7. Analysis and Design for Torsion

INTRODUCTION

TORSION IN PLAIN CONCRETE MEMBERS

TORSION IN REINFORCED CONCRETE MEMBERS

TORSION PLUS SHEAR

KCI CODE PROVISIONS FOR TORSION DESIGN

447.328
Theory of Reinforced Concrete and Lab. II
Fall 2007

7. Analysis and Design for Torsion

INTRODUCTION

- Reinforced concrete members are subjected to
  - bending moment
  - transverse shear
  - axial force
  - torsion : seldom acts alone
- In the past, torsion effect was embedded in the overall CONSERVATIVE factor of safety

  ⇦ Not considered explicitly in design

7. Analysis and Design for Torsion

INTRODUCTION

- Current design/analysis methods result in less conservatism leading to that small members must be REINFORCED to avoid torsional failure.
- Change of design method is based on

  i) advanced structural analysis and precise design result in reduction of safety factor and smaller cross section.

  ii) Increasing use of structural members of which behavior is torsion-dominant.

7. Analysis and Design for Torsion

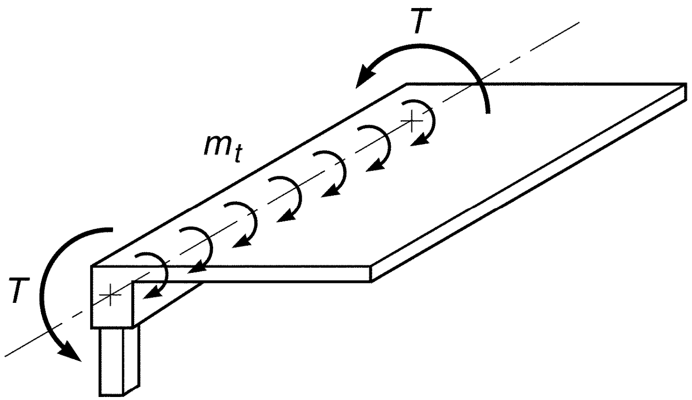
⇨ curved bridge girders, eccentrically loaded box girder, helical stairway slabs

7. Analysis and Design for Torsion

INTRODUCTION

I. Primary torsion : equilibrium torsion,
statically determinate torsion

⇦ The external load has no alternative load path but must be supported by torsion.



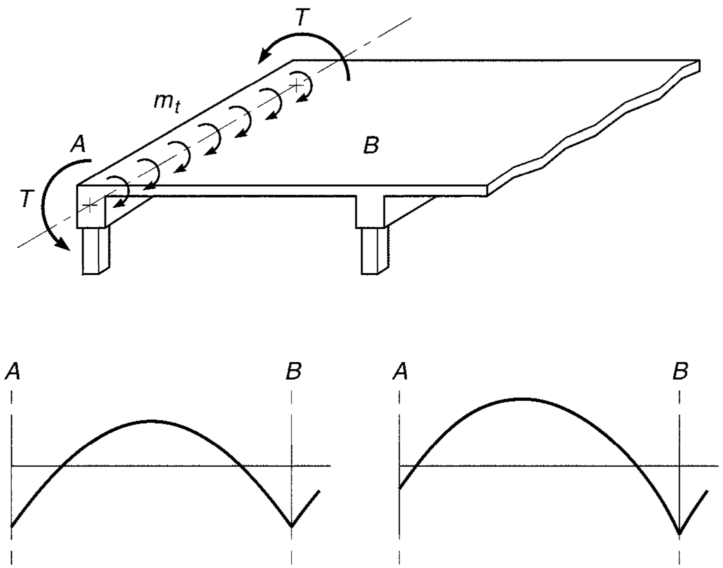
<cantilever slab>

Load applied to the slab surface which cause twisting moment m_t are equilibrated by the resisting torque T provided by columns.

7. Analysis and Design for Torsion

II. Secondary torsion : compatibility torsion,
statically indeterminate torsion

⇦ compatibility of deformation between adjacent parts of a structure.



<spandrel slab>

Torsional moment cannot be obtained based on static equilibrium alone.

i) stiff and suitably reinforced edge beam

ii) less stiff and inadequately reinforced edge beam.

7. Analysis and Design for Torsion

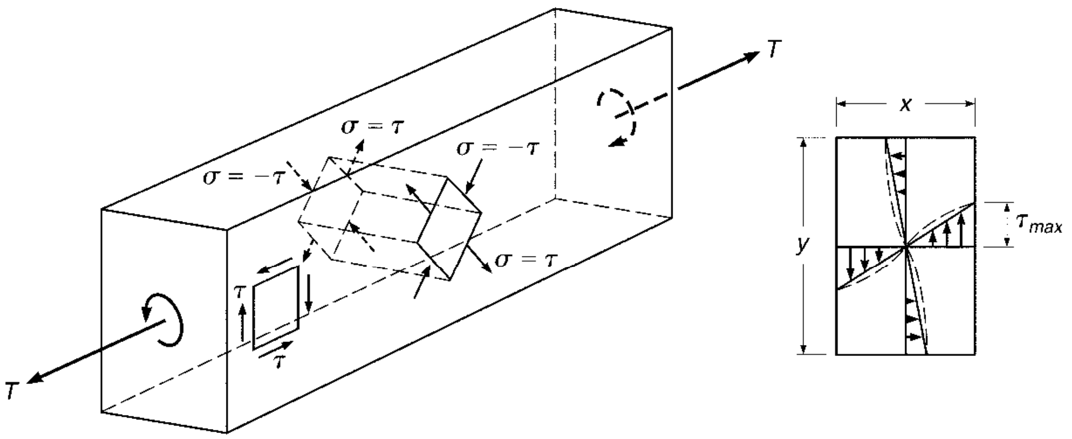
INTRODUCTION

- Current techniques for analysis permit the realistic evaluation of torsional moments even for statically indeterminate conditions.

  ⇨ But, secondary torsional effects are often neglected when torsional stresses are low and alternative equilibrium states are possible.

