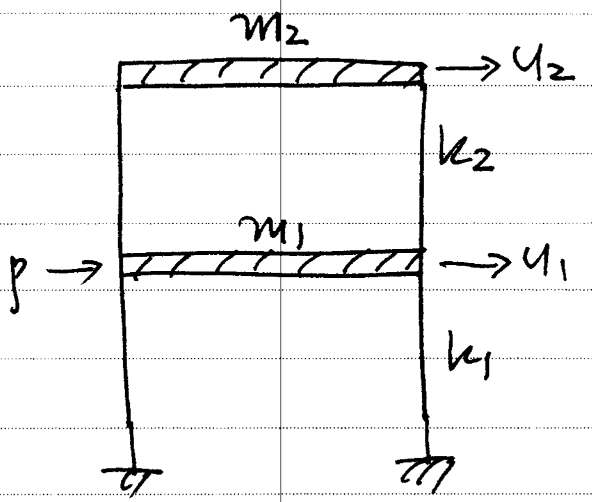
제 12장 선형시스템의 동력해석과 응답

12.1 무감쇠 2-DOF System



$p(t) = p_0 \sin \omega t$

$p(t) = p_0 e^{i\omega t}$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} p \\ 0 \end{Bmatrix}$$

$$(\underset{\sim}{k} - \omega^2 \underset{\sim}{m}) \underline{\hat{u}} = \underline{\hat{p}} \qquad \underline{u}_0 = \begin{Bmatrix} u_{10} \\ u_{20} \end{Bmatrix}$$

$$(\underset{\sim}{k} - \omega^2 \underset{\sim}{m}) \underline{u}_0 = \underline{p}_0$$

$$\underline{u}_0 = (k - \omega^2 m)^{-1} \underline{p}_0$$

$$= \frac{Adj(\underset{\sim}{k} - \omega^2 \underset{\sim}{m})}{\det(\underset{\sim}{k} - \omega^2 \underset{\sim}{m})} \begin{Bmatrix} p_0 \\ 0 \end{Bmatrix}$$

$$\det(\underset{\sim}{k}-\overset{=\lambda}{\omega^2}\underset{\sim}{m}) = (k_1+k_2-\lambda m_1)(k_2-\lambda m_2)-k_2^2$$

$$= m_1 m_2 \left\{ \left(\frac{k_1+k_2}{m_1}-\lambda\right)\left(\frac{k_2}{m_2}-\lambda\right)-\frac{k_2^2}{m_1 m_2} \right\}$$

$$\det(\underset{\sim}{k}-\omega^2\underset{\sim}{m}) = m_1 m_2(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)$$

$\omega_1 =$ 1차 고유각진동수

$\omega_2 =$ 2차 〃

$$u_{10} = \frac{p_0(k_2-m_2\omega^2)}{m_1 m_2(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)}$$

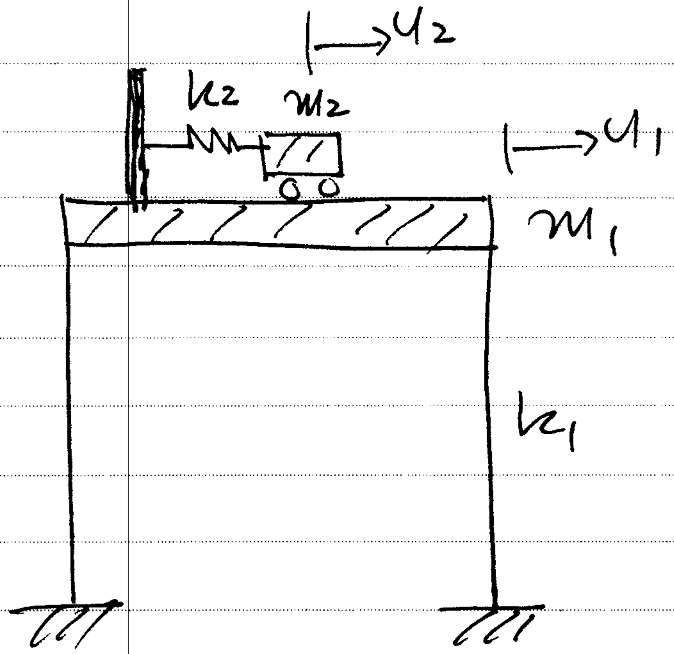
$$u_{20} = \frac{p_0 k_2}{m_1 m_2(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)}$$

