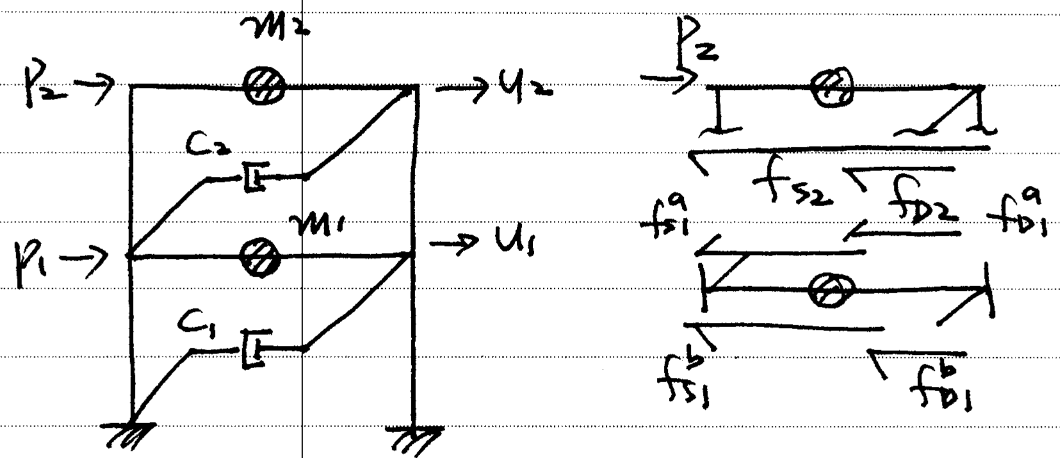
제9장. 운동방정식, 문제설정, 해석방법

9.1. 단순한 시스템 : 2층 전단구조물



9.1.1 Newton's Law

$$p_j - f_{Sj} - f_{Dj} = m_j \ddot{u}_j$$

$$m\ddot{u}_j + f_{Dj} + f_{Sj} = p_j$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{Bmatrix} f_{D1} \\ f_{D2} \end{Bmatrix} + \begin{Bmatrix} f_{S1} \\ f_{S2} \end{Bmatrix} = \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix}$$

$$\begin{Bmatrix} f_{S1} \\ f_{S2} \end{Bmatrix} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\begin{Bmatrix} f_{D1} \\ f_{D2} \end{Bmatrix} = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix}$$

$$\underline{m}\ddot{\underline{u}} + \underline{c}\dot{\underline{u}} + \underline{k}\underline{u} = \underline{p}$$

$\underline{m}$: 질량행렬
$\underline{c}$: 감쇠 행렬
$\underline{k}$: 강성 행렬

$$m\ddot{\underline{u}} + \underline{f_D} + \underline{f_S} = \underline{p}$$

$\underline{f_D}$: 감쇠 저항력 vector
$\underline{f_S}$: 탄성 저항력 vector
$\underline{p}$: 하중 vector

9.1.2 운동평형

$$\underline{f_I} = \begin{Bmatrix} f_{I1} \\ f_{I2} \end{Bmatrix} = \begin{bmatrix} m_1\ddot{u}_1 \\ m_2\ddot{u}_2 \end{bmatrix} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix}$$

↑ 관성력 vector

9.1.3. ~ 9.1.4

$$\underline{f_I} + \underline{f_D} + \underline{f_S} = \underline{p}$$

↑ 비탄성시스템에도 적용

9.2 선형시스템에 대한 일반적인 접근법

9.2.1. Discretization

- 유한요소법
- matrix 구조해석법
- 유한한 DOF system

9.2.2 탄성력

$$f_s = \underline{k}\,\underline{u}$$

- Direct stiffness method
- Finite element method
- 강성영향계수 k_{ij}

9.2.3. 감쇠력

- 감쇠행렬은 구조물의 치수나 부재의 크기로 부터 계산하기 어렵다
- 실험, 계측, Data에 근거하여 행렬구성
- 감쇠영향계수 : c_{ij}

9.2.4. 관성력

- lumped mass matrix : tributary Area
- Consistent mass matrix
  : finite element method; virtual work principle
- 질량영향계수 m_{ij}
- 비구조적 요소의 기여로 포함

