Lecture 7:

Device examples (2): electrothermal transduction

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By Joule's law, $I^2 R_x = Q_x$, $I^2 \frac{\Delta x}{\sigma_i A_i} = -\left(k_i A_i \left.\frac{dT}{dx}\right|_{x+\Delta x} - k_i A_i \left.\frac{dT}{dx}\right|_{x} \right)$

therefore, we get $\frac{I^2}{\sigma_i A_i} = -k_i A_i \frac{d^2 T}{dx^2}$

then if we integral both side, $-k_i A_i \frac{dT}{dx} = \frac{I^2}{\sigma_i A_i} x + a$ --- (1st integral)

$-k_i A_i T(x) = \frac{I^2}{\sigma_i A_i} \frac{x^2}{2} + ax + b$ --- (2nd integral)

when $x = 0$, $T = T_c$ and $x = l$, $T = T_h$

so $\therefore b = -k_i A_i T_c$, $a = -\frac{k_i A_i (T_h - T_c)}{l} - \frac{l}{2} \frac{I^2}{\sigma_i A_i}$

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$R = \frac{l_1}{\sigma_1 A_1} + \frac{l_2}{\sigma_2 A_2}$, $K = \frac{k_1 A_1}{l_1} + \frac{k_2 A_2}{l_2}$

$RK = \left(\frac{l_1}{\sigma_1 A_1} + \frac{l_2}{\sigma_2 A_2} \right) \cdot \left(\frac{k_1 A_1}{l_1} + \frac{k_2 A_2}{l_2} \right) = \frac{k_1}{\sigma_1} + \frac{k_2}{\sigma_2} + \frac{k_1}{\sigma_2} \frac{l_2}{l_1} \frac{A_1}{A_2} + \frac{k_2}{\sigma_1} \frac{l_1}{l_2} \frac{A_2}{A_1}$

$\geq \frac{k_1}{\sigma_1} + \frac{k_2}{\sigma_2} + 2\sqrt{\frac{k_1}{\sigma_2} \frac{k_2}{\sigma_1}}$

when $\frac{k_1}{\sigma_2} \frac{l_2}{l_1} \frac{A_1}{A_2} = \frac{k_2}{\sigma_1} \frac{l_1}{l_2} \frac{A_2}{A_1}$

that is $\left(\frac{l_2}{l_1} \frac{A_1}{A_2} \right)^2 = \frac{k_2 / \sigma_1}{k_1 / \sigma_2}$, so we have $\underline{\frac{A_1 / l_2}{A_2 / l_1} = \sqrt{\frac{k_2 / \sigma_1}{k_1 / \sigma_2}} = \sqrt{\frac{k_2 \sigma_2}{k_1 \sigma_1}}}$