12.2. Tuned mass damper (동조질량감쇠기)

$$\omega_1^* = \sqrt{\frac{k_1}{m_1}}, \quad \omega_2^* = \sqrt{\frac{k_2}{m_2}}, \quad \mu = \frac{m_2}{m_1}$$

$$u_{10} = -\frac{p_0(\omega^2-\omega_2^{*2})}{m_1(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)}$$

$$u_{20} = \frac{p_0\,\omega_2^{*2}}{m_1(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)}$$



$$u_{10} = \frac{p_0}{k_1} \frac{1-(\omega/\omega_2^*)^2}{\left(1+\mu(\omega_2^*/\omega_1^*)^2-(\omega/\omega_1^*)^2\right)\left(1-(\omega/\omega_2^*)^2\right)-\mu\left(\omega_2^*/\omega_1^*\right)^2}$$

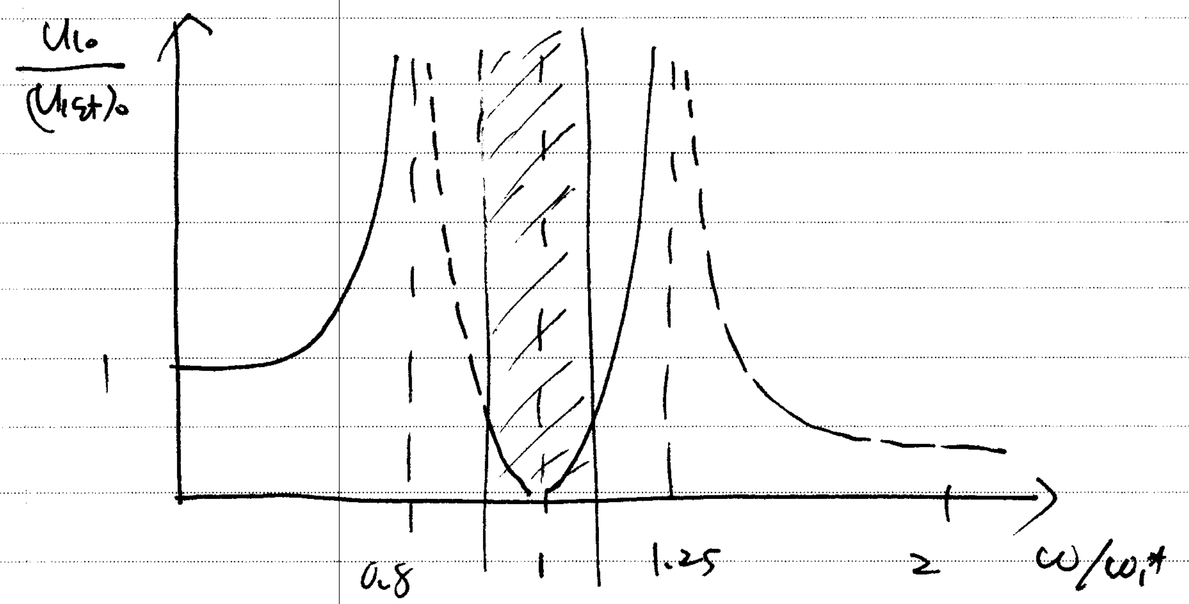
$$u_{20} = \frac{p_0}{k_1} \frac{1}{\left(1+\mu(\omega_2^*/\omega_1^*)^2-(\omega/\omega_1^*)^2\right)\left(1-(\omega/\omega_2^*)^2\right)-\mu\left(\omega_2^*/\omega_1^*\right)^2}$$

