Shape memory alloys

❖ Shape Memory Alloys

- has an internal solid state phase transformation mechanisms which allows 2 stable states depending on of applied stress and temperatures

➢ Nickel – Titanium ("Nitinol")

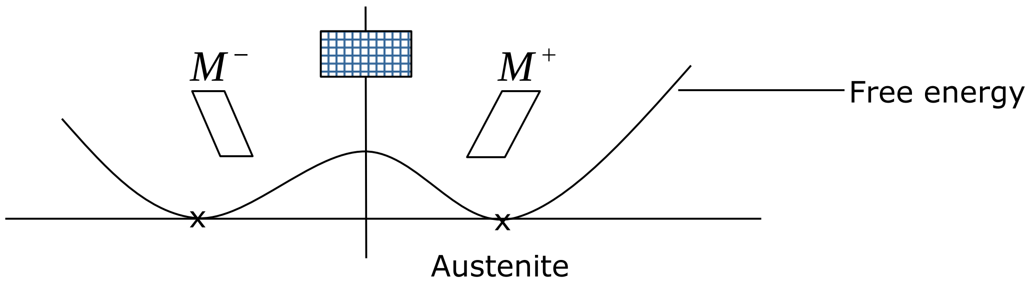
- utilized in robot applications –
  hose clamps
  Large space structure vibration control
  adaptive acoustics
- current :
  - Slow adoptive structures
    - twist control of rotors & propellers
    - adoptive fixed-wing lifting surfaces
    - airfoil twist control

➢ The phenomena

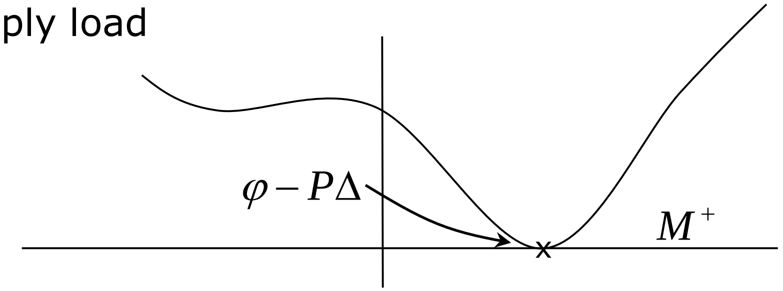
- stress + temperature induced martensitic phase transformation
- depends on comp, temperature, stress, history temperature

Shape memory alloys

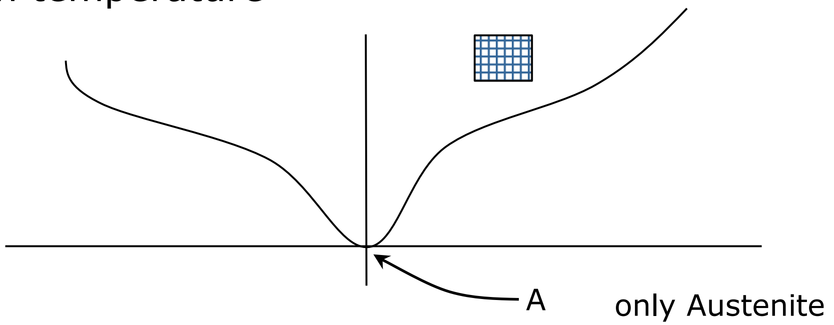
- Heuristic phenomology
  - room temperature



  - apply load



  - high temperature



Shape memory alloys

- Temperature + stress induced phase transformation

- constitutive Relation

$$\sigma - \sigma_0 = d\left(\varepsilon - \varepsilon_0\right) + \theta\left(T - T_0\right) + \Omega\left(\xi - \xi_0\right)$$