7. Analysis and Design for Torsion

TORSION IN PLAIN CONCRETE MEMBERS



– St. Venant torsion theory

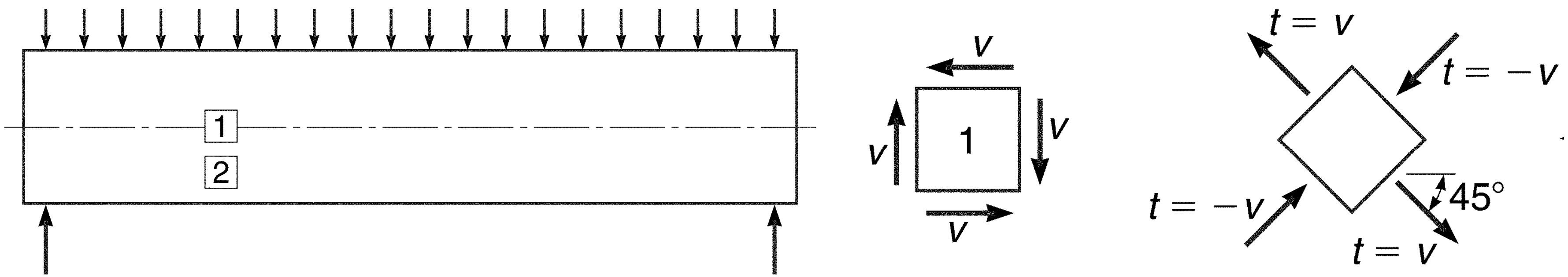
– The largest shear stress occurs at the wide faces.

– In reality, the stress distribution is not linear

7. Analysis and Design for Torsion

TORSION IN PLAIN CONCRETE MEMBERS

Comparison with Transverse Shear



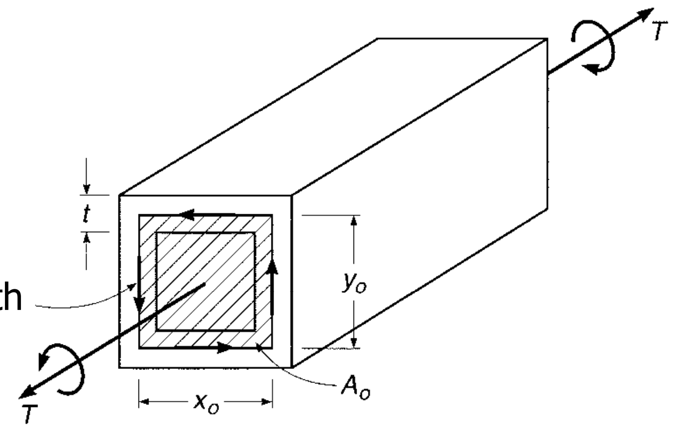
Inclined tension stresses due to torsional shear are of the same kind as those cause by transverse shear.

BUT, the torsional shear stresses are opposite sign. ⇨ corresponding diagonal tension stresses are at right angles to each other.

TORSION IN PLAIN CONCRETE MEMBERS

Thin-Walled Tube, Space Truss Analogy

Representative analyzing method of members subjected to torsion.



Assumptions

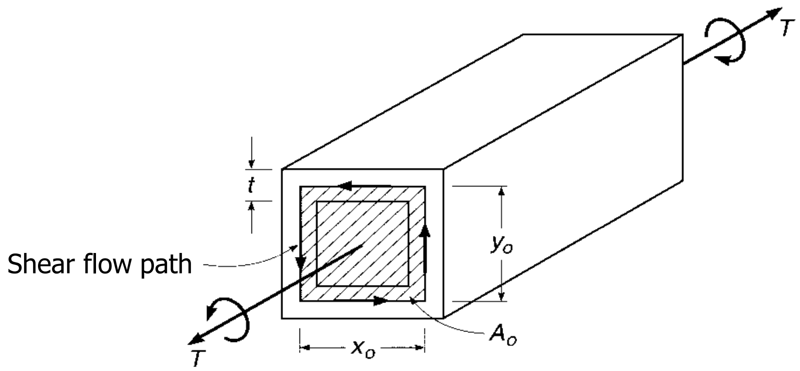
- Shear stresses are treated as a CONSTANT over a finite thickness *t*.
- Beam member is regarded as an equivalent TUBE.

7. Analysis and Design for Torsion

Relation between the applied torque and the shear flow

summing the moments about the axial centerline of the tube.

$$T = \underbrace{2qx_0 \frac{y_0}{2}}_{\text{contribution of the horizontal walls to the resisting torque.}} + \underbrace{2qy_0 \frac{x_0}{2}}_{\text{contribution of the vertical walls to the resisting torque.}} \qquad (1)$$



where q is the shear flow (force per unit length)

$$T = 2qx_0 y_0 \qquad (2)$$

$$= 2qA_0 \qquad (3)$$

where A_0 is the shear flow path

Q) In case of hollow box section, A_0?

7. Analysis and Design for Torsion

The unit shear stress acting within the walls of tube

$$\tau = \frac{q}{t} = \frac{T/2A_0}{t} = \frac{T}{2A_0 t} \qquad (4)$$

Observations

1. As shown in previous figure, the PRINCIPAL tensile stress $\sigma = \tau$
2. Concrete will crack when $\tau = \sigma = f_t = 0.33\sqrt{f_{ck}}$

   Q) why not the modulus of rupture $f_r = 0.65\sqrt{f_{ck}}$

Cracking torque T_{cr} is

$$T_{cr} = \tau_{cr} 2A_0 t = \frac{1}{3}\sqrt{f_{ck}}(2A_0 t) \qquad (5)$$

7. Analysis and Design for Torsion

TORSION IN PLAIN CONCRETE MEMBERS

Thin-Walled Tube, Space Truss Analogy

Tube thickness t

- Recalling that A_0 is the area enclosed by the shear flow path, A_0 must be some fraction of the area A_{cp}, which is the full area of cross section.
- Tube thickness t can, in general, be approximated as a fraction of the ratio A_{cp}/p_{cp}, where p_{cp} is the perimeter of the cross section

7. Analysis and Design for Torsion

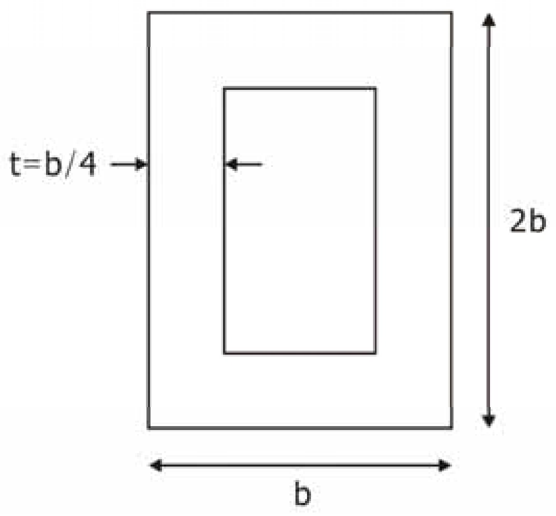
Example – Solid member with rectangular cross section

i) t is typically 1/6 ~ 1/4 of the minimum width.