$\omega = \omega_2^*$ 에서 $u_{10} = 0$

(예)

$\mu = 0.2$, $m_2 = 0.2\, m_1$

$$\underbrace{\omega_1^* = \omega_2^* \;,\; \frac{k_1}{m_1}}_{\text{tuned}} = \frac{k_2}{m_2}$$



$\omega = \omega_1^*$, 즉 ω_1^* 의 가진진동수 에서는 응답의 크기가 영에 가깝다.

$\omega = \omega_1^*$ 에서 $u_{20} = -\frac{p_0}{k_2}$

$k_2 u_{20} = -p_0$

m_2는 외력 p_0에게 반대되는 방향으로 힘을 m_1에 가한다.

12.3. 부감쇠시스템에 대한 강제응답.

$$\underset{\sim}{m}\ddot{\underline{u}} + \underset{\sim}{k}\underline{u} = \underline{p}(t)$$

초기조건, $t=0$, $u=u(0)$, $\dot{u}=\dot{u}(0)$

$$\underline{u} = \sum_{r=1}^{N} \underline{\phi}_r q_r(t) = \underline{\Phi}\,\underline{q}(t)$$

$$\underset{\sim}{m}\underline{\Phi}\ddot{\underline{q}}(t) + \underset{\sim}{k}\underline{\Phi}\underline{q}(t) = \underline{p}(t)$$

$$m\sum_{r=1}^{N} \underline{\phi}_r \ddot{q}_r(t) + \underline{k}\sum_{r=1}^{N} \underline{\phi}_r q_r(t) = \underline{p}(t)$$

$$\underline{\phi}_n^T \underset{\sim}{m} \sum_{r=1}^{N} \underline{\phi}_r \ddot{q}_r + \underline{\phi}_n^T \underset{\sim}{k} \sum_{r=1}^{N} \underline{\phi}_r q_r = \underline{\phi}_n^T \underline{p}(t)$$

A 직교조건에 의해서

$$\underbrace{(\underline{\phi}_n^T \underset{\sim}{m} \underline{\phi}_n)}_{M_n}\ddot{q}_n + \underbrace{(\underline{\phi}_n^T \underset{\sim}{k} \underline{\phi}_n)}_{K_n} q_n = \underbrace{\phi_n^T p(t)}_{P_n}$$

$$M_n \ddot{q}_n(t) + K_n q_n = P_n$$

$t=0$, $q_n = q_n(0)$, $\dot{q}_n = \dot{q}_n(0)$

$M_n = \underset{\sim}{\phi}_n^T \underset{\sim}{m} \underset{\sim}{\phi}_n$

: nth 모드의 일반화 질량

$K_n = \underset{\sim}{\phi}_n^T \underset{\sim}{k} \underset{\sim}{\phi}_n$

: nth 모드의 일반화 강성

$P_n = \underset{\sim}{\phi}_n^T \underset{\sim}{p}$

: nth 모드의 일반화 하중

$n = 1, \cdots, N$: N개의 비연계 방정식

$$\underbrace{\underset{\sim}{M} \ddot{\underset{\sim}{q}}}_{\text{대각행렬},\ M_n} + \underbrace{\underset{\sim}{K} \underset{\sim}{q}}_{K_n} = \underbrace{\underset{\sim}{P}(t)}_{\text{vector},\ P_n}$$

$$\underset{\sim}{u}(t) = \sum_{r=1}^{N} \underset{\sim}{\phi}_r q_r(t) = \sum_{r=1}^{N} \underset{\sim}{u}_r$$