Mechanical ↓ Thermal ↓ Phase change ↓

$\xi = \%$ of martensite

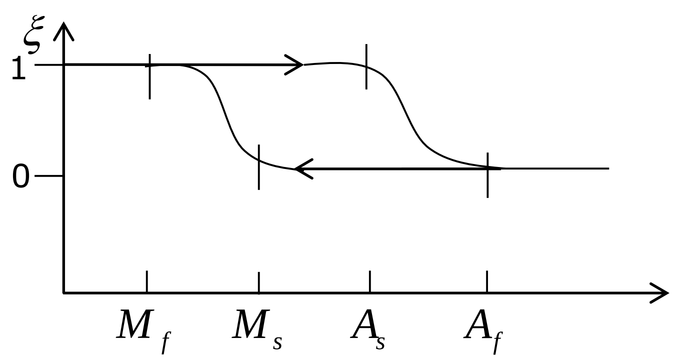
- martensite function

$$\begin{cases} \xi = 1 & \text{All martensite} \\ \xi = 0 & \text{All austenite} \end{cases}$$

- look at phase transformation

$$\xi = f\left(T, \sigma\right)$$

First, $\xi = f\left(T\right)$



Transformation is characterized by 4 temperatures

M_f : martensite finish

M_s : martensite start

A_s : Austenite start

A_f : Austenite finish

Shape memory alloys

two types of material

1) $A_s > M_s$

2) $A_s < M_s$

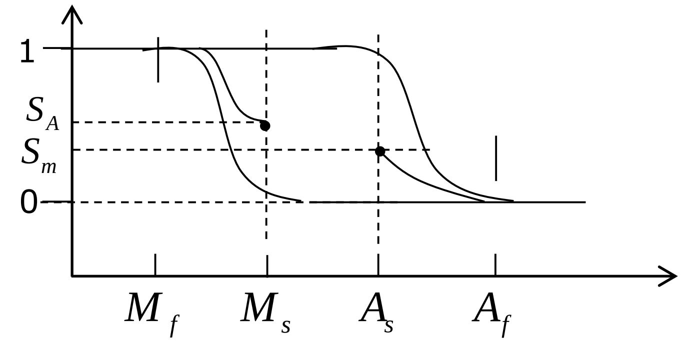
where, in room temperature

$$M \rightarrow A : \xi = \frac{1}{2}\left\{\cos\left[a_A\left(T - A_s\right)\right] + 1\right\}$$

$$A \rightarrow M : \xi = \frac{1}{2}\left\{\cos\left[a_M\left(T - M_f\right)\right] + 1\right\}$$

$$A_s < T < A_f$$

$$M_f < T < M_s$$



$say \;\; M - A \quad \xi_0 = \xi_m$

$$\xi = \xi/_2 \left\{\cos\left[a_A\left(T - A_S\right)\right] + 1\right\}$$

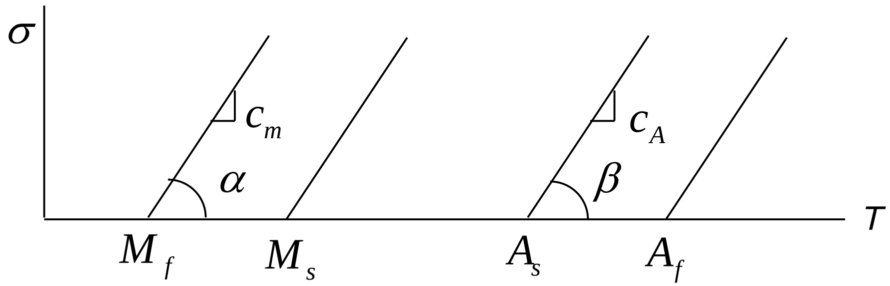
$A - M \quad \xi_0 = \xi_m$

$$\xi = \frac{1-\xi_A}{2}\left\{\cos\left[a_M\left(T - M_f\right)\right]\right\} + \frac{1+\xi_A}{2}$$

