Introduction to Nuclear Fusion

Prof. Dr. Yong-Su Na

Force balance in a tokamak

Tokamak Equilibrium

- **The Grad-Shafranov Equation**

$$\Delta^* \psi = -\mu_0 R^2 \frac{dp}{d\psi} - F \frac{dF}{d\psi}$$

J_ΦxB_p: strength of **B_p**

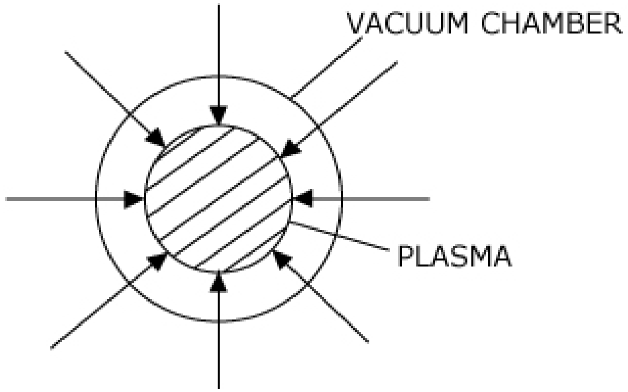
∇p: plasma load on a magnetic flux surface

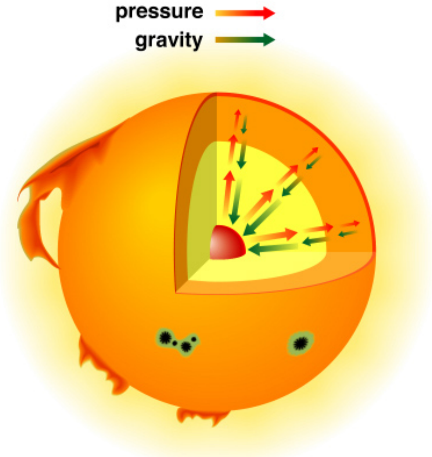
J_pxB_Φ: strength of **B_Φ**

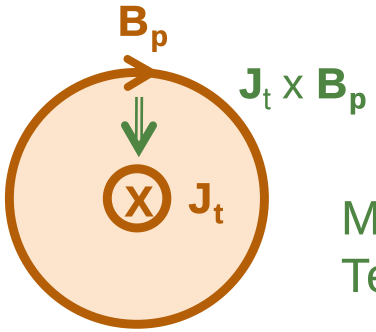
What kind of forces does a plasma have regarding equilibrium?

Tokamak Equilibrium

- **Basic Forces Acting on Tokamak Plasmas**
  - Radial force balance

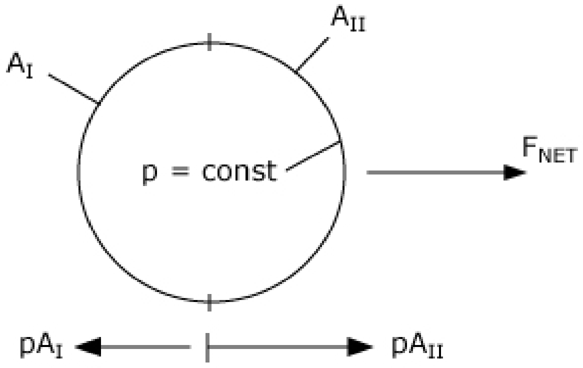






Magnetic pressure,
Tension force

  - Toroidal force balance: Tire tube force

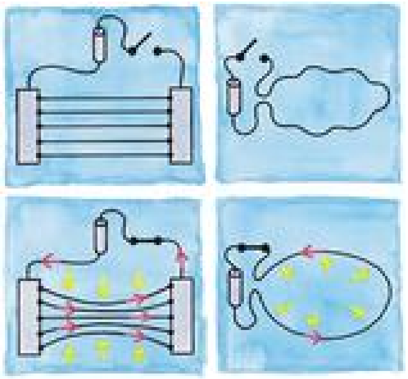


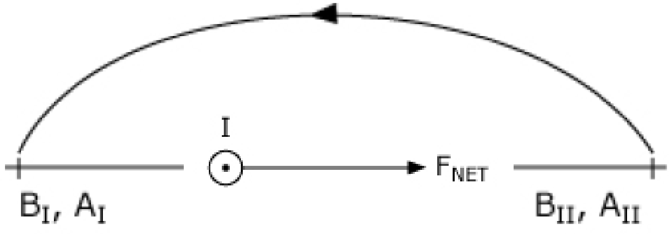
$$F_{NET} \sim -e_R (pA_I - pA_{II})$$

J.P. Freidberg, "Ideal Magneto-Hydro-Dynamics", lecture note

Tokamak Equilibrium

- **Basic Forces Acting on Tokamak Plasmas**
  - Toroidal force balance: Hoop force





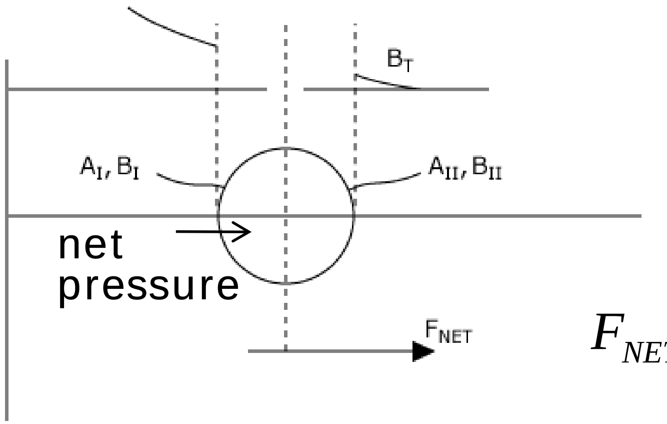
$$F_{NET} \sim e_R (B_I^2 A_I - B_{II}^2 A_{II}) / 2\mu_0$$

$$\phi_I = \phi_{II}$$

$$B_I > B_{II}, \quad A_I < A_{II}$$

$$B_I^2 A_I > B_{II}^2 A_{II}$$

  - Toroidal force balance: $1/R$ force



$$\phi_I = \phi_{II}$$

$$B_I > B_{II}, \quad A_I < A_{II}$$

$$B_I^2 A_I > B_{II}^2 A_{II}$$

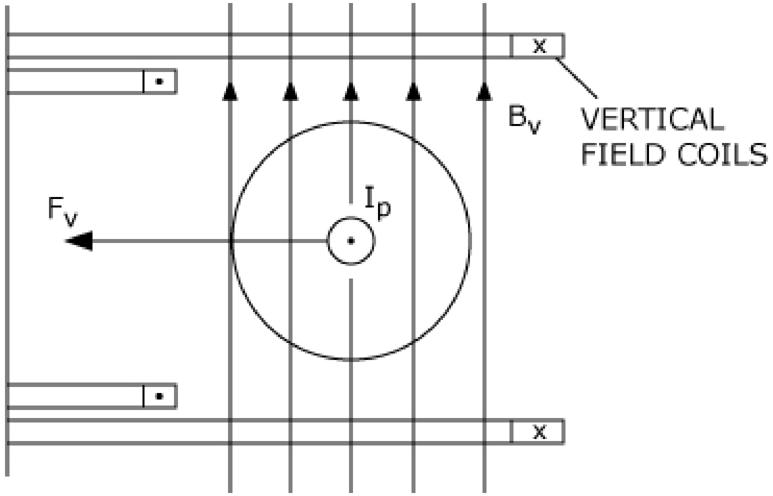
$$F_{NET} = 2\pi^2 a^2 \frac{B^2}{2\mu_0}$$

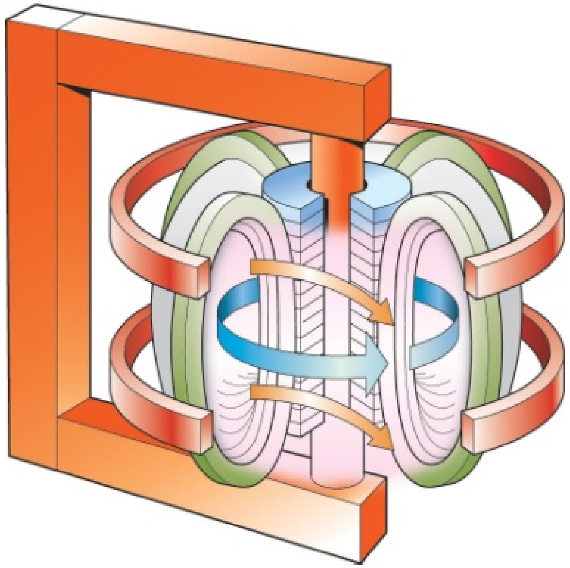
J.P. Freidberg, "Ideal Magneto-Hydro-Dynamics", lecture note

Tokamak Equilibrium

- **Basic Forces Acting on Tokamak Plasmas**
  - External coils required to provide the force balance

