+\frac{1}{\varepsilon(y)}\right]=c'(y)$$

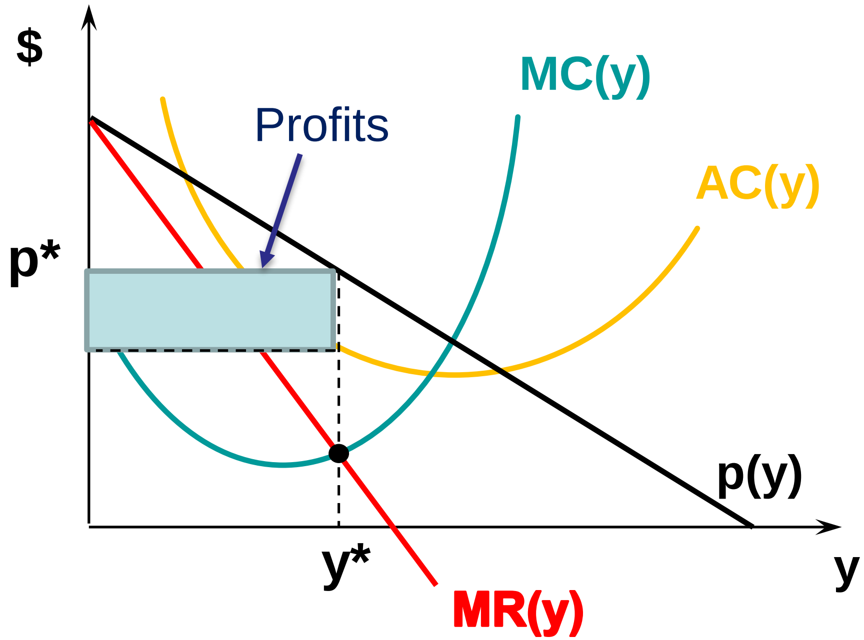
  - Since $\varepsilon(y)\le 0$ and $c'(y)\ge 0$, at the optimal it should be

$$\left|\frac{1}{\varepsilon(y)}\right|<1 \Rightarrow |\varepsilon(y)|>1$$

Monopolist would never choose to produce the output level where the demand is price-inelastic!

Linear demand curve and Monopoly

- Linear demand curve $p(y) = a - by$
- Revenue: $r(y) = p(y)y = ay - by^2$
- Marginal revenue: $MR(y) = a - 2by$
  - MR curve has the same y-intercept and the double slope with corresponding demand curve



Linear demand curve and Monopoly

- Case 1: Linear cost $c(y)=cy$

  MR=MC $\Rightarrow a - 2by = c$

  Thus, the monopolist's optimal output and price are

  $$y^* = \frac{a-c}{2b},\ p^* = \frac{a+c}{2}$$

- Case 2: Quadratic cost $c(y) = F + \alpha y + \beta y^2$

  MR=MC $\Rightarrow a - 2by = \alpha + 2\beta y$

  Thus, the monopolist's optimal output and price are

  $$y^* = \frac{a-\alpha}{2(b+\beta)},\ p^* = a - b\left(\frac{a-\alpha}{2(b+\beta)}\right)$$

Mark up pricing

- Mark up pricing: Output price is the marginal cost plus a "mark up."
  - How big is a monopolist's markup and how does it change with the own-price elasticity of demand?
- Since at the optimal point of monopolist

$$MR = p(y)\left[1 - \frac{1}{|\varepsilon(y)|}\right] = MC(y)$$

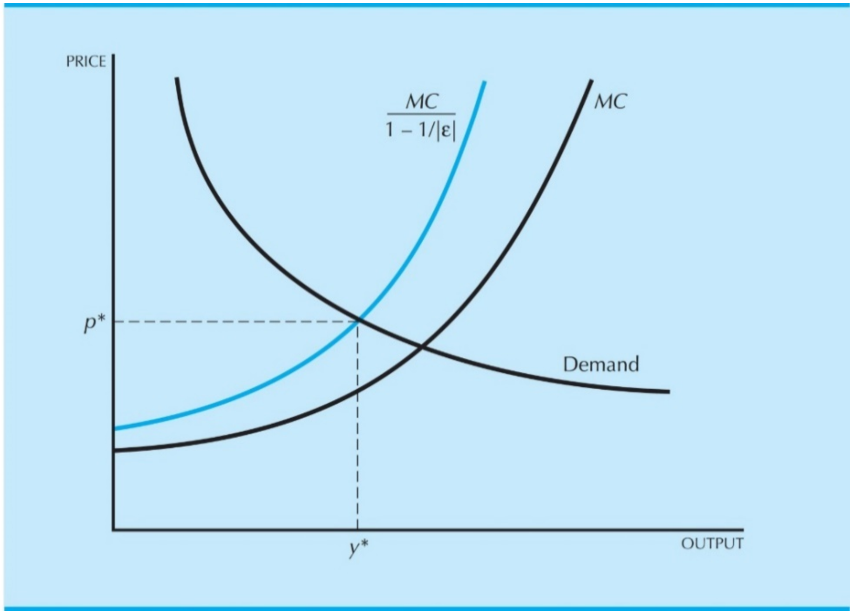
$$\Rightarrow \quad p(y) = \frac{MC(y)}{1 - 1/|\varepsilon(y)|}$$ (Inverse) Supply function of monopolist

- Mark up = $\dfrac{1}{1 - 1/|\varepsilon(y)|}$
  - Since at the optimal, $|\varepsilon(y)| > 1$, markup is greater than 1.
  - Markup depends on the elasticity of demand.