등가정적하중

$$\underset{\sim}{f}_s = \sum_{r=1}^{N} (\underset{\sim}{f}_s)_r = \sum_{r=1}^{N} \underset{\sim}{k} \underset{\sim}{u}_r$$

$$= \sum \underset{\sim}{k} \underset{\sim}{\phi}_r q_r = \sum \omega_r^2 \underset{\sim}{m} \underbrace{\underset{\sim}{\phi}_r q_r}_{\underset{\sim}{u}_r}$$

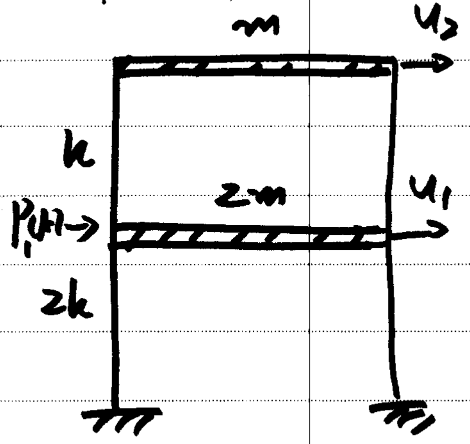
$$M_n \ddot{q}_n(t) + K_n q_n(t) = P_n$$

$$t=0,\quad q_n = q_n(0),\quad \dot{q}_n = \dot{q}_n(0)$$

$$q_n(t) = q_n(0)\cos\omega_n t + \frac{\dot{q}_n(0)}{\omega_n}\sin\omega_n t + \frac{1}{M_n\omega_n}\int_0^t P_n(\tau)\sin\omega_n(t-\tau)\,d\tau$$

$$\omega_n^2 = K_n/M_n$$

<예제>



$P_1(t) = 30\times10^3$ kg-f, $t \geq 0$
$\quad\;\; = 0 \qquad\qquad t<0$

$m = 98\times10^3$ kg

$k = 720\times10^3$ kg-f/m

$$\begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} 3\times720 & -720 \\ -720 & 720 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 30 \\ 0 \end{Bmatrix}$$

힘의 단위는 ton, 변위 단위 = m

$\omega_1 = 6$ rad/sec, $\omega_2 = 12$ rad/sec

$$\Phi = [\phi_1, \phi_2] = \begin{bmatrix} \frac{1}{2} & 1 \\ 1 & -1 \end{bmatrix}$$

$$M_1 = \phi_1^T m \phi_1 = 1.5, \quad M_2 = \phi_2^T m \phi_2 = 3.0$$

$$K_1 = (36)(1.5), \quad K_2 = (144)(3.0)$$

$$p_1 = \phi_1^T p = (\tfrac{1}{2} \quad 1) \begin{Bmatrix} 3 \\ 0 \end{Bmatrix} = 1.5$$

$$p_2 = \phi_2^T p = (1 \quad -1) \begin{Bmatrix} 3 \\ 0 \end{Bmatrix} = 3.0$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} 3\times 72 & -72 \\ -72 & 72 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 0 \end{Bmatrix}$$

1st mode의 지배방정식

$$1.5\,\ddot{q}_1 + (36)(1.5)\,q_1 = 1.5$$

$$q_1(0) = 0, \quad \dot{q}_1(0) = 0$$

$$q_1(t) = \frac{1}{\omega_1} \int_0^t (1.0) \sin \omega_1 (t - \tau)\, d\tau$$

$$= \left(\frac{1}{36}\right)(1 - \cos 6t)$$

2nd mode

$$3.0\,\ddot{q}_2 + (144)(3.0)\,q_2 = 3.0$$

$$\ddot{q}_2 + 144\,q_2 = 1.0$$

$$q_2(0) = 0, \quad \dot{q}_2(0) = 0$$

$$q_2(t) = \frac{1}{\omega_2}\int_0^t (1.0)\sin\omega_2(t-\tau)\,d\tau$$

$$= \frac{1}{144}(1-\cos 12t)$$

변위응답

$$\underline{u}(t) = \underline{u}_1(t) + \underline{u}_2(t) = \underline{\phi}_1 q_1(t) + \underline{\phi}_2 q_2(t)$$

