YCU4

제4장 임의 가진력에 대한 응답.

$$m\ddot{u} + c\dot{u} + ku = p(t)$$

$$u(0) = 0, \quad \dot{u}(0) = 0$$

Linear System

$$\underset{\text{input}}{p(t)} \longrightarrow \boxed{\text{Linear System}} \longrightarrow \underset{\text{output}}{u(t)}$$

physical system is causal

$$L\{p(t)\} = u(t)$$

↑

Linear System.

Impulse Response function

$$p(t) = \int_0^\infty p(\tau)\,\delta(t-\tau)\,d\tau$$

↑ Delta function

$$L\{p(t)\} = L\int_0^\infty p(\tau)\,\delta(t-\tau)\,d\tau$$

2/CH4

$$= \int_0^\infty p(\tau)\, L\{\delta(t-\tau)\}\, d\tau$$

$$= \int_0^\infty p(\tau)\, h(t-\tau)\, d\tau$$

$$h(t-\tau) = L\{\delta(t-\tau)\}$$

Impulse Response function.

Laplace Transform

For given $f(t)$

Laplace Transform $F(s)$

$$F(s) = \int_0^\infty e^{-st} f(t)\, dt = \mathcal{L}\{f(t)\} = \hat{f}(s)$$

Inverse transform

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\{\hat{f}(s)\}$$

3/cu4

Convolution (표절)

$$F(s) = \mathcal{L}\{f(t)\}$$

$$H(s) = \mathcal{L}\{h(t)\}$$

$$G(s) = F(s)H(s) = \mathcal{L}\{g(t)\}$$

$$g(t) = \int_0^t f(\tau)h(t-\tau)d\tau$$

$$= \int_0^t h(\tau)f(t-\tau)d\tau$$

$$= \mathcal{L}^{-1}\{F(s)H(s)\}$$

$$= f(t) * h(t)$$

↑ Convolution 기호

Laplace Transformal 의한 Impulse Response function

$$m\ddot{u} + c\dot{u} + ku = f(t)$$

$$\mathcal{L}\{m\ddot{u} + c\dot{u} + ku\} = \mathcal{L}\{f(t)\} = 1$$

$$\mathcal{L}\{\dot{u}\} = s\hat{u}(s) - u(0)$$

4/C4강

$$\mathcal{L}\{\ddot{u}\} = s^2\hat{u}(s) - su(0) - \dot{u}(0)$$

$$\mathcal{L}\{u^{(n)}\} = s^n\hat{u}(s) - s^{n-1}u(0) - s^{n-2}\dot{u}(0) - \cdots - u^{(n-1)}(0)$$

$u(0)=0$, $\dot{u}(0)=0$ 라고 가정

$$(ms^2+cs+k)\hat{u} = 1$$

$$\hat{h}(s) = \hat{u}(s) = \frac{1}{ms^2+cs+k}$$

$$= \left(\frac{1}{m}\right)\frac{1}{s^2+2\omega_n\zeta s+\omega_n^2}$$

$$= \left(\frac{1}{m}\right)\frac{1}{(s+\omega_n\zeta)^2+\omega_n^2-\omega_n^2\zeta^2}$$

$$= \left(\frac{1}{m}\right)\frac{1}{(s+\omega_n\zeta)^2+\omega_D^2}$$

$$= \frac{1}{m\omega_D}\cdot\frac{\omega_D}{(s+\omega_n\zeta)^2+\omega_D^2}$$

$$\mathcal{L}^{-1}\left\{\frac{\omega_D}{(s+\omega_n\zeta)^2+\omega_D^2}\right\} = e^{-\omega_n\zeta t}\sin\omega_D t$$

5/CNB

$$h(t) = \mathcal{L}^{-1}\{\hat{h}(s)\}$$
$$= \left(\frac{1}{m\omega_D}\right) e^{-\omega_n \zeta t} \sin \omega_D t$$

$$\hat{h}(s) \longleftrightarrow h(t)$$

$$m\ddot{u} + c\dot{u} + ku = p(t)$$
$u(0)=0$, $\dot{u}(0)=0$ 에 대한 해

$$u(t) = p(t) * h(t)$$
$$= \int_0^\infty p(\tau) h(t-\tau) d\tau$$
(why ?)
$$= \int_0^t p(\tau) h(t-\tau) d\tau$$
$$= \left(\frac{1}{m\omega_D}\right) \int_0^t p(\tau) e^{-\omega_n \zeta (t-\tau)} \sin \omega_D (t-\tau) d\tau$$
$$= \int_0^t p(\tau) \underbrace{\frac{1}{m\omega_D} e^{-\omega_n \zeta (t-\tau)} \sin \omega_D (t-\tau)}_{h(t-\tau)} d\tau$$

