

## 제 10장 자주진동

$$\underline{m}\ddot{\underline{u}} + \underline{c}\dot{\underline{u}} + \underline{k}\underline{u} = \underline{P}(t)$$

M: symmetric positive definite matrix

$$\underline{u}^T \underline{M} \underline{u} > 0, \quad \underline{u} \neq \{0\}$$

↑  
Kinetic energy 등 관찰된

K: symmetric positive matrix

$$\underline{u}^T \underline{K} \underline{u} \geq 0, \quad \underline{u} \neq \{0\}$$

↑  
potential energy

$$\underline{u}^T \underline{K} \underline{u} = 0 \rightarrow \text{rigid body motion}$$

positive definite system

M: positive definite

K: positive definite

positive semi-definite system

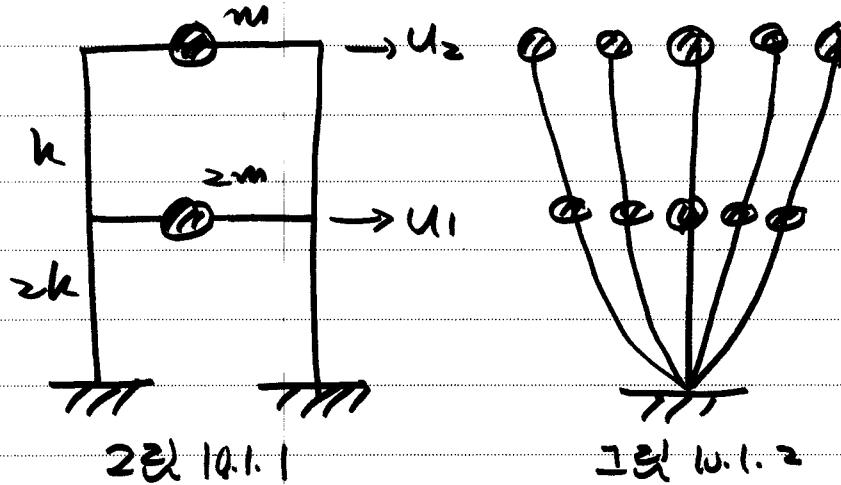
M: positive definite

K: positive

## Undamped system (무진동계 시스템)

$$\frac{m}{\tilde{u}} \dot{\tilde{u}} + \frac{k}{\tilde{u}} \dot{u} = 0$$

$U(0)$ ,  $J(0)$  at  $t=0$



- 일반적으로  $u_1(t)$ ,  $u_2(t)$ 는 시계성이 있으므로

- v. 그러나- 특징은 나머지의  $U_1(u)$ ,  $U_2(u)$ 이 같아서  
 $U_1(t)$ ,  $U_2(t)$ 는 초기 진동을 하-고  $U(t)$ 는  
형상이 변하지 않고 진폭만 시간에 따라-리-서  
줄라진다-

→ 고체물

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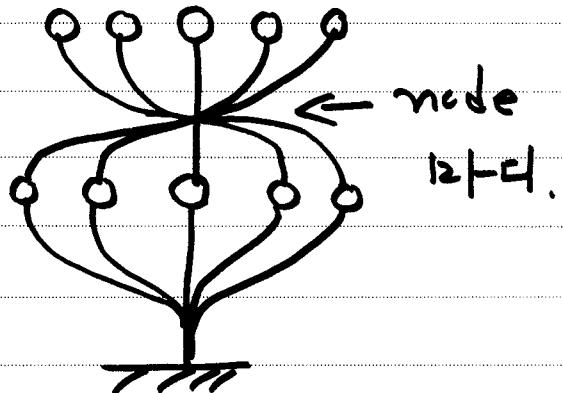
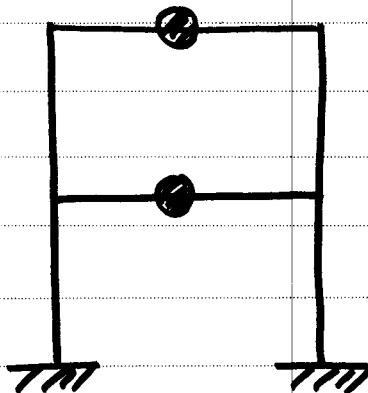
## · 고체 전동주기 ( 고속주기 ) T<sub>h</sub>

## · 고속 각진률수

C. W. M.

## · 고유 진동수

fm



## 10.2. 고주진동수, 고주진동보드

$$\underline{m}\ddot{\underline{u}} + \underline{k}\underline{u} = \underline{0} \quad (1)$$

Separation of variables

$$\underline{u}(t) = \underline{q}_n(t) \underline{d}_n$$

$$\dot{\underline{u}}(t) = \dot{\underline{q}}_n(t) \underline{d}_n \quad (2)$$

$$\ddot{\underline{u}}(t) = \ddot{\underline{q}}_n(t) \underline{d}_n$$

$$(2) \rightarrow (1)$$

$$\cancel{m} \cancel{d}_n \ddot{\underline{q}}_n(t) + \cancel{k} \cancel{d}_n \underline{q}_n(t) = \underline{0} \quad (3)$$

$\underline{d}_n$  를 일비급한다.