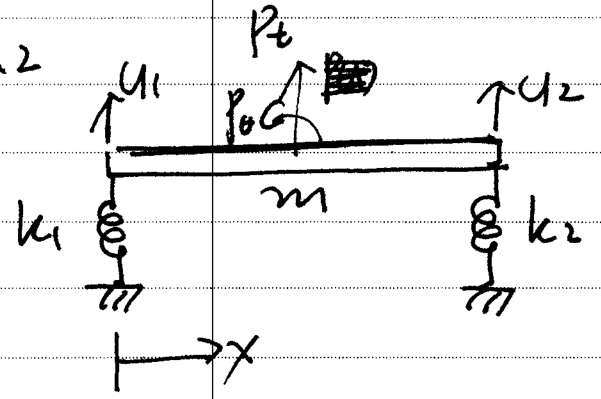
9.2.5. 운동방정식 : 외부하중

$$\underline{f_I} + \underline{f_D} + \underline{f_S} = \underline{p}$$

방정식의 연계

- $\underline{m}$, $\underline{c}$, $\underline{k}$ 의 비대각항은 연계항

<u>예제 9.2</u>



$$u_x = \frac{x}{L} u_2 + \frac{L-x}{L} u_1$$

$$\dot{u}_x = \frac{x}{L} \dot{u}_2 + \frac{L-x}{L} \dot{u}_1$$

$$T = \int_0^L \frac{1}{2}\left(\frac{m}{L} dx\right)(\dot{u}_x)^2$$

$$= \left(\frac{m}{L}\right) \int_0^L \frac{1}{2}\left(\frac{L-x}{L} \dot{u}_1 + \frac{x}{L} \dot{u}_2\right)^2 dx$$

$$V = \frac{1}{2} k_1 u_1^2 + \frac{1}{2} k_2 u_2^2$$

$$W_{nc} = p_t \left(\frac{u_1 + u_2}{2}\right) + p_\theta \cdot \frac{u_2 - u_1}{L}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{u}_1} = \left(\frac{m}{L}\right)\int_0^L \left(\frac{L-x}{L}\ddot{u}_1 + \frac{x}{L}\ddot{u}_2\right)\left(\frac{L-x}{L}\right)dx$$

$$= \frac{m}{3}\ddot{u}_1 + \frac{m}{6}\ddot{u}_2$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{u}_2} = \frac{m}{6}\ddot{u}_1 + \frac{m}{3}\ddot{u}_2$$

$$\frac{\partial V}{\partial u_1} = k_1 u_1, \quad \frac{\partial V}{\partial u_2} = k_2 u_2$$

$$\frac{\partial W_{nc}}{\partial u_1} = \frac{P_t}{2} - \frac{P_\theta}{L}$$

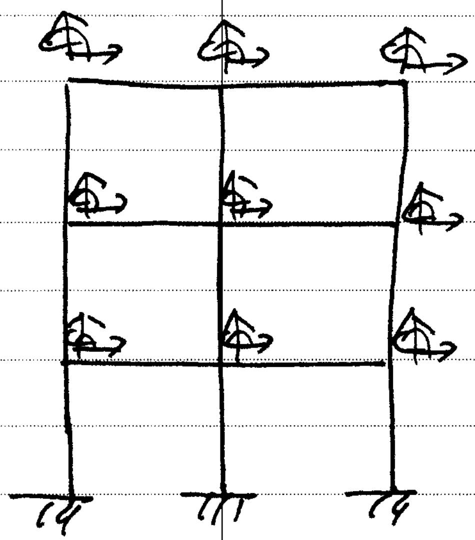
$$\frac{\partial W_{nc}}{\partial u_2} = \frac{P_t}{2} + \frac{P_\theta}{L}$$

$$\frac{m}{3}\ddot{u}_1 + \frac{m}{6}\ddot{u}_2 + k_1 u_1 = \frac{P_t}{2} - \frac{P_\theta}{L}$$

$$\frac{m}{6}\ddot{u}_1 + \frac{m}{3}\ddot{u}_2 + k_2 u_2 = \frac{P_t}{2} + \frac{P_\theta}{L}$$

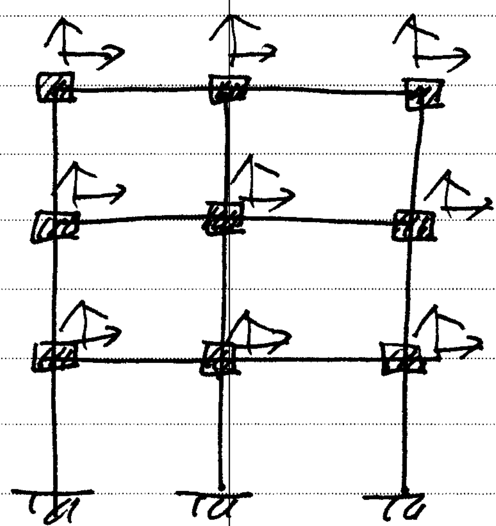
$$\begin{bmatrix} \frac{m}{3} & \frac{m}{6} \\ \frac{m}{6} & \frac{m}{3} \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} \frac{P_t}{2} - \frac{P_\theta}{L} \\ \frac{P_t}{2} + \frac{P_\theta}{L} \end{Bmatrix}$$

9.3 Static Condensation



. Static degrees of freedom

. 정적변형을 잘 묘사할 수 있을 만큼의 충분한 수의 DOF가 필요하다



◦ Dynamic degrees of freedom

◦ 동적효과, 관성효과를 잘 반영할 수 있을 만큼의 충분한 수의 DOF.