ii) using 1/4 for a member with a width-to-depth ratio 0.5,

$$A_0 \approx \frac{2}{3} A_{cp}$$

iii) for the same member $t = \frac{3}{4} A_{cp} / p_{cp}$



$$A_0 = \left(b - \frac{1}{4} b \right) \left(2b - \frac{1}{4} b \right) = \left(\frac{3}{4} b \right) \left(\frac{7}{4} b \right) = \frac{21}{16} b^2$$

$$A_{cp} = 2b^2 \quad \Rightarrow \quad A_0 = \frac{21}{16} b^2 \approx \frac{2}{3} \left(2b^2 \right) = \frac{2}{3} A_{cp}$$

$$t = \frac{b}{4} = \frac{3}{4} \left(\frac{2b^2}{6b^2} \right) = \frac{3}{4} \frac{A_{cp}}{p_{cp}}$$

7. Analysis and Design for Torsion

Example (cont.)

$$\Rightarrow \quad T_{cr} = \frac{1}{3}\sqrt{f_{ck}}\left[2\left(\frac{2}{3}A_{cp}\right)\left(\frac{3}{4}\frac{A_{cp}}{P_{cp}}\right)\right]$$

$$= \frac{1}{3}\sqrt{f_{ck}}\frac{A_{cp}^2}{p_{cp}} \qquad (6)$$

; It has been found that Eq.(6) gives a reasonable estimate of the cracking torque of SOLID reinforced concrete members REGARDLESS of the cross-sectional shape.

cf.) For hollow sections with the gross section A_g

$$T_{cr} = \frac{1}{3}\sqrt{f_{ck}}\frac{A_{cp}^2}{p_{cp}}\frac{A_g}{A_{cp}} \qquad (7)$$

7. Analysis and Design for Torsion

TORSION IN REINFORCED CONCRETE MEMBERS

In case of $T \geq T_{cr}$, stirrups and longitudinal bars are needed.

- longitudinal bar alone : at most 15% improvement of torsional strength

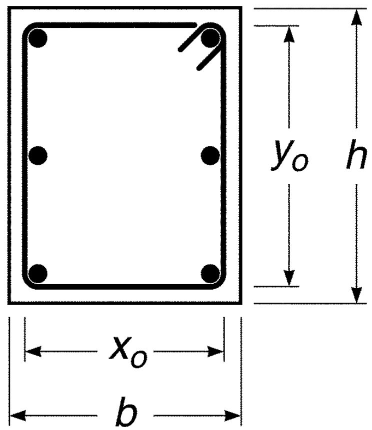
⇦ only dowel action

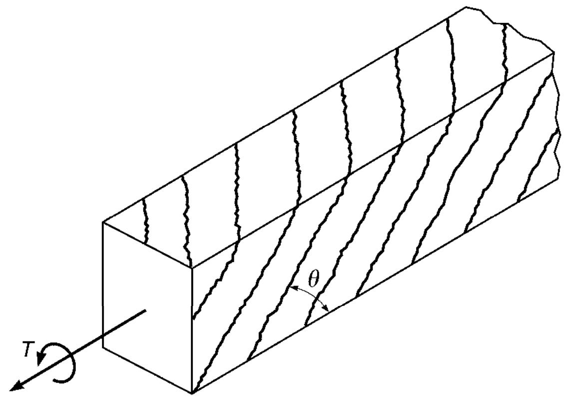
⇨ Eqs.(6)&(7) are satisfactory and somewhat conservative

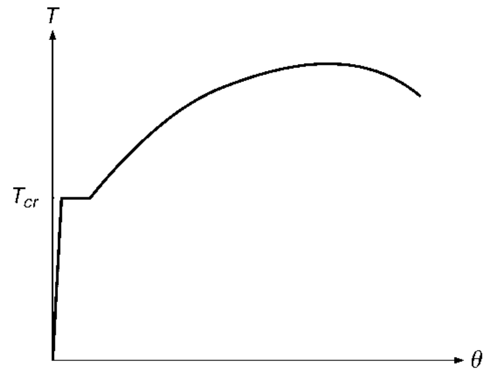
, which is particularly weak and unreliable if longitudinal splitting along bars is not restrained by transverse reinforcement.

7. Analysis and Design for Torsion

TORSION IN REINFORCED CONCRETE MEMBERS







- Torsional cracks initiate at a torque that is equal or only somewhat larger than that in an unreinforced members.
- Spiral crack patterns
- Upon cracking, torsional resistance of the concrete drops to about 50%
- Torque-twist curve shows a plateau. *(yielding?)*

7. Analysis and Design for Torsion

TORSION IN REINFORCED CONCRETE MEMBERS

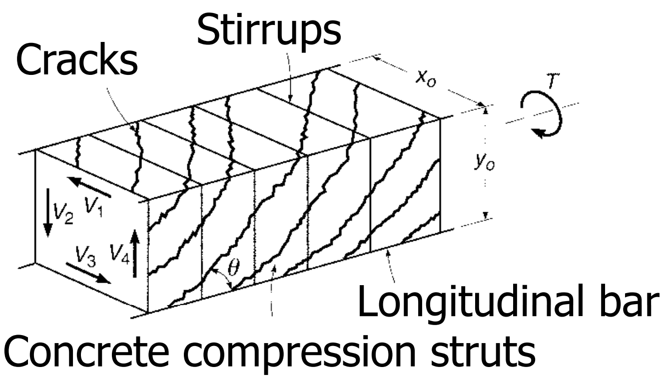
Space Truss Analogy

treats the member as a SPACE TRUSS consisting of

– spiral *concrete diagonals*

– transverse *tension tie members* ; closed stirrups and ties

– *tension chords* ; longitudinal reinforcing bars.



TORSION IN REINFORCED CONCRETE MEMBERS

Space Truss Analogy

The hollow-tube, space truss analogy represents a simplification of actual behavior.