$$\underline{u}_1(t) = \underline{\phi}_1 q_1 = \begin{Bmatrix} 1/2 \\ 1 \end{Bmatrix}\left(\frac{1}{36}\right)(1-\cos 6t)$$

$$\underline{u}_2(t) = \underline{\phi}_2 q_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}\left(\frac{1}{144}\right)(1-\cos 12t)$$

$$\underline{u}(t) = \underline{u}_1(t) + \underline{u}_2(t) = \begin{Bmatrix} \frac{1}{72}(1-\cos 6t) + \frac{1}{144}(1-\cos 12t) \\ \frac{1}{36}(1-\cos 6t) - \frac{1}{144}(1-\cos 12t) \end{Bmatrix}$$

$$\underline{u}(t) = \begin{Bmatrix} \frac{3}{144} \\ \frac{3}{144} \end{Bmatrix} + \begin{Bmatrix} -\frac{1}{72}\cos 6t - \frac{1}{144}\cos 12t \\ -\frac{1}{36}\cos 6t + \frac{1}{144}\cos 12t \end{Bmatrix}$$

등가정적하중

$$\underline{f}_s = \underset{\sim}{k}\underline{u} = \sum_{n=1}^{2} \underset{\sim}{k}\underline{u}_n = \sum_{n=1}^{2} (\underline{f}_s)_n$$

$$= \sum_{n=1}^{2} \underset{\sim}{k}\underline{\phi}_n q_n = \sum_{n=1}^{2} \omega_n^2 \underset{\sim}{m}\underline{\phi}_n q_n$$

$$(\underline{f}_s)_1 = (36)\begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix}\begin{Bmatrix} \frac{1}{2} \\ 1 \end{Bmatrix}\left(\frac{1}{36}\right)(1-\cos 6t)$$

$$= \begin{Bmatrix} 10 \\ 10 \end{Bmatrix}(1-\cos 6t)$$

$$(\underline{f}_s)_2 = (144)\begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix}\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}\left(\frac{1}{144}\right)(1-\cos 12t)$$

$$= \begin{Bmatrix} 20 \\ -10 \end{Bmatrix}(1-\cos 12t)$$

$$\underline{f}_s = (\underline{f}_s)_1 + (\underline{f}_s)_2$$

$$= \begin{Bmatrix} 10 \\ 10 \end{Bmatrix}(1-\cos 6t) + \begin{Bmatrix} 20 \\ -10 \end{Bmatrix}(1-\cos 12t)$$

$$= \begin{Bmatrix} 30 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -10\cos 6t - 20\cos 12t \\ -10\cos 6t + 10\cos 12t \end{Bmatrix}$$

12.4. 감쇠시스템의 강제응답

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{p}(t)$$

At $t=0$, $\quad \mathbf{u} = \mathbf{u}(0)$, $\quad \dot{\mathbf{u}} = \dot{\mathbf{u}}(0)$

$\boldsymbol{\phi}_r$: 무감쇠 시스템의 고유벡터

$$\mathbf{u}(t) = \sum_{r=1}^{N} \boldsymbol{\phi}_r \, q_r(t)$$

$$\dot{\mathbf{u}}(t) = \sum_{r=1}^{N} \boldsymbol{\phi}_r \, \dot{q}_r(t)$$

$$\ddot{\mathbf{u}}(t) = \sum_{r=1}^{N} \boldsymbol{\phi}_r \, \ddot{q}_r(t)$$

$$\mathbf{M}\sum_{r=1}^{N} \boldsymbol{\phi}_r \ddot{q}_r(t) + \mathbf{C}\sum_{r=1}^{N} \boldsymbol{\phi}_r \dot{q}_r(t) + \mathbf{k}\sum_{r=1}^{N} \boldsymbol{\phi}_r q_r(t) = \mathbf{p}(t)$$