Mark up pricing

- When demand elasticity is constant, the monopolist's supply function becomes

$$p(y^*) = \frac{MC}{1 - 1/|\varepsilon|}$$



- Example

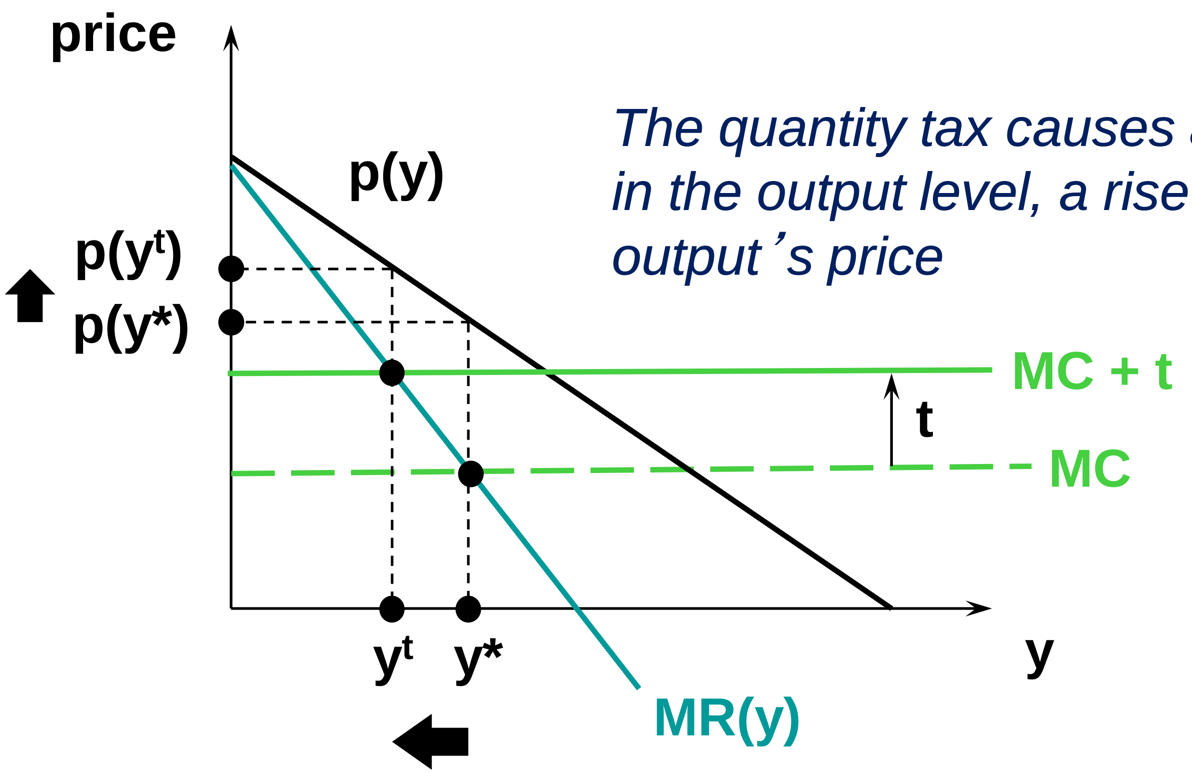
$$D(p) = y = Ap^{-b}$$

$$\varepsilon(y) = \frac{p}{y}\frac{dy}{dp} = \frac{p}{Ap^{-b}}(-b)Ap^{-b-1} = -b$$

$$Thus,\ p^* = \frac{MC}{1 - 1/b}$$

Quantity Tax Levied on a Monopolist

- Example: linear demand & constant MC
  - Quantity tax is equivalent to the increase in MC
  - What happens to the price charged when a quantity tax is imposed?



Quantity Tax Levied on a Monopolist

- Linear demand: $p(y) = a - by$
  - MR = MC (after tax)

$$a - 2by = c + t$$

$$y^t = \frac{a - c - t}{2b}, \; p^t = \frac{a + c + t}{2}$$

  - The change in output due to tax

$$\frac{dy^t}{dt} = -\frac{1}{2b} \leq 0$$

  - The change in price due to tax

$$\frac{dp^t}{dt} = \frac{1}{2}$$

    - The price rises by less than the tax increase

Quantity Tax Levied on a Monopolist

- Constant elasticity demand
  - We know that

$$p^t = \frac{mc + t}{1 - 1/|\varepsilon|}$$

  - The change in price due to tax

$$\frac{dp^t}{dt} = \frac{1}{1 - 1/|\varepsilon|} > 1 \quad \text{since } |\varepsilon| > 1$$

    - The price rises by more than the amount of tax

Comparative Statics

- The effect of cost change on the monopolist's output
  - Profit-max. $\max_{y} p(y)y - c(y)$
  - F.O.C. $p(y) + p'(y)y - c'(y) = 0$
  - Totally differentiating F.O.C.

$$p'dy + (p'' \cdot y + p')dy - c''(y)dy - dc = 0$$

  - Therefore

$$\frac{dy}{dc} = \frac{1}{2p' + p''y - c''(y)} \leq 0 \text{ by S.O.C.}$$

    - Profit-max monopolist will always reduce its output when its cost increases

Comparative Statics

- The effect of cost change on the monopolist's price

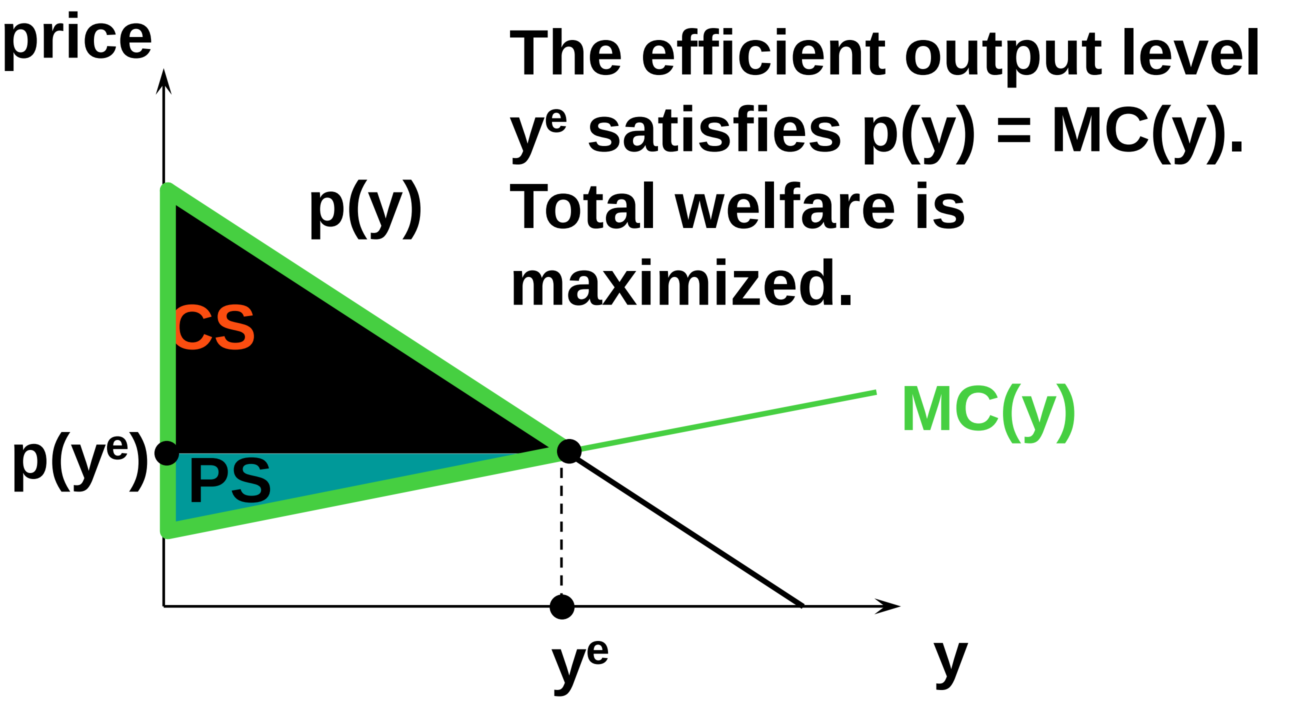
$$\frac{dp}{dc} = \frac{dp}{dy} \cdot \frac{dy}{dc}$$