$$\vec{I}_p \times \vec{B}_v$$





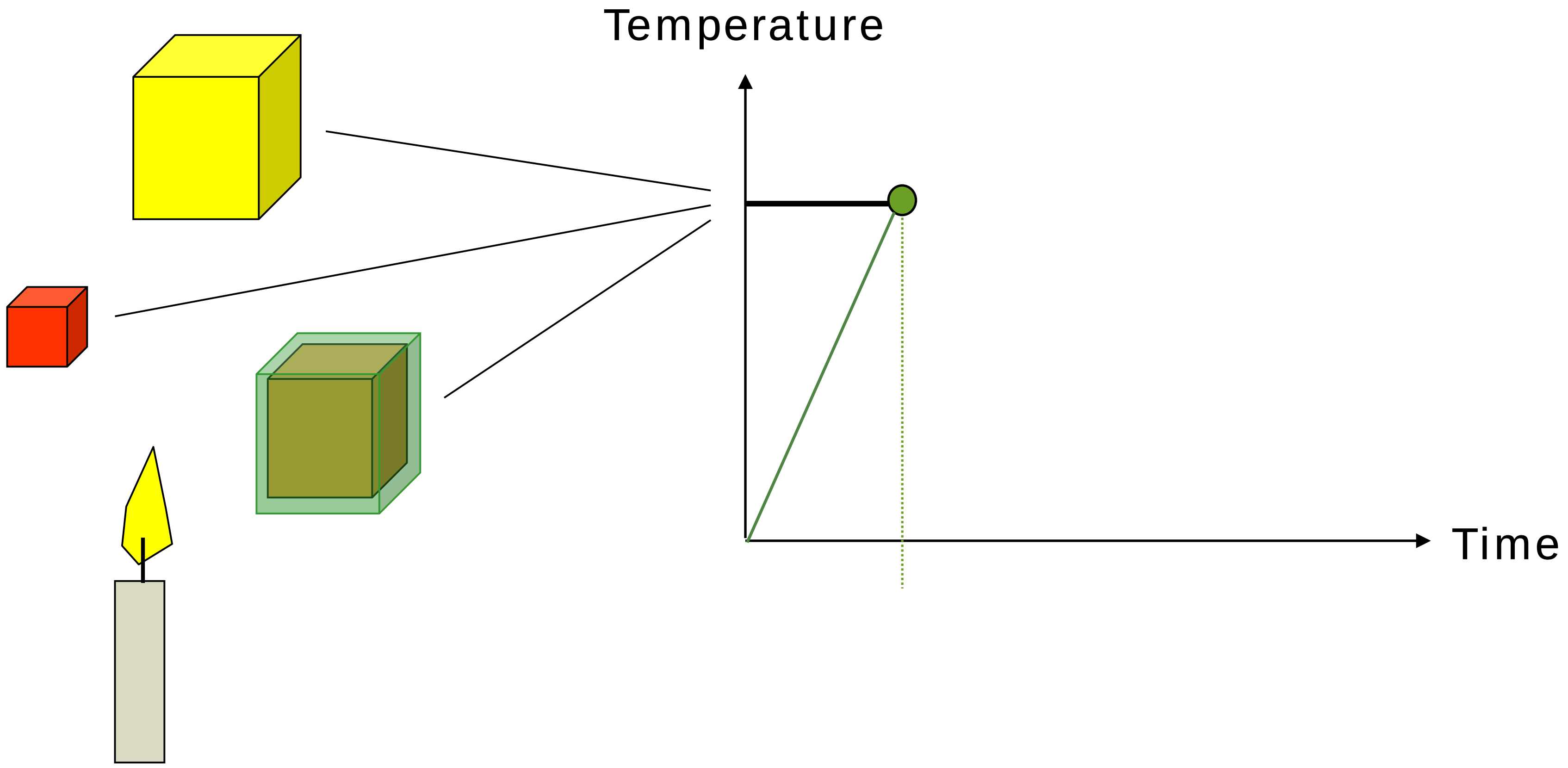
$$F_v = BIL = 2\pi R_0 I_p B_v$$

$$B_\phi > B_\theta > B_v$$

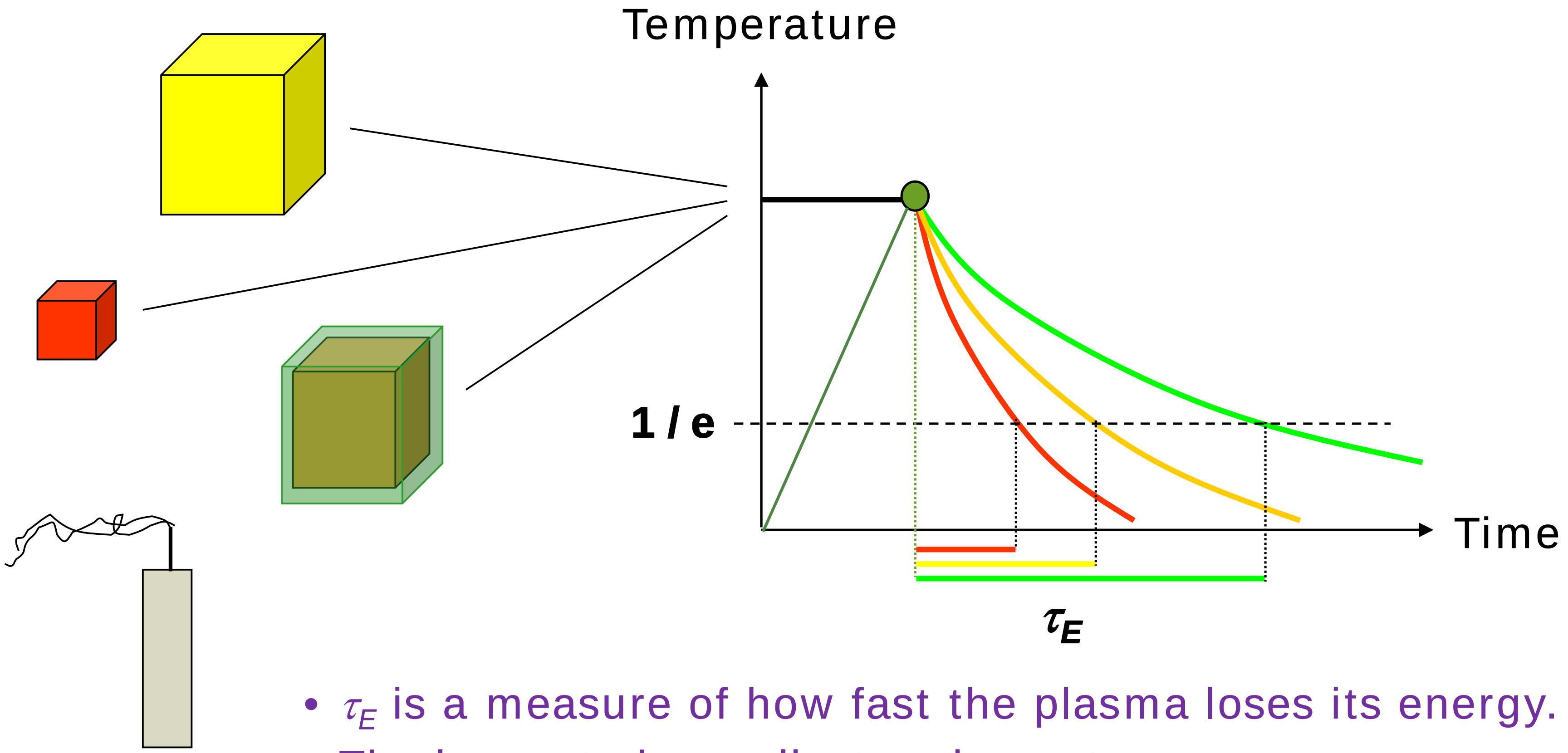
How about vertical movement?

Plasma transport in a Tokamak

Energy Confinement Time



Energy Confinement Time



- τ_E is a measure of how fast the plasma loses its energy.
- The loss rate is smallest, τ_E largest
  if the fusion plasma is big and well insulated.

Tokamak Transport

- **Transport Coefficients**

$\Gamma = -D\nabla n$: Fick's law

$D = \frac{(\Delta x)^2}{2\tau}$: diffusion coefficient (m^2/s)

$q = -\kappa \nabla T$: Fourier's law

Thermal diffusivity

$$\chi \equiv \frac{\kappa}{n} \approx D \approx \frac{(\Delta x)^2}{\tau} \approx \frac{a^2}{\tau_E} \rightarrow \tau_E \approx \frac{a^2}{\chi}$$

- Particle transport in fully ionised plasmas with magnetic field

$$D_\perp = \frac{\eta_\perp n \sum kT}{B^2}$$

Tokamak Transport

• Classical Transport

- Classical thermal conductivity (expectation): $\chi_i \sim 40\chi_e$
- Typical numbers expected: ~10^{-4} m^2/s
- Experimentally found: ~1 m^2/s, $\chi_i \sim \chi_e$

Bohm diffusion (1946): $$D_\perp = \frac{1}{16}\frac{kT_e}{eB}$$

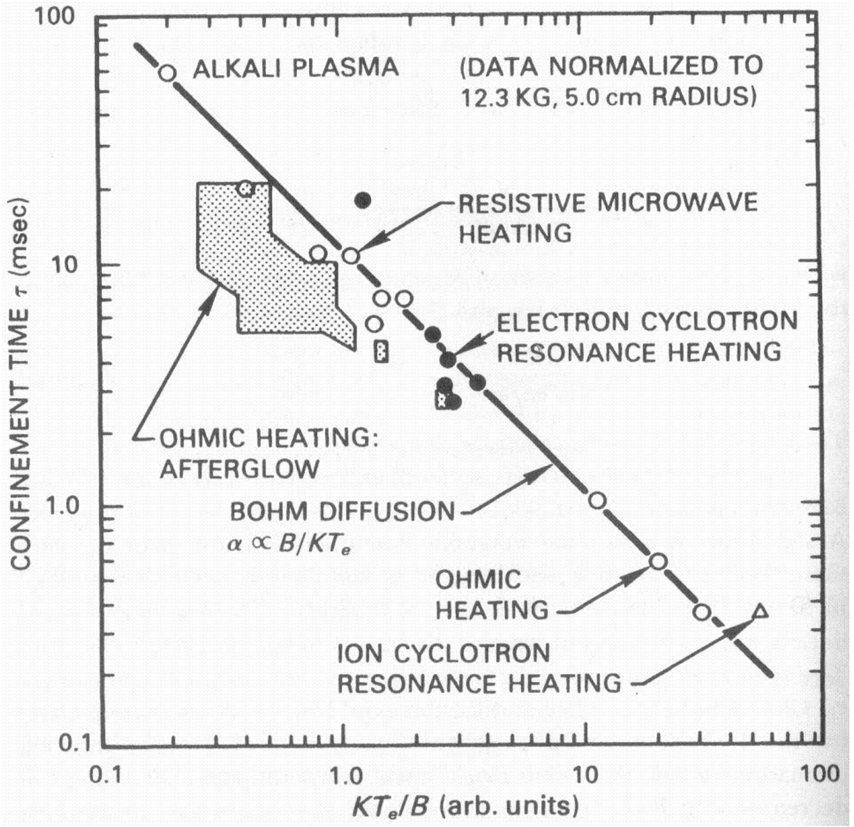
David Bohm (1917-1992)

WIKIPEDIA The Free Encyclopedia

Tokamak Transport

- **Classical Transport**

Bohm diffusion: $$D_\perp = \frac{1}{16}\frac{kT_e}{eB}$$



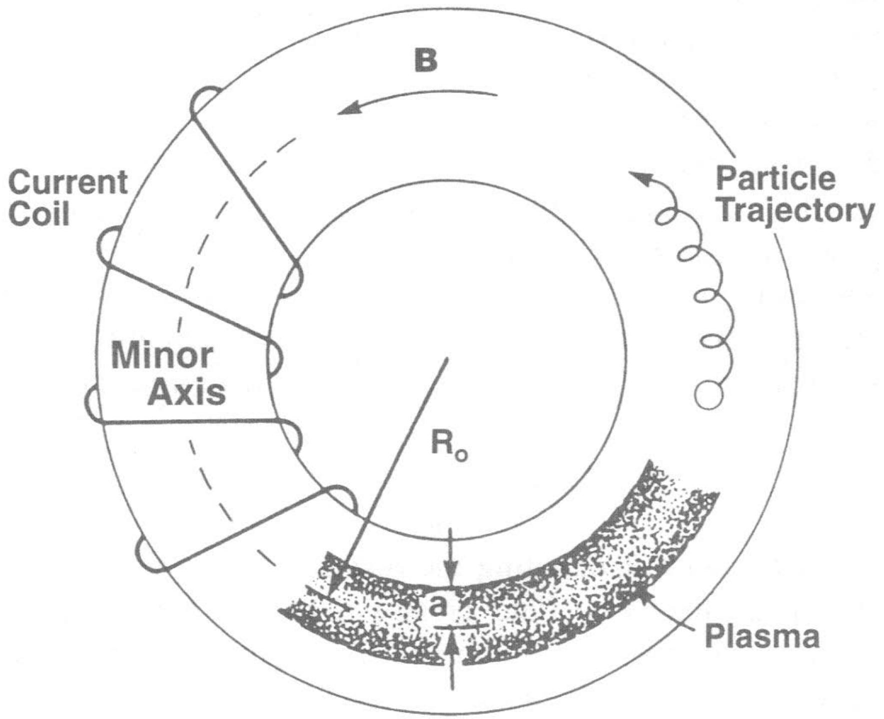
τ_E in various types of discharges in the Model C Stellarator

F. F. Chen, "Introduction to Plasma Physics and Controlled Fusion" (2006)

Tokamak Transport

- **Neoclassical Transport**

  - Major changes arise from toroidal effects characterised by inverse aspect ratio, $\varepsilon = a/R_0$

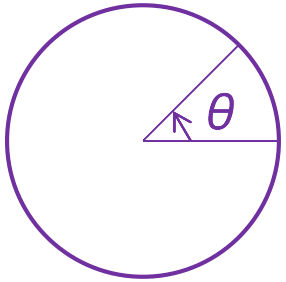


Tokamak Transport

- **Particle Trapping**

$$\nabla \cdot \vec{B} = 0$$

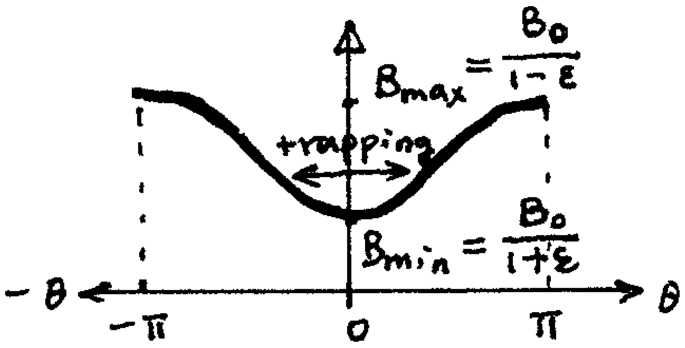
HW. Derive this.



$$\Rightarrow \frac{1}{1+\varepsilon\cos\theta}\left\{\frac{1}{r}\frac{\partial}{\partial r}(rB_r)+\frac{1}{r}\frac{\partial}{\partial\theta}\left[(1+\varepsilon\cos\theta)B_\theta\right]+\frac{1}{R_0}\frac{\partial B_\phi}{\partial\phi}\right\}=0$$

$$\Rightarrow B_\theta(r,\theta)=\frac{B_\theta^0(\theta=0)}{1+\varepsilon\cos\theta}$$

$$\left|B(r,\theta)\right|=\left|B_\theta(r,\theta)\hat{\theta}+B_\phi(r,\theta)\hat{\phi}\right|=\frac{B_0}{1+\varepsilon\cos\theta}$$



- Particle trapping by magnetic mirrors
  trapped particles with banana orbits
  untrapped particles with circular orbits

Discuss the particle motion

- Trapped fraction: $f_{trap}=\sqrt{1-\frac{1}{R_m}}=\sqrt{1-\frac{B_{\min}}{B_{\max}}}=\sqrt{1-\frac{1-\varepsilon}{1+\varepsilon}}=\sqrt{\frac{2\varepsilon}{1+\varepsilon}}$

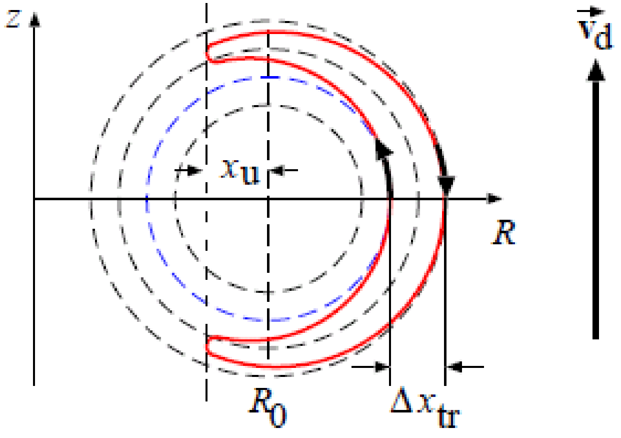
for a typical tokamak, ε ~ 1/3 → f_{trap} ~ 70%

Tokamak Transport

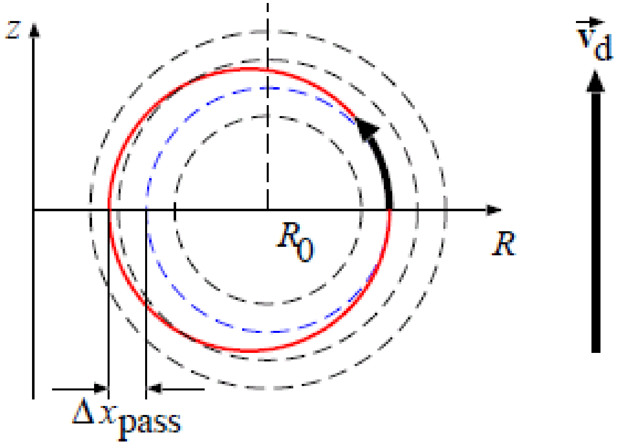
$$\mathbf{v}_{D,\nabla B} = \pm\frac{1}{2}v_\perp r_L \frac{\mathbf{B}\times\nabla B}{B^2}$$

$$\mathbf{v}_{D,R} = \frac{mv_\parallel^2}{qB_0^2}\frac{\mathbf{R}_0\times\mathbf{B}_0}{R^2}$$