$$\boldsymbol{\phi}_n^T \mathbf{M}\sum_{r=1}^{N} \boldsymbol{\phi}_r \ddot{q}_r(t) + \boldsymbol{\phi}_n^T \mathbf{C}\sum_{r=1}^{N} \boldsymbol{\phi}_r \dot{q}_r(t) + \boldsymbol{\phi}_n^T \mathbf{k}\sum_{r=1}^{N} \boldsymbol{\phi}_r q_r(t)$$
$$= \boldsymbol{\phi}_n^T \mathbf{p}(t)$$

$$M_n \ddot{q}_n + \sum_{r=1}^{N} C_{nr} \dot{q}_r + K_n q_n = P_n$$

$$C_{nr} = \boldsymbol{\phi}_n^T \mathbf{C} \boldsymbol{\phi}_r$$

$$\underset{\sim}{M}\ddot{\underline{q}} + \underset{\sim}{C}\dot{\underline{q}} + \underset{\sim}{K}\underline{q} = \underline{P}$$

$$\underset{\sim}{M} = \begin{bmatrix} M_1 & & \\ & \ddots & \\ & & M_n \end{bmatrix}, \quad \underset{\sim}{K} = \begin{bmatrix} K_1 & & \\ & \ddots & \\ & & K_n \end{bmatrix}$$

$$\underset{\sim}{C} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1N} \\ \vdots & & & \\ C_{N1} & C_{N2} & \cdots & C_{NN} \end{bmatrix}, \quad \underline{P} = \begin{Bmatrix} P_1 \\ \vdots \\ P_n \end{Bmatrix}$$

고전적 감쇠

$$\underset{\sim}{C} = \begin{bmatrix} C_{11} & & \\ & \ddots & \\ & & C_{NN} \end{bmatrix} = \begin{bmatrix} C_1 & & \\ & \ddots & \\ & & C_N \end{bmatrix}$$

$$M_n \ddot{q}_n + C_n \dot{q}_n + K_n q_n = P_n$$

At $t=0$, $q_n = q_n(0)$, $\dot{q}_n = \dot{q}_n(0)$

$$2\omega_n \zeta_n = C_n / M_n$$

$$\omega_{nD} = \omega_n \sqrt{1-\zeta_n^2}$$

$$q_n(t) = e^{-\omega_n \zeta_n t}\left[q_n(0)\cos\omega_{nD} t + \frac{\dot{q}_n(0) + q_n(0)\zeta_n\omega_n}{\omega_{nD}} \sin\omega_{nD} t \right]$$

$+$

$$+\frac{1}{M_n \omega_{nD}}\int_0^t P_n(\tau) e^{-\omega_n \zeta_n (t-\tau)} \sin \omega_{nD}(t-\tau)\, d\tau$$

$$\underline{u} = \sum_{n=1}^{N} \underline{u}_n = \sum_{n=1}^{N} \underline{\phi}_n q_n(t)$$

$$(\underline{f}_s) = \sum_{n=1}^{N} (\underline{f}_s)_n = \sum_{n=1}^{N} \underset{\sim}{k}\, \underline{u}_n$$

$$= \sum_{n=1}^{N} \underset{\sim}{k}\, \underline{\phi}_n q_n(t) = \sum_{n=1}^{N} \omega_n^2 m\, \underline{\phi}_n q_n(t)$$

(Ⅱ.요소력)