Shape memory alloys

- Stress dependence of ξ

- Transformation temperatures increased with applied stress



$$\begin{cases} c_m = \tan(\alpha) \\ c_A = \tan(\beta) \end{cases}$$

$$\Rightarrow \begin{cases} M_f' = M_f + \dfrac{\sigma}{\sigma_m} \\ A_s' = A_s + \dfrac{\sigma}{c_m} \end{cases}$$

plugging in stress effects

$$M-A \qquad \zeta = \frac{\zeta_m}{2}\left\{\cos\left[a_A\left(T-A_s\right)-b_A\sigma\right]+1\right\}$$

$$A-M \qquad \zeta = \frac{1-\zeta_A}{2}\cos\left[a_M\left(T-M_f\right)+b_M\sigma\right]+\frac{1+\zeta_A}{2}$$

$$b_A = -\frac{a_A}{c_A} \quad , \quad b_M = -\frac{a_M}{c_A}$$

$$M-A \qquad c_A\left(T-A_s\right)-\frac{\pi}{|b_A|} \le \sigma \le c_A\left(T-A_s\right)$$

$$A-M \qquad c_M\left(T-M_f\right)-\frac{\pi}{|b_M|} \le \sigma \le c_M\left(T-M_f\right)$$

$c_M\left(T-M_s\right)$

Shape memory alloys

Constitutive Modeling

1-D

Assume $M_f < M_s < T_R < A_S < A_f$

- Case A
  - Isothermal loading
    - all austenite (*S=0*)
  - initial conditions

$$\sigma_0 = 0, \;\; \zeta_0 = 0, \;\; \zeta = 0, \;\; T = T_0 \text{ isothermal}$$

$$\sigma - \sigma_0 = d\left(\varepsilon - \varepsilon_0\right) + \theta\left(T - T_0\right) + \Omega\left(\zeta - \zeta_0\right)$$

to start

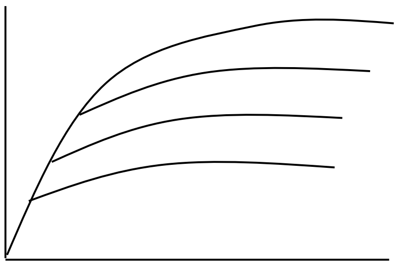
$\sigma = D\varepsilon$: linear elastic Austenite

→ fire until stresses reaches range where martensite start

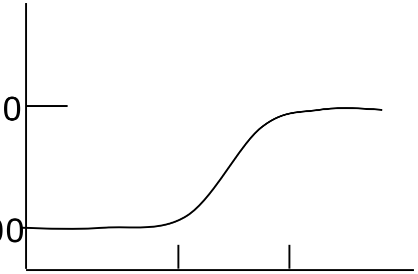
$$\sigma_{1,M} = c_M\left(T_0 - M_s\right) \rightarrow \varepsilon_{1,M} = \sigma_{1,M} / D$$

Shape memory alloys

Stress-strain



Yield strength



Transformation

once it begins, $\sigma - \sigma_0 = D(\varepsilon - \varepsilon_0) + \Omega(\zeta - \zeta_0)$

$$\begin{cases} \sigma_0 = \sigma_{\lim} \\ \varepsilon = \varepsilon_{\lim} \\ \zeta_0 = 0 \end{cases} \Rightarrow \sigma = D\varepsilon + \Omega(\zeta)$$

where,

$$\zeta = \frac{1-\zeta_A}{2}\cos\left[a_M\left(T - \left(M_f + \frac{\sigma}{c_M}\right)\right)\right] + \frac{1+S_A}{2}$$