◦ 일반적으로

$$NDOF(Dynamic) \le NDOF(Static)$$

•

$\underline{u}_s$: secondary DOF, to be condensed
$\underline{u}_p$: primary DOF, not to be 〃 .

$$\begin{bmatrix} \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{M}_{pp} \end{bmatrix} \begin{Bmatrix} \ddot{\underline{u}}_s \\ \ddot{\underline{u}}_p \end{Bmatrix} + \begin{bmatrix} \underset{\sim}{K}_{ss} & \underset{\sim}{K}_{sp} \\ \underset{\sim}{K}_{ps} & \underset{\sim}{K}_{pp} \end{bmatrix} \begin{Bmatrix} \underline{u}_s \\ \underline{u}_p \end{Bmatrix} = \begin{Bmatrix} \underline{0} \\ \underline{P}_p \end{Bmatrix}$$

$$\begin{cases} \underset{\sim}{K}_{ss} \underline{u}_s + \underset{\sim}{K}_{sp} \underline{u}_p = \underline{0} \\ \underset{\sim}{M}_{pp} \ddot{\underline{u}}_p + \underset{\sim}{K}_{ps} \underline{u}_s + \underset{\sim}{K}_{pp} \underline{u}_p = \underline{P}_p \end{cases}$$

↑

$$\underline{u}_s = - \underset{\sim}{K}_{ss}^{-1} \underset{\sim}{K}_{sp} \underline{u}_p$$

$$\underset{\sim}{M}_{pp} \ddot{\underline{u}}_p + \underbrace{(\underset{\sim}{K}_{pp} - \underset{\sim}{K}_{ps} \underset{\sim}{K}_{ss}^{-1} \underset{\sim}{K}_{sp})}_{= \hat{\underset{\sim}{K}}_{pp}} \underline{u}_p = \underline{P}_p$$

$$\underset{\sim}{M}_{pp} \ddot{\underline{u}}_p + \hat{\underset{\sim}{K}}_{pp} \underline{u}_p = \underline{P}_p$$

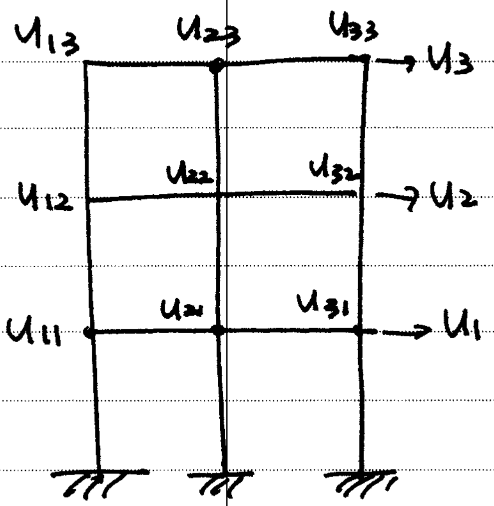
$$\begin{bmatrix} \underset{\sim}{K}_{ss} & \underset{\sim}{K}_{sp} \\ \underset{\sim}{K}_{ps} & \underset{\sim}{K}_{pp} \end{bmatrix} \begin{Bmatrix} \underline{u}_s \\ \underline{u}_p \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

↑ Gauss elimination

$$\begin{bmatrix} \mathbf{I} & \mathbf{T} \\ \mathbf{0} & \hat{\mathbf{K}} \end{bmatrix} \begin{Bmatrix} \underline{u}_s \\ \underline{u}_p \end{Bmatrix} = \begin{Bmatrix} 0 \\ \mathbf{P}_p \end{Bmatrix}$$

$$\mathbf{T} = +\mathbf{k}_{ss}^{-1} \mathbf{k}_{sp}$$

Kinematic condensation



$$u_{13} = u_{23} = u_{33} = u_3$$

$$u_{12} = u_{22} = u_{32} = u_2$$

$$u_{11} = u_{21} = u_{31} = u_1$$

Condensation 전 : $\mathbf{K} \underline{u} = \underline{P}$

$$\begin{Bmatrix} u_{11} \\ u_{21} \\ u_{31} \\ u_{12} \\ u_{22} \\ u_{32} \\ u_{13} \\ u_{23} \\ u_{33} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\underline{u} = \mathbf{L} \underline{u}^*$$