6/CH13

Non zero 초기조건에 대한 해

$$m\ddot{u} + c\dot{u} + ku = p(t)$$

$$\mathcal{L}\{m\ddot{u} + c\dot{u} + ku\} = \mathcal{L}\{p(t)\}$$

$$(ms^2 + cs + k)\hat{u}(s) = (ms + c)u(0) + m\dot{u}(0) + \hat{p}(s)$$

$$\hat{u}(s) = \frac{ms + c}{ms^2 + cs + k}u(0) + \frac{m}{ms^2 + cs + k}\dot{u}(0) + \frac{\hat{p}(s)}{ms^2 + cs + k}$$

Laplace Transform Formula

$$e^{at}\sin bt \Leftrightarrow \frac{b}{(s-a)^2 + b^2}$$

$$e^{at}\cos bt \Leftrightarrow \frac{s-a}{(s-a)^2 + b^2}$$

$$\frac{ms + c}{ms^2 + cs + k} = \frac{s + \omega_n\zeta}{(s + \omega_n\zeta)^2 + \omega_D^2} + \frac{\omega_n\zeta}{(s + \omega_n\zeta)^2 + \omega_D^2}$$

$$\frac{m}{ms^2 + cs + k} = \left(\frac{1}{\omega_D}\right)\frac{\omega_D}{(s + \omega_n\zeta)^2 + \omega_D^2}$$

17/04/13

$$u(t) = \left\{ e^{-\omega_n \zeta t} \cos\omega_D t + \frac{\omega_n \zeta}{\omega_D} e^{-\omega_n \zeta t} \sin\omega_D t \right\} u(0)$$

$$+ \left\{ \left(\frac{1}{\omega_D}\right) e^{-\omega_n \zeta t} \sin\omega_D t \right\} \dot{u}(0)$$

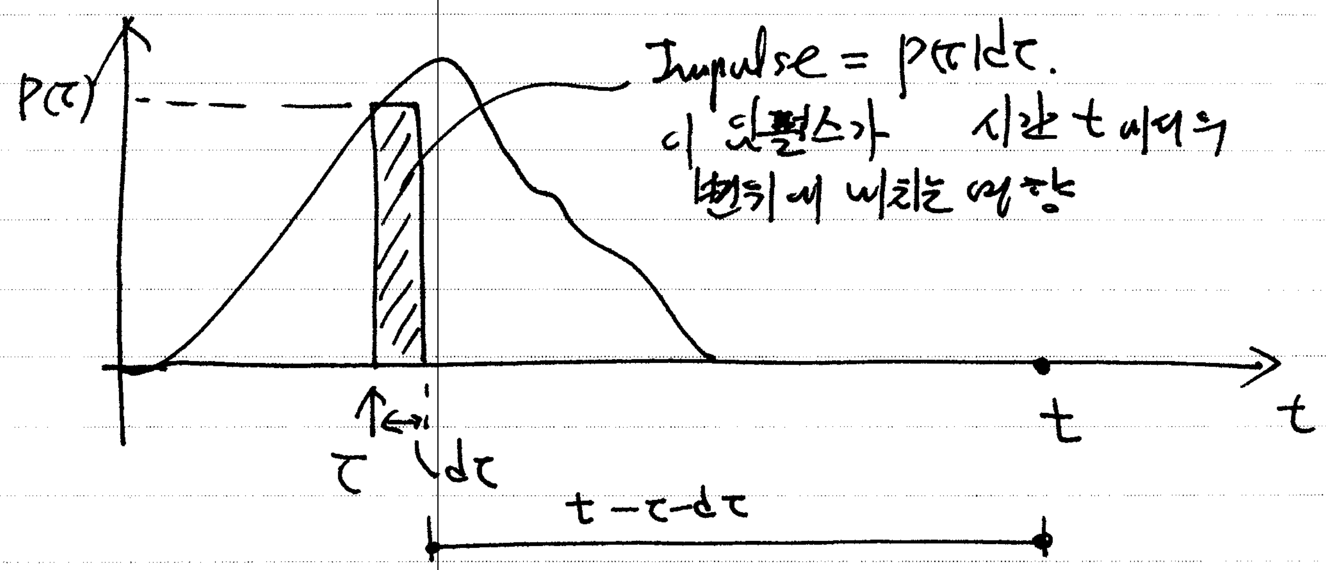
$$+ \int_0^t p(\tau) \frac{1}{m\omega_D} e^{-\omega_n \zeta (t-\tau)} \sin\omega_D (t-\tau)\, d\tau$$

complementar component
natural component

+

particular component
forced component

4.1. 임의의 펄스에 대한 응답.