This progresses until $\zeta = 1$ or wherever

$$\zeta = 1 \quad \Rightarrow \quad \sigma = c_M\left(T_0 - M_f\right)$$

Fiber Optics Sensor

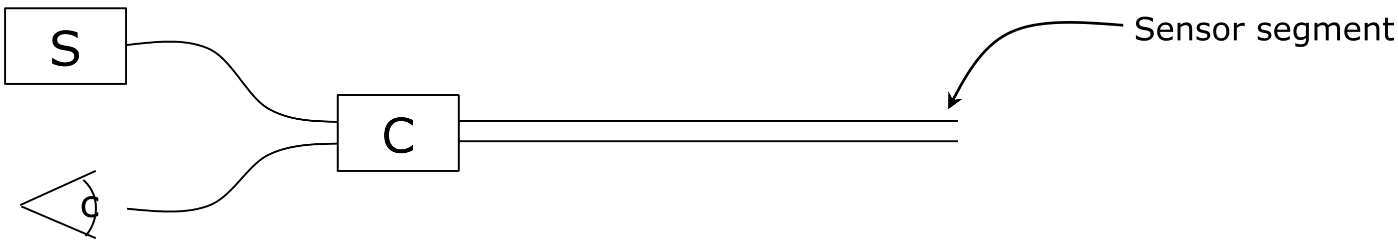
Fiber optic sensor

- Intensoretric
- Interferometric
- Polarmetric
- Modalmetric
- Spectral
- OTDR

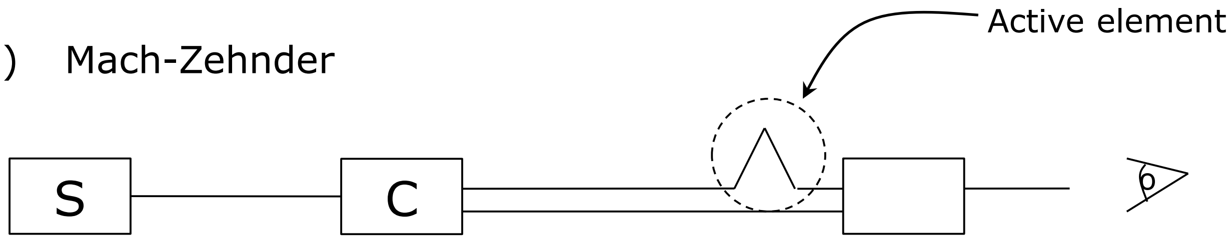
▪ Interferometric

- 3 types

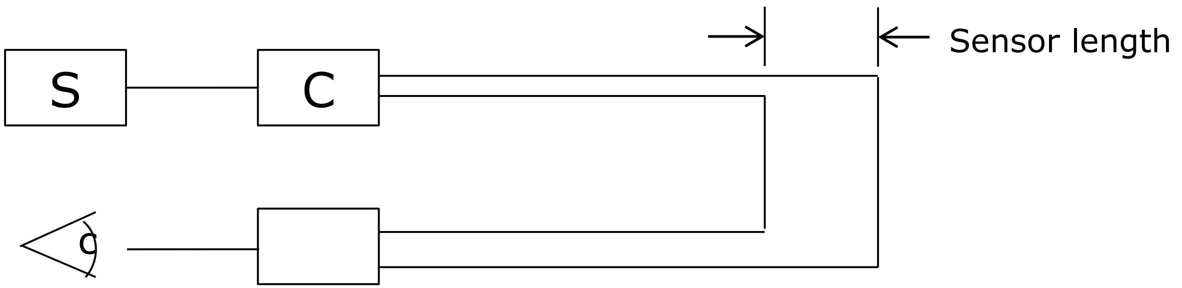
i) Michelson



ii) Mach-Zehnder

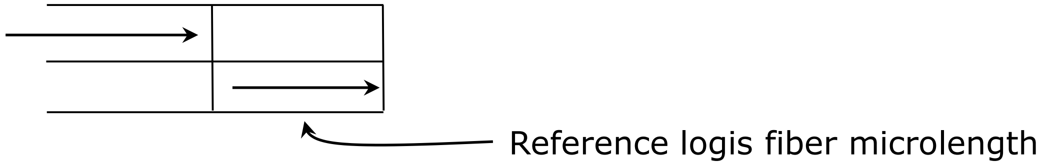


Fiber Optics Sensor

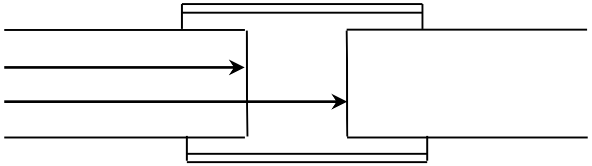


iii) Fabrey-Revot

- intrinsic



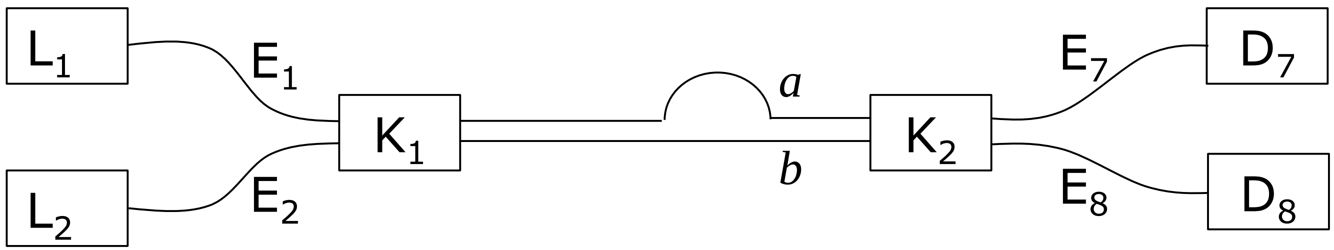
- extrinsic



Fiber Optics Sensor

Simple model

Model of Interferometric Sensor



Transmission matrices

$$E_{in} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \qquad E_{out} = \begin{bmatrix} E_7 \\ E_8 \end{bmatrix} = \underbrace{[K_2]}_{2x2}[T]\underbrace{[K_1]}_{2x2}E_{in}$$

For standard 3db coupler,

- coupler