Potential Energy V

$$V = \frac{1}{2} \underline{u}^T \underset{\sim}{k} \underline{u}$$

$$= \frac{1}{2} \underline{u}^{*T} \underset{\sim}{L}^T \underset{\sim}{k} \underset{\sim}{L} \underline{u}^*$$

$$= \frac{1}{2} \underline{u}^{*T} \underset{\sim}{k}^* \underline{u}^*$$

$$W_{nc} = \underline{u}^T \underline{P}$$

$$= \underline{u}^{*T} \underset{\sim}{L}^* \underline{P} = \underline{u}^{*T} \underline{P}^*$$

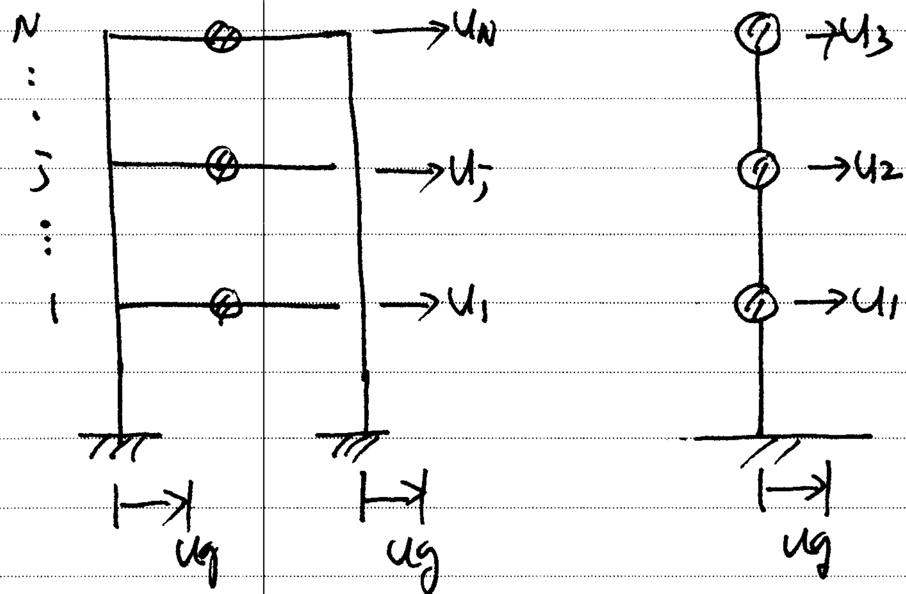
$$\delta V = \delta W_{nc}$$

$$\underset{\sim}{k}^* \underline{u}^* = \underset{\sim}{L}^* \underline{P} = \underline{P}^*$$

minimum Potential Energy principle

9.4. 평면시스템 : 지반운동

9.4.1 평면시스템 평행이동지반운동



$$\underline{u}^t(t) = \underline{u}(t) + u_g(t)\underline{1}$$

$$\underline{f}_I + \underline{f}_D + \underline{f}_S = \underline{0}$$

$$\underline{f}_I = \underline{m}\ddot{\underline{u}}^t = \underline{m}(\ddot{\underline{u}} + u_g(t)\underline{1})$$

$$\underline{f}_D = \underline{c}\dot{\underline{u}}$$

$$\underline{f}_S = \underline{k}\underline{u}$$

$$\underline{m}\ddot{\underline{u}} + \underline{c}\dot{\underline{u}} + \underline{k}\underline{u} = -\underline{m}u_g(t)\underline{1} = -\underline{m}\underline{1}u_g(t)$$

$$\underline{P}_{eff} = -\underline{m}\,\underline{1}\,\ddot{u}_g(t)$$

$$\underline{m}\ddot{\underline{u}} + \underline{c}\dot{\underline{u}} + \underline{k}\underline{u} = \underline{P}_{eff}$$