1. $(\underline{f}_s)_n$ 을 정적하중으로 가한다

2. 정적해석을 통해 r_n 을 구한다

3. 전체 응/요력은 중첩으로 구한다

$$r(t) = \sum_{n=1}^{N} r_n(t)$$

12.8. 가진벡터 $\underline{p}(t) = \underline{s}\,p(t)$ 의 모드전개

$$\underline{p}(t) = \underline{s}\,p(t)$$

$p(t)$ ← 시간의존 함수

$\underline{s}$ ← 공간력분포

$$\underline{s} = \sum_{r=1}^{N} \underline{s}_r = \sum_{r=1}^{N} \Gamma_r \underset{\sim}{m} \underline{\phi}_r$$

$$\underline{\phi}_n^T \underline{s} = \underline{\phi}_n^T \sum_{r=1}^{N} \Gamma_r \underset{\sim}{m} \underline{\phi}_r$$

$$= (\underline{\phi}_n^T \underset{\sim}{m} \underline{\phi}_n)\,\Gamma_n = M_n \Gamma_n$$

$$\boxed{\Gamma_n = \frac{\underline{\phi}_n^T \underline{s}}{M_n}}$$

$$\underline{s}_n = \Gamma_n \underset{\sim}{m} \underline{\phi}_n$$

$$(f_I)_n = -m\ddot{u}_n = -\underset{\sim}{m} \underline{\phi}_n \ddot{q}_n(t)$$

$$= (-\ddot{q}_n(t))\, \underset{\sim}{m} \underline{\phi}_n$$

$$\Updownarrow$$

$$\Gamma_n \underset{\sim}{m} \underline{\phi}_n \Leftarrow \underline{s}_n$$

12.9 $\underline{p}(t) = \underline{s}\, p(t)$에 대한 모드 해석법

$$P_n(t) = \underline{\phi}_n^T \underline{p}(t) = \underline{\phi}_n^T \left(\sum_{r=1}^{N} \Gamma_r \underset{\sim}{m} \underline{\phi}_r \right) p(t)$$

$$= \Gamma_n M_n \, p(t)$$

$$M_n \ddot{q}_n(t) + C_n \dot{q}_n(t) + K_n q_n(t) = \Gamma_n M_n \, p(t)$$

$$\ddot{q}_n(t) + 2\zeta_n \omega_n \dot{q}_n + \omega_n^2 q_n = \Gamma_n \, p(t)$$

$$\ddot{D}_n(t) + 2\zeta_n \omega_n \dot{D}_n + \omega_n^2 D_n = p(t)$$

$$q_n(t) = \Gamma_n D_n(t)$$

$$\underline{u}(t) = \sum_{n=1}^{N} \underline{u}_n = \sum_{n=1}^{N} \underline{\phi}_n q_n(t)$$

$$= \sum_{n=1}^{N} \Gamma_n D_n(t) \underline{\phi}_n$$

$$\underline{f}_s = \sum_{n=1}^{N} \underset{\sim}{k} \underline{u}_n = \sum_{n=1}^{N} (\underline{f}_s)_n$$

$$= \sum_{n=1}^{N} \underset{\sim}{k} \underline{\phi}_n \Gamma_n D_n = \sum_{n=1}^{N} \omega_n^2 \overset{\underset{\sim}{m}}{\cancel{\underset{\sim}{k}}} \underline{\phi}_n \Gamma_n D_n$$

$$f_s = \sum_{n=1}^{N} \underbrace{(\Gamma_n \mathbf{m} \phi_n)}_{s_n} \omega_n^2 D_n(t)$$

$$= \sum_{n=1}^{N} (\omega_n^2 D_n(t))\, s_n = \sum_{n=1}^{N} \underbrace{A_n(t)\, s_n}_{(f_s)_n}$$

<u>응답해석</u> $r(t) = \sum_{n=1}^{N} r_n(t)$

1. s_n을 정해석 시스템에 정적으로 작용시켜 정적 응답 r_n^{st} ~~(t)~~을 얻는다.