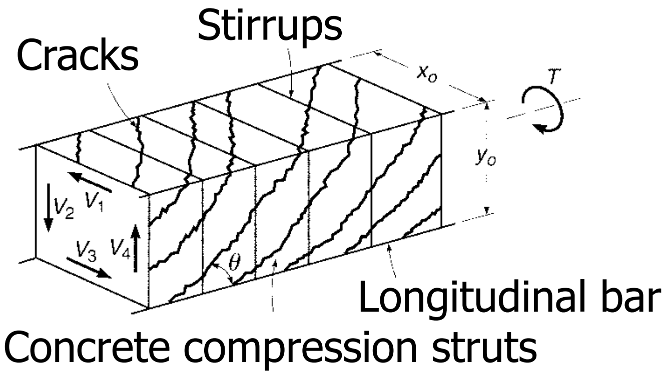
Limitations

1. Calculations of torsional strength is controlled only by the strength of reinforcement, independent of concrete strength.
   ⇨ does not reflect the effect of higher concrete strength.
2. Greatly underestimates torsional capacity

TORSION IN REINFORCED CONCRETE MEMBERS

Space Truss Analogy

- Torsional resistance can be represented as the sum of the contributions of the shears in each of the FOUR walls of the equivalent hollow tube.



The contribution of the right-hand vertical wall is

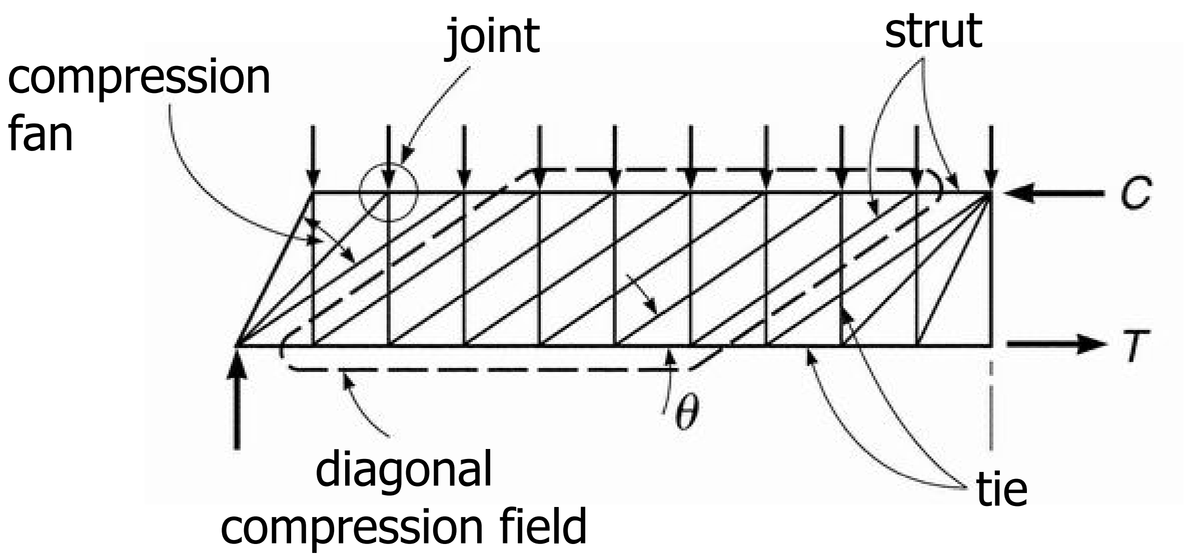
$$T_4 = \frac{V_4 x_0}{2} \qquad (8)$$

<Review>

4. Shear & Diagonal Tension in Beams

Variable Angle Truss Model

- Recently the truss concept has been greatly extended by Schlaich, Thurlimann, Marti, Collins, MacGregor, etc.
- It was realized that the angle of inclination of the concrete strut may range between 25° and 65°



<Review>

4. Shear & Diagonal Tension in Beams

Variable Angle Truss Model

Improved model components

1) Strut or concrete compression member uniaxially loaded.
2) Ties or steel tension member
3) Pin connected joint at the member intersection
4) Compression fans, which forms at the supports or under concentrated loads, transmitting the forces into the beam.
5) Diagonal compression field, occurring where parallel compression struts transmit forces from one stirrup to another.

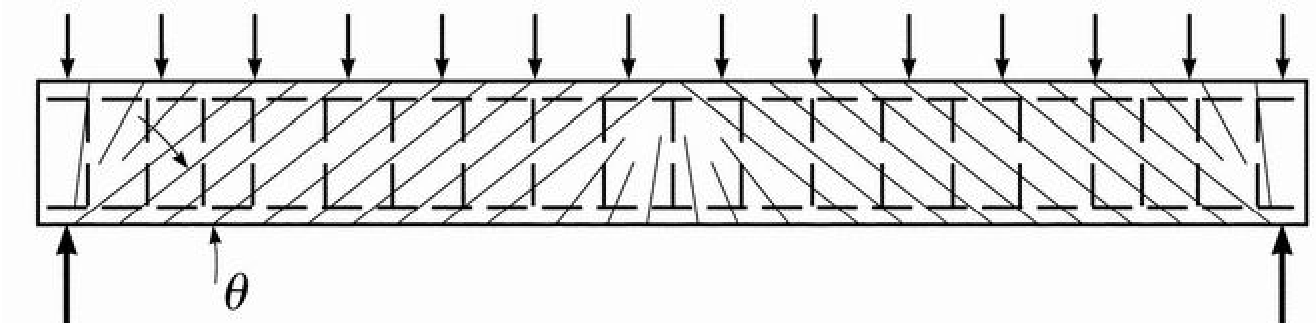
4. Shear & Diagonal Tension in Beams

Compression Field Theory

- is mandatory for shear design in AASHTO LRFD Bridge Design Specification of the U.S.

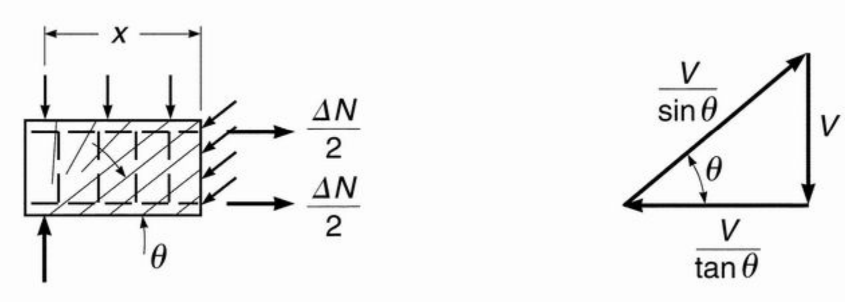
- accounts for requirements of compatibility as well as equilibrium and incorporates stress-strain relationship of material.

⇨ can predict not only the failure load but also the complete load-deformation response.



Compression Field Theory

- the net shear V at a section a distance x from the support is resisted by the vertical component of the diagonal compression force in the concrete struts.