일반화

$$\underline{u}^t = \underline{u}^s + \underline{u}$$

$$\underline{u}^s = \underline{\iota}\, u_g$$

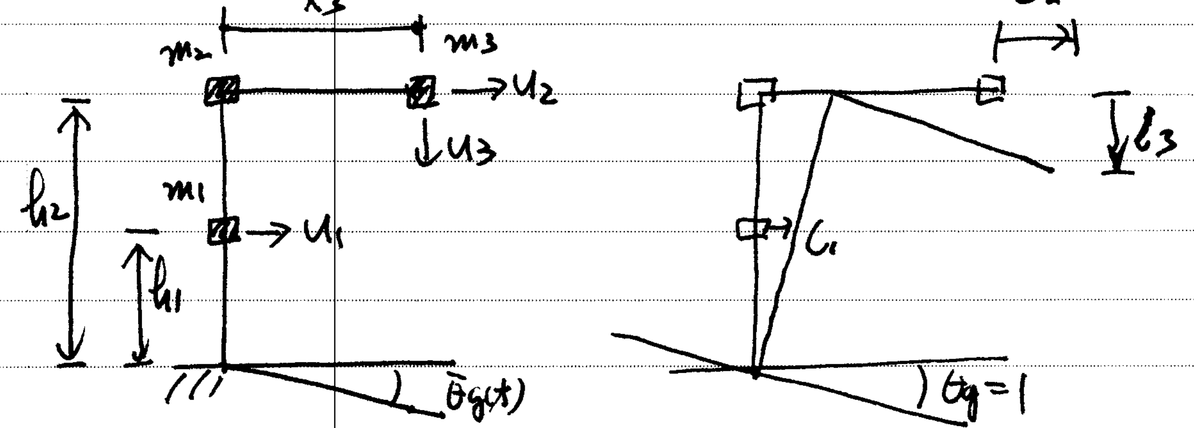
$$\underline{u}^t = \underline{\iota}\, u_g + \underline{u}$$

$$\underline{m}\ddot{\underline{u}} + \underline{c}\dot{\underline{u}} + \underline{k}\underline{u} = -\underline{m}\,\underline{\iota}\,\ddot{u}_g = \underline{P}_{eff}$$

9.4.2. 대칭평면 건물 : 평행 지반운동

그림 9.4.5 참조

9.4.3. 평면시스템 : 회전지반운동



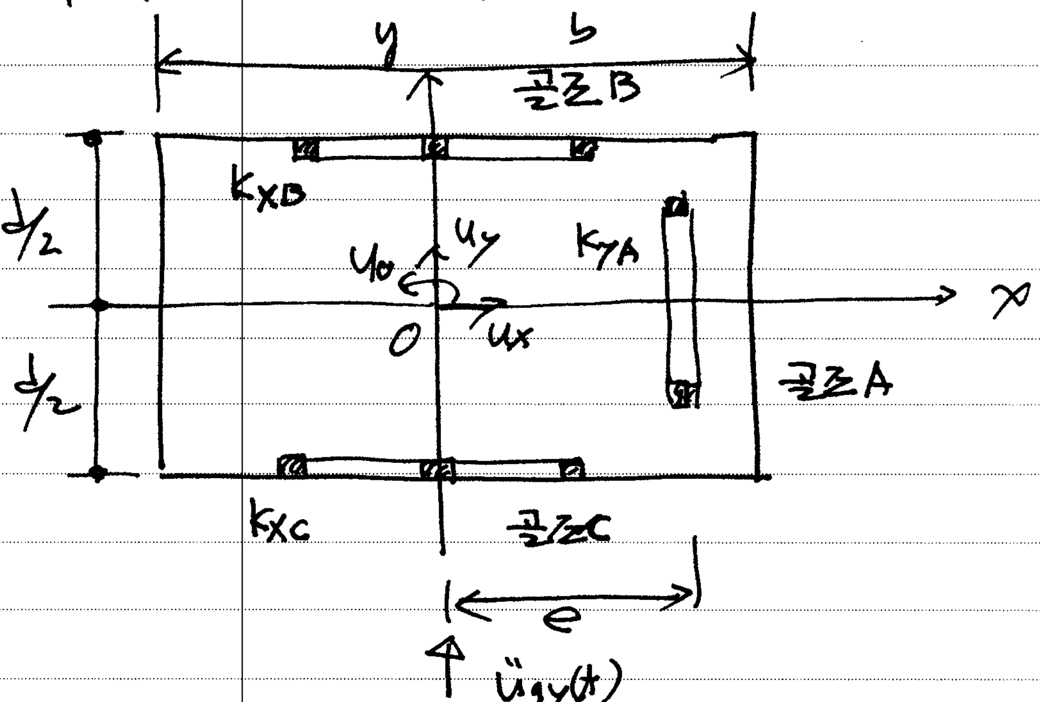
$$\underline{u}^t = \underline{u} + \underline{\iota}\,u_g$$

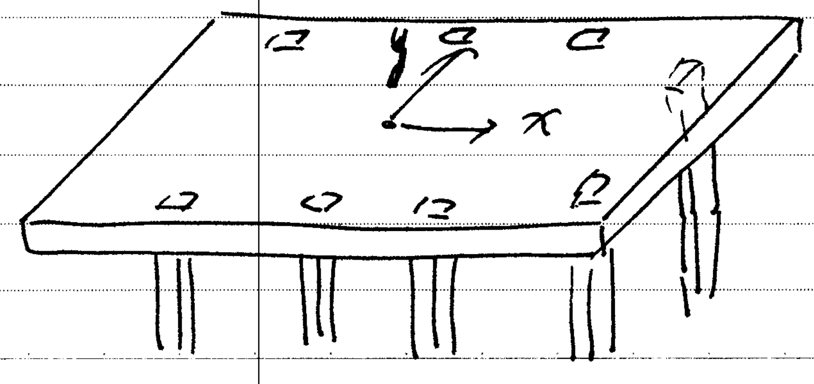
$$\underline{\iota} = \langle h_1, h_2, h_3 \rangle^T$$