뉴턴의 제2 법칙

$$\frac{d}{dt}(m\dot{u}) = p(t)$$

$m = \text{const}$

$$p(t) = m\ddot{u}$$

$$\int_{t_1}^{t_2} p\,dt = m(\underset{\uparrow\,\dot{u}(t_2)}{\dot{u}_2} - \underset{\uparrow\,\dot{u}(t_1)}{\dot{u}_1}) = m\Delta\dot{u}$$

시간 τ 에서 $\tau + d\tau$ 까지 작용하는 $p(\tau)$ 에 의해서

시간 $\tau + d\tau$ 에서의 속도의 변화

$$p(\tau)d\tau = m(\dot{u}(\tau + d\tau) - \dot{u}(\tau))$$

$$= m\,\Delta\dot{u}(\tau)$$

시간 τ 에서 $\tau + d\tau$ 까지 작용하는 $p(\tau)$에 의해서 발생하는 변위변화. $\Delta u(\tau)$

$$\Delta u(\tau) = u(\tau + d\tau) - u(\tau)$$

$$= \frac{1}{2}\frac{p(\tau)}{m}(d\tau)^2$$

$$\simeq 0$$

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$\Delta \dot{u}(\tau) = \frac{1}{m} p(\tau) d\tau$ 에 의해서 생긴

자유진동에 의한 시간 t에서의 변위의 증가.

$$\Delta u(t) = \frac{\Delta \dot{u}(\tau)}{\omega_D} e^{-\omega_n \zeta (t-\tau-d\tau)} \sin \omega_D (t-\tau-d\tau)$$

$$= \frac{p(\tau)}{m\omega_D} e^{-\omega_n \zeta (t-\tau-d\tau)} \sin \omega_D (t-\tau-d\tau) d\tau$$

$$= \frac{p(\tau)}{m\omega_D} e^{-\omega_n \zeta (t-\tau)} \sin \omega_D (t-\tau) d\tau$$

이 영향을 시간 $t=0$ 에서 t 까지 모으면

$$u(t) = \int_0^t \frac{p(\tau)}{m\omega_D} e^{-\omega_n \zeta (t-\tau)} \sin \omega_D (t-\tau) d\tau$$

만약 $\zeta = 0$ 이면

$$u(t) = \int_0^t \frac{p(\tau)}{m\omega_n} \sin \omega_n (t-\tau) d\tau$$

10/CH15

Forced component의 속도와 가속도

Leibnitz Formula

$$g(x) = \int_{\alpha(x)}^{\beta(x)} f(x,\tau)\,d\tau$$

$$g'(x) = \int_{\alpha(x)}^{\beta(x)} \frac{\partial}{\partial x} f(x,\tau)\,d\tau$$

$$+ f[x,\beta(x)]\beta'(x) - f[x,\alpha(x)]\alpha'(x)$$

(증명)

$$g'(x) = \lim_{h\to 0} \frac{1}{h} \left\{ \int_{\alpha(x+h)}^{\beta(x+h)} f(x+h,\tau)\,dx \right.$$

$$\left. - \int_{\alpha(x)}^{\beta(x)} f(x,\tau)\,d\tau \right\}$$

$$= \lim_{h\to 0} \frac{\int_{\alpha(x+h)}^{\beta(x+h)} f(x+h,\tau)\,d\tau - \int_{\alpha(x)}^{\beta(x)} f(x+h,\tau)\,d\tau}{h}$$

$$+ \lim_{h\to 0} \frac{\int_{\alpha(x)}^{\beta(x)} f(x+h,\tau)\,d\tau - \int_{\alpha(x)}^{\beta(x)} f(x,\tau)\,d\tau}{h}$$

NO. 11/cyB

$$= \lim_{h\to 0} \frac{\int_{\beta(x)}^{\beta(x+h)} f(x+h,\tau)\,d\tau}{h}$$

$$- \lim_{h\to 0} \frac{\int_{\alpha(x)}^{\alpha(x+h)} f(x+h,\tau)\,d\tau}{h}$$

$$+ \int_{\alpha(x)}^{\beta(x)} \frac{\partial f(x,\tau)}{\partial x}\,d\tau$$

$$= \lim_{h\to 0} f(x+h,\tau) \lim_{h\to 0} \frac{\beta(x+h)-\beta(x)}{h}$$

$$- \lim_{h\to 0} f(x+h,\alpha(x)) \lim_{h\to 0} \frac{\alpha(x+h)-\alpha(x)}{h}$$

$$+ \int_{\alpha(x)}^{\beta(x)} \frac{\partial f(x,\tau)}{\partial x}\,d\tau$$

$$= \int_{\alpha(x)}^{\beta(x)} \frac{\partial f(x,\tau)}{\partial x}\,d\tau + f(x,\beta(x))\frac{d\beta}{dx} - f(x,\alpha(x))\frac{d\alpha}{dx}$$

$$\dot{u}(t) = \frac{1}{m}\int_0^t p(\tau)\, e^{-\omega_n \zeta (t-\tau)} \cos\omega_D(t-\tau)\, d\tau$$