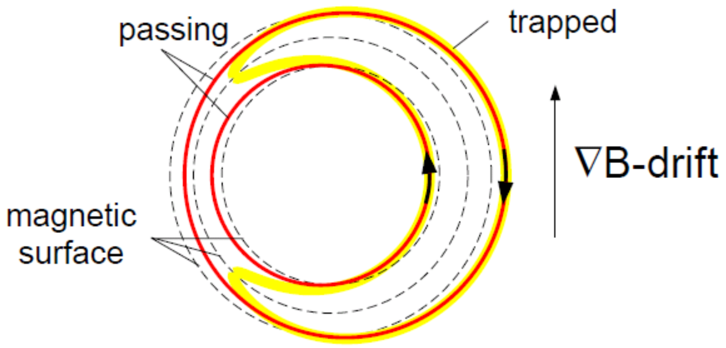
- **Particle Trapping**

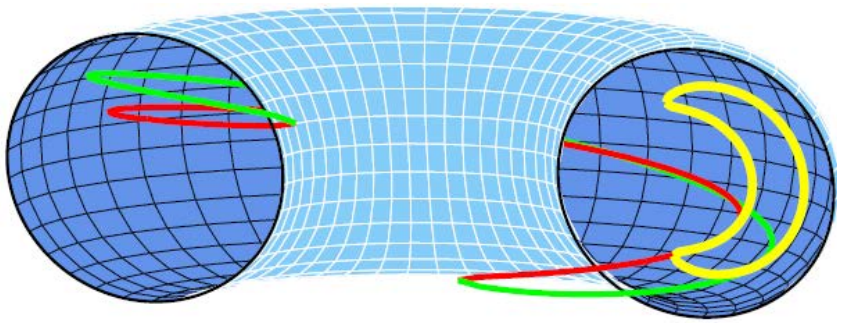


trapped particles



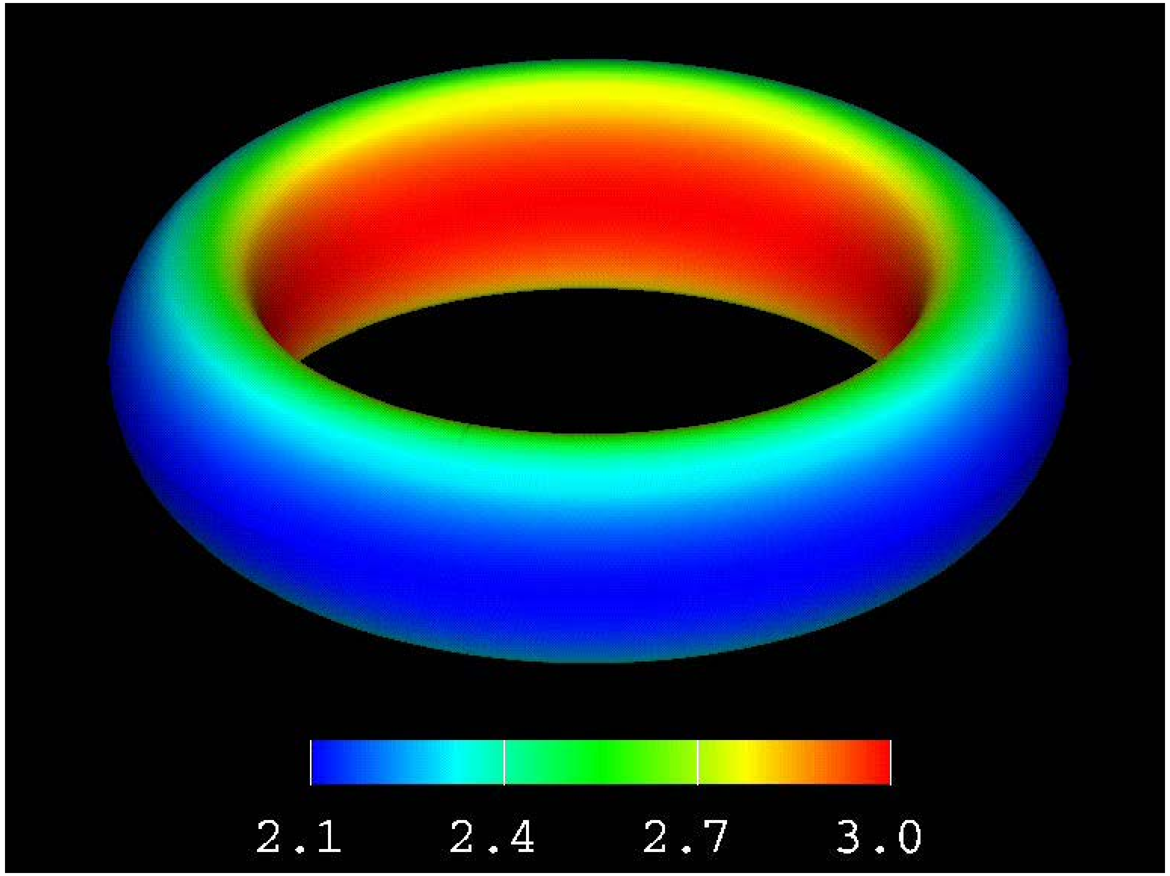
passing particles





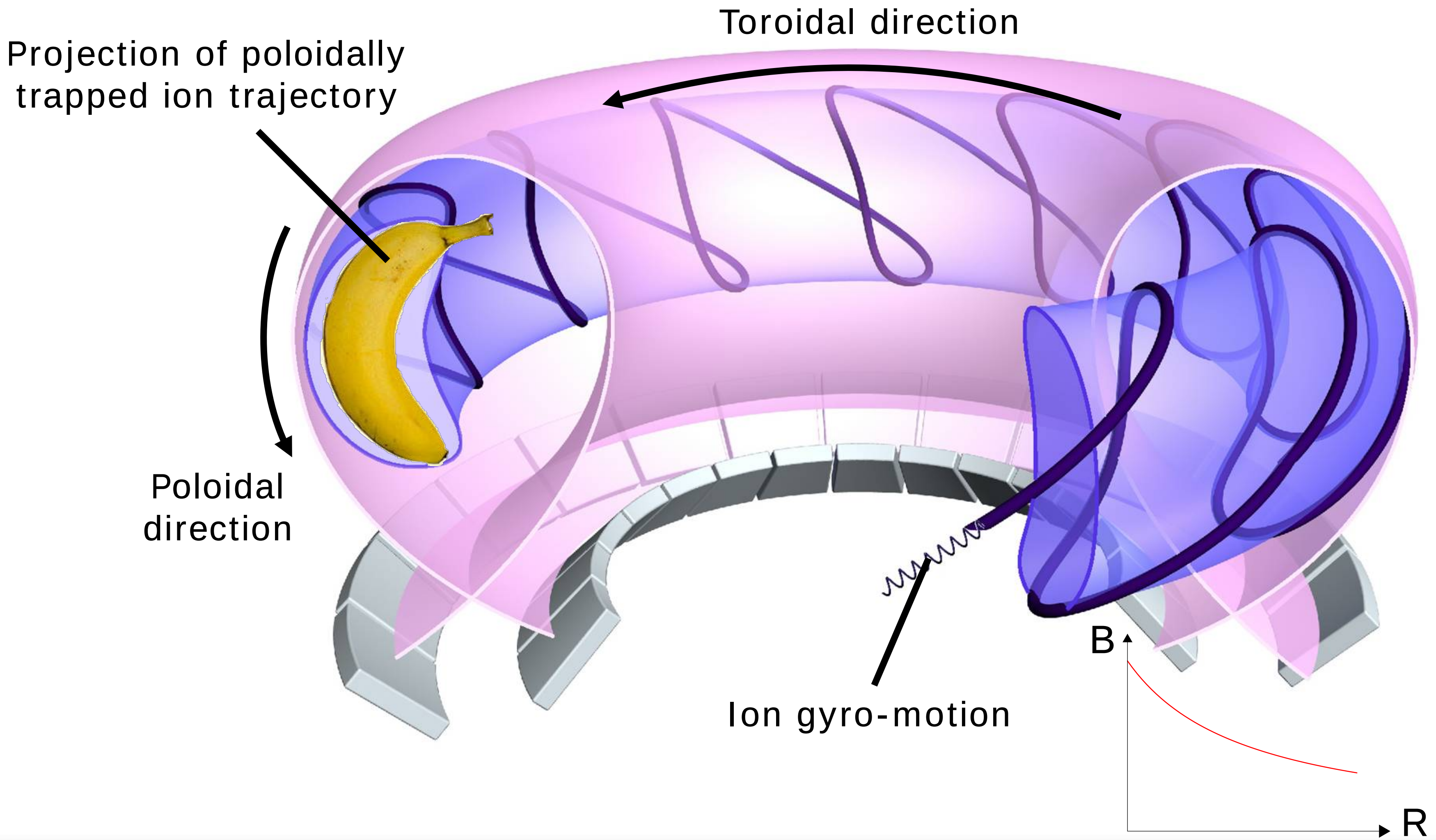
Tokamak Transport

- **Particle Trapping**



Tokamak Transport

- **Neoclassical Bootstrap current**



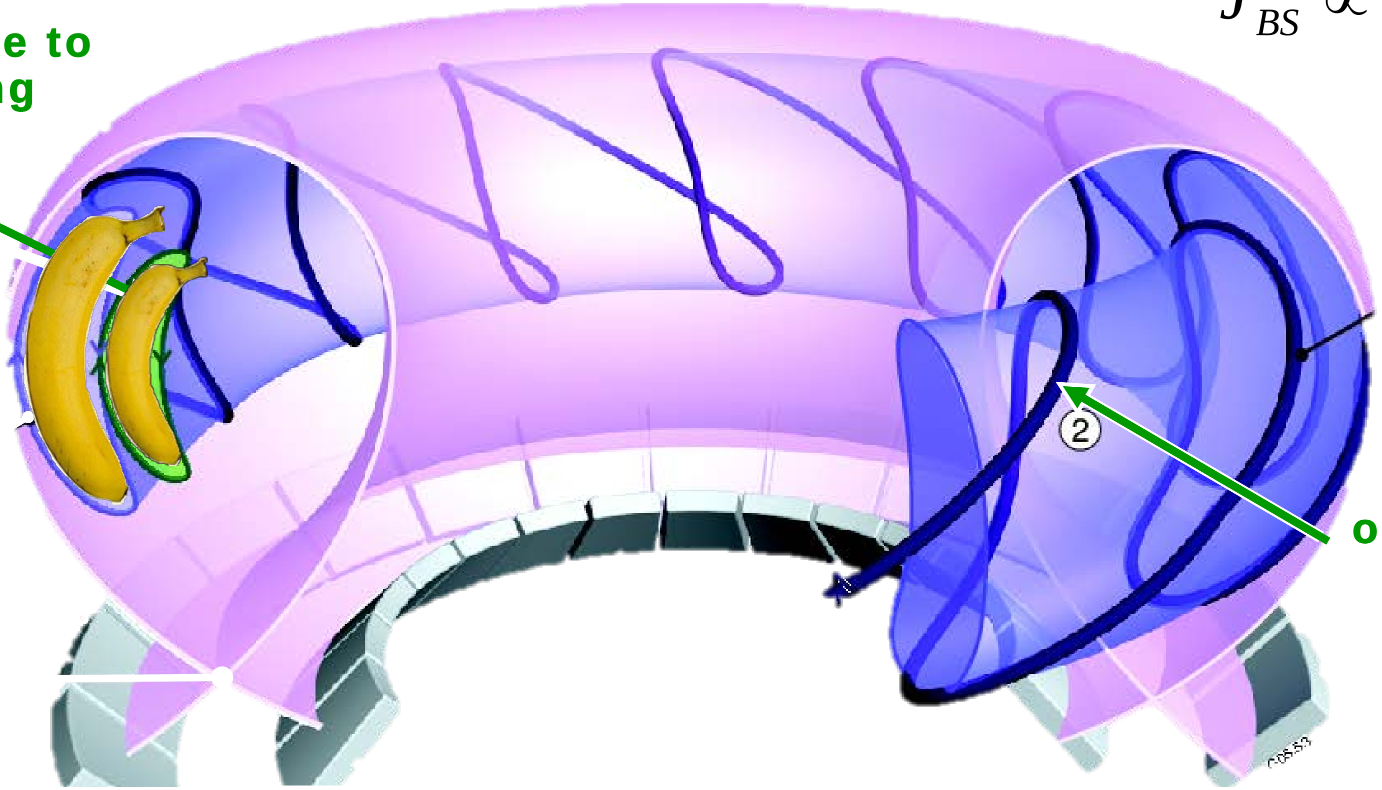
Tim Hender, "Neoclassical Tearing Modes in Tokamaks", KPS/DPP, Daejun, Korea, 24 April 2009

http://tfy.tkk.fi/fusion/research/

Tokamak Transport

- **Neoclassical Bootstrap current**

$$J_{BS} \propto \nabla p$$



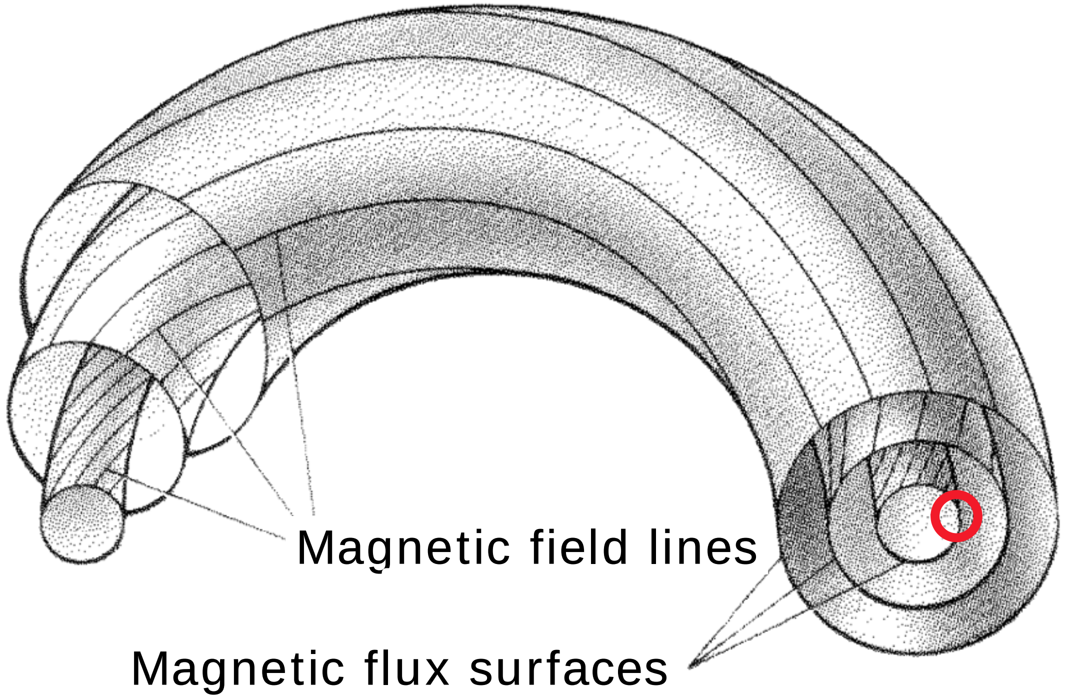
- More & faster particles on orbits nearer the core (green .vs. blue) lead to a net “banana current”.
- This is transferred to a helical bootstrap current via collisions.