$$= \frac{p'}{2p' + p''y - c''(y)} \geq 0 \quad \text{since } p' < 0$$

  - Profit-max monopolist will raise its price when its cost increases

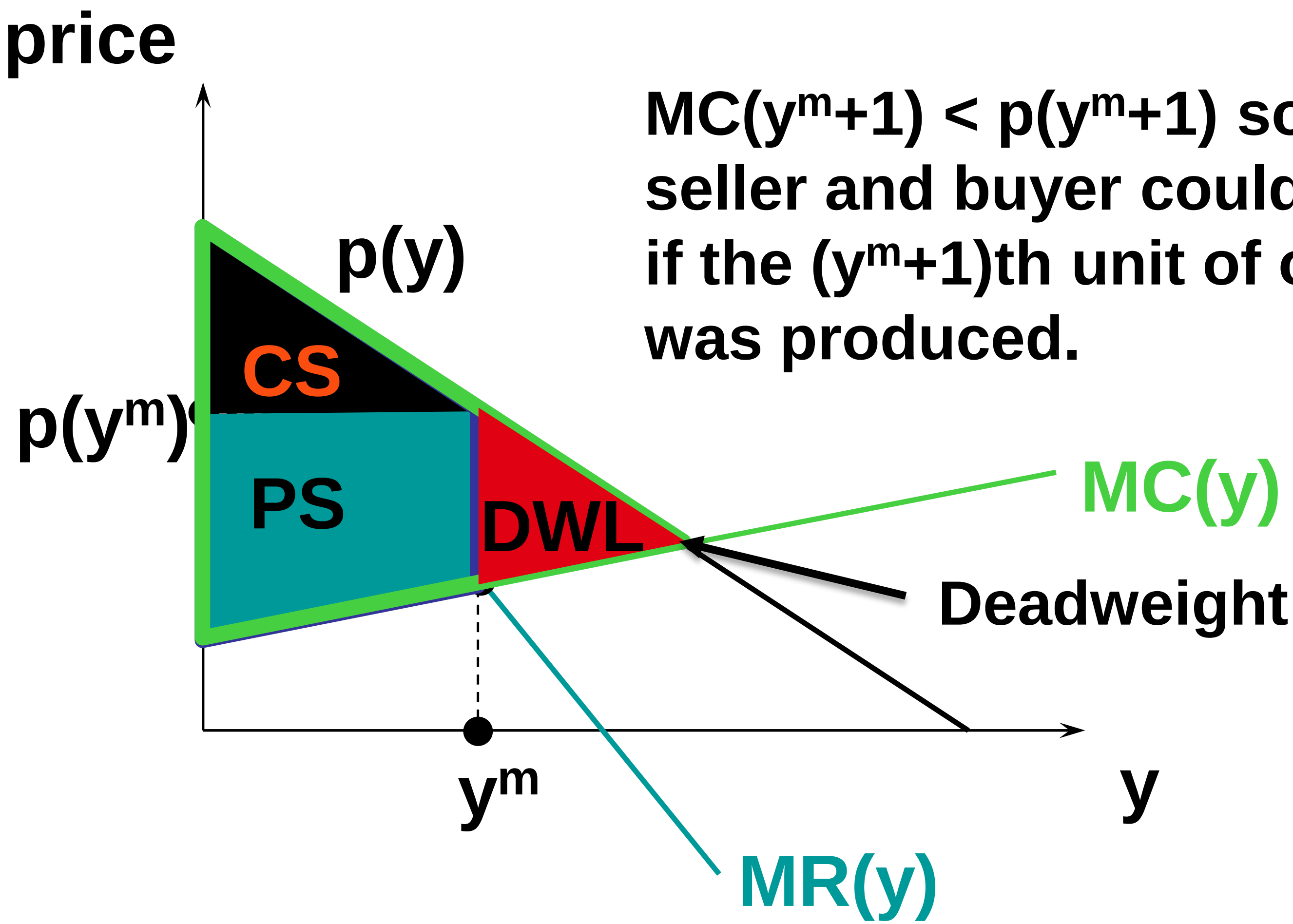
The Inefficiency of Monopoly

- A market is Pareto efficient if it achieves the maximum possible total welfare (gains-to-trade).
- Competitive market (p=MC) is Pareto efficient.



The Inefficiency of Monopoly

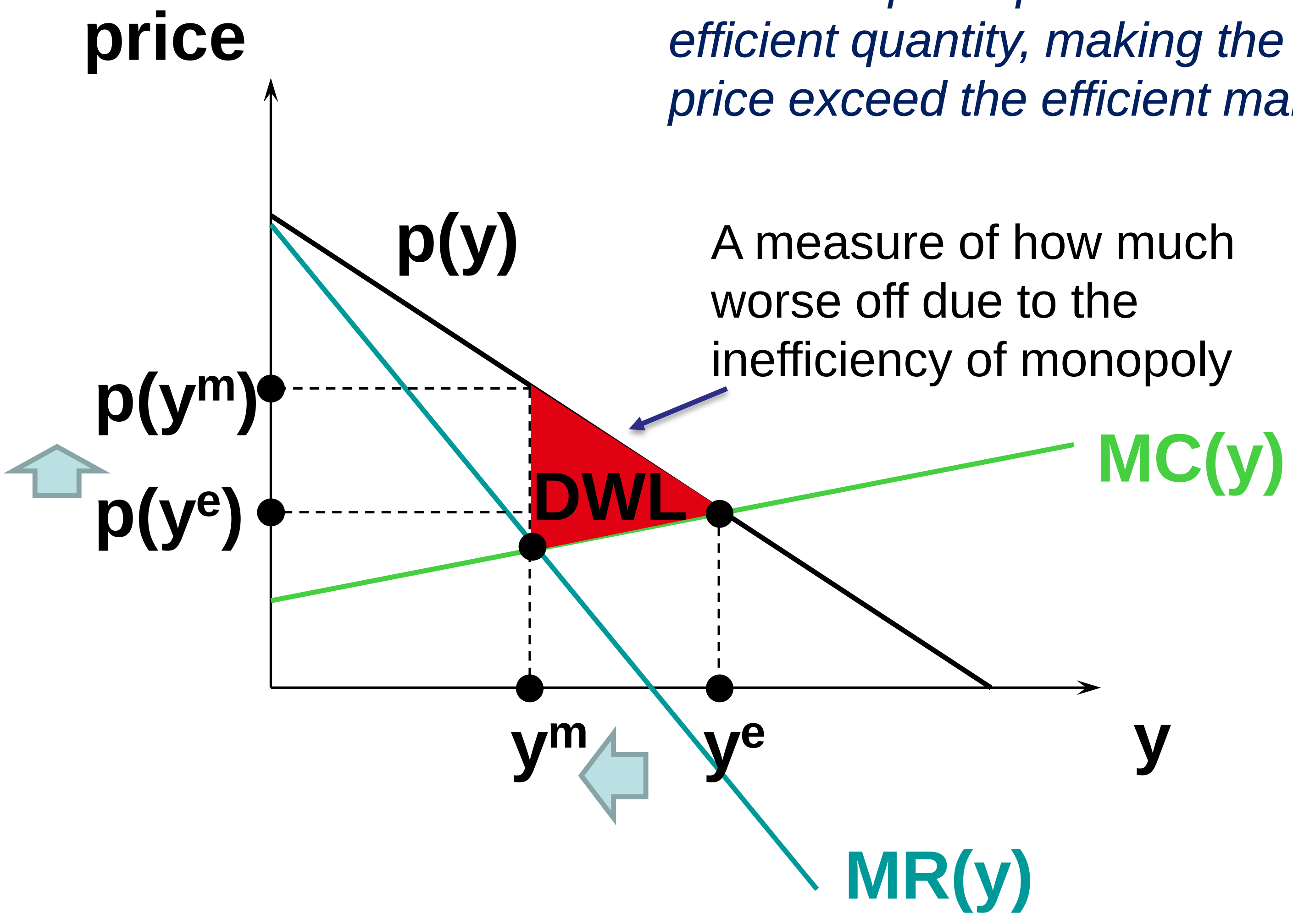
- Monopoly is Pareto inefficient.