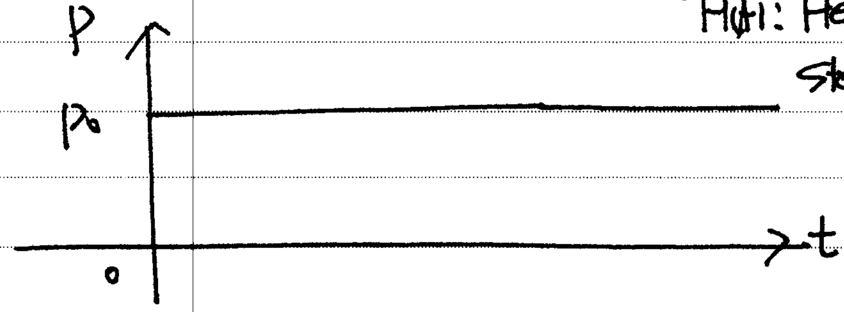
$$- \frac{1}{m}\frac{\omega_n \zeta}{\omega_D}\int_0^t p(\tau)\, e^{-\omega_n \zeta (t-\tau)} \cos\omega_D(t-\tau)\, d\tau$$

$$\ddot{u}(t) = \frac{p(t)}{m} - \omega_n^2 u(t) - 2\omega_n \zeta \dot{u}(t)$$

4.3. 계단 하중

$$p(t) = \begin{cases} 0, & t < 0 \\ p_0, & t > 0 \end{cases}, \quad p(t) = p_0 H(t)$$

$H(t)$: Heaviside step function



무감쇠 시스템

$$u(t) = \frac{1}{m\omega_n}\int_0^t p(\tau) \sin\omega_n(t-\tau)\, d\tau$$

$$= \frac{p_0}{m\omega_n}\int_0^t \sin\omega(t-\tau)\, d\tau$$

$$= \frac{p_0}{m\omega_n^2}(1-\cos\omega_n t)$$

$$= \frac{p_0}{k}(1-\cos\omega_n t)$$

$$= (u_{st})_0 (1-\cos\omega_n t)$$

최대 변위 u_0

$$u_0 = 2(u_{st})_0, \quad t = \frac{jT_n}{2}, \; j=1, 3, 5, \cdots$$

감쇠시스템

$$m\ddot{u} + c\dot{u} + ku = p_0 H(t)$$

$$u(t) = e^{-\omega_n \zeta t}(A\cos\omega_D t + B\sin\omega_D t) + p_0/k$$

$$A = -\frac{p_0}{k}, \quad B = -\frac{p_0}{k}\cdot\frac{\zeta}{\sqrt{1-\zeta^2}}$$

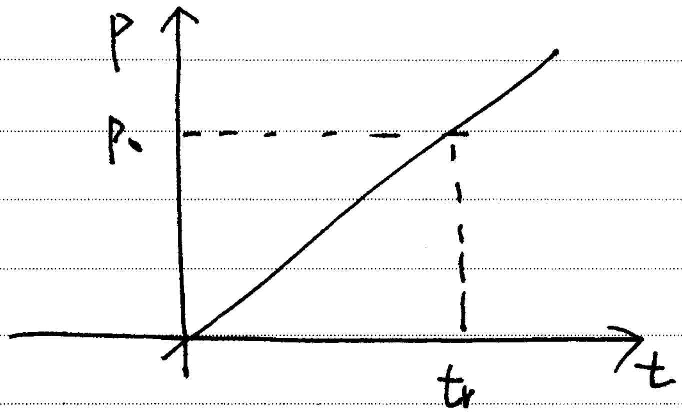
$$u(t) = (u_{st})_0\left[1 - e^{-\omega_n \zeta t}\left(\cos\omega_D t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_D t\right)\right]$$