$$\dot{f}_n^T M \ddot{f}_n + f_n^T K f_n \ddot{q}_n(t) = 0 \quad (4)$$

$$\frac{f_n^T K f_n}{f_n^T M f_n} = -\frac{\ddot{q}_n(t)}{\ddot{q}_n(t)} = \lambda_n \quad (5)$$

$$\ddot{q}_n(t) + \lambda_n q_n(t) = 0 \quad (6)$$

- $\lambda > 0$  : positive definite system

$$q_n(t) = A e^{\sqrt{\lambda_n} t} + B e^{-\sqrt{\lambda_n} t}$$

: Time Harmonic motion

- $\lambda = 0$  : rigid body motion

$$q_n(t) = At + B$$

:  $t \rightarrow \infty$ ,  $q_n(t)$  diverges

- $\lambda < 0$  : energy is not finite

$$q_n(t) = A e^{\sqrt{-\lambda_n} t} + B e^{-\sqrt{-\lambda_n} t}$$

(6)  $\rightarrow$  (3)

$$K \dot{q}_n q_n(t) - \gamma_n \ddot{q}_n q_n(t) = 0$$

$$(K \ddot{q}_n + \gamma_n \ddot{q}_n) q_n(t) = 0 \quad (7)$$

$\Rightarrow$  상 성립하기 위해선

(Eigenvalue problem)  $\rightarrow (K - \gamma_n \ddot{q}_n) \ddot{q}_n = 0 \quad (8)$

자유해석 - 아닌 경우로  $\Rightarrow$  유기 출해석  
상태에서는

$$|K - \gamma_n \ddot{q}_n| = 0$$

$$\det(K - \gamma_n \ddot{q}_n) = 0 \quad (9)$$

: characteristic determinant  
characteristic equation

characteristic equation의 해  
characteristic value 해는 eigenvalue

$\lambda_n$ : 고주파

$\omega_n = \sqrt{\lambda_n}$ : 고주파 진동수

$f_n = \omega_n / 2\pi$ : 고주 진동수

$T_n = 2\pi / \omega_n = 1/f_n$ : 고주기

$$\omega_1 \geq \omega_2 \geq \dots \geq \omega_n$$

$$\omega_1 \leq \omega_2 \leq \dots \leq \omega_n$$

Eigenvector, characteristic vector  
normal mode,  $\underline{\phi}_n$

$$(K - \lambda_n m) \underline{\phi}_n = 0 \quad (10)$$

$$\lambda_n \Leftrightarrow \underline{\phi}_n$$

방정식 (10)의 비자-비중은  $\underline{\phi}_n$

각 모드는 청상을 변하지 않으면서 그 고주파수를 가지고 진동하기 때문이다.

### 10.3. 보스행렬과 소极力행렬

$$\underline{\phi} = [\underline{\phi}_1 \ \underline{\phi}_2 \ \dots \ \underline{\phi}_n]$$

$$= \begin{bmatrix} \underline{\phi}_{11} & \underline{\phi}_{12} & \dots & \underline{\phi}_{1N} \\ \vdots & & & \\ \underline{\phi}_{n1} & \underline{\phi}_{n2} & \dots & \underline{\phi}_{nN} \end{bmatrix} : \text{보스행렬}$$

$$\underline{\omega}^2 = \begin{bmatrix} \omega_1^2 & & & \\ & \omega_2^2 & & \\ & & \ddots & \\ & & & \omega_N^2 \end{bmatrix} : \text{소极力행렬}$$

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No.

## Generalized eigenvalue problem

- Hildebrand, "methods of applied mathematics"

$$A\mathbf{x} = \lambda B\mathbf{x}$$

$A, B$  are real, symmetric, positive definite matrices.

- ① All the eigenvalues are real and positive.
- ② For each eigenvalue  $\lambda_n$  or natural frequency  $\omega_n$  of multiplicity 1, there is an associated linearly independent eigenvector (mode shape vector)
- ③ If  $\lambda_n$  is a root of multiplicity  $r$ , then there are  $r$  linearly independent eigenvectors associated with  $\lambda_n$ .
- ④ If  $\lambda_m$  and  $\lambda_n$  are two distinct eigenvalues, then the corresponding eigenvectors  $\mathbf{f}_m$  and  $\mathbf{f}_n$  are orthogonal with respect to  $A$  and  $B$  matrices.

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no

$$\underline{\phi}_m = \begin{Bmatrix} \phi_{1m} \\ \vdots \\ \phi_{Nm} \end{Bmatrix}, \underline{\phi}_n = \begin{Bmatrix} \phi_{1n} \\ \vdots \\ \phi_{Nn} \end{Bmatrix}$$

$$\underline{\phi}_m^T A \underline{\phi}_m = 0, \underline{\phi}_n^T A \underline{\phi}_n = 0$$

$$\underline{\phi}_m^T B \underline{\phi}_m = 0, \underline{\phi}_n^T B \underline{\phi}_n = 0$$

( $\frac{\text{正}}{\text{反}}$ )

$$A \underline{\phi}_m = \lambda_m \underline{\phi}_m$$

$$A \underline{\phi}_n = \lambda_n \underline{\phi}_n$$

$$\underline{\phi}_m^T A \underline{\phi}_m = \lambda_m \underline{\phi}_m^T B \underline{\phi}_m$$

$$\underline{\phi}_m^T A \underline{\phi}_n = \lambda_n \underline{\phi}_m^T B \underline{\phi}_n$$

$$(\lambda_m - \lambda_n) \underline{\phi}_m^T B \underline{\phi}_n = 0$$

$$\lambda_m \neq \lambda_n \rightarrow \underline{\phi}_m^T B \underline{\phi}_n = 0$$

↓

$$\underline{\phi}_m^T A \underline{\phi}_n = 0$$

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- ⑤ If  $\lambda_n$  is an eigenvector of multiplicity  $r$ , the eigenvectors associated with  $\lambda_n$  can always be chosen so that they are mutually orthogonal with respect to  $A$  and  $B$ .