$MC(y^m+1) < p(y^m+1)$ so both seller and buyer could gain if the (y^m+1)th unit of output was produced.

The Inefficiency of Monopoly

The monopolist produces less than the efficient quantity, making the market price exceed the efficient market price.



Chap. 26

MONOPOLY BEHAVIOR

Introduction

- So far a monopoly has been thought of as a firm which has to sell its product at the same price to every customer (uniform pricing).
  - Can price-discrimination earn a monopoly higher profits?
- Price discrimination
  - selling different units of the same good at different prices, either to the same or different consumers
  - In order for price discrimination to be a viable strategy for the firm, the company must have the ability to sort consumers and to prevent resale

Introduction

- First-degree price discrimination
  - the price charged for each unit is equal to the max. willingness to pay for that unit.
  - Perfect discrimination
- Second-degree price discrimination
  - Prices differ depending on the number of units of the good bought, but not across consumers.
  - Nonlinear pricing. Quantity discounts or premium
- Third-degree price discrimination
  - Different purchasers are charged different prices, but each purchaser pays a constant amount for each unit of good bought.
  - Student discounts.

First-degree Price Discrimination

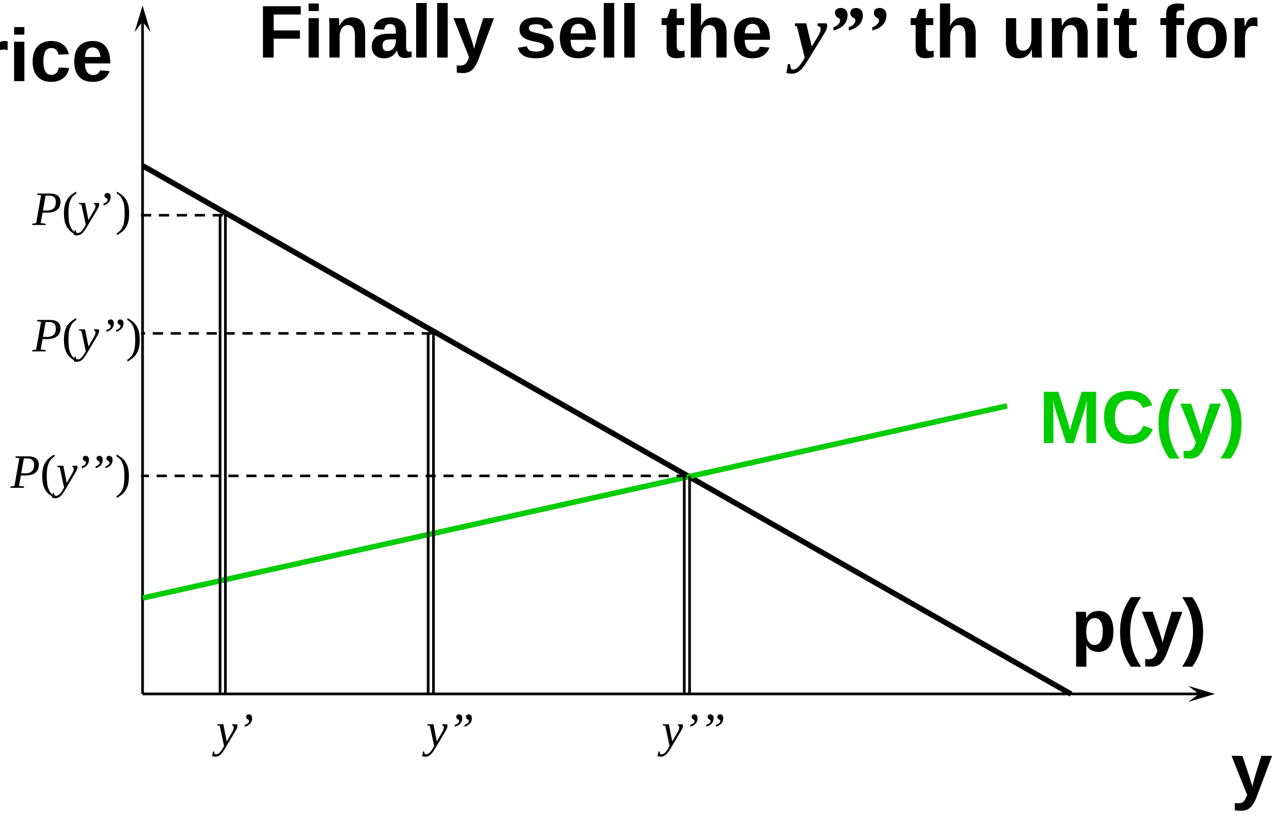
- Each output unit is sold at a different price. Price may differ across buyers.
- It requires that the monopolist can discover the buyer with the highest valuation of its product, the buyer with the next highest valuation, and so on.