$$\underset{\sim}{m}\ddot{\underline{u}} + \underset{\sim}{c}\dot{\underline{u}} + \underset{\sim}{k}\underline{u} = \underline{P}_{eff}$$

$$\underline{P}_{eff} = -\underset{\sim}{m}\,\underline{\iota}\,\ddot{u}_g = -\ddot{u}_g \begin{Bmatrix} m_1 h_1 \\ (m_2+m_3)h_2 \\ m_3 x_3 \end{Bmatrix}$$

9.5. 비대칭평면 : 지반운동





Lagrange Equation을 적용한다.

$$T = \frac{1}{2} m \dot{u}_x^2 + \frac{1}{2} m \dot{u}_y^2 + \frac{1}{2} I_o \dot{u}_\theta^2$$

$$= \frac{1}{2} m \dot{u}_x^2 + \frac{1}{2} m (\dot{u}_y + \dot{u}_{gy})^2 + \frac{1}{2} I_o \dot{u}_\theta^2$$

$$V = \frac{1}{2} k_{yA} (u_y + e u_\theta)^2$$

$$+ \frac{1}{2} k_{xB} (u_x - \frac{d}{2} u_\theta)^2 + \frac{1}{2} k_{xC} (u_x + \frac{d}{2} u_\theta)^2$$

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{u}_x} = m \ddot{u}_x, \quad \frac{d}{dt} \frac{\partial T}{\partial \dot{u}_y} = m (\ddot{u}_y + \ddot{u}_{gy})$$

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{u}_\theta} = I_o \ddot{u}_\theta$$

$$\frac{\partial V}{\partial u_x} = k_{xB} (u_x - \frac{d}{2} u_\theta) + k_{xC} (u_x + \frac{d}{2} u_\theta)$$

$$\frac{\partial V}{\partial u_y} = k_{yA} (u_y + e u_\theta)$$

$$\frac{\partial V}{\partial u_\theta} = k_{yA} (u_y + e u_\theta) e + k_{xB} (u_x - \frac{d}{2} u_\theta)(-\frac{d}{2})$$

$$+ k_{xC} (u_x + \frac{d}{2} u_\theta)(\frac{d}{2})$$

$$m\ddot{u}_x + (k_{xB} + k_{xC})u_x + \frac{d}{2}(k_{xC} - k_{xB})u_\theta = 0$$

$$m\ddot{u}_y + k_{yA}u_y + e k_{yA} u_\theta = -m\ddot{u}_{gy}$$

$$I_o\ddot{u}_\theta + \frac{d}{2}(k_{xC} - k_{xB})u_x + k_{yA}u_y + \left((k_{yA} e^2) + \frac{d^2}{4}(k_{xB} + k_{xC})\right)u_\theta = 0$$

$$\begin{bmatrix} m & & \\ & m & \\ & & I_o \end{bmatrix}\begin{Bmatrix} \ddot{u}_x \\ \ddot{u}_y \\ \ddot{u}_\theta \end{Bmatrix} + \begin{bmatrix} k_{xB}+k_{xC} & 0 & \frac{d}{2}(k_{xC}-k_{xB}) \\ 0 & k_{yA} & e k_{yA} \\ \frac{d}{2}(k_{xC}-k_{xB}) & e k_{yA} & \bigcirc \end{bmatrix}\begin{Bmatrix} u_x \\ u_y \\ u_\theta \end{Bmatrix}$$

$\bigcirc$ ← $e^2 k_{yA} + \frac{d^2}{4}(k_{xB}+k_{xC})$

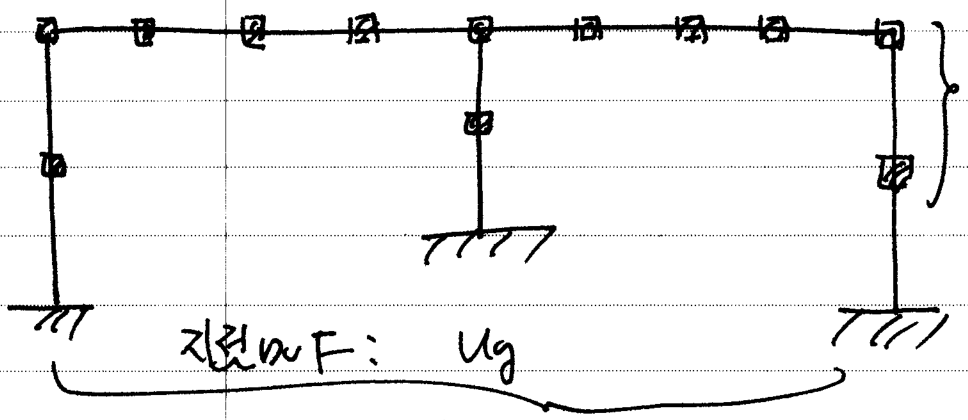
$$= \begin{Bmatrix} 0 \\ -m\ddot{u}_{gy} \\ 0 \end{Bmatrix} \doteq P_{eff.} = -\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_o \end{bmatrix}\begin{Bmatrix} 0 \\ \ddot{u}_{gy} \\ 0 \end{Bmatrix}$$