- The horizontal component of the compression in the struts must be equilibrated by the total tension force ΔN in the longitudinal steel.

$$\Delta N = \frac{V}{\tan\theta} = V\cot\theta \qquad (25)$$

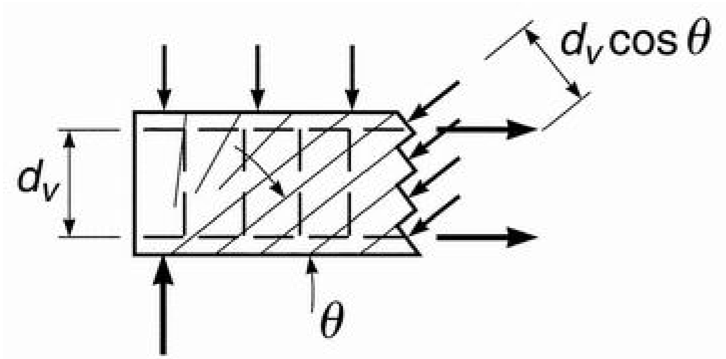
⇨ These forces superimpose on the longitudinal forces due to flexure.

<Review>

4. Shear & Diagonal Tension in Beams

Compression Field Theory

- letting the effective depth for shear calculation d_v, the distance between longitudinal force resultants.



- The diagonal compressive stress in a web having b_v is,

$$f_d = \frac{V}{b_v d_v \sin\theta \cos\theta} \tag{26}$$

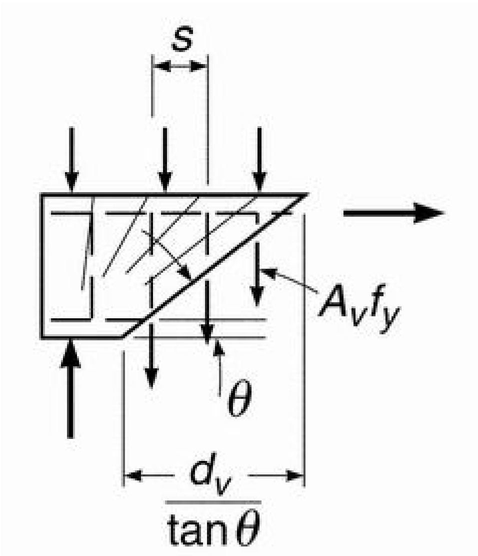
$$\Leftarrow \quad \frac{1}{\sin\theta} = d_v \cos\theta \cdot f_d \cdot b_v$$

4. Shear & Diagonal Tension in Beams

Compression Field Theory

- The tensile force in the vertical stirrups, each having area A_v and assumed to act at the yield stress f_y and uniformly spaced at s,

$$A_v f_y = \frac{V \cdot s \cdot \tan\theta}{d_v} \qquad (27)$$



4. Shear & Diagonal Tension in Beams

Compression Field Theory

Note

1. Vertical stirrups within the length $d_v/\tan\theta$ can be designed to resist the lowest shear that occurs within this length, i.e., the shear at the right end.
2. The angle θ range from 20° to 75°, but it is economical to use an angle θ somewhat less than 45°.
3. If a lower slope angle is selected, less vertical reinforcement but more longitudinal reinforcement will be required, and the compression in the concrete diagonals will be increased.

<Review>

4. Shear & Diagonal Tension in Beams

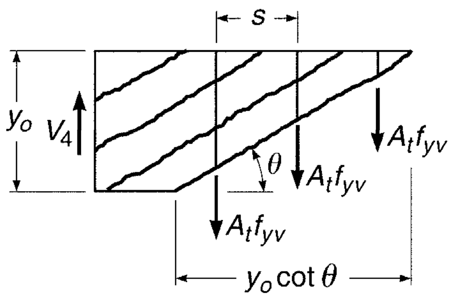
Modified Compression Field Theory *(Handout 4-2)*

- The cracked concrete is treated as a new material with its own stress-strain relationships including the ability to carry tension following crack formation.
- As the diagonal tensile strain in the concrete INCREASES, the compressive strength and s-s curve of the concrete in the diagonal compression struts DECREASES.
- Equilibrium Compatibility, Constitutive relationship are formulated in terms of average stress and average strains.
- Variability of inclination angle and stress-strain softening effects are considered.

TORSION IN REINFORCED CONCRETE MEMBERS

Space Truss Analogy

- The equilibrium of a section of the vertical wall - with one edge parallel to a torsional crack with angle θ - can be evaluated using the following figure



- Assuming the stirrups crossing the crack are yielding, the shear in the wall

$$V_4 = \underline{A_t} f_{yv} n \qquad (9)$$

area of one leg of a closed stirrup

7. Analysis and Design for Torsion

Space Truss Analogy

- Eq.(9) can be rewritten as,

$$V_4 = \frac{A_t f_{yv} y_0}{s} \cot\theta \qquad (10)$$

- Combining Eq.(10) and (8)

$$T_4 = \frac{A_t f_{yv} y_0 x_0}{2s} \cot\theta \qquad (11)$$

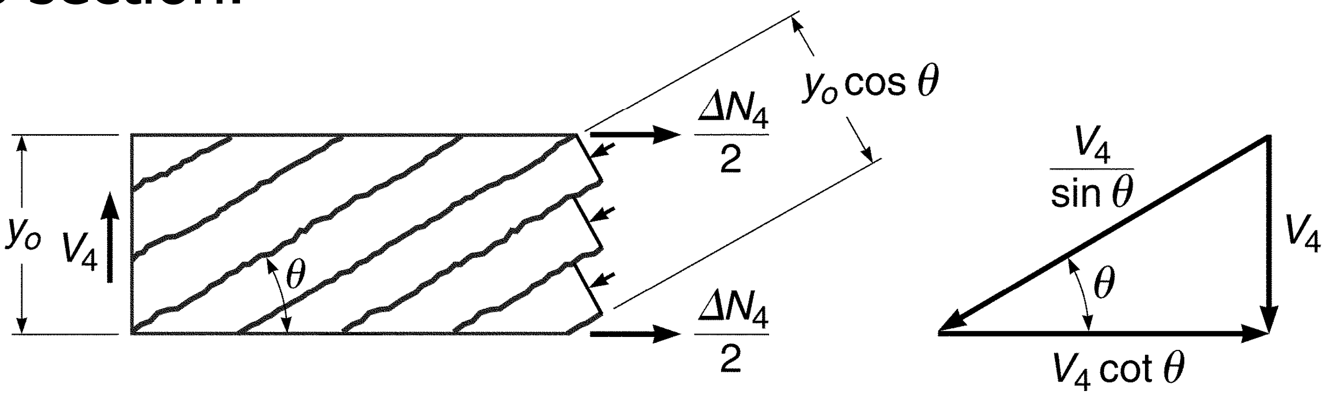
- In the same manner for four wall, identical expressions can be obtained. Finally, summing over all four sides, the nominal torsional capacity of the section is

$$T_n = \sum_{i=1}^{4} T_i = \frac{2A_t f_{yv} y_0 x_0}{s} \cot\theta = \frac{2A_{oh} A_t f_{yv}}{s} \cot\theta \qquad (12)$$

7. Analysis and Design for Torsion

Space Truss Analogy

- The diagonal compression struts that form parallel to the torsional cracks are necessary for the equilibrium of the cross section.