First-degree Price Discrimination

Sell the *y'* th unit for $ *p*(*y'*)

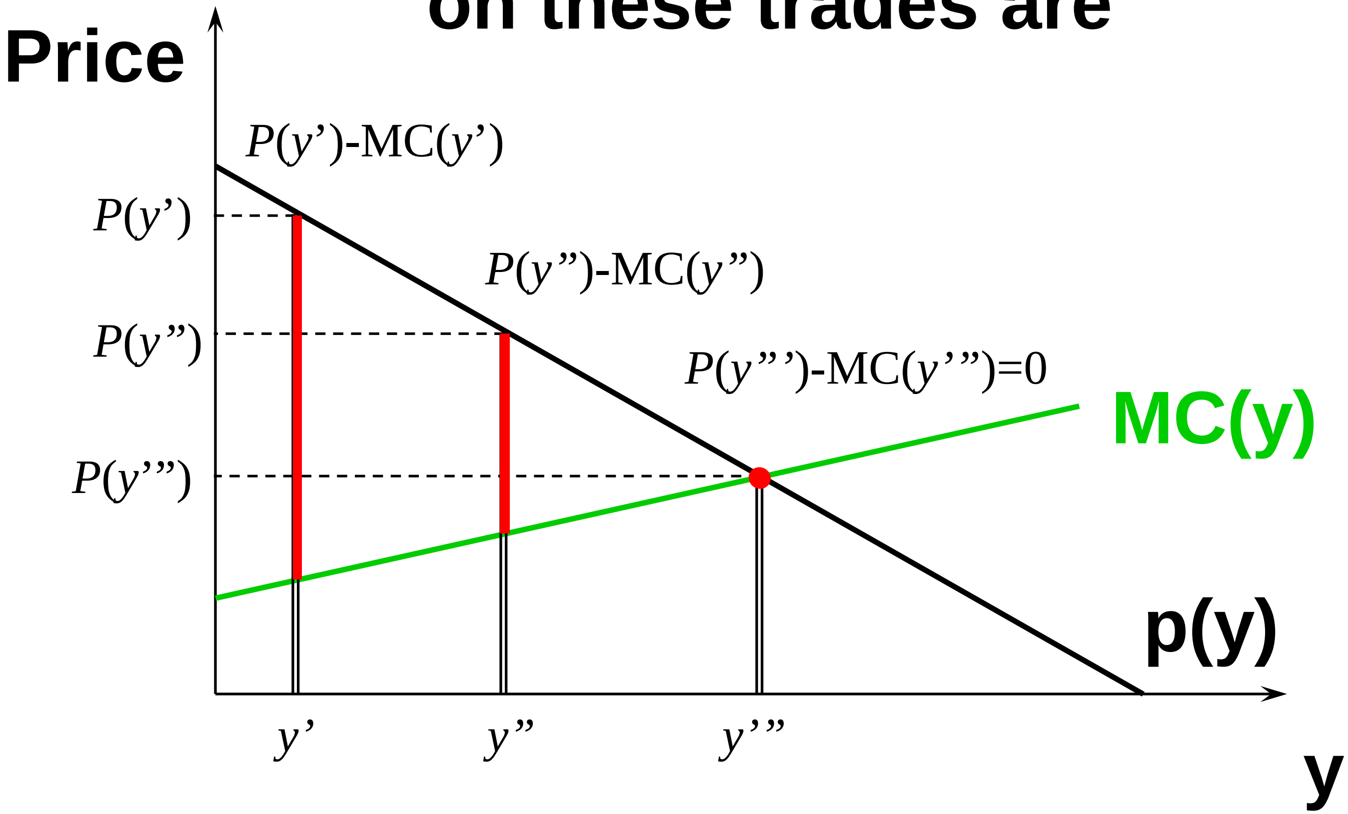
Later on sell the *y"* th unit for $ *p*(*y"*)

Finally sell the *y"'* th unit for $ *p*(*y'"*)



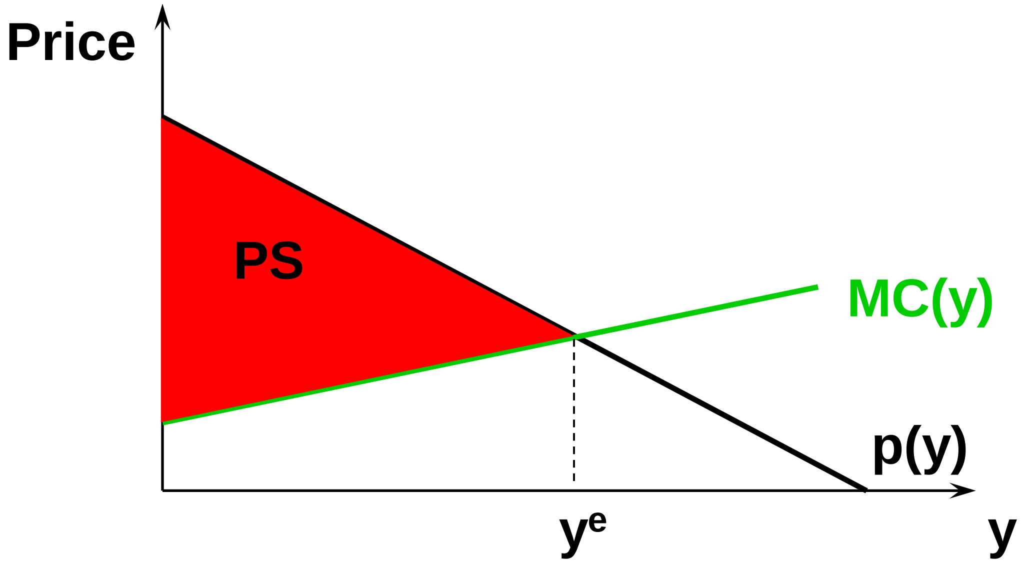
First-degree Price Discrimination

The gains to the monopolist on these trades are



Note that the consumers' gains are zero.

First-degree Price Discrimination



- The sum of the gains to the monopolist on all trades is the maximum possible total gains-to-trade
- First-degree price discrimination is Pareto-efficient.
- First-degree price discrimination gives a monopolist all of the possible gains-to-trade, leaves the buyers with zero surplus, and supplies the efficient amount of output.