만약 $\ddot{u}_{gx}$, $\ddot{u}_{g\theta}$ 가 존재한다면

$$P_{eff} = -\begin{Bmatrix} \ddot{u}_{gx} \\ \ddot{u}_{gy} \\ \ddot{u}_{g\theta} \end{Bmatrix} = -\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_o \end{bmatrix}\begin{Bmatrix} \ddot{u}_{gx} \\ \ddot{u}_{gy} \\ \ddot{u}_{g\theta} \end{Bmatrix}$$

9.7. 다지점가진

+ 장경간 교량과 같이 지점마다 입력 지반운동이 다를 수 있는 경우가 있다.



$$\begin{bmatrix} \underset{\sim}{M} & \underset{\sim}{M}_g \\ \underset{\sim}{M}_g^T & \underset{\sim}{M}_{gg} \end{bmatrix} \begin{Bmatrix} \ddot{u}^t \\ \ddot{u}_g \end{Bmatrix} + \begin{bmatrix} \underset{\sim}{c} & \underset{\sim}{c}_g \\ \underset{\sim}{c}_g^T & \underset{\sim}{c}_{gg} \end{bmatrix} \begin{Bmatrix} \dot{u}^t \\ \dot{u}_g \end{Bmatrix} + \begin{bmatrix} k & k_g \\ k_g^T & k_{gg} \end{bmatrix} \begin{Bmatrix} u^t \\ u_g \end{Bmatrix} = \begin{Bmatrix} 0 \\ p_g(t) \end{Bmatrix}$$

$\underline{P}(t) = 0$

미지량 : u^t , $p_g(t)$
기지량 : u_g , $\underline{P}(t) = 0$

$$\begin{Bmatrix} \underline{u}^t \\ \underline{u}_g \end{Bmatrix} = \begin{Bmatrix} \underline{u}^s \\ \underline{u}_g \end{Bmatrix} + \begin{Bmatrix} \underline{u} \\ \underline{0} \end{Bmatrix}$$

$$\begin{bmatrix} k & k_g \\ k_g & k_{gg} \end{bmatrix} \begin{Bmatrix} u_s \\ u_g \end{Bmatrix} = \begin{Bmatrix} 0 \\ p_g^s \end{Bmatrix}$$

u^s : 준정적 변위, $u_g(t)$를 부질량 구조물에 가하여 생기는 변위 벡터

p_g^s : 준정적 변위에 의하여 생기는 지점 반력

$$m\ddot{u}^t + m_g\ddot{u}_g + c\dot{u}^t + c_g\dot{u}_g + ku^t + k_gu_g = 0$$

$$m\ddot{u} + c\dot{u} + ku = P_{eff}$$

$$P_{eff} = -(m\ddot{u}^s + m_g\ddot{u}_g) - (c\dot{u}^s + c_g\dot{u}_g) - (ku^s + k_gu_g)$$

그런데 $ku^s + k_gu_g = 0$

$$u^s = -k^{-1}k_g u_g$$

$$= \underset{\sim}{\iota}\, u_g$$

↑영향행렬

$$P_{eff} = -(\underset{\sim}{m}\underset{\sim}{\iota} + m_g)\ddot{u}_g - (\underset{\sim}{c}\underset{\sim}{\iota} + c_g)\dot{u}_g(t)$$

강성비례감쇠 행렬 : $c = a_1 k$

↳ $(\underset{\sim}{c}\underset{\sim}{\iota} + \underset{\sim}{c_g})\dot{u}_g(t) = 0$

$$P_{eff} = -(\underset{\sim}{m}\underset{\sim}{\iota} + m_g)\ddot{u}_g \rightarrow -\underset{\sim}{m}\underset{\sim}{\iota}\ddot{u}_g$$

↳ 비대각행렬 = 0

9.8. 비탄성 시스템

$$\underline{f_S} = \underline{f_S}(\underline{u}, \underline{\dot{u}})$$

$$m\ddot{u} + c\dot{u} + f_S(u; \dot{u}) = -\underline{m}\,\underline{\iota}\,\ddot{u}_g(t)$$