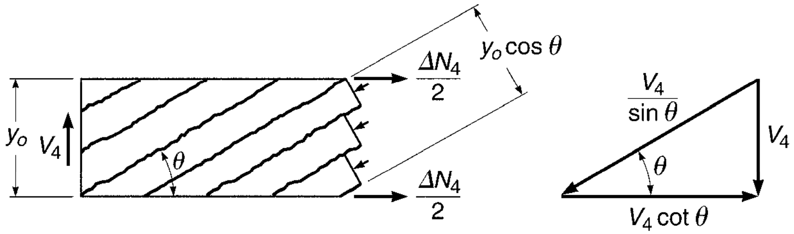
i) the horizontal component of compression in the struts must be equilibrated by an axial tensile for ΔN_4

ii) the diagonal stresses in the struts must be uniformly distributed.

7. Analysis and Design for Torsion

Space Truss Analogy

- The total contribution of the right-hand vertical wall to the change in axial force due to the TORSION.



$$\Delta N_4 = V_4 \cot\theta = \frac{A_t f_{yv} y_0}{s} \cot^2\theta \qquad (13)$$

The total increase in axial force for the member is

$$\Delta N = \sum_{i=1}^{4} \Delta N_i = \frac{A_t f_{yv}}{s} 2(x_0 + y_0) \cot^2\theta \qquad (14)$$

$$= \frac{A_t f_{yv} p_h}{s} \cot^2\theta \qquad (15)$$

7. Analysis and Design for Torsion

- Longitudinal reinforcement must be provided to carry the increased axial force ΔN

$$A_l f_{yl} = \frac{A_t f_{yv} p_h}{s} \cot^2 \theta \qquad (16)$$

$$\Rightarrow \quad A_l = \frac{A_t}{s} p_h \frac{f_{yv}}{f_{yl}} \cot^2 \theta \qquad (17)$$

<u>Notes</u>

1. After cracking, the effective area enclosed by the shear flow path is somewhat less than A_{oh}.
2. KCI adopted $A_0=0.85A_{oh}$ instead of A_{oh} , where A_{oh} is the area enclosed by the centerline of the transverse reinforcement.
3. The thickness of the equivalent tube at loads near ultimate is approximated by $t=A_{oh}/p_h$, p_h is perimeter of A_{oh}.

TORSION PLUS SHEAR

- Member are rarely subjected to torsion alone.

  ⇨ Bending moment+Transverse Shear+Torsional moment

- Uncracked member,

  Shear forces and torque produce shear stresses.

  Cracked member,

  Both shear and torsion increase the forces in the diagonal struts.

  ⇨ increase the width of diagonal crack

  ⇨ increase the forces required in the transverse reinforcement.

TORSION PLUS SHEAR

- The nominal shear stress caused by

Shear force V is $\tau_v = \dfrac{V}{b_w d}$ (18)

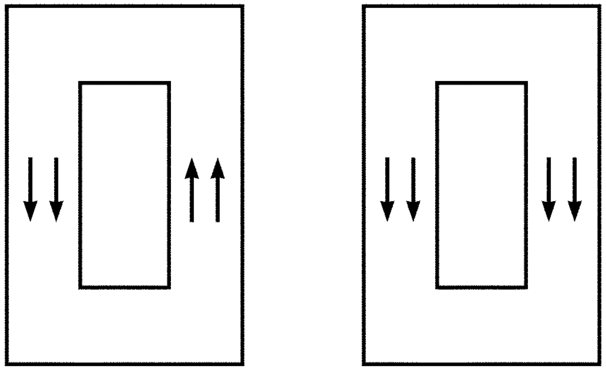
Torsion T is $\tau_t = \dfrac{T}{2A_0 t}$ (19)

- The Maximum shear stress

for hollow section

$$\tau = \tau_v + \tau_t = \frac{V}{b_w d} + \frac{T p_h}{1.7 A_{oh}^2} \quad (20)$$

where, $A_0 = 0.85 A_{oh}$ and $t = \dfrac{A_{oh}}{p_h}$



Torsional stresses

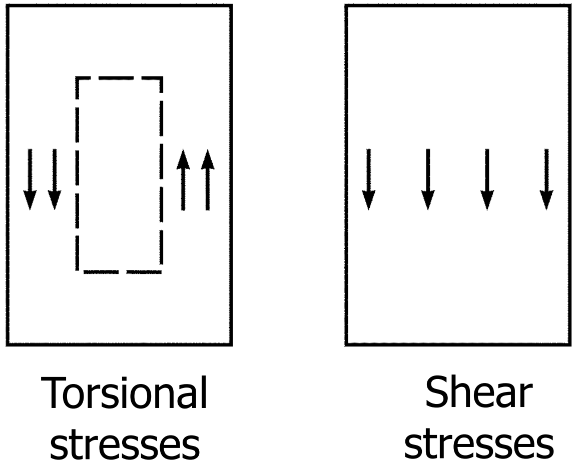
Shear stresses

TORSION PLUS SHEAR

for solid section

τ_t is predominately distributed around the perimeter, as represented by the hollow tube analogy.

Experimental result indicates that eq.(17) is somewhat conservative



$$\tau = \sqrt{\left(\frac{V}{b_w d}\right)^2 + \left(\frac{T p_h}{1.7 A_{oh}^2}\right)^2} \qquad (21)$$

Eqs.(18) & (19) are valid for service and ultimate stage.

7. Analysis and Design for Torsion

KCI CODE PROVISIONS FOR TORSION

KCI Code provisions (7.5) are based on the thin wall tube, space truss analogy.

KCI Code 7.6.2 requires that

$$T_u \le \phi T_n \tag{22}$$

The strength reduction factor ϕ=0.80, but 0.85 for precast members.