Second-degree Price Discrimination

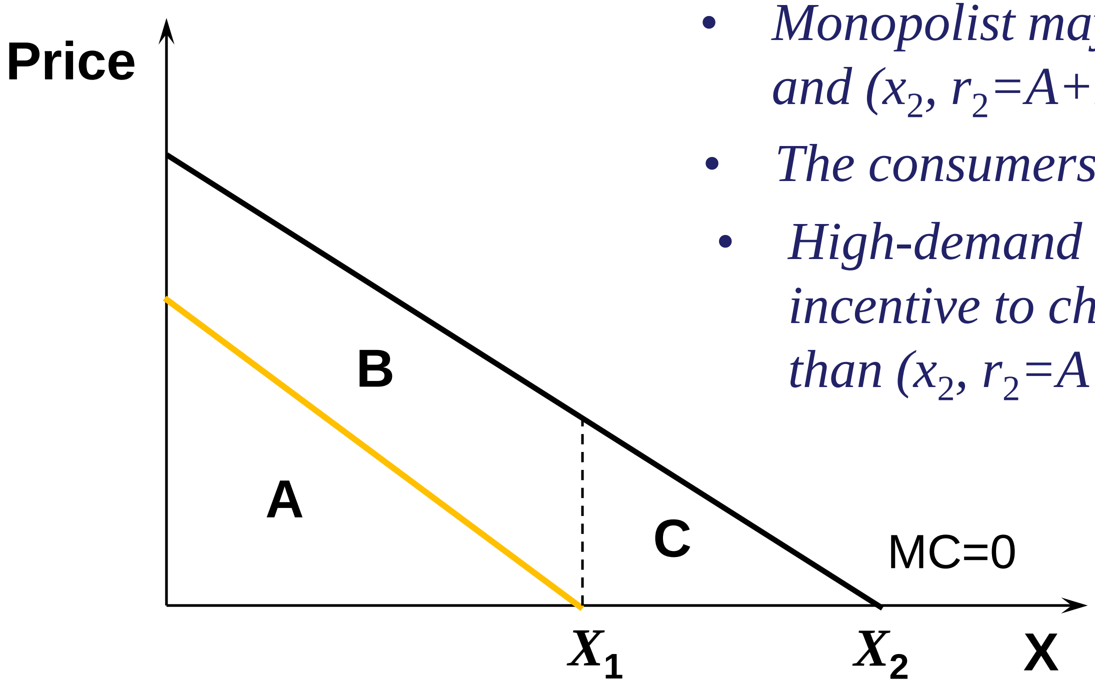
- Second-degree price discrimination
  - Prices differ depending on the number of units of the good bought, but not across consumers.
  - Nonlinear pricing. Quantity discounts or premium
- A monopolist produces the good *x*, and let all other goods be denoted by *y* (numeraire): $u(x)+y$
  - Suppose there are two consumers

    $$u_1(x_1),\ u_2(x_2) \quad \text{with } u_2(x) > u_1(x),\ u_2'(x) > u_1'(x) \ \text{ for all } x$$

    - consumer 1: low demand, consumer 2: high demand
  - Suppose that the monopolist chooses a nonlinear price function $p(x)$

Second-degree Price Discrimination

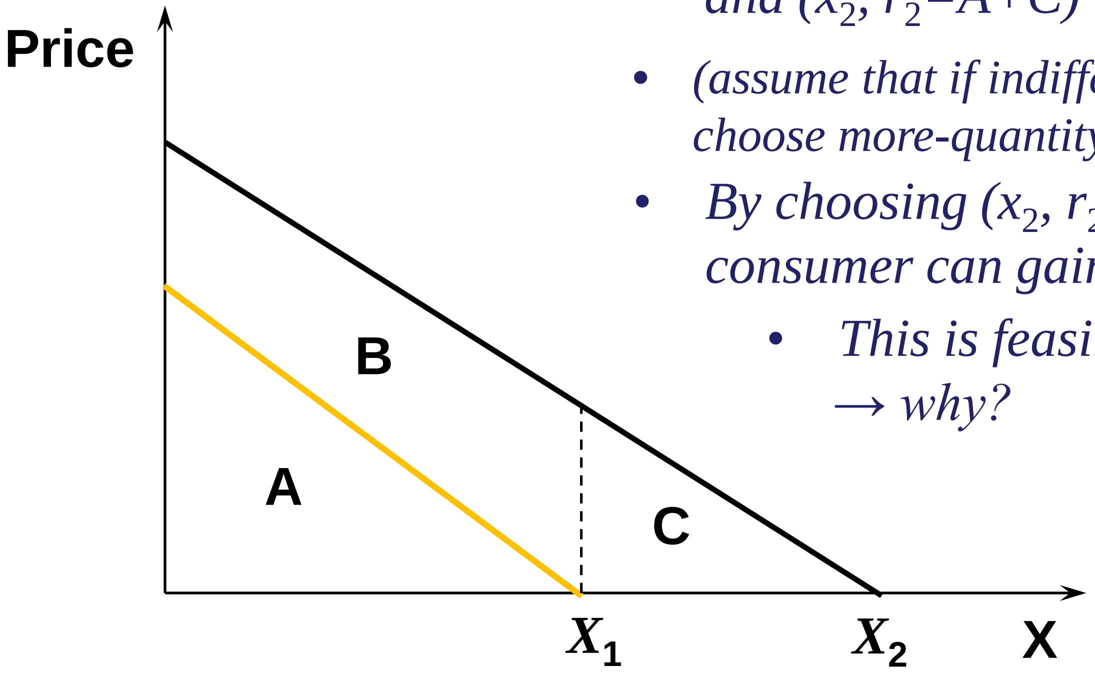
- If consumer i consumes x_i, let $r_i = p(x_i)x_i$
- Then the choice of the function $p(x_i)$ reduces the choice of the price schedule such that $\begin{cases} (r_1, x_1) & \text{for consumer 1} \\ (r_2, x_2) & \text{for consumer 2} \end{cases}$



- *Monopolist may want to offer (x_1, r_1=A) and (x_2, r_2=A+B+C)*
- *The consumers' gains are zero.*
- *High-demand consumer has an incentive to choose (x_1, r_1=A) rather than (x_2, r_2=A+B+C) since he gains B.*

Second-degree Price Discrimination

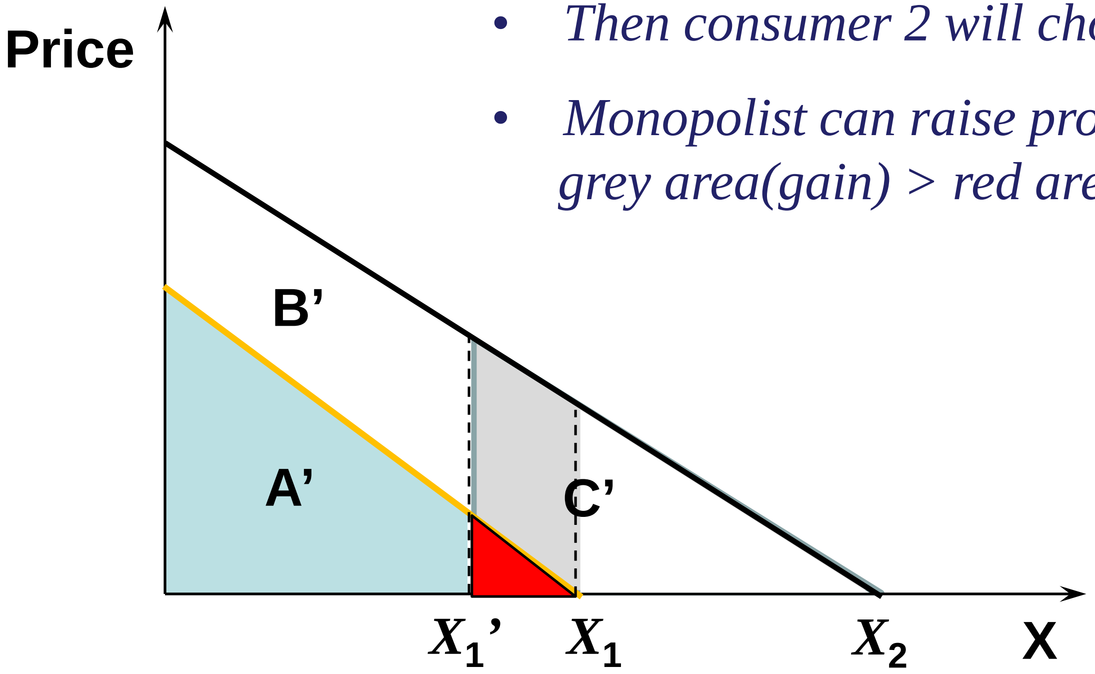
- Need to consider self-selection problem