$$K_1 = K_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$$

- path (no attenuation)

$$T = \begin{bmatrix} e^{i\varphi_A} & 0 \\ 0 & e^{i\varphi_B} \end{bmatrix}$$

φ_A : phase difference through path a

φ_B : phase difference through path b

Fiber Optics Sensor

Substitution

- $E_2 = 0$

$$\begin{bmatrix} E_7 \\ E_8 \end{bmatrix} = \begin{bmatrix} E_1 & \left\{ e^{i\varphi_A} - e^{i\varphi_B} \right\} / 2 \\ iE_1 & \left\{ e^{i\varphi_A} - e^{i\varphi_B} \right\} / 2 \end{bmatrix}$$

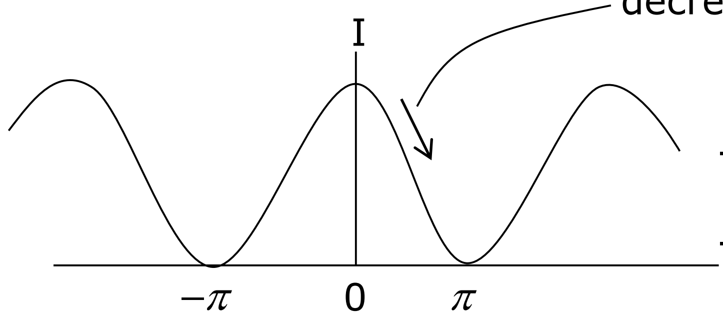
intensities u (for attenuation)

$$I_7 = I_1 \left[1 - \cos\left(\varphi_A - \varphi_B \right) \right] / 2 = I_1 \sin^2 \left(\frac{\Delta\varphi}{2} \right)$$

$$I_8 = I_1 \left[1 + \cos\left(\varphi_A - \varphi_B \right) \right] / 2 = I_1 \cos^2 \left(\frac{\Delta\varphi}{2} \right)$$

u

decreasing intensities



- ambiguity between $+\varphi$ and $-\varphi$
  cos is even function
- Solution - either