Tokamak Transport

- **Particle Trapping**

  - Collisional excursion across flux surfaces

    untrapped particles: $2r_g$ $(2r_{Li})$



Tokamak Transport

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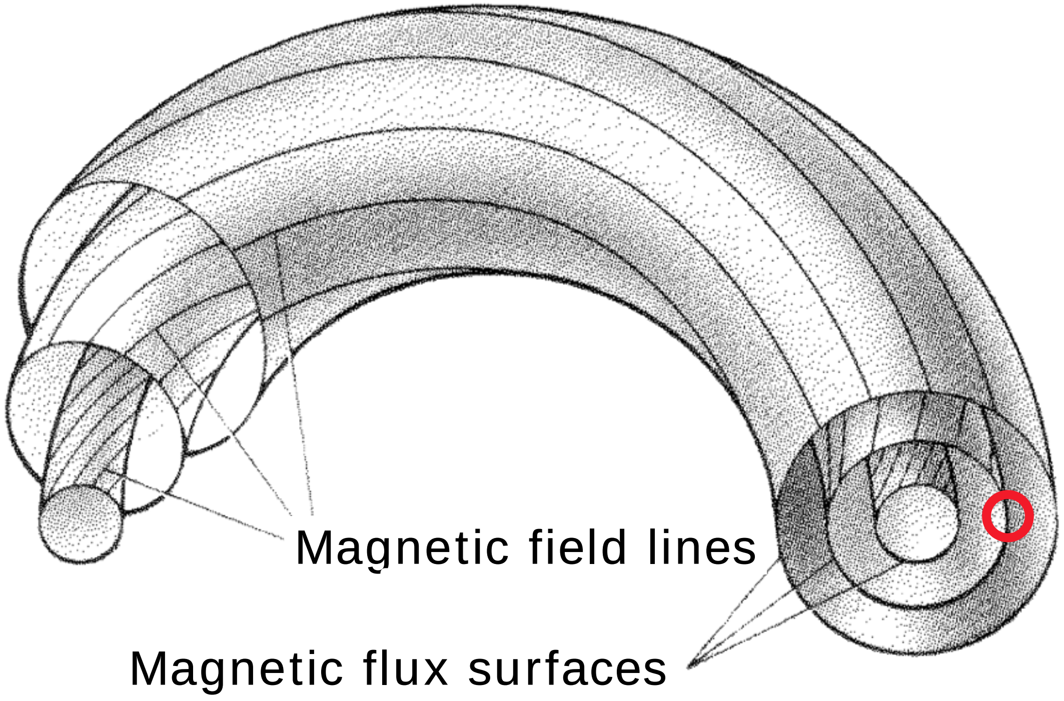


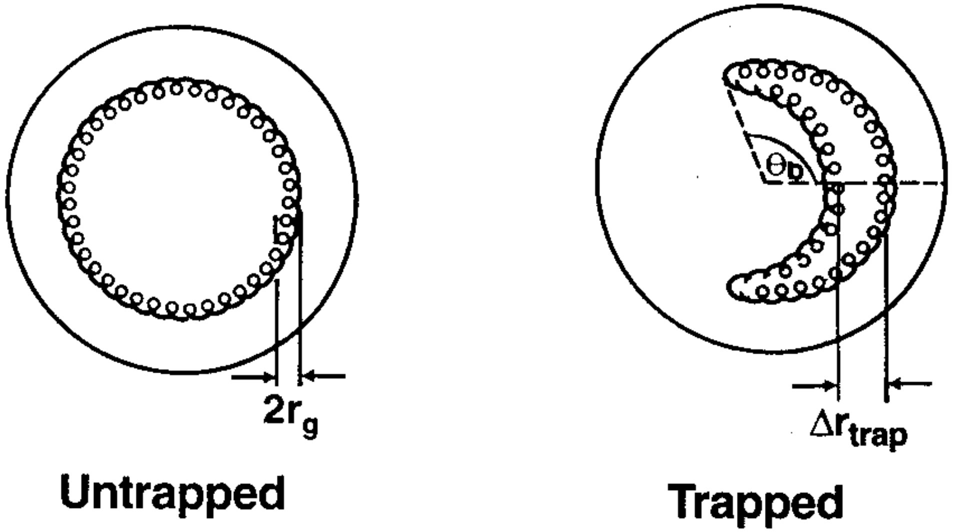
Tokamak Transport

- **Particle Trapping**

  - Collisional excursion across flux surfaces

    untrapped particles: $2r_g$ ($2r_{Li}$)

    trapped particles: $\Delta r_{trap} >> 2r_g$ – enhanced radial diffusion across the confining magnetic field



  - If the fraction of trapped particle is large, this leakage enhancement constitutes a substantial problem in tokamak confinement.

Tokamak Transport

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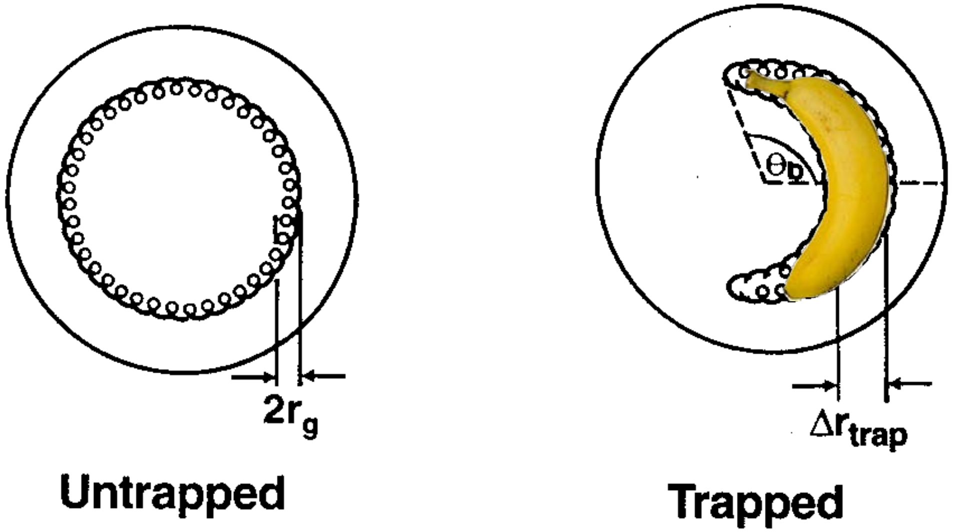
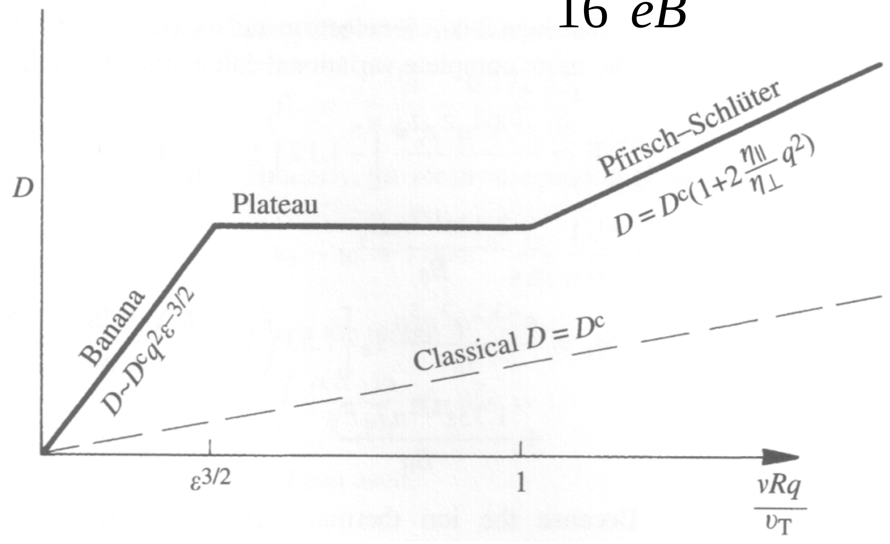


  - If the fraction of trapped particle is large, this leakage enhancement constitutes a substantial problem in tokamak confinement.

Tokamak Transport

- **Neoclassical Transports**
  - May increase D, χ up to two orders of magnitude:
    - χ_i 'only' wrong by factor 3-5
    - D, χ_e still wrong by up to two orders of magnitude!

$$D_\perp = \frac{1}{16}\frac{kT_e}{eB}$$ Bohm diffusion



$$D_\perp = \frac{\eta_\perp n \sum kT}{B^2}$$

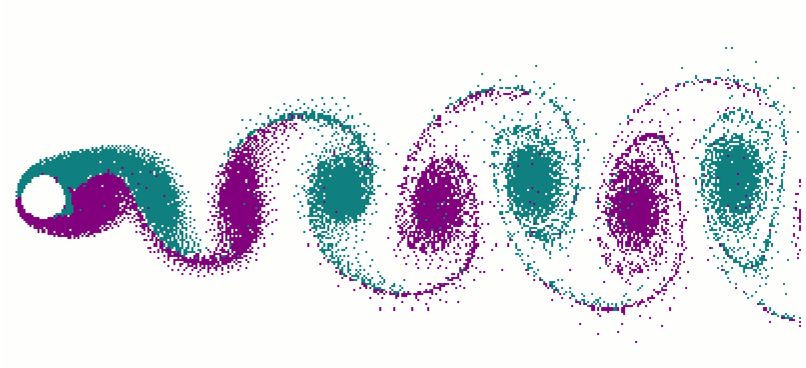
J. Wesson, Tokamaks (2004)

Tokamak Transport

- **Transport in fusion plasmas is ‘anomalous’**
  - Normal (water) flow: Hydrodynamic equations can develop nonlinear turbulent solutions (Reynolds, 1883)

$$\mathrm{Re} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho v L}{\mu} = \frac{vL}{\nu}$$

ρ: density of the fluid (kg/m³)
v: mean velocity of the object relative to the fluid (m/s)
L: a characteristic linear dimension, (travelled length of the fluid; hydraulic diameter when dealing with river systems) (m)
μ: dynamic viscosity of the fluid (Pa·s or N·s/m² or kg/(m·s))
ν: kinematic viscosity (μ/ρ) (m²/s)



A vortex street around a cylinder. This occurs around cylinders, for any fluid, cylinder size and fluid speed, provided that there is a Reynolds number of between ~40 and 103

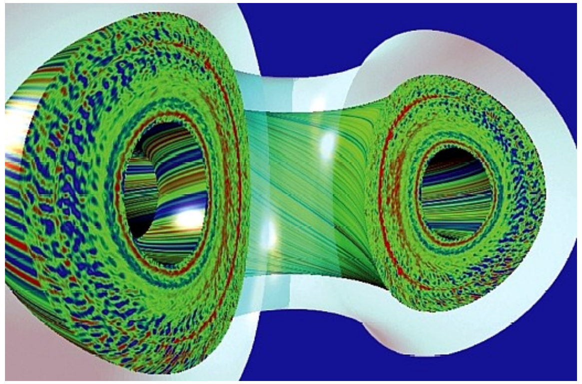
Wikipedia
The Free Encyclopedia

Tokamak Transport

- **Transport in fusion plasmas is 'anomalous'**

  - Normal (water) flow: Hydrodynamic equations can develop nonlinear turbulent solutions (Reynolds, 1883)

  - Transport mainly governed by turbulence:
    radial extent of turbulent eddy: 1 - 2 cm
    typical lifetime of turbulent eddy: 0.5 - 1 ms



  - Anomalous transport coefficients are of the order 1 m^2/s

$$D \sim \frac{(\Delta x)^2}{\tau}$$: diffusion coefficient (m^2/s)

http://scidacreview.org/0801/html/fusion.html

Tokamak Transport

- **Microinstabilities**
  - often associated with non-Maxwellian velocity distributions: deviation from thermodynamic equilibrium (nonuniformity, anisotropy of distributions) → free energy source which can drive instabilities
  - kinetic approach required: limited MHD approach