∨ 정적평형위치를 중심으로 감쇠 진동

∨ 궁극적으로 정적 평형 위치에 도달

4.4 램프하중

$$p(t) = p_0 (t/t_r)$$



undamped system

$$u(t) = \frac{1}{m\omega_n} \int_0^t \left(\frac{p_0}{t_r}\tau\right) \sin(\omega_n(t-\tau)) d\tau$$

$$= \frac{1}{m\omega_n} \frac{p_0}{t_r} \left\{ \left[\frac{\tau}{\omega} \cos\omega_n(t-\tau)\right]_0^t - \int_0^t \left(\frac{1}{\omega_n}\right) \cos\omega_n(t-\tau) d\tau \right\}$$

$$= \left(\frac{p_0}{m\omega_n^2}\right)\left(\frac{t}{t_r} - \frac{1}{\omega_n t_r} \sin\omega_n t\right)$$

$$= (u_{st})_0 \left(\frac{t}{t_r} - \frac{1}{\omega_n t_r} \sin\omega_n t\right)$$

4.5. 유한한 상승시간을 갖는 계단하중

$$p(t) = \begin{cases} p_0(t/t_r) & t \le t_r \text{ : 상승단계} \\ p_0 & t \ge t_r \text{ : 일정단계} \end{cases}$$

$t \le t_r$: 상승단계

$$u(t) = (u_{st})_0 \left(\frac{t}{t_r} - \frac{1}{\omega_n t_r} \sin \omega_n t \right)$$

$t > t_r$: 일정하중단계

$$u(t) = \frac{1}{m\omega_n} \int_0^{t_r} \left(\frac{p_0}{t_r} \tau\right) \sin \omega_n (t-\tau)\, d\tau$$

$$+ \frac{1}{m\omega_n} \int_{t_r}^{t} p_0 \sin \omega_n (t-\tau)\, d\tau$$

$$= (u_{st})_0 \left\{ \cos \omega_n (t_r - t_r) + \frac{1}{\omega_n t_r} (\sin \omega_n (t-t_r) - \sin \omega_n t) \right\}$$

$$u(t) = (u_{st})_0 \left\{ 1 + \frac{1}{\omega_n t_r} [\sin \omega_n (t-t_r) - \sin \omega_n t] \right\}$$

$$\dot{u}(t) = (u_{st})_o \left\{ \frac{1}{t_r} \left[\cos\omega_n(t-t_r) - \cos\omega_n t \right] \right\}$$

$$\omega_n t_r = \frac{2\pi}{T_n} t_r = 2\pi \text{ 이면 } \dot{u}(t) = 0$$

$$\frac{t_r}{T_n} = 1, 2, 3, \ldots, \infty. \text{ 이면}$$

$$\dot{u}(t) = 0 \qquad u(t) = (u_{st})_o$$

observations

1. 하중상승단계 : 주기 T_n으로 정적변위를 중심으로 진동
2. 일정하중단계 : 주기 T_n으로 정적 변위를 중심으로 진동
3. 램프하중 끝에서 $\dot{u}(t_r) = 0$ 이면 진동중단.
4. $t_r/T_n \ll 1$: 계단하중응답과 유사
5. $t_r/T_n \gg 1$: 정적해에 근접해서 진동

Trigonometric Identity

$$\sin\omega_n(t-t_r) = \sin\omega_n t \cos\omega_n t_r - \cos\omega_n t \sin\omega_n t_r$$

$$u(t) = (u_{st})_0 \left\{ 1 + \frac{1}{\omega_n t_r} \left[\sin\omega_n t (\cos\omega_n t_r - 1) - \cos\omega_n t \sin\omega_n t_r \right] \right\}$$

$$R_d = \frac{u_0}{(u_{st})_0} = 1 + \frac{1}{\omega_n t_r}\sqrt{(\cos\omega_n t_r - 1)^2 + \sin^2\omega_n t_r}$$

$$= 1 + \frac{1}{\omega_n t_r}\sqrt{2 - 2\cos\omega_n t_r}$$

$$= 1 + \frac{1}{2\pi t_r/T_n}(2)\left|\sin\frac{\omega_n t_r}{2}\right|$$

$$= 1 + \frac{1}{\pi(t_r/T_n)}\sin\frac{\pi t_r}{T_n}$$

계단하중의 응답스펙트럼

✓ Rd를 (t_r/T_n)의 함수로 표현한 그래프

1. $t_r < T_n/4$: $u_0 \simeq 2(u_{st})_0$, 갑작기가해지는하중
2. $t_r > 3T_n$: $u_0 \simeq (u_{st})_0$, 정적하중의 一 ~~~
3. $t_r/T_n = 1, 2, 3, \ldots$, $\dot{u}(t_r) = 0$, $u_0 = (u_{st})_0$
   일정하중 단계에서 진동하지 않음.