Slope + sensitivity as well as mean

$\Delta\varphi$

- how changes in environment effect $\Delta\varphi$

$$\Delta\varphi = K \left[n\Delta L + L\Delta n \right]$$

$K = \dfrac{\lambda}{2\pi}$

Fiber Optics Sensor

Effects

$\Delta L = \varepsilon_{11} L$: elongation in change in path length

$\Delta\varphi = K_n \Delta L$

2) $\Delta n = f(\varepsilon)$:photoelastic effect

$$\Delta n = -n^3 \left[P_{11}\varepsilon_{33} + P_{12}\varepsilon_{22} + P_{12}\varepsilon_{11} \right] / 2$$

Photoelastic constants

putting it together

$$\Delta\varphi = K_n \Delta L \left\{ \varepsilon_{11}^f - \frac{1}{2} n^2 \left[P_{11}\varepsilon_{33}^f + P_{12}\varepsilon_{22}^f + P_{12}\varepsilon_{11}^f \right] + \alpha \Delta T \right\}$$

long strain photoelastic thermal

- Typical : silica core fibers

$P_{11} = 0.113, \quad P_{12} = 0.252$

$\varepsilon_{22} = \varepsilon_{33} = \nu\varepsilon_{11}$

$$\Delta\varphi = KnL\varepsilon_{11} \left[1 - \frac{n^2}{2} \left\{ (1-\nu) P_{12} - \nu P_{11} \right\} \right]$$

$\Delta\varphi = SL\varepsilon_{11}, \qquad S = 1.13 \times 10^7$ rad/strain-m

Scale factor

Fiber Optics Sensor

1 cm given length

1 at $\frac{I_8}{I_1} = 0.9975$

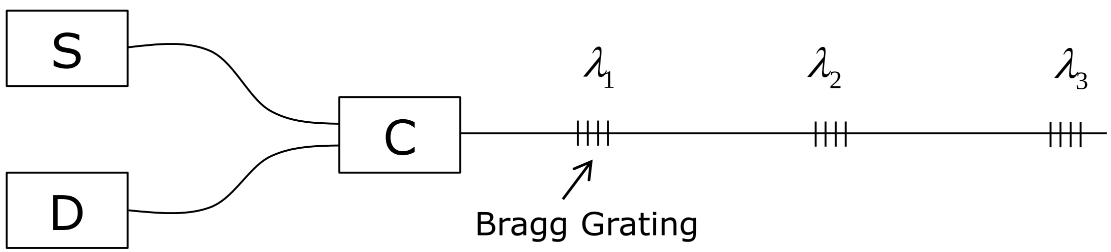
other application - polimetry + multi-mode fibers

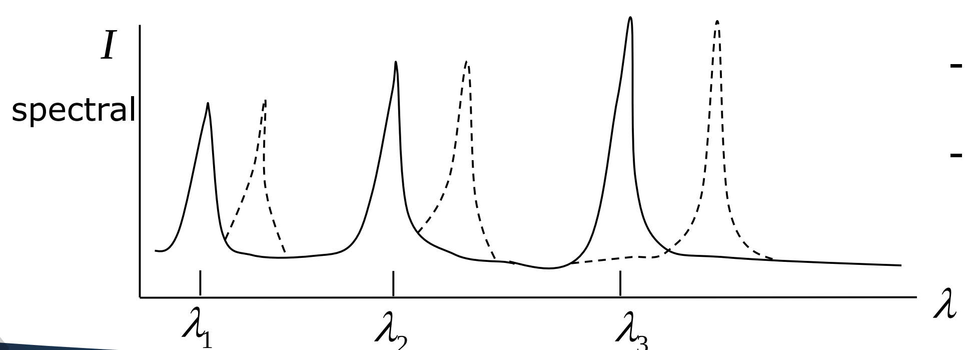
… 2 orthogonal polarization

- 2 propagating modes

… beating

- Bragg Grating Based sensors





- Changes in grating wavelengths due to strain
- Multiplexing