- *To induce high-demand consumer to select x_2, it should be to offer (x_1, r_1=A) and (x_2, r_2=A+C)*
- *(assume that if indifferent, consumer will choose more-quantity option)*
- *By choosing (x_2, r_2=A+C), the high-demand consumer can gain B too.*
  - *This is feasible but not optimal ! → why?*

Second-degree Price Discrimination

- Need to find profit-maximizing price schedule

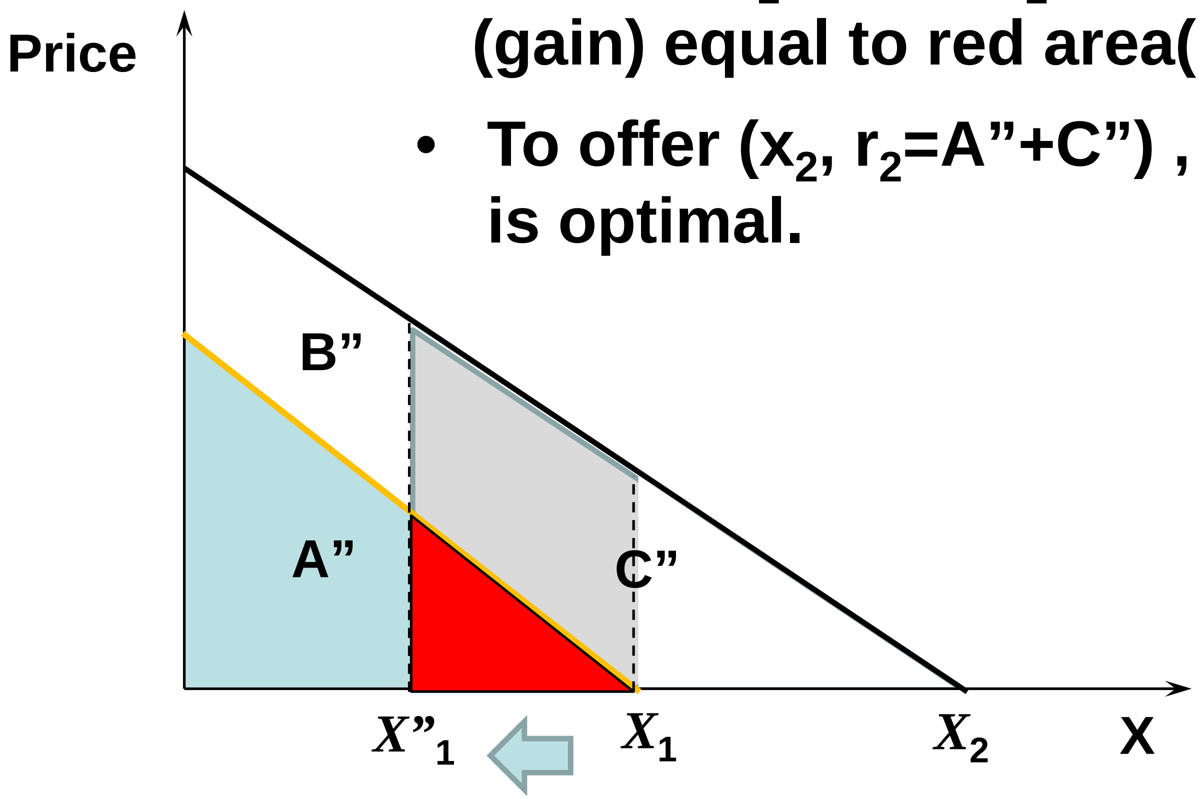


- *Offer (x_2, r_2=A'+C') and (x_1', r_1=A')*
- *Then consumer 2 will choose (x_2, r_2=A'+C')*
- *Monopolist can raise profits.*
  grey area(gain) > red area(loss)

Second-degree Price Discrimination

- Need to find profit-maximizing price schedule

- **Reduce x_1 until x''_1 where grey area (gain) equal to red area(loss).**
- **To offer (x_2, r_2=A''+C'') , (x''_1, r_1=A'') is optimal.**



Second-degree Price Discrimination

- If consumer i consumes x_i, let $r_i = p(x_i)x_i$
- the choice of the price schedule

$$\begin{cases} (r_1, x_1) & \text{for consumer 1} \\ (r_2, x_2) & \text{for consumer 2} \end{cases}$$

- Participation constraints

$$u_1(x_1) - r_1 \geq 0$$

$$u_2(x_2) - r_2 \geq 0$$

- Self-selection constraints

$$u_1(x_1) - r_1 \geq u_1(x_2) - r_2$$

$$u_2(x_2) - r_2 \geq u_2(x_1) - r_1$$

$$\Rightarrow \begin{cases} r_1 \leq u_1(x_1) & \& \quad r_1 \leq u_1(x_1) - u_1(x_2) + r_2 \\ r_2 \leq u_2(x_2) & \& \quad r_2 \leq u_2(x_2) - u_2(x_1) + r_1 \end{cases}$$

- Since the monopolist wants to choose r_1 and r_2 to be large as possible, one of the two inequalities will be binding!

Second-degree Price Discrimination

- As for r_2 first, suppose that $r_2 = u_2(x_2)$
- Then

$$r_2 \le r_2 - u_2(x_1) + r_1$$
$$\Rightarrow u_2(x_1) \le r_1$$
$$\Rightarrow u_1(x_1) < u_2(x_1) \le r_1$$

Contradicts to $r_1 \le u_1(x_1)$

$$\Rightarrow \quad r_2 = u_2(x_2) - u_2(x_1) + r_1$$

- As for r_1, suppose that $r_1 = u_1(x_1) - u_1(x_2) + r_2$
- Then

$$r_1 = u_1(x_1) - u_1(x_2) + \left(u_2(x_2) - u_2(x_1) + r_1\right)$$
$$\Rightarrow u_2(x_2) - u_2(x_1) = u_1(x_2) - u_1(x_1)$$
$$\Rightarrow \int_{x_1}^{x_2} u_2'(t)\,dt = \int_{x_1}^{x_2} u_1'(t)\,dt$$

Contradicts to $u_1'(x) < u_2'(x)$

$$\Rightarrow \quad r_1 = u_1(x_1)$$

Second-degree Price Discrimination

- Optimal price schedule must satisfy

$$r_1 = u_1(x_1), \quad r_2 = u_2(x_2) - u_2(x_1) + r_1$$

  - Low-demand consumer will be charged his max. WTP.
  - High-demand consumer will be charged the highest price that will induce him to choose (r_2, x_2) rather than (r_1, x_1) .