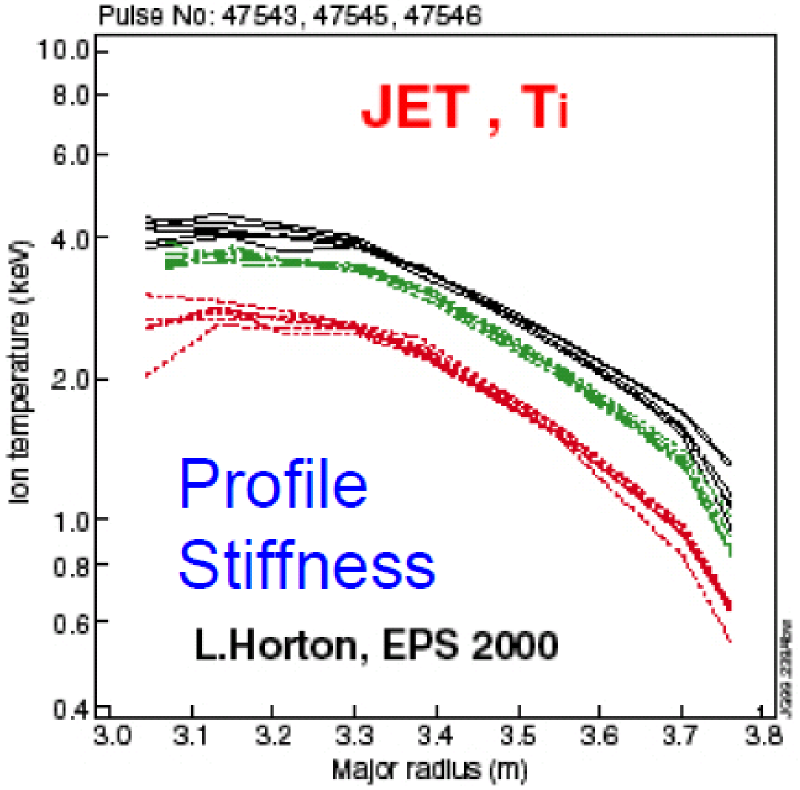
- **Two-stream or beam-plasma instability**
  - Particle bunching → **E** perturbation → bunching↑ → unstable

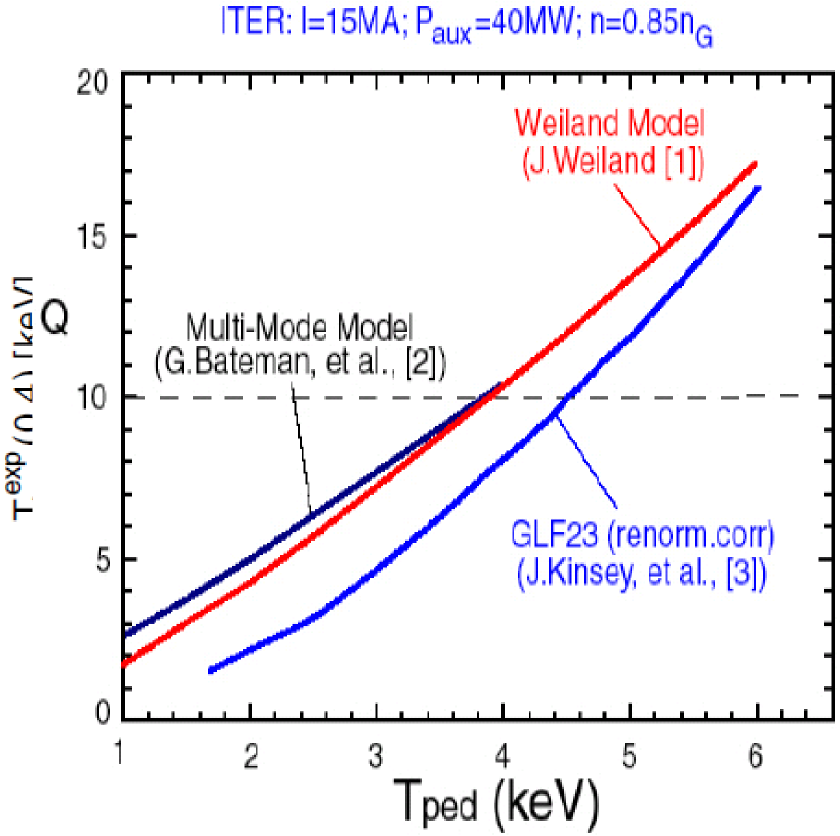
- **Drift (or Universal) instability**
  - driven by ∇p (or ∇n) in magnetic field
  - excited by drift waves with a phase velocity of v_{De} with a very short wavelength
  - most unstable, dominant for anomalous transport

- **Trapped particle modes**
  - Preferably when the perturbation frequency < bounce frequency
  - drift instability enhanced by trapped particle effects
  - Trapped Electron Mode (TEM), Trapped Ion Mode (TIM)

Tokamak Transport

- **Profile consistency (or profile resilience or stiffness)**





$$D^{exp} = D^{NC} + D^{anomalous} > D^{NC}$$

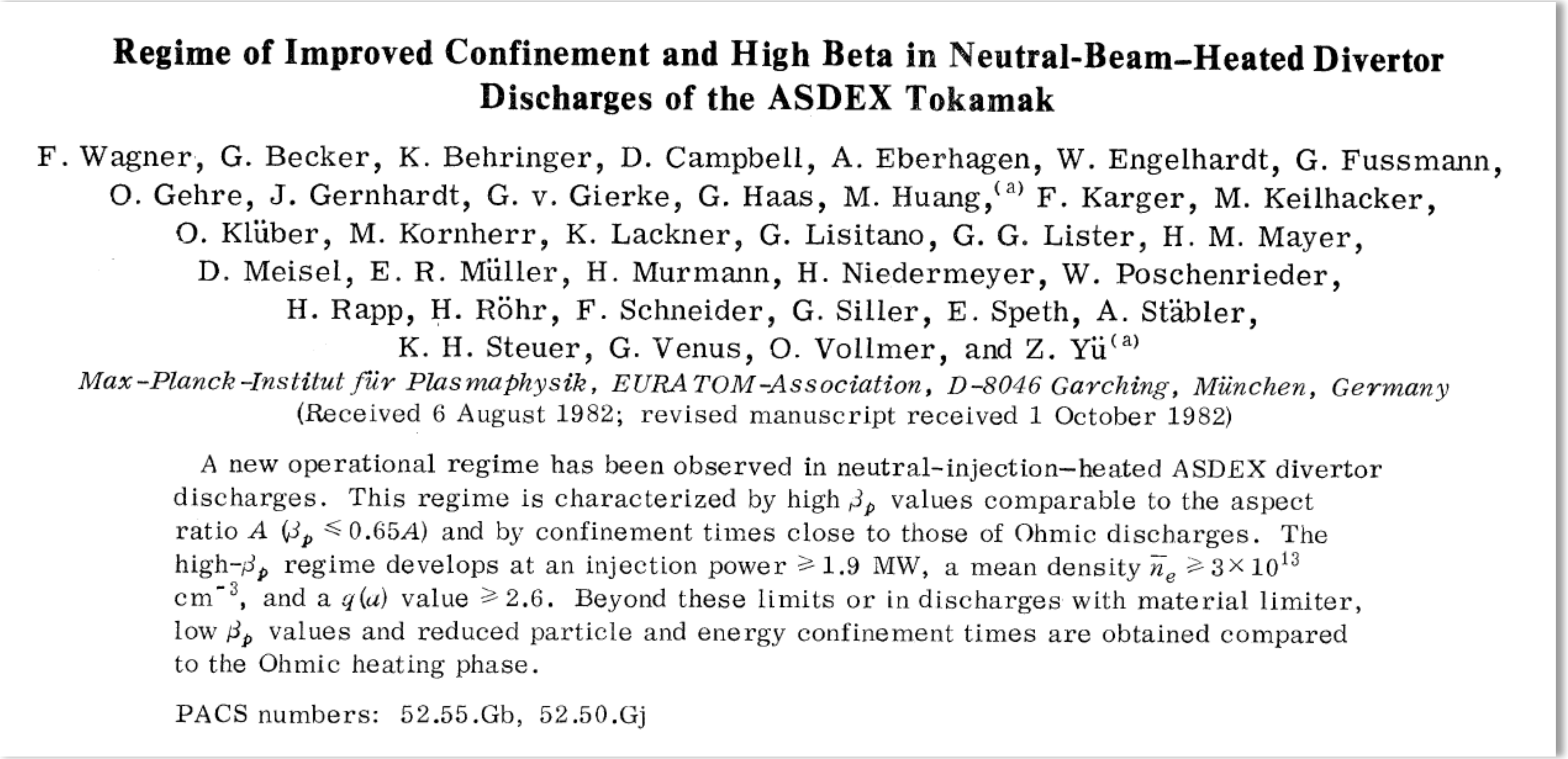
$$\chi^{exp} = \chi^{NC} + \chi^{anomalous} > \chi^{NC}$$

ASDEX Upgrade

How to reduce plasma transport?

Tokamak Transport

- **Suppression of Anomalous Transport: H-mode**
  - 1982 IAEA FEC, F. Wagner et al. (ASDEX, Germany)
  - Transition to H-mode: state with reduced turbulence at the plasma edge
  - Formation of an edge transport barrier: steep pressure gradient at the edge



Regime of Improved Confinement and High Beta in Neutral-Beam-Heated Divertor Discharges of the ASDEX Tokamak

F. Wagner, G. Becker, K. Behringer, D. Campbell, A. Eberhagen, W. Engelhardt, G. Fussmann, O. Gehre, J. Gernhardt, G. v. Gierke, G. Haas, M. Huang,[a] F. Karger, M. Keilhacker, O. Klüber, M. Kornherr, K. Lackner, G. Lisitano, G. G. Lister, H. M. Mayer, D. Meisel, E. R. Müller, H. Murmann, H. Niedermeyer, W. Poschenrieder, H. Rapp, H. Röhr, F. Schneider, G. Siller, E. Speth, A. Stäbler, K. H. Steuer, G. Venus, O. Vollmer, and Z. Yü[a]

Max-Planck-Institut für Plasmaphysik, EURATOM-Association, D-8046 Garching, München, Germany

(Received 6 August 1982; revised manuscript received 1 October 1982)

A new operational regime has been observed in neutral-injection–heated ASDEX divertor discharges. This regime is characterized by high β_p values comparable to the aspect ratio A ($\beta_p \leq 0.65A$) and by confinement times close to those of Ohmic discharges. The high-β_p regime develops at an injection power ≥ 1.9 MW, a mean density $\bar{n}_e \geq 3\times10^{13}$ cm^{-3}, and a $q(a)$ value ≥ 2.6. Beyond these limits or in discharges with material limiter, low β_p values and reduced particle and energy confinement times are obtained compared to the Ohmic heating phase.

PACS numbers: 52.55.Gb, 52.50.Gj

Tokamak Transport

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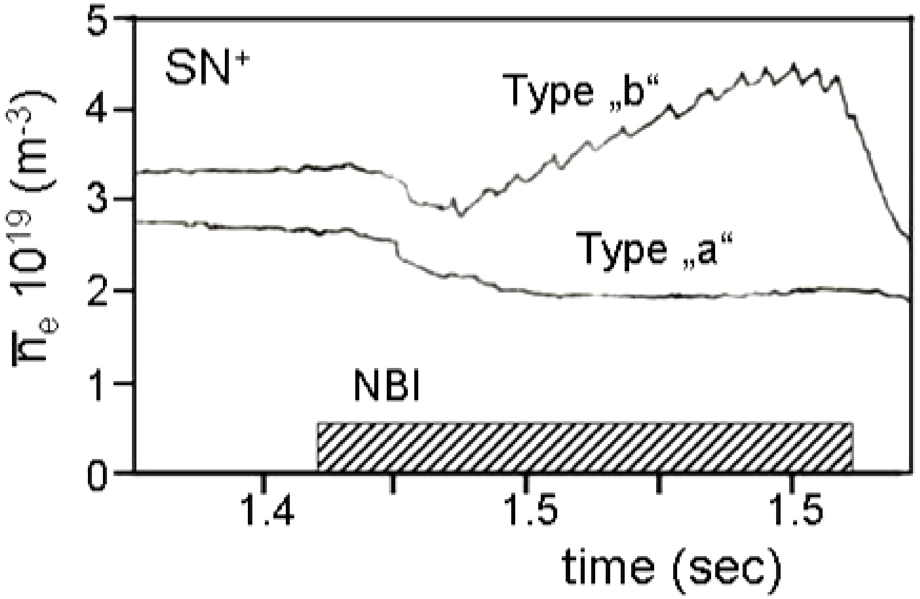
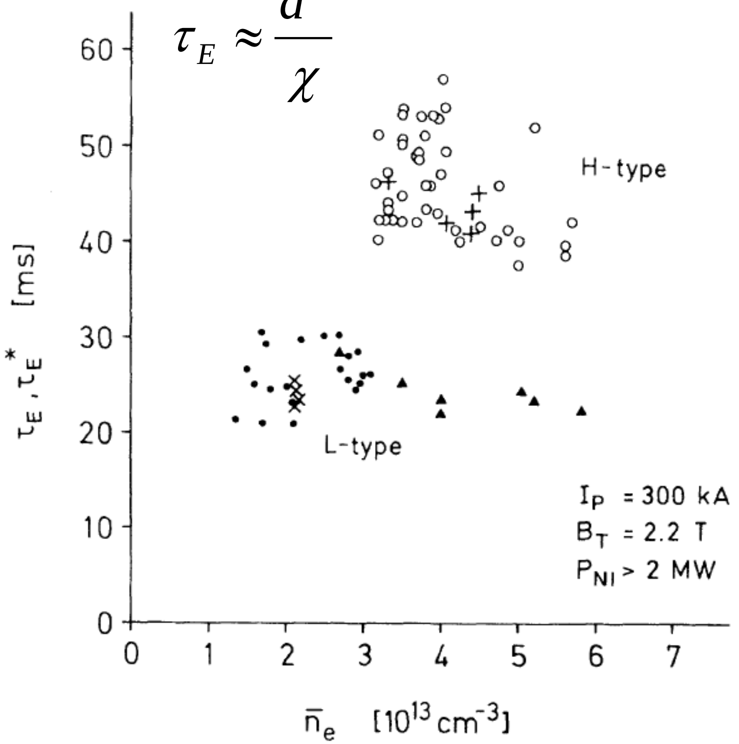