T_n is based on Eq.(12) with A_0 substituted for A_{oh}

$$T_n = \frac{2A_0 A_t f_{yv}}{s}\cot\theta \tag{23}$$

7. Analysis and Design for Torsion

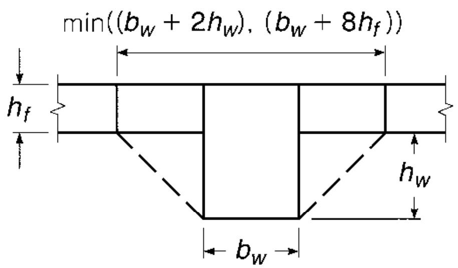
KCI CODE PROVISIONS FOR TORSION

T Beams and Box Sections

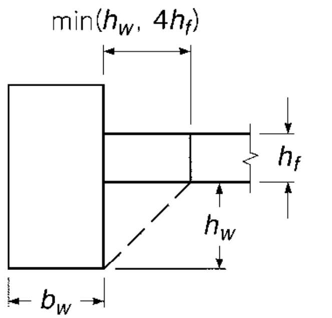
- The contributing width of the overhanging flange on either side of the web is equal to the smaller of (KCI Code 7.6.2)

  i) the projection of the beam above or below the slab, whichever is greater.

  ii) four times the slab thickness.



<Symmetric slab>

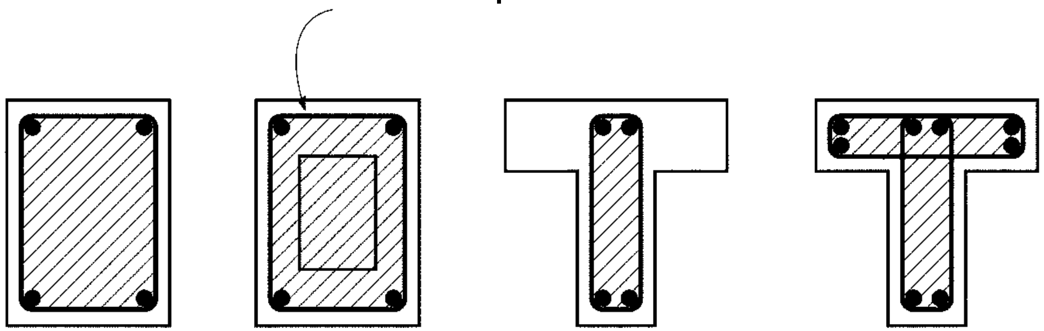


<Single side slab>

KCI CODE PROVISIONS FOR TORSION

T Beams and Box Sections

After torsional cracking, the applied torque is resisted by the portion of the section represented by A_{oh}(the area enclosed by the center line of the outermost closed transverse torsional reinforcement.



A_{oh} = Shaded area

7. Analysis and Design for Torsion

<u>Minimum torsion</u>

$$T_u < \phi \frac{\sqrt{f_{ck}}}{12} \frac{A_{cp}^2}{p_{cp}} \tag{24}$$

Then, torsional effects may be neglected.

Recall the cracking Torque (Eq.(6))

$$T_{cr} = \frac{1}{3}\sqrt{f_{ck}} \frac{A_{cp}^2}{p_{cp}} \tag{6}$$

This lower limit is 25 percent of the cracking torque.

- For members subjected to an axial loan N_u

$$T_u < \phi \frac{\sqrt{f_{ck}}}{12} \frac{A_{cp}^2}{p_{cp}} \sqrt{1 + \left(\frac{3}{\sqrt{f_{ck}}}\right)\left(\frac{N_u}{A_g}\right)} \tag{25}$$

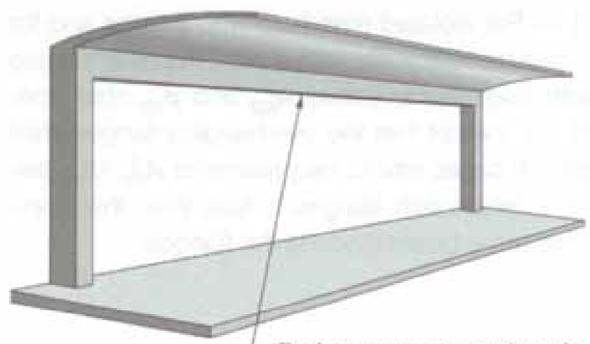
7. Analysis and Design for Torsion

Equilibrium vs. Compatibility Torsion

KCI Code 7.5.2 provides that two conditions may be identified in designing for torsion

i) Primary Torsion : The torsional moment cannot be reduced by redistribution of internal force.

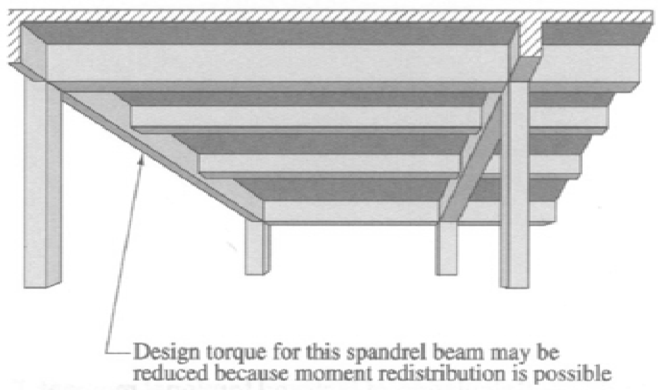
This is referred to as equilibrium torsion, since the torsional moment is required for the structure to be in equilibrium



⇨ The supporting member must be designed to provide the torsional resistance required by static equilibrium. (according to KCI Code 7.6)

7. Analysis and Design for Torsion

ii) Secondary Torsion : The torsional moment can be reduced by redistribution of internal forces after cracking, if the torsions arises from the member twisting to maintain compatibility of deformation.



In this case, factored torsional moment can be reduced to

$$T_u = \phi \frac{\sqrt{f_{ck}}}{3} \frac{A_{cp}^2}{p_{cp}} \quad (26)$$

<u>Note</u>

For hollow sections, A_{cp} shall not be replaced with A_g.

7. Analysis and Design for Torsion

Limitation on Shear Stress

- Extensive empirical observations indicate that the width of diagonal cracks caused by combined shear and torsion under SERVICE LOADS can be controlled by limiting the calculated shear stress under factored shear and torsion, so that

$$v_{\max} \le \phi\left(\frac{V_c}{b_w d} + \frac{2}{3}\sqrt{f_{ck}}\right) \tag{27}$$

where, v_{max} corresponds to the upper limits on shear capacity (Eq.(17) & (18))

7. Analysis and Design for Torsion

Limitation on Shear Stress

- For solid sections

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + \frac{2}{3}\sqrt{f_{ck}}\right) \tag{28}$$

For hollow sections

$$\frac{V_u}{b_w d} + \frac{T_u p_h}{1.7 A_{oh}^2} \le \phi\left(\frac{V_c}{b_w d} + \frac{2}{3}\sqrt{f_{ck}}\right) \tag{29}$$

⇨ If the Eq.(28) & (29) are not satisfied, either member size or concrete strength must be increased.