: Gram-Schmidt orthogonalization procedure

→ Hilderbrand

- ⑥ the eigenvectors are normalized with respect to  $B$ , if  $\underline{\phi}_n^T B \underline{\phi}_n = 1$ .

- ⑦ the set of  $N$  eigenvectors is complete.  
Any vector of dimension  $N$  can be expressed as a linear combination of the  $N$  eigenvectors.

$$x = \sum_{i=1}^N a_i \underline{\phi}_i$$

$$Bx = \sum_{i=1}^N a_i B \underline{\phi}_i$$

$$\underline{\phi}_j^T B x = \sum_{i=1}^N a_i \underline{\phi}_j^T B \underline{\phi}_i = a_j \underline{\phi}_j^T B \underline{\phi}_j$$

$$a_j = \frac{\underline{\phi}_j^T B x}{\underline{\phi}_j^T B \underline{\phi}_j}$$

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NO

## Dynamical Systems

$$\underline{\ddot{m}u} + \underline{k}\underline{u} = \underline{0}$$

### Restrained structure

- $\underline{m}, \underline{k}$  are symmetric positive definite
- All eigenvalues are real positive.

### Unrestrained structure

- $\underline{m}$ : positive definite
- $\underline{k}$ : positive semi-definite
- All the eigenvalues are real and greater than or equal to zero.
- At least one eigenvalue is zero that is associated with rigid body motion

### Orthogonality

$$\underline{\phi}_n^T \underline{m} \underline{\phi}_m = \begin{cases} 0 & m \neq n \\ \text{nonzero} & m = n \end{cases}$$

$$K \underline{\phi}_n = \omega_n^2 \underline{m} \underline{\phi}_n$$

$$\underline{\phi}_n^T K \underline{\phi}_n = \omega_n^2 \underline{\phi}_n^T \underline{m} \underline{\phi}_n$$

## 10.5 모드 적고성의 의미

### <의미 1>

$\gamma$  벌집의 모드의 변화를 통하여  $n$  번째 모드가  
관성역학이라는 원인에 의해 (zero)이다.

$$\underline{u}_n(t) = \underline{q}_n(t) \underline{\phi}_n$$

$$\dot{\underline{u}}_n(t) = \ddot{\underline{q}}_n(t) \underline{\phi}_n$$

$$(\underline{f}_I)_n = -m \dot{\underline{u}}_n(t) = -m \underline{\phi}_n \ddot{\underline{q}}_n(t)$$

$$\underline{u}_r(t) = \underline{q}_r(t) \underline{\phi}_r$$

$$(\underline{f}_I)_n^T \underline{u}_r = -(\underline{\phi}_n^T m \underline{\phi}_r) \ddot{\underline{q}}_n(t) \underline{q}_r(t)$$

$$= 0, \text{ if } n \neq r$$

### <의미 2>

$\gamma$  벌집의 모드의 변화를 관찰하면서  $n$  번째  
모드와 관계로 힘에 저항하는 힘은  
0이 (zero)이다.

$$(\underline{f}_S)_n = k \underline{u}_n(t) = k \underline{\phi}_n \underline{q}_n(t)$$

$$(\underline{f}_S)_n^T \underline{u}_r = (\underline{\phi}_n^T k \underline{\phi}_r) \underline{q}_n(t) \underline{q}_r(t)$$

$$= 0, \text{ if } n \neq r$$

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10.6 보드의 전극화.

$$M_n = \underline{\phi}_n^T \underline{m} \underline{\phi}_n = 1$$

$$\underline{\phi}_n^T \underline{m} \underline{\phi}_n = \pm 1$$

$$K_n = \underline{\phi}_n^T K \underline{\phi}_n = \omega_n^2 M_n = \omega_n^2$$

$$K = \underline{\phi}^T K \underline{\phi} = \frac{J^2}{l}$$

10.7. 벡터의 보조원자

$$U = \sum_{r=1}^N \underline{\phi}_r q_r = \text{전지 } \leftarrow$$

$$\underline{\phi}_n^T \underline{m} U = \sum_{r=1}^N \underline{\phi}_n^T \underline{m} \underline{\phi}_r q_r$$

$$= (\underline{\phi}_n^T \underline{m} \underline{\phi}_n) q_n$$

$$q_n = \frac{\underline{\phi}_n^T \underline{m} U}{\underline{\phi}_n^T \underline{m} \underline{\phi}_n}$$

$$q_r = \frac{\underline{\phi}_r^T \underline{m} U}{\underline{\phi}_r^T \underline{m} \underline{\phi}_r}$$