$$\tau_E \approx \frac{a^2}{\chi}$$



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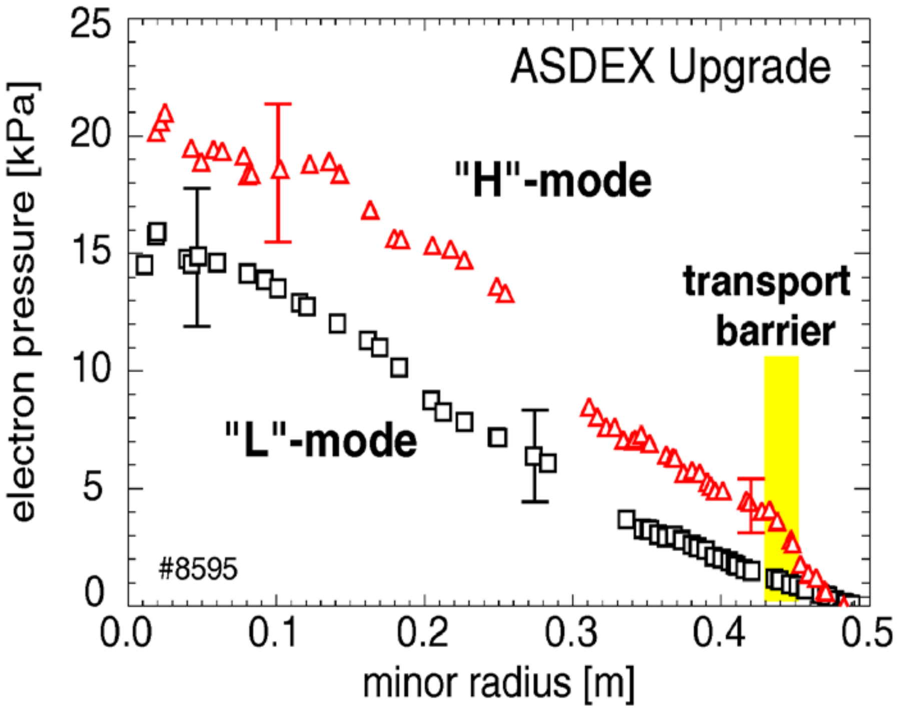

Hoover dam

Tokamak Transport

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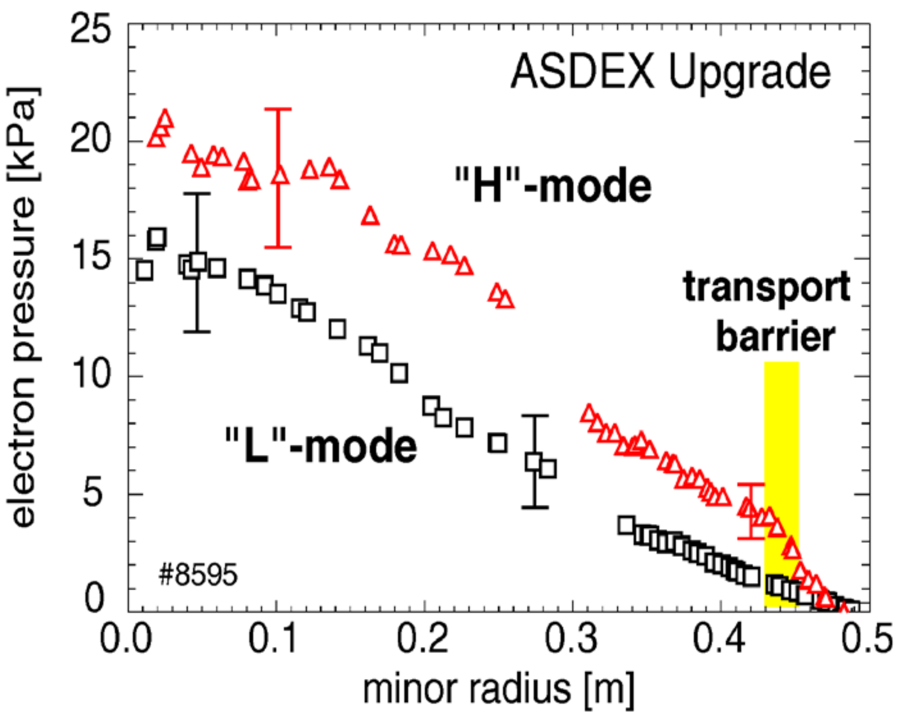
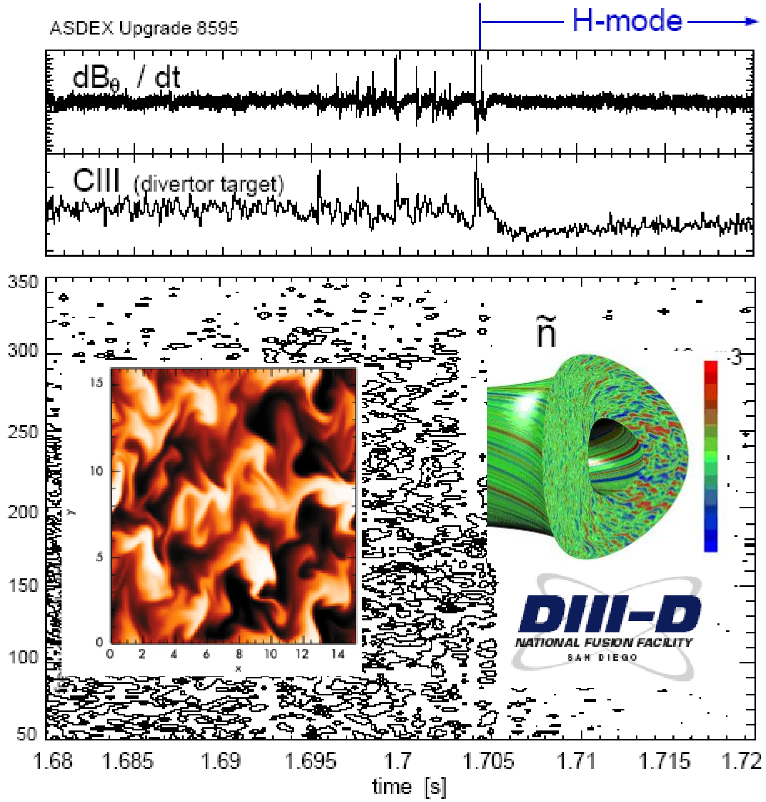
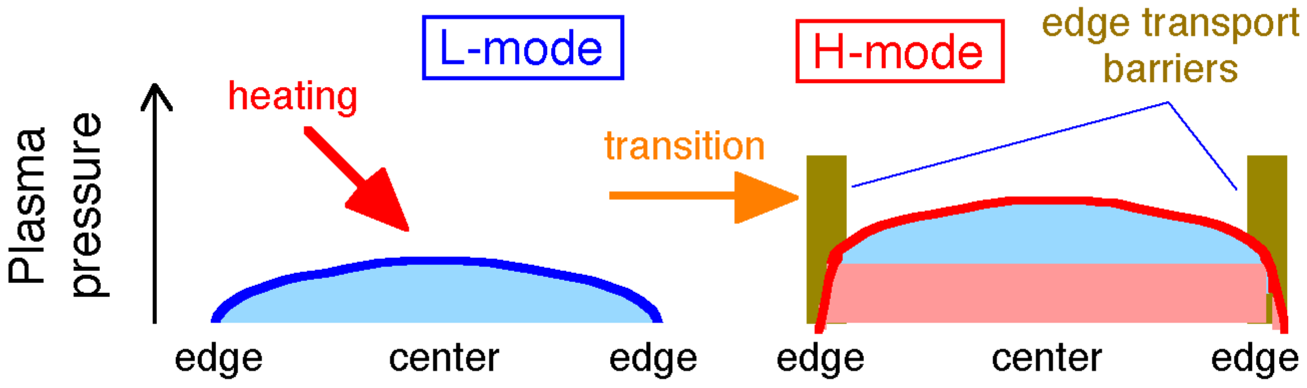
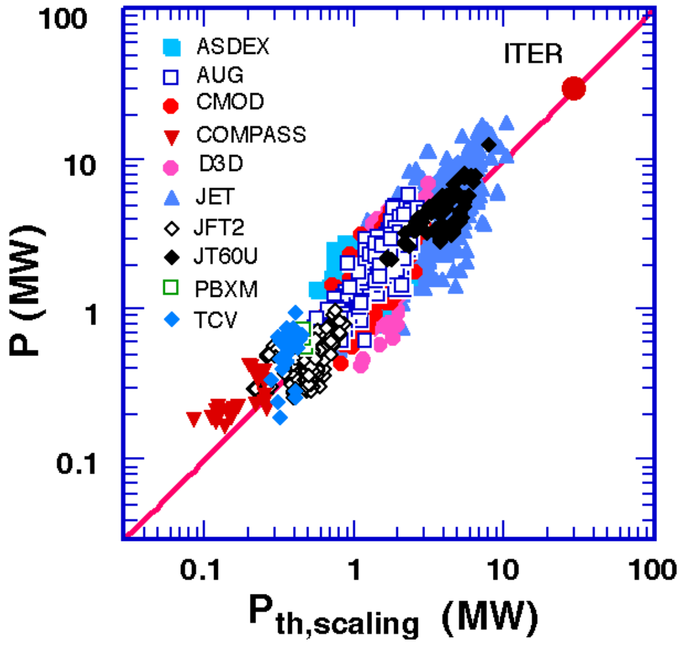




L-H transition threshold power

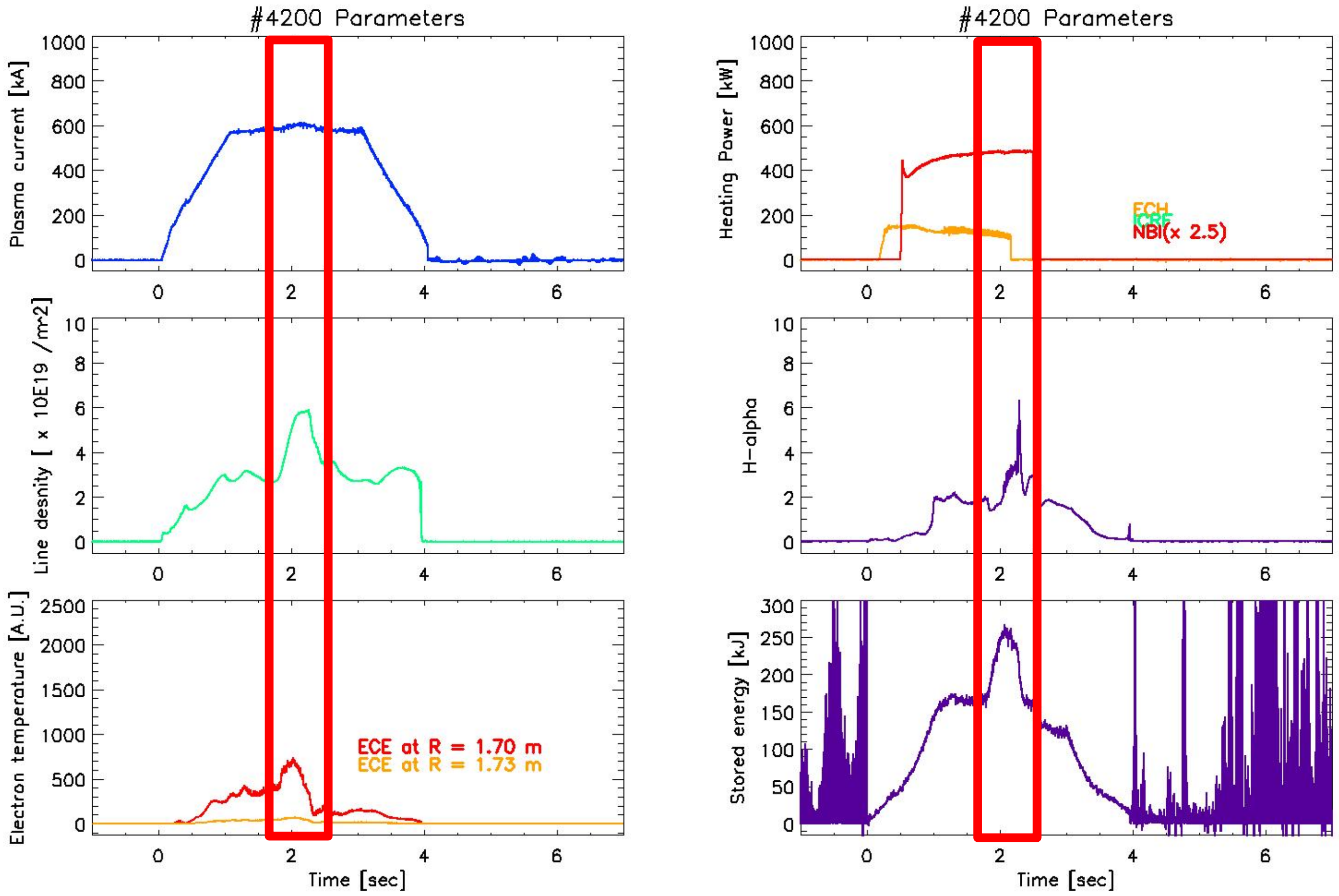




$$P_{th} = 2.84 M^{-1} B_t^{0.82} n_{20}^{0.58} R^{1.0} a^{0.81}$$

Tokamak Transport

- **First H-mode Transition in KSTAR (November 8, 2010)**



- B_0 = 2.0 T, Heating = 1.5 MW (NBI : 1.3 MW, ECH : 0.2 MW)
  After Boronisation on November 7, 2010