9.9. 문제선언

$\underline{m}$, $\underline{k}$, $\underline{f_S}(u, \dot{u})$, $\underline{c}$, $p(t)$, $\underline{\ddot{u}_g}(t)$

가 주어지면 MDOF system의 응답 결정

응답 : 변위, 속도, 가속도, 내력, 응력

9.10 요소력

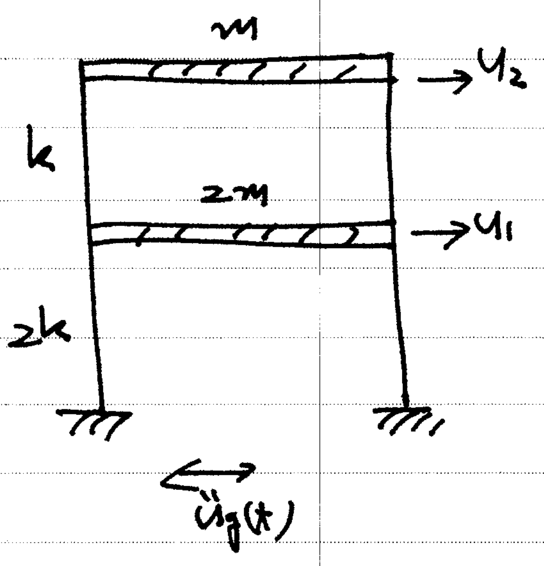
1. $\underline{u}(t)$: 모든 자유도의 변위 이력

↘ 요소강성행렬 → 응력/내력

2. $\underline{u}(t)$ → $f_S(t) = \underline{k}\,u(t)$ → 정적해석 모델

↑ 변위 ↑ 등가정적하중 ↑ 상세한 정적해석

<예제>



$m = 98 \times 10^3$ kg

$k = 1720 \times 10^3$ kg-f/m

$$\underline{m} = \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix}$$

$$\underline{k} = \begin{bmatrix} 3\times1720 & -1720 \\ -1720 & 1720 \end{bmatrix}$$

$$\begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} 3\times1720 & -1720 \\ -1720 & 1720 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= -\begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \ddot{u}_g(t)$$

힘의 단위는 ton-f , 변위 단위는 m

Eigenvalue Analysis

$\omega_1 = 6$ rad/sec , $\omega_2 = 12$ rad/sec

$$\underline{\phi}_1 = \begin{Bmatrix} \frac{1}{2} \\ 1 \end{Bmatrix}, \quad \underline{\phi}_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

$$M_1 = \phi_1^T \underset{\sim}{m} \phi_1 = \begin{bmatrix} \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix} \begin{Bmatrix} \frac{1}{2} \\ 1 \end{Bmatrix}$$

$$= 20 \left(\frac{1}{2}\right)^2 + 10 (1)^2 = 15$$

$$M_2 = \phi_2^T \underset{\sim}{m} \phi_2 = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= 20 (1)^2 + 10 (1)^2 = 30$$

$$K_1 = \omega_1^2 M_1 = (6^2)(15) = 540$$

$$K_2 = \omega_2^2 M_2 = (12^2)(30) =$$

$\xi_1 = 0.05$, $\xi_2 = 0.05$ $\Leftarrow$ 가정

$$s = \underset{\sim}{m}\{1\}, \quad \Gamma_1 = \frac{L_1}{M_1}, \quad \Gamma_2 = \frac{L_2}{M_2}$$

$$s_1 = \Gamma_1 \underset{\sim}{m} \phi_1, \quad s_2 = \Gamma_2 \underset{\sim}{m} \phi_2$$

$$L_1 = \{1\}^T \underset{\sim}{m} \phi_1 = \phi_1^T \underset{\sim}{m} \{1\}$$

$$= \begin{bmatrix} \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$= \left(\frac{1}{2}\right)(20) + (1)(10) = 20$$

$$L_2 = \Phi_2^T \underline{M}\{1\}$$

$$= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = 10$$

$\Gamma_1 = \frac{L_1}{M_1} = \frac{20}{15}$, $\Gamma_2 = \frac{L_2}{M_2} = \frac{\text{[redacted]}}{30} = \frac{10}{30}$

$$\underline{S}_1 = \left(\frac{20}{15}\right) \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = \begin{Bmatrix} \frac{200}{15} \\ \frac{200}{15} \end{Bmatrix}$$

$$\underline{S}_2 = \left(\frac{10}{30}\right) \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix} \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} = \begin{Bmatrix} \frac{200}{30} \\ -\frac{100}{30} \end{Bmatrix}$$