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## 10.8. 두 가지의 시스템의 자유진동

$$m\ddot{y} + k_y = 0$$

$$u = u(0), \dot{u} = \dot{u}(0), t = 0$$

$$\underline{u}(t) = \sum_{r=1}^N \underline{\phi}_r q_r(t), \quad \ddot{\underline{u}}(t) = \sum_{r=1}^N \underline{\phi}_r \ddot{q}_r(t)$$

$$m \sum_{r=1}^N \underline{\phi}_r \ddot{q}_r(t) + k \sum_{r=1}^N \underline{\phi}_r q_r(t) = 0$$

$$\underline{\phi}_n^T m \sum_{r=1}^N \underline{\phi}_r \ddot{q}_r(t) + \underline{\phi}_n^T k \sum_{r=1}^N \underline{\phi}_r q_r(t) = 0$$

$$(\underline{\phi}_n^T m \underline{\phi}_n) \ddot{q}_n(t) + (\underline{\phi}_n^T k \underline{\phi}_n) q_n(t) = 0$$

$$M_n \ddot{q}_n(t) + K_n q_n(t) = 0$$

$$\ddot{q}_n(t) + \left(\frac{K_n}{M_n}\right) q_n(t) = 0$$

$$\ddot{q}_n(t) + \omega_n^2 q_n(t) = 0$$

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NO

3주 3주간

$$\underline{U}(t) = \sum_{r=1}^N \phi_r q_r(t), \quad \dot{\underline{U}}(t) = \sum_{r=1}^N \phi_r \dot{q}_r(t)$$

$$\underline{\Phi}_n^T \underline{M} \underline{U}(t) = \sum_{r=1}^N \underline{\Phi}_n^T \underline{M} \phi_r q_r(t)$$

$$q_m(t) = \frac{\underline{\Phi}_n^T \underline{M} \underline{U}(t)}{\underline{\Phi}_n^T \underline{M} \underline{\Phi}_n}$$

$$\underline{\Phi}_n^T \underline{M} \dot{\underline{U}}(t) = \sum_{r=1}^N \underline{\Phi}_n^T \underline{M} \phi_r \dot{q}_r(t)$$

$$\dot{q}_m(t) = \frac{\underline{\Phi}_n^T \underline{M} \dot{\underline{U}}(t)}{\underline{\Phi}_n^T \underline{M} \underline{\Phi}_n}$$

Decoupled normal coordinate equations

$$M_n \ddot{q}_m(t) + K_m q_m(t) = 0$$

$$\ddot{q}_m(t) + \omega_m^2 q_m(t) = 0$$

$$(q_m(t) = \frac{\underline{\Phi}_n^T \underline{M} \underline{U}(t)}{\underline{\Phi}_n^T \underline{M} \underline{\Phi}_n})$$

$$(\dot{q}_m(t) = \frac{\underline{\Phi}_n^T \underline{M} \dot{\underline{U}}(t)}{\underline{\Phi}_n^T \underline{M} \underline{\Phi}_n})$$

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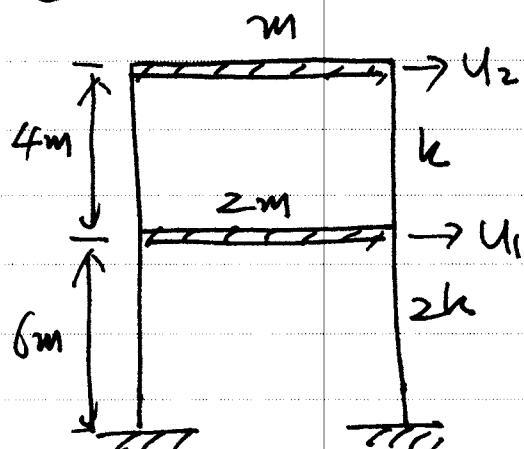
(Solution)

$$q_n(t) = q_n(0) \cos \omega_n t$$

$$+ \frac{1}{\omega_n} \dot{q}_n(0) \sin \omega_n t$$

$$U(t) = \sum_{n=1}^N \phi_n q_n(t)$$

(Q3)



$$m = 98 \times 10^3 \text{ kg}$$

$$k = 720 \times 10^3 \text{ N/m}$$

$$\bar{z} > |z_2|$$

$$u_1(0) = 0.1 \text{ m}$$

$$(u_2(0) = 0.1 \text{ m})$$

$$\dot{u}_1(0) = \dot{u}_2(0) = 0$$

$$\tilde{M} = \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix}, \quad \tilde{k} = \begin{bmatrix} 3k & -k \\ -k & k \end{bmatrix}$$

$$\begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} 3k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

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no

회의 단위로서  $\tan-f$  를 사용한다

$$\tilde{m} = \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix}, \quad \tilde{k} = \begin{bmatrix} 3 \times 720 - 720 & -720 \\ -720 & 720 \end{bmatrix}$$

회의 단위로서  $k\text{-Newton}\cdot f$  를 사용한다

$$\tilde{m} = \begin{bmatrix} 2 \times 9.8 & 0 \\ 0 & 9.8 \end{bmatrix}, \quad \tilde{k} = \begin{bmatrix} 3 \times 720 \times 9.8 - 720 \times 9.8 & -720 \times 9.8 \\ -720 \times 9.8 & 720 \times 9.8 \end{bmatrix}$$

자비방정식

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} + \begin{bmatrix} 3 \times 72 - 72 & -72 \\ -72 & 72 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