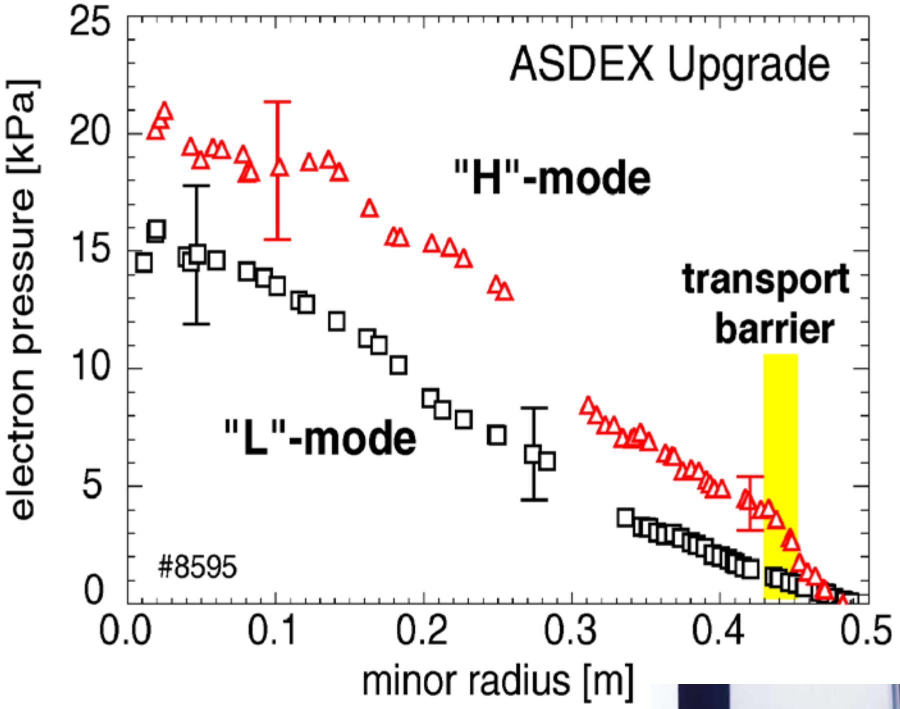
Tokamak Transport

- First H-mode Transition in KSTAR (November 8, 2010)