2. $\ddot{D}_n + 2\zeta_n \omega_n \dot{D}_n + \omega_n^2 D_n = p(t)$ 에서 D_n을 구한다

3. $A_n = \omega_n^2 D_n$을 구한다

4. $r_n(t) = r_n^{st} A_n(t)$

<2권, 12.9.1>

12.10 모드기여계수

$$r^{st} = \sum_{n=1}^{N} r_n^{st}$$

$$r_n^{st} \Longleftrightarrow s_n$$

$$\bar{r}_n = \frac{r_n^{st}}{r^{st}}, \quad \sum_{n=1}^{N} \bar{r}_n = 1$$

12.11. 모드응답과 요구되는 모드의 수

$$D_{n0} \equiv \max|D_n(t)|$$

$$r_{n0} = r^{st}\,\bar{r}_n\,\omega_n^2 D_{n0}$$

$$R_{dn} = \frac{D_{n0}}{(D_{n,st})_0}, \quad D_{n,st} = \frac{p(t)}{\omega_n^2}$$

$$(D_{n,st})_0 = \frac{p_0}{\omega_n^2}$$

$$r_{n0} = r^{st}\,\bar{r}_n\,\cancel{\omega_n^2}\,R_{dn}\cdot\frac{p_0}{\cancel{\omega_n^2}}$$

$$= p_0\,r^{st}\,\bar{r}_n\,R_{dn}$$

$\gamma(t)$에 대한 nth 모드의 기여는

$\bar{\gamma}_n$과 R_{dn}에 의해서 표현될 수 있다.

⊕ $\bar{\gamma}_n$: 모드기여계수, 정적응답의 기여

⊕ R_{dn} : 동적응답계수

○ T_n/T, ζ_n에 의해서 결정된다.

<그림 12.11.4>, <표 12.11.1>

12.12. 정적보정법

$\omega/\omega_n \rightarrow 0$ 이면 정적응답

$T/T_n \rightarrow \infty$

$R_{dn} \simeq 1$

ω, T : 가진진동수, 가진주기

○ $1 \leftrightarrow N_d$: 동적효과가 크다

○ $N_d+1 \leftrightarrow N$: 정적응답

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$$r(t) = \sum_{n=1}^{Nd} r_n(t) + \sum_{n=Nd+1}^{N} r_n(t)$$

$Nd+1 \Longleftrightarrow N$ th모드의 경우

$$\omega_n^2 D_n(t) = p(t)$$

$$r(t) = r^{st} \sum_{n=1}^{Nd} \bar{r}_n \left[\omega_n^2 D_n(t)\right]$$

$$+ r^{st} \sum_{n=Nd+1}^{\cancel{Nd+1}\,N} \bar{r}_n \underbrace{\left[\omega_n^2 D_n(t)\right]}_{\to\, p(t)}$$

$$= r^{st} \sum_{n=1}^{Nd} \bar{r}_n \left[\omega_n^2 D_n(t)\right]$$

$$+ r^{st} p(t) \sum_{n=Nd+1}^{N} \bar{r}_n$$

$$= r^{st} \sum_{n=1}^{Nd} \bar{r}_n \left[\omega_n^2 D_n(t)\right]$$

$$+ r^{st} p(t) \left(1 - \sum_{n=1}^{Nd} \bar{r}_n\right)$$

12.13. 모드가속도 중첩법

$$r(t) = r^{st} \sum_{n=1}^{N} \bar{r}_n \left[\omega_n^2 D_n(t) \right]$$

$$\omega_n^2 D_n = p(t) - \ddot{D}_n - 2\zeta_n \omega_n \dot{D}_n$$

$$r(t) = r^{st} \sum_{n=1}^{N} \bar{r}_n \left(p(t) - \ddot{D}_n - 2\zeta_n \omega_n \dot{D}_n \right)$$

$$= r^{st} p(t) - r^{st} \sum_{n=1}^{N} \bar{r}_n \left(\ddot{D}_n + 2\zeta_n \omega_n \dot{D}_n \right)$$

$$= r^{st} \left(p(t) - \sum_{n=1}^{N_d} \bar{r}_n \left(\ddot{D}_n + 2\zeta_n \omega_n \dot{D}_n \right) \right)$$

⇒ 정적보정법이 실질적으로 동일하다.