특성방정식

$$|k - \omega^2 \tilde{m}| = 0$$

$$\begin{vmatrix} 3 \times 72 - 2\lambda & -72 \\ -72 & 72 - \lambda \end{vmatrix} = 0$$

$$(72 - \lambda)(3 \times 72 - 2\lambda) - 72^2 = 0$$

$$(2\lambda - 72)(\lambda - 2 \times 72) = 0$$

$$\lambda_1 = 36, \quad \lambda_2 = 144$$

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$$\omega_1 = \sqrt{\gamma_1} = 6, \omega_2 = \sqrt{\gamma_2} = 12$$

$$f_1 = 6/2\pi \text{ Hz}, f_2 = (12/2\pi) \text{ Hz}$$

$$T_1 = \frac{2\pi}{6} \text{ sec}, T_2 = \frac{\pi}{6} \text{ sec}$$

$\omega_1$  and  $\omega_2$  가는 특성 백터, 고득 백터  $\pm$

$$\begin{pmatrix} 3\pi/2 - \gamma_1 & -12 \\ -12 & \gamma_2 - \gamma_1 \end{pmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$(-12)\phi_{11} + (\gamma_2 - 3\pi)\phi_{21} = 0$$

$$\phi_{21} = 1, \phi_{11} = \frac{1}{2}$$

$$\{\underline{\phi}\} = \left\{ \begin{array}{c} \frac{1}{2} \\ 1 \end{array} \right\} = \begin{Bmatrix} \phi_{11} \\ \phi_{21} \end{Bmatrix}$$

$\omega_2$  and  $\omega_2$  가는 고득 백터  $\pm$

$$\begin{pmatrix} 3\pi/2 - 2\gamma_2 & -12 \\ -12 & \gamma_2 - \gamma_2 \end{pmatrix} \begin{Bmatrix} \phi_{12} \\ \phi_{22} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$(-12)\phi_{12} + (\gamma_2 - 144)\phi_{22} = 0$$

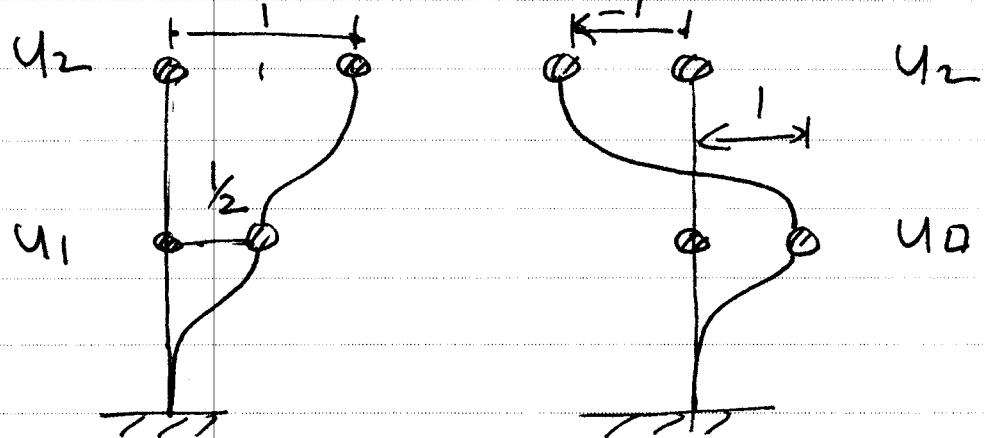
$$\phi_{12} = 1, \phi_{22} = -1$$

$$\underline{\phi} = \left\{ \begin{array}{c} \phi_{12} \\ \phi_{22} \end{array} \right\} = \left\{ \begin{array}{c} 1 \\ -1 \end{array} \right\}$$

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NO

$$\underline{\Phi} = [\underline{\phi}_1, \underline{\phi}_2] = \begin{bmatrix} \frac{1}{2} & 1 \\ 1 & -1 \end{bmatrix}$$



$$x_1 | B \equiv$$

$$x_2 | B \equiv$$

$$M_1 = \underline{\phi}_1^T M \underline{\phi}_1 = (1/2, 1) \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= (2)(\frac{1}{2})^2 + (1)(1)^2 = 1.5$$

$$M_2 = \underline{\phi}_2^T M \underline{\phi}_2 = (1, -1) \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= (2)(1)^2 + (1)(-1)^2 = 3.0$$

$$k_1 = \omega_1^2 M_1 = (36)(1.5)$$

$$k_2 = \omega_2^2 M_2 = (144)(3.0)$$

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3> 3 건

$$\underline{\Phi}_1^T \underline{M} \underline{U}(0) = (-1) \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} 0.1 \\ 0.1 \end{Bmatrix}$$

$$= 0.2$$

$$\underline{\Phi}_2^T \underline{M} \underline{U}(0) = (1 - 1) \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} 0.1 \\ 0.1 \end{Bmatrix}$$

$$= 0.1$$

$$q_{1(0)} = \frac{\underline{\Phi}_1^T \underline{M} \underline{U}(0)}{M_1} = \frac{0.2}{1.5} = \frac{2}{15}$$

$$q_{2(0)} = \frac{\underline{\Phi}_2^T \underline{M} \underline{U}(0)}{M_2} = \frac{0.1}{3} = \frac{1}{30}$$

Decoupled equation

$$\ddot{q}_{1(t)} + 36 q_{1(t)} = 0 \quad ) \quad B \leq 1$$
$$\dot{q}_{1(0)} = \frac{2}{15}, \quad q_{1(0)} = 0$$

$$\ddot{q}_{2(t)} + 144 q_{2(t)} = 0 \quad ) \quad B \leq 2$$
$$\dot{q}_{2(0)} = \frac{1}{30}, \quad q_{2(0)} = 0$$