Part C : 펄스하중에 대한 응답

4.6. 풀이 방법

✓ 초기조건 영향 고려 : 안정상태에 도달하지 못함

✓ 해석 방법

(1) 고전적 방법 : 미분 방정식의 해

(2) 듀아멜 적분의 주력

(3) 중첩 법

고려 pulse 형상

(1) 직사각형 pulse

(2) 삼각형 pulse

(3) sine pulse

응답 평가

(1) 고전적 방법

(2) 강제진동단계, 자유진동단계

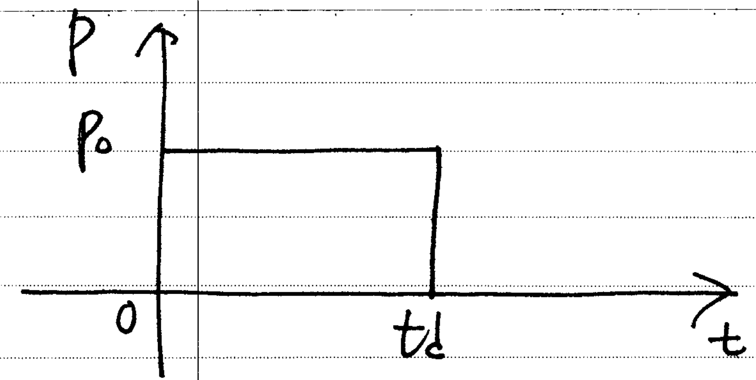
(3) 감쇠의 영향 경시

⟶ 무감쇠 시스템의 응답

4.7 직사각형 펄스

$$p_0 = \begin{cases} 0 & t<0 \\ p_0 & 0 \le t \le t_d \\ 0 & t>t_d \end{cases}$$

NO. 19/cu4



$$m\ddot{u} + ku = p(t)$$

$$u(0) = 0, \quad \dot{u}(0) = 0$$

1. 강제진동 단계, $t \le t_d$

$$\frac{u(t)}{(u_{st})_0} = 1 - \cos\omega_n t = 1 - \cos\frac{2\pi t}{T_n},$$

2. 자유진동 단계, $t \ge t_d$

$$u(t) = (u_{st})_0 \left\{ \cos\omega_n(t - t_d) - \cos\omega_n t \right\}$$

$$= (u_{st})_0 \left\{ \cos\omega_n t \cos\omega_n t_d + \sin\omega_n t \sin\omega_n t_d - \cos\omega_n t \right\}$$

$$= (u_{st})_0 \left\{ \cos\omega_n t (\cos\omega_n t_d - 1) + \sin\omega_n t \sin\omega_n t_d \right\}$$

$$= (u_{st})_0 \left\{ \cos\omega_N t\,(-2)\sin^2\frac{\omega_N t_d}{2} + \sin\omega_N t\,(2)\sin\frac{\omega_N t_d}{2}\cos\frac{\omega_N t_d}{2} \right\}$$

$$= (u_{st})_0 (-2)\sin\frac{\omega_N t_d}{2} \left\{ \cos\omega_N t \sin\frac{\omega_N t_d}{2} - \sin\omega_N t \cos\frac{\omega_N t_d}{2} \right\}$$

$$= (u_{st})_0 (2) \sin\frac{\omega_N t_d}{2} \left\{ \sin\left(\omega_N t - \frac{\omega_N t_d}{2}\right) \right\}$$

$$\frac{u(t)}{(u_{st})_0} = 2\sin\frac{\pi t_d}{T_N} \left\{ \sin 2\pi\left(\frac{t}{T_N} - \frac{t_d}{2T_N}\right) \right\}$$

<u>응답이력</u>

✓ $u(t)/(u_{st})_0$는 t_d/T_N의 함수

✓ 하중 작용 시 : $(u_{st})_0$만큼 이동한 위치에서
고유진주기 T_N으로 진동

✓ 하중 조료후 : 본래의 평형위치를 중심으로 진동

✓ $t_d/T_N = 1, 2, 3, \cdots$: pulse 종료후
~~$u=0$, $\dot{u}=0$~~ $u(t_d)=0$, $\dot{u}(t_d)=0$
→ 변형전 위치에서 정지