- The profit of the monopolist with constant marginal cost c

$$\begin{aligned} \pi &= [r_1 - cx_1] + [r_2 - cx_2] \\ &= [u_1(x_1) - cx_1] + [u_2(x_2) - u_2(x_1) + u_1(x_1) - cx_2] \end{aligned}$$

- To find x_1 and x_2 which maximize the profit

$$\frac{\partial \pi}{\partial x_1} = u_1'(x_1) - c - u_2'(x_1) + u_1'(x_1) = 0$$

$$\frac{\partial \pi}{\partial x_2} = u_2'(x_2) - c = 0$$

Second-degree Price Discrimination

- Optimal price schedule must satisfy

$$u_1'(x_1) = \left[u_2'(x_1) - u_1'(x_1)\right] + c$$
$$u_2'(x_2) = c$$

- Utility maximization of quasi-linear utility

$$\max_x \quad u_i(x) + y$$
$$s.t. \quad p(x)x + y = m$$
$$\Downarrow$$
$$\max_x \quad u_i(x) + m - px$$

$$\text{F.O.C.: } p = u_i'(x)$$

- Thus, low-demand consumer consumes an inefficiently small amount of the good,
- while the high-demand consumer consumes socially optimal amount!

Third-degree Price Discrimination

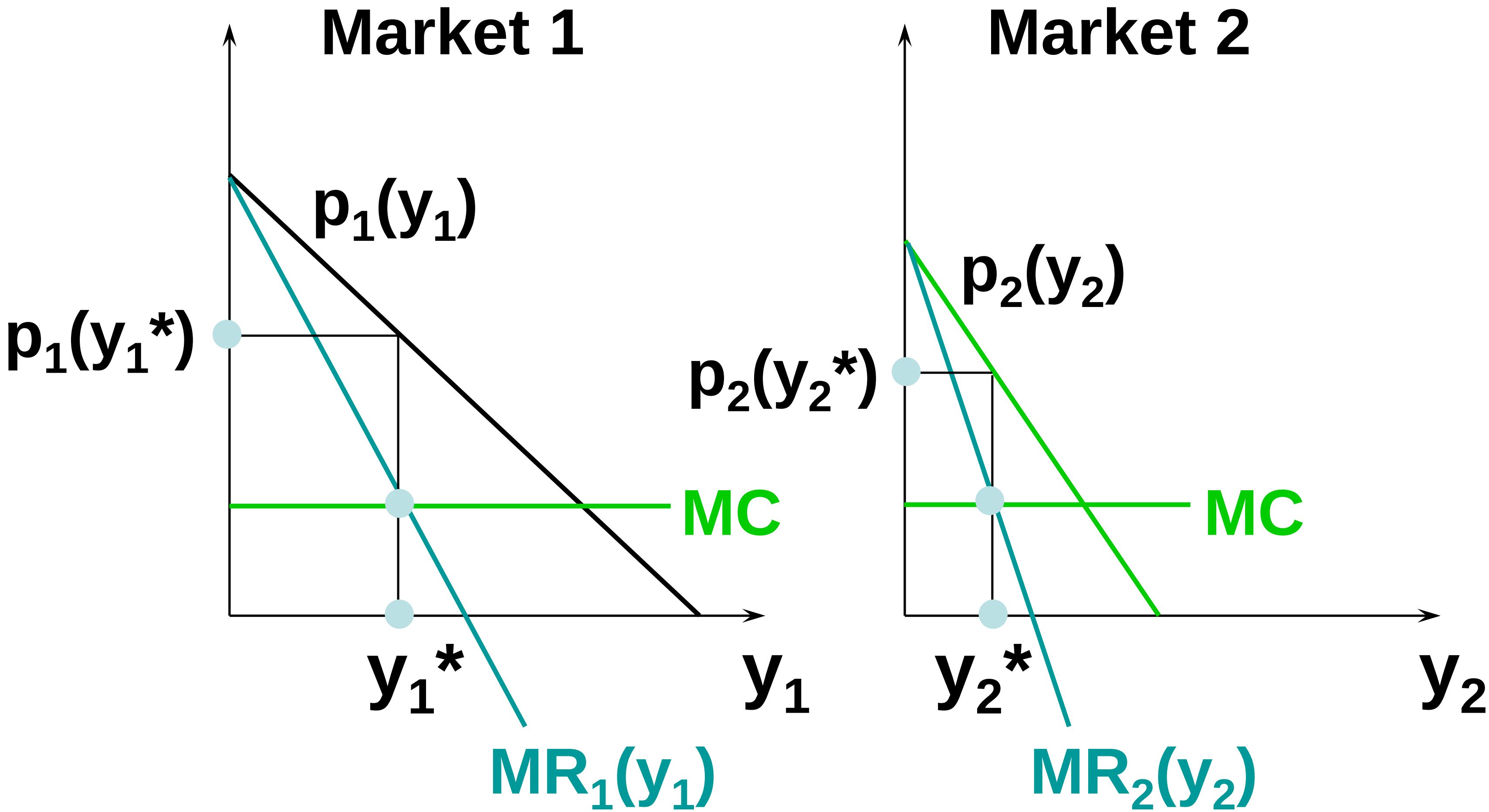
- Third-degree Price discrimination
  - When consumers are charged different prices, but each consumer faces a constant price for all units of output
- Suppose that there are two separate markets (by age, by time, by region…)
  - Let $p_i(x)$ be the inverse demand function for group i.
  - Then monopolist's profit-max. problem

$$\max_{x_1, x_2} p_1(x_1)x_1 + p_2(x_2)x_2 - cx_1 - cx_2$$

  - F.O.C.

$$p_1(x_1) + p_1'(x_1) \cdot x_1 = c$$
$$p_2(x_2) + p_2'(x_2) \cdot x_2 = c$$

Third-degree Price Discrimination



$MR_1(y_1^*) = MR_2(y_2^*) = MC$ but $p_1(y_1^*) \neq p_2(y_2^*)$.

Third-degree Price Discrimination

- Let ε_i be the elasticity of demand in market *i*.
- F.O.C.

$$p_1(x_1)\left[1-\frac{1}{|\varepsilon_1|}\right]=c$$

$$p_2(x_2)\left[1-\frac{1}{|\varepsilon_2|}\right]=c$$

- Note that

$$p_1(x_1)\geq p_2(x_2) \text{ iff } |\varepsilon_1|<|\varepsilon_2|$$

- Thus, at the third-degree price discrimination, the market with the more elastic demand is charged the lower price.