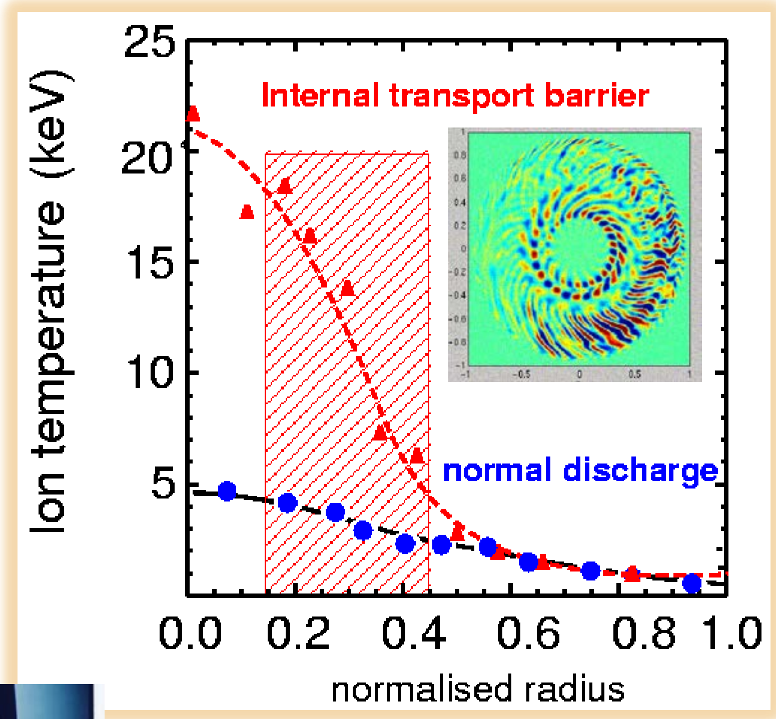
Tokamak Transport

- **Suppression of Anomalous Transport: ITBs**

H-mode

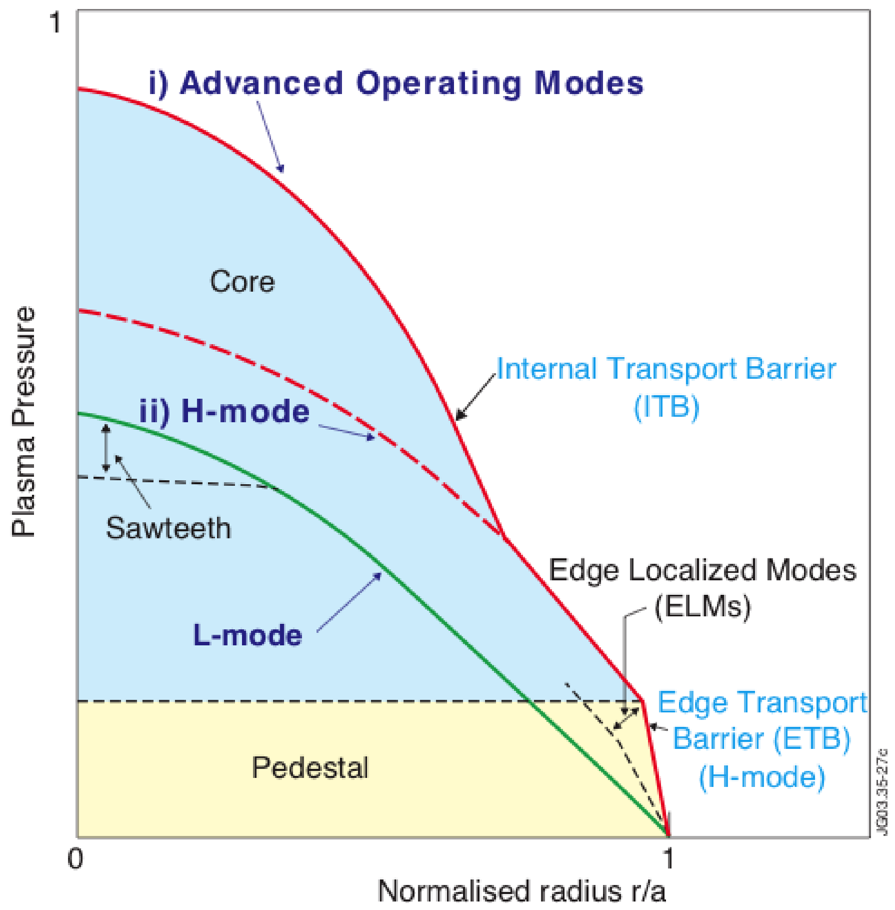


Reversed shear mode

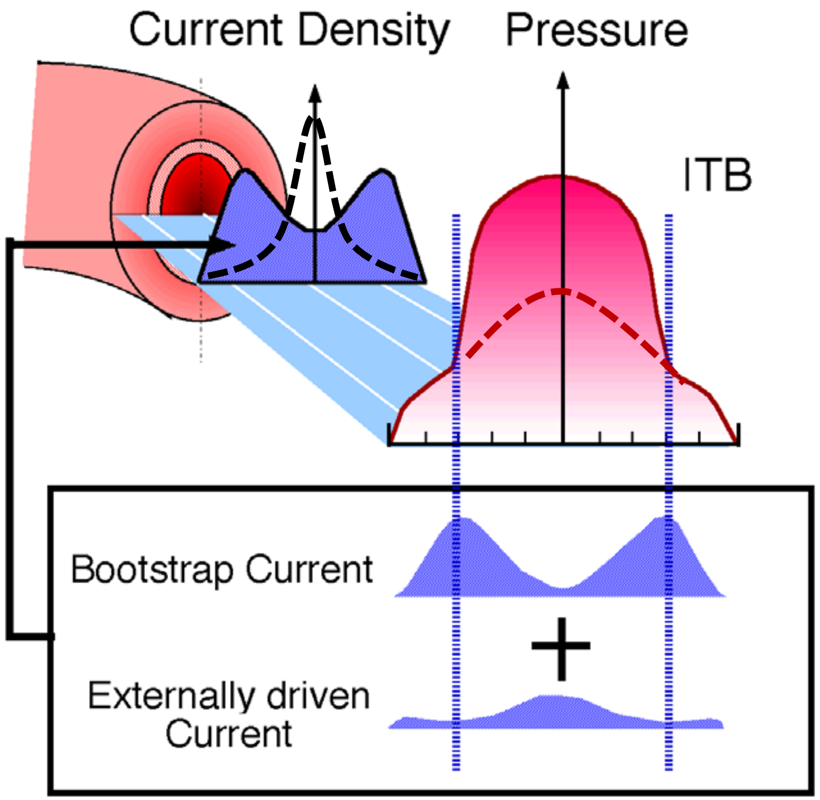


Tokamak Transport

- **Suppression of Anomalous Transport**



Improved confinement suitable for the steady-state operation



Non-monotonic current profile

↓

Turbulence suppression

↓

High pressure gradients

↓

Large bootstrap current

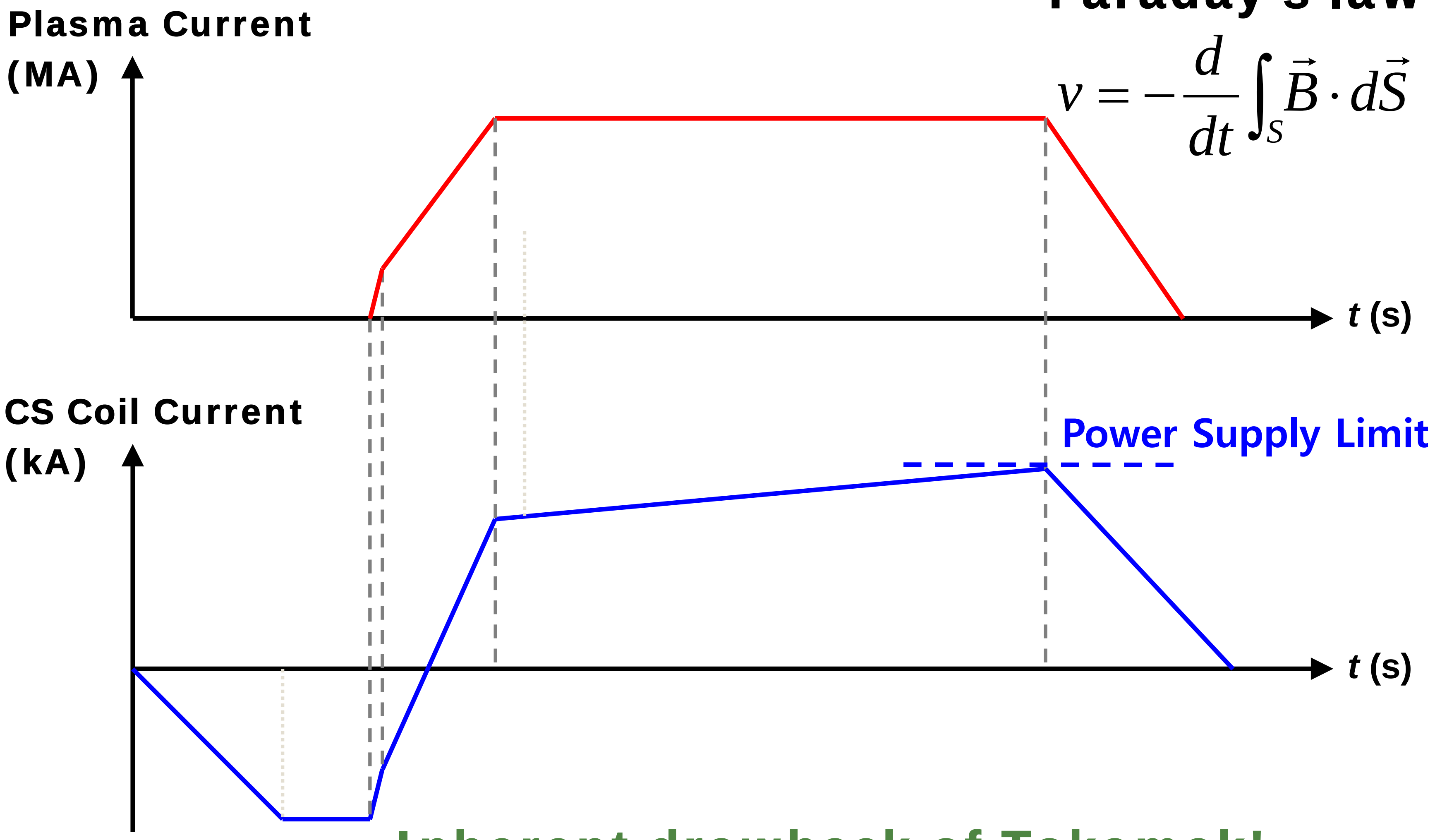
↓

Non-inductive current drive

Pulsed Operation

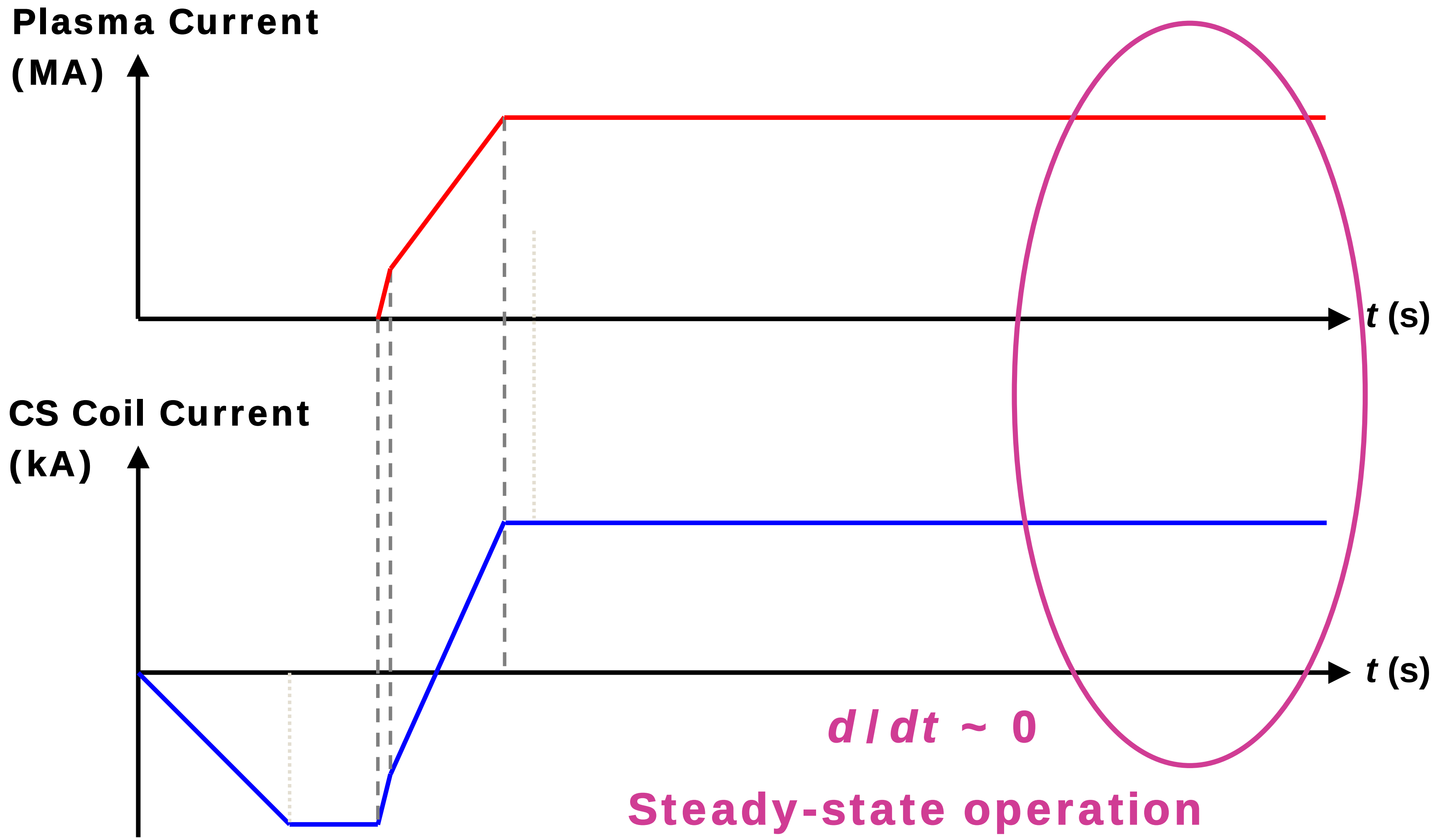
Faraday's law

$$v = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S}$$



Inherent drawback of Tokamak!

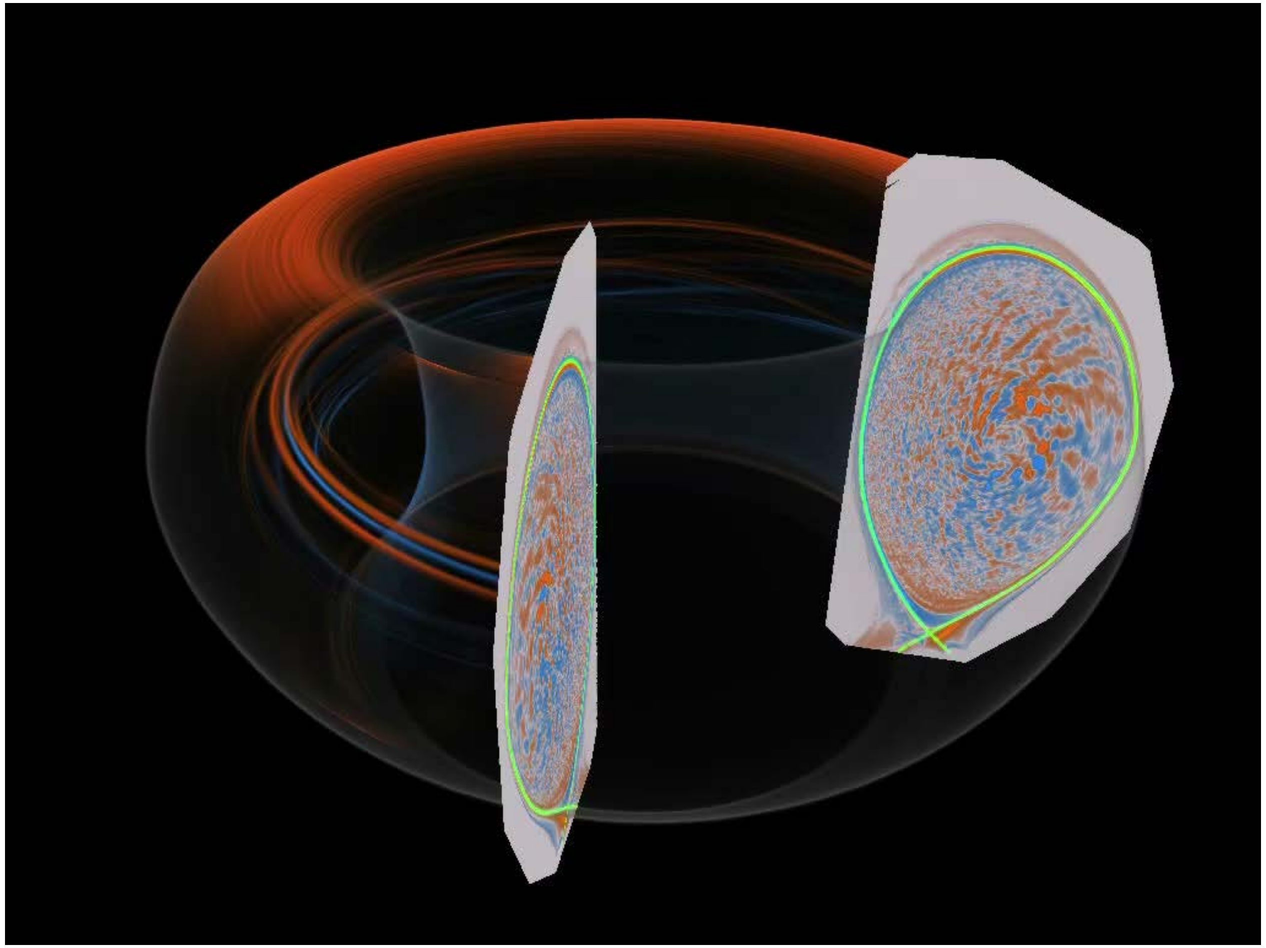
Steady-State Operation



Steady-state operation
by self-generated and externally driven current

Turbulence Simulations

- XGC1 simulation



Courtesy of S. H. Ku (PPPL)

Energy Confinement Time

$$\tau_E = \frac{W}{P_{in} - \frac{\partial W}{\partial t}} \approx \frac{W}{P_{in}} = \frac{\text{stored energy}}{\text{applied heating power}}$$

In steady conditions, neglecting radiation loss, Ohmic heating replaced by total input power

- Since tokamak transport is anomalous, empirical scaling laws for energy confinement are necessary.
- To predict the performance of future devices, the energy confinement time is one of the most important parameter.
- Empirical scaling laws: regression analysis from available experimental database.

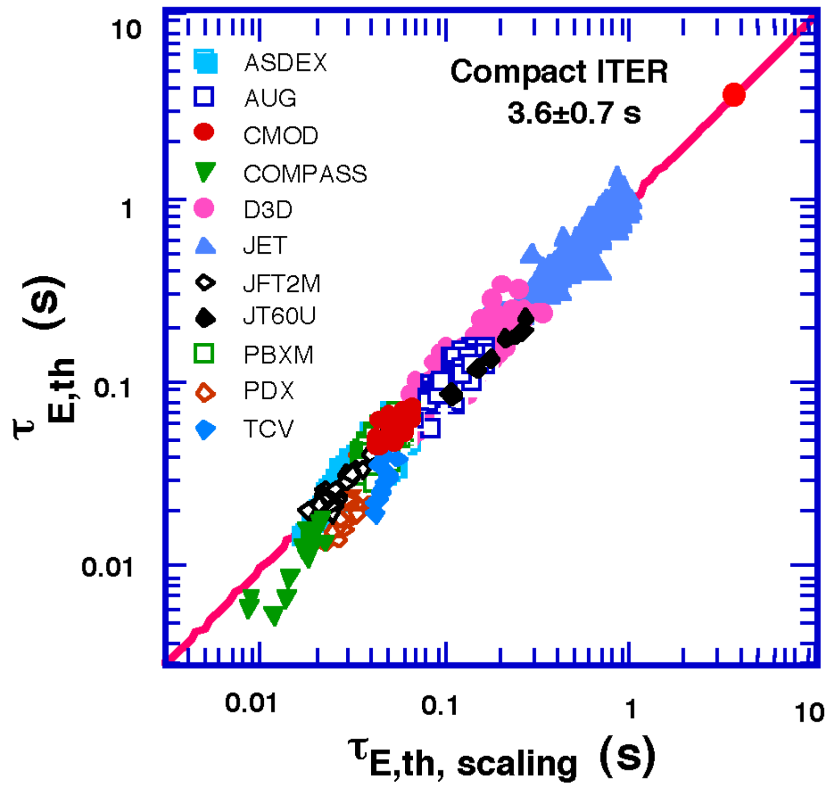
$$\tau_{th,E}^{fit} = C I^{\alpha I} B^{\alpha B} P^{\alpha P} n^{\alpha n} M^{\alpha M} R^{\alpha R} \varepsilon^{\alpha \varepsilon} \kappa^{\alpha \kappa}$$

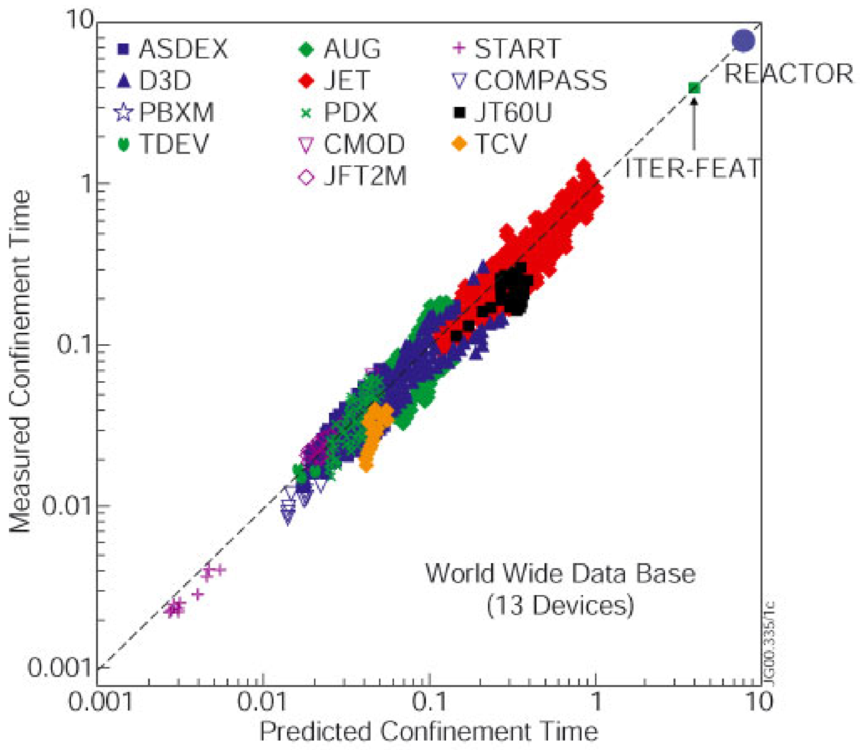
in engineering variables

Energy Confinement Time

- **H-mode confinement scaling**

$$\tau_{th,E}^{IPB98(y,2)} = 0.0562 I^{0.93} B^{0.15} P^{-0.69} n^{0.41} M^{0.19} R^{1.97} \varepsilon^{0.58} \kappa_a^{0.78}$$





τ_E in KSTAR and ITER?
Why should ITER be large?