Q1) If the wall thickness varies around perimeter?

Q2) If the wall thickness is less than the assumed value A_{oh}/p_h?

7. Analysis and Design for Torsion

KCI CODE PROVISIONS FOR TORSION

Reinforcement for Torsion

- Nominal torsional strength is given by Eq.(23)

$$T_n = \frac{2A_0 A_t f_{yv}}{s} \cot\theta \tag{23}$$

where angle θ shall be taken between 30° and 60° and it shall be permitted to take θ equal to 45° for nonprestressed members.

A_0 can be determined by ANALYSIS* except that it shall be permitted to take A_0 equal to $0.85A_{oh}$.

∗ T.T.C. Hsu, "Shear flow zone in Torsion of Reinforced Concrete", Journal of structural Engineering, V.116, No.11, 1990, 3206-3226.

7. Analysis and Design for Torsion

Reinforcement for Torsion

- The required cross-sectional area of one stirrup leg for torsion is

$$A_t = \frac{T_u s}{2\phi A_0 f_{yv} \cot\theta} \tag{30}$$

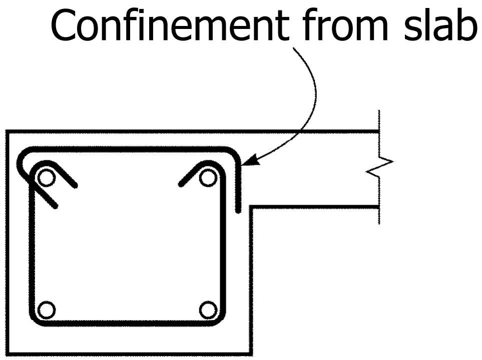
where $f_{yv} \leq 400$MPa for reasons of crack control.

- Combined with requirement for shear.(2-leg stirrup)

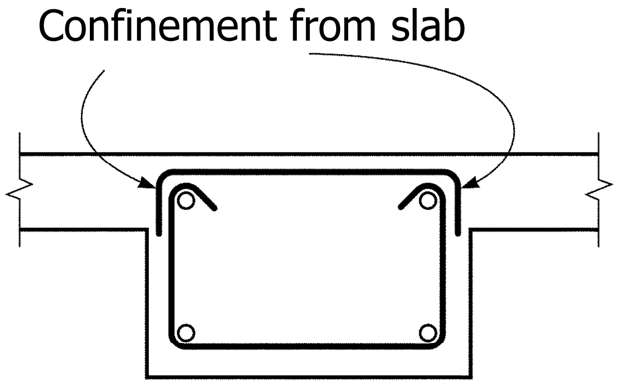
$$\left(\frac{A_{v+t}}{s}\right) = \frac{A_v}{s} + 2\frac{A_t}{s} \tag{31}$$

7. Analysis and Design for Torsion

Reinforcement for Torsion



<Spandrel beam>



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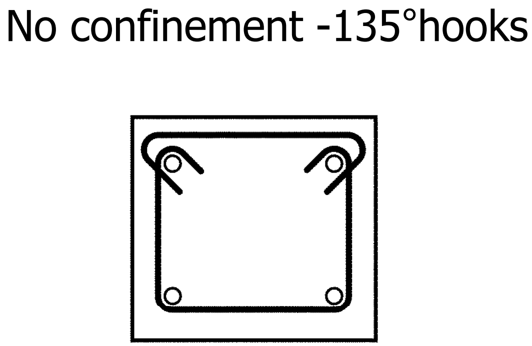

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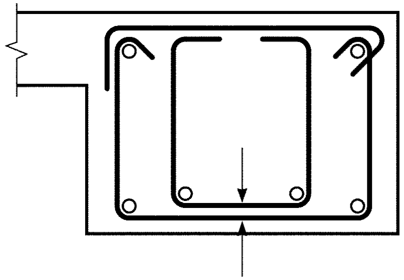
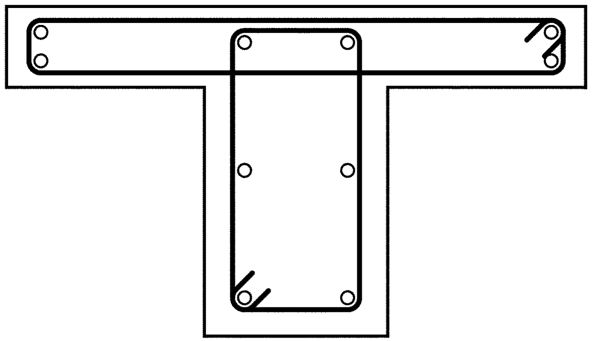


<Wide spandrel beam>



<T beam with torsional reinforcement in flanges>

7. Analysis and Design for Torsion

Reinforcement for Torsion

- Maximum spacing of torsional stirrup

  ⇦ to control spiral cracking

  $s_{max}=p_h/8$ or 300mm, whichever is smaller (32)

- Minimum area of closed stirrup for members requiring both shear and torsion reinforcement

$$(A_v + 2A_t) = 0.063\sqrt{f_{ck}}\,\frac{b_w s}{f_{yv}} \geq \frac{0.35 b_w s}{f_{yv}} \tag{33}$$

- Minimum total area of longitudinal torsional reinforcement

$$A_{l,\min} = \frac{0.42\sqrt{f_{ck}}\,A_{cp}}{f_y} - \underbrace{\left(\frac{A_t}{s}\right)}_{\geq 0.175 b_w / f_{yv}} p_h \frac{f_{yv}}{f_y} \tag{34}$$

7. Analysis and Design for Torsion

Reinforcement for Torsion

- The spacing of the longitudinal bars ≤ 300mm and they should be distributed around the perimeter of the cross section to control cracking.
- Longitudinal bars shall have a diameter at least 0.042 times the stirrup spacing, but not less than a D10.
- Torsional reinforcement shall be provided for a distance of at least (b_t+d) beyond the point required by analysis.

⇦ it is larger than that used for shear and flexural reinforcement because torsional diagonal cracks develop in a helical shape.

7. Analysis and Design for Torsion

Homework #1