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$$q_1(t) = q_{1(0)} \cos \theta t + \frac{1}{2} \dot{q}_{1(0)} \sin \theta t$$

$$= \left(\frac{2}{15}\right) \cos \theta t$$

$$q_2(t) = q_{2(0)} \cos 12t + \frac{1}{12} \dot{q}_{2(0)} \sin 12t$$

$$= \left(\frac{1}{30}\right) \cos 12t$$

$$\underline{u}(t) = \sum_{n=1}^2 \underline{d}_n q_n(t) = \sum_{n=1}^2 \underline{u}_n$$

$$= \begin{cases} \frac{1}{2} \left\{ \left(\frac{2}{15}\right) \cos \theta t + \left(\frac{1}{30}\right) \cos 12t \right\} \\ 1 \end{cases}$$

$$= \begin{cases} \frac{1}{15} \cos \theta t + \frac{1}{30} \cos 12t \\ \frac{2}{15} \cos \theta t - \frac{1}{30} \cos 12t \end{cases}$$

증가-성장화증

$$(f_s) = (f_s)_1 + (f_s)_2$$

$$(f_s)_1 = k \underline{u}_1 = \omega_1^2 M \underline{\phi}_1 q_1(t)$$

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$$(\underline{f_s})_2 = \underline{k} \underline{y}_2 = \omega^2 M \underline{f}_2 \cos \theta$$

$$(\underline{f_s})_1 = (36) \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix} \left\{ \frac{1}{2} \right\} \left( \frac{2}{15} \right) \cos \theta t$$

$$= \begin{cases} 48 \\ -48 \end{cases} \left\{ \cos \theta t \right\} \tan f$$

$$(\underline{f_s})_2 = (144) \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix} \left\{ \frac{1}{4} \right\} \left( \frac{1}{30} \right) \cos 12t$$

$$= \begin{cases} 96 \\ -48 \end{cases} \left\{ \cos 12t \right\} \tan f$$

$t=0$   $\alpha/4$

$$(\underline{f_s})_1 = \begin{cases} 48 \\ -48 \end{cases}, (\underline{f_s})_2 = \begin{cases} 96 \\ -48 \end{cases}$$

$$(\underline{f_s}) = (\underline{f_s})_1 + (\underline{f_s})_2 = \begin{cases} 144 \\ 0 \end{cases}, \tan f$$

(check)

$$2k = 2 \times 1720 \tan f / m$$

$$(0.1) \times (2 \times 1720 \tan f) = 144 \tan f$$

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## 10.9 강의를 가진 시스템의 자동진동

$$\underline{m}\ddot{\underline{x}} + \underline{c}\dot{\underline{x}} + \underline{k}\underline{x} = \underline{0}$$

At  $t=0$ ,  $\underline{x}=\underline{x}(0)$ ,  $\dot{\underline{x}}=\dot{\underline{x}}(0)$ 

$$\underline{x} = \underline{\Phi} \underline{q} = \sum_{n=1}^N \phi_n q_n$$

 $\underline{\Phi}$ :  $B \leq$  행렬, 무간단 시스템

$$\underline{q} = \begin{Bmatrix} q_1 \\ \vdots \\ q_N \end{Bmatrix}$$

$$\underline{m}\underline{\Phi}\ddot{\underline{q}} + \underline{c}\underline{\Phi}\dot{\underline{q}} + \underline{k}\underline{\Phi}\underline{q} = \underline{0}$$

$$\underbrace{\underline{\Phi}^T \underline{m} \underline{\Phi} \ddot{\underline{q}}}_{\downarrow} + \underbrace{\underline{\Phi}^T \underline{c} \underline{\Phi} \dot{\underline{q}}}_{\leftarrow} + \underbrace{\underline{\Phi}^T \underline{k} \underline{\Phi} \underline{q}}_{\swarrow} = \underline{0}$$

 $M, K$ : 대각행렬.

$C$  대각행렬: 고전적 관리, 단수 대각  
 $N$  개의 비연계 미분방정식

$D$  가  $B$  대각행렬: 비고전적 관리, 단수 대각화된 연계된 미분방정식

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## 고전적 진동시스템

NDUF system의 경우 N개의 대연결  
→ 힘과 질량

$$M_n \ddot{q}_n + C_n \dot{q}_n + K_n q_n = 0$$

$$M_n = \Phi_n^T M \Phi_n, \quad K_n = \Phi_n^T K \Phi_n$$

$$C_n = \Phi_n^T C \Phi_n$$

$$\ddot{q}_n + \frac{C_n}{M_n} \dot{q}_n + \frac{K_n}{M_n} q_n = 0$$

$$\ddot{q}_n + 2\zeta_n \omega_n \dot{q}_n + \omega_n^2 q_n = 0$$

$$\zeta_n = \frac{C_n}{2M_n \omega_n}$$

↑  
nth mode 진동(12)

## 10.10 고전적 진동시스템의 자극진동

$$c_{nd} = \omega_n \sqrt{1 - \zeta_n^2}$$

$$q_n(t) = e^{-\zeta_n \omega_n t} \left[ q_n^{(0)} \cos \omega_{nd} t + \frac{\dot{q}_n^{(0)} + \zeta_n \omega_n q_n^{(0)}}{\omega_{nd}} \right] \times \sin \omega_{nd} t$$

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NO

$$\underline{u}(t) = \sum_{n=1}^N \phi_n g_n(t) = \sum_{n=1}^N u_n \underline{\phi}_n$$
$$u_n = \phi_n g_n(t)$$