NO. 21*/CH4

최대응답

1. 강제진동 단계

① $t_d/T_n \le \frac{1}{2}$, $t_d < \frac{1}{2}T_n$
작용시간 끝 $t=t_d$에서 최대응답

$$u(t_d) = (u_{st})_0 \left(1 - \cos\frac{2\pi t_d}{T_n}\right)$$

$$R_d = 1 - \cos\frac{2\pi t_d}{T_n}$$

② $t_d/T_n \ge \frac{1}{2}$, $t_d \ge T_n/2$

강제진동구간에서 최고점에 도달

$$u_0 = 2(u_{st})_0$$

$$R_d = 2$$

2. 자유진동 단계

$$R_d = \frac{u_0}{(u_{st})_0} = 2\left|\sin\frac{\pi t_d}{T_n}\right|$$

3. 최대응답

① $t_d/T_n \le \frac{1}{2}$

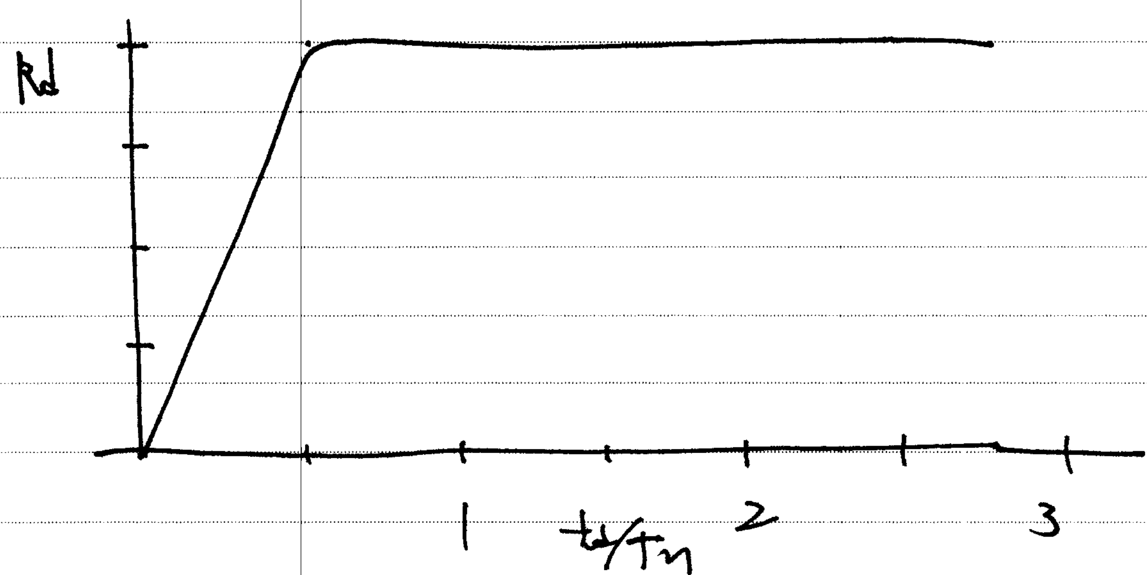
- ✓ 자유진동응답이 지배적
- ✓ $R_d = 2\sin \pi t_d/T_n$

② $t_d/T_n \ge \frac{1}{2}$

- ✓ 강제 진동응답이 지배적
- ✓ $R_d = 2$

4. 충격스펙트럼 (응답스펙트럼)

단자유도 시스템의 최대 변위를 시스템의 고유주기의 함수로 표현한 그림.



T_n, k, p_0, t_d 가 주어지면

$$u_0 = (u_{st})_0 R_d = \left(\frac{p_0}{k}\right) R_d$$

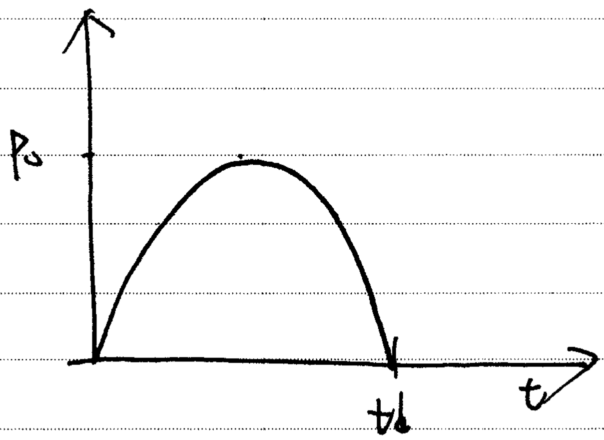
NO. 22/8/014

등가정적하중의 최대값

$$f_{s0} = k u_0 = p_0 R_d$$

4.8. 반사이클 sine 펄스하중

$$p = \begin{cases} 0 & t \le 0 \\ p_0 \sin(\pi t / t_d), & 0 \le t \le t_d \\ 0, & t \ge t_d \end{cases}$$