## 6.11 고주파 놓개의 해석.

$$k \underline{\phi} = \lambda \underline{m} \underline{\phi}$$

$$p(\lambda) = \det(k - \lambda m) = 0$$

(1) Vector Iteration (벡터 반복법)

:  $k \underline{\phi} = \lambda m \underline{\phi}$  이용

(2) 명회수법: 노드의 적포성 이용

(3) 대각식 반복법:  $p(\lambda) = 0$  이용

✓ 모든 고주파 풀이법은 반복적

✓ 고주파 ( $\lambda_m, \phi_m$ )

◦ 반복법에 의해  $\lambda_m$  계산

$$(k - \lambda_m m) \underline{\phi}_m = 0$$

$\phi_m$ 은 해석적으로 계산

25%

• 베クトル이 드는 풀이법

$$\lambda_n = \frac{\phi^T K \phi}{\phi^T M \phi} \quad (\text{Rayleigh quotient})$$

$\lambda_n \approx$  Rayleigh 지수를 이용하여 계산

◦ 구조공학분야: 수치의 저차모드가 가지는 특성

◦ 역설계 베クトル: | 흐름 분석

◦ 부설경기 베クトル: Lanczos 방법을

-| 흐름에서 흐름을 가지는 c |.

◦ 변환변수 대체로 베クトル을 계산

## 10-12. Rayleigh quotient (제일가장 2/4)

$$\phi^T K \phi = \lambda \phi^T M \phi$$

$$\lambda = \frac{\phi^T K \phi}{\phi^T M \phi}$$

: Rayleigh quotient .

$$1. \phi = \phi_n, \lambda = \lambda_n$$

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2. 4는 4나이 대로 12세 8주년  
으차에서-

$\rightarrow x \in \lambda n a | ca \Downarrow 2 m | \frac{12}{2} \frac{1}{2}$

9月2日

→ 레이블과 지수는 고윳값 터 트쳐에서

# Stationary

$$3. \quad x_1 \leq \frac{x}{\|x\|} \leq \gamma_n$$

$\|x\|^2$       Rayleigh       $\|x\|^2$   
 $x_1 \leq$

### 10.13. Vector iteration method

the fundamental frequency & mode

1st  $\underline{x}_1$  = Assumed mode shape

$$R = \sum x_i$$

$$k\bar{X}_2 = \underline{R} \quad \text{solve for } \bar{X}_2$$

normalize  $\vec{x}_3$

$$\bar{x}_2 = \frac{\bar{x}_2}{(\bar{x}_2 + m\bar{x}_2)Y_2}$$

$$\lambda^{(2)} = \frac{\bar{x}_2 \text{ } \underline{\text{v}} \text{ } \bar{x}_2}{\bar{x}_2^T \text{ } \underline{\text{m}} \text{ } \bar{x}_2} = \frac{\bar{x}_2 \text{ } \underline{\text{m}} \text{ } x_1}{\bar{x}_2^T \text{ } \underline{\text{m}} \text{ } \bar{x}_2}$$

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NO

$$\frac{x^{(2)} - \bar{x}^{(1)}}{x^{(3)}} \leq \text{하용도차-} (?)$$

수렴기준- 불균형회귀 단一手 전구화-

$$\underline{x}_2 = \frac{\bar{x}_2}{(\bar{x}_2^\top \mathbf{m} \bar{x}_2)^{1/2}}$$

2nd

$$R_2 = \mathbf{m} \underline{x}_2$$

$$R_2 \bar{x}_3 = R_2 \rightarrow \bar{x}_3$$

$$x^{(3)} = \frac{\bar{x}_3^\top R_2 \bar{x}_3}{\bar{x}_3^\top \mathbf{m} \bar{x}_3} = \frac{\bar{x}_3^\top \mathbf{m} \underline{x}_3}{\bar{x}_3^\top \mathbf{m} \bar{x}_3}$$

$$\frac{x^{(3)} - x^{(2)}}{x^{(3)}} \leq \text{하용도차-} (?)$$

수렴기준- 불균형회귀 정구화

$$\bar{x}_s$$

$$\underline{x}_s = \frac{(\bar{x}_s^\top \mathbf{m} \bar{x}_s)^{1/2}}{\bar{x}_s^\top \mathbf{m} \bar{x}_s}$$

3rd

•  
•  
•

4th

$$R_j = \mathbf{m} \underline{x}_j$$

$$R_j x_{j+1} = R_j \rightarrow x_{j+1}$$

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$$\lambda^{(j+1)} = \frac{\bar{x}_{j+1} \leq \bar{x}_{j+1}}{\bar{x}_{j+1}^T m \bar{x}_{j+1}} = \frac{\bar{x}_{j+1} m x_j}{\bar{x}_{j+1}^T m \bar{x}_{j+1}}$$

$$\frac{\lambda^{(\infty)} - \lambda^{(j)}}{\lambda^{(j+1)}} \stackrel{?}{\rightarrow} \text{정수인 } \bar{x}$$

$$x_{j+1} = \frac{\bar{x}_{j+1}}{(\bar{x}_{j+1}^T m x_{j+1})^{1/2}}$$

o

o

o

$\boxed{l+h}$  단위에서 허용오차- 만족시

$$x_1 \doteq x^{(l+1)}$$

$$\phi_1 \doteq \frac{\bar{x}_{l+1}}{(\bar{x}_{l+1}^T m \bar{x}_{l+1})^{1/2}}$$

10. 13. 2  $\Phi$ 의 최적화의 수렴

$$X_1 = \sum_{i=1}^N a_i \phi_i$$

$$R_1 = \sum_{i=1}^N a_i \phi_i = \sum_{i=1}^N a_i x_i \phi_i$$

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NO.

$$\underline{R} \bar{x}_2 = \underline{R}_1 = \sum_{i=1}^N a_i \underline{m} \underline{\phi}_i$$

$$\begin{aligned}\bar{x}_2 &= \sum_{i=1}^N a_i \underline{k}^{-1} \underline{m} \underline{\phi}_i \\ &= \sum_{i=1}^N a_i \left( \frac{1}{\lambda_i} \right) \underline{\phi}_i\end{aligned}$$

$$\text{Assume } x_2 = \bar{x}_2$$

$$\underline{x}_3 = \sum_{i=1}^N a_i \left( \frac{1}{\lambda_i} \right)^2 \underline{\phi}_i$$

⋮

$$\underline{x}_{k+1} = \sum_{i=1}^N a_i \left( \frac{1}{\lambda_i} \right)^r \underline{\phi}_i$$

$$= \left( \frac{1}{\lambda} \right)^r \sum_{i=1}^N a_i \left( \frac{\lambda_i}{\lambda} \right)^r \underline{\phi}_i$$

$$\lambda_1 < \lambda_2 < \dots < \lambda_N$$

$$\left( \frac{\lambda_i}{\lambda} \right)^r \rightarrow 0 \text{ as } r \rightarrow \infty$$

$$\text{as } r > 1$$

$$\underline{x}_{r+1} = \left( \frac{1}{\lambda} \right)^r a_i \underline{\phi}_i$$

$$\text{반복} \quad a_1 = 0 \text{ 이면} \quad x_{r+1} \rightarrow \underline{\phi}_2$$

$$\begin{array}{l} \text{반복} \\ a_1 = 0 \end{array} \quad > \text{이면} \quad x_{r+1} \rightarrow \underline{\phi}_3$$

### 10.B.3. 2nd and higher modes

2nd mode  $\Rightarrow$  고주파 (  $\lambda_2, \underline{\phi}_2$  )

- ✓  $\lambda_1$  과  $\underline{\phi}_1$  은 주파수이다.
- ✓  $x_1$  은 주파수이다.
- ✓  $x_1$ 에서 1차로는 성분을 소거한다.

$$\hat{x}_1 = x_1 - a_1 \underline{\phi}_1$$

$$x_1 = a_1 \underline{\phi}_1 + \dots + a_N \underline{\phi}_N$$

$$\underline{\phi}_1^T \underline{\phi}_1 = a_1$$

$$a_1 = \frac{\underline{\phi}_1^T \underline{\phi}_1}{\underline{\phi}_1^T \underline{\phi}_1}$$

- ✓ 이 반복 단계이다  $a_1 \underline{\phi}_1$  을 소거하여 한다.  $\rightarrow$  1차 성분을 뺀다.

Higher mode 고주파 (  $\lambda_n, \underline{\phi}_n$  )

$$\hat{x} = x - \sum_{j=1}^{n-1} (\underline{\phi}_j^T \underline{\phi}_n) \underline{\phi}_j$$

\* the highest mode  $\underline{x}_N (\lambda_N, \phi_N)$

$\underline{x}_1$  = Assumed mode shape

$$\underline{R}_1 = K \underline{x}_1$$

$$\underline{M} \underline{x}_2 = \underline{R}_1 = K \underline{x}_1$$

$$\underline{x}_2 = \underline{M}^{-1} K \underline{x}_1$$

$$\underline{R}_2 = K \underline{x}_2$$

$$\underline{M} \underline{x}_3 = \underline{R}_2$$

$$\underline{x}_3 = \underline{M}^{-1} K \underline{x}_2$$

⋮

$$\underline{x}_{N+1} = \underline{M}^{-1} K \underline{x}_N$$

$\underline{x}_{N+1} \rightarrow \phi_N$  at 수렴함.

$$\frac{\underline{x}_{N+1}^T K \underline{x}_{N+1}}{\underline{x}_{N+1}^T \underline{M} \underline{x}_{N+1}} \rightarrow \lambda_N \text{ at 수렴함.}$$

$(\underline{x}^n)$ 

$$\underline{x}_1 = \sum_{i=1}^N a_i \underline{\phi}_i$$

$$\underline{R}_1 = K \underline{x}_1 = \sum_{i=1}^N a_i K \underline{\phi}_i$$

$$\underline{x}_2 = m^{-1} \underline{R}_1 = \sum_{i=1}^N a_i m^{-1} K \underline{\phi}_i$$

$$= \sum_{i=1}^N a_i (\gamma_i) \underline{\phi}_i$$

$$\underline{x}_3 = \sum_{i=1}^N a_i (\gamma_i)^2 \underline{\phi}_i$$

..  
..  
..

$$\underline{x}_{r+1} = \sum_{i=1}^N a_i (\gamma_i)^r \underline{\phi}_i$$

$$= (\gamma_N)^r \sum_{i=1}^N a_i \left(\frac{\gamma_i}{\gamma_N}\right)^r \underline{\phi}_i$$

$$= (\gamma_N)^r a_N \underline{\phi}_i$$