1. Case 1 : $t_d / T_n \ne \frac{1}{2}$

① 강제진동단계, $t \le t_d$

$$\frac{u(t)}{(u_{st})_0} = \frac{1}{1 - (T_n / 2t_d)^2} \left[\sin\left(\pi \frac{t}{t_d}\right) - \frac{T_n}{2t_d} \sin\left(2\pi \frac{t}{T_n}\right) \right]$$

② 자유진동단계, $t > t_d$

$$\frac{u(t)}{(u_{st})_0} = \frac{(T_n / t_d) \cos(\pi t_d / T_n)}{(T_n / 2t_d)^2 - 1} \sin\left[2\pi \left(\frac{t}{T_n} - \frac{1}{2} \frac{t_d}{T_n} \right) \right]$$

2. Case 2 : $t_d/T_n = \frac{1}{2}$

① 강제진동 단계, $t \le t_d$

$$\frac{u(t)}{(u_{st})_0} = \frac{1}{2}\left(\sin\frac{2\pi t}{T_n} - \frac{2\pi t}{T_n}\cos\frac{2\pi t}{T_n}\right)$$

② 자유진동 단계, $t \ge t_d$

$t = t_d$ 에서 $u(t_d)$, $\dot{u}(t_d)$

$$\frac{u(t_d)}{(u_{st})_0} = \frac{\pi}{2}, \quad \dot{u}(t_d) = 0$$

$$\frac{u(t)}{(u_{st})_0} = \frac{\pi}{2}\cos 2\pi\left(\frac{t}{T_n} - \frac{1}{2}\right)$$

3. 응답이력

① t_d/T_n 이 작을때
동력해는 정력해와 큰 차이를 보인다

② t_d/T_n 이 큰 값을 가질때
동력해와 정력해는 비슷하다.
→ 하중이 T_n에 비하여 느리게 변한다.

③ 강제진동 구간에서 ω 와 ω_n의 상관이 없다.
$u(t) > 0$

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④ $t_d/T_n = 1.5, 2.5, \cdots$ 이면
펄스 종료시 $u(t_d)=0$, $\dot{u}(t_d)=0$

4. 최대응답

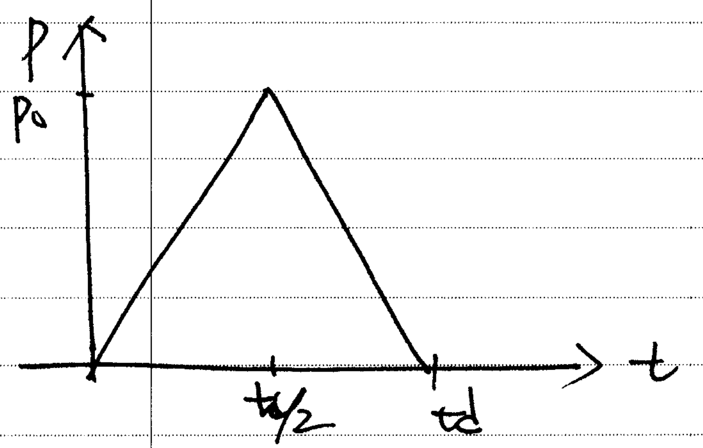
그림 4.8.3

① $t_d > T_n/2$ 이면 전체영역의 최대값이 강제진동단계에서 발생

② $t_d < T_n/2$ 이면 전체영역에서의 최대값이 자유진동단계에서 발생

③ $t_d = T_n/2$ 이면 각각의 최대값이 같다.

4-9 대칭 삼각형 펄스하중



1. $0 \le t \le t_d/2$

$$\frac{u(t)}{(u_{st})_0} = 2\left(\frac{t}{t_d} - \frac{T_n}{2\pi t_d}\sin 2\pi\frac{t}{T_n}\right)$$

2. $t_d/2 \le t \le t_d$

$$\frac{u(t)}{(u_{st})_0} = 2\left\{1 - \frac{t}{t_d} + \frac{T_n}{2\pi t_d}\left[2\sin\frac{2\pi}{T_n}\left(t - \frac{t_d}{2}\right) - \sin 2\pi\frac{t}{T_n}\right]\right\}$$

3. $t \ge t_d$

$$\frac{u(t)}{(u_{st})_0} = 2\left\{\frac{T_n}{2\pi t_d}\left[2\sin\frac{2\pi}{T_n}\left(t - \frac{t_d}{2}\right) - \sin\frac{2\pi}{T_n}(t - t_d) - \sin 2\pi\frac{t}{T_n}\right]\right\}$$

4. 전체영역에서의 최대응답

① $t_d > T_n/2$
강제진동단계에서 최고값

② $t_d < T_n/2$
자유진동 단계에서 최고값

③ $t_d = T_n/2$: 같다.