제 15장 M-DOF system의 수치적 평가

15.1 시간증분법, Direct Time Integration

$$\underset{\sim}{m}\ddot{\underset{\sim}{u}} + \underset{\sim}{c}\dot{\underset{\sim}{u}} + \underset{\sim}{f_s}(u, \dot{u}) = \underset{\sim}{p}(t)$$

$t=0$, $\underset{\sim}{u} = \underset{\sim}{u}(0)$, $\dot{\underset{\sim}{u}} = \dot{\underset{\sim}{u}}(0)$ (초기조건)

시간 t에 대한 해 : 변위 vector $u(t)$

✓ 일정간격 Δt의 시간단계 t_i

$$t_i = i\Delta t$$

$$\underset{\sim}{u}_i = \underset{\sim}{u}(t_i), \quad \dot{\underset{\sim}{u}}_i = \dot{\underset{\sim}{u}}(t_i), \quad \ddot{\underset{\sim}{u}}_i = \ddot{\underset{\sim}{u}}(t_i)$$

$$\underset{\sim}{p}_i = \underset{\sim}{p}(t_i)$$

$$\underset{\sim}{m}\ddot{\underset{\sim}{u}}_i + \underset{\sim}{c}\dot{\underset{\sim}{u}}_i + (\underset{\sim}{f_s})_i = \underset{\sim}{p}_i$$

$$\underset{\sim}{m}\ddot{\underset{\sim}{u}}_{i+1} + \underset{\sim}{c}\dot{\underset{\sim}{u}}_{i+1} + (\underset{\sim}{f_s})_{i+1} = \underset{\sim}{p}_{i+1}$$

✓ 양해법 $\Leftrightarrow$ 음해법

✓ 요구조건
- 수렴성
- 안정성
- 정확성

15.2 비고전적 감쇠를 가진 선형시스템의 해석

N DOF System

$$\underset{\sim}{m}\ddot{\underline{u}} + \underset{\sim}{c}\dot{\underline{u}} + \underset{\sim}{k}\underline{u} = \underline{p}$$

무감쇠 시스템의 모드벡터 $\phi_1, \cdots, \phi_N$

$$\underline{u}(t) \cong \sum_{n=1}^{J} \phi_n q_n(t) = \underset{\sim}{\Phi}\,\underline{q}(t)$$

$(N \times J)\ (J \times 1)$

저차 J개의 모드를 고려한다.

$$\underset{\sim}{M}\ddot{\underline{q}} + \underset{\sim}{C}\dot{\underline{q}} + \underset{\sim}{K}\underline{q} = \underline{P}$$

$$\underset{\sim}{M} = \underset{\sim}{\Phi}^T \underset{\sim}{m} \underset{\sim}{\Phi},\quad \underset{\sim}{C} = \underset{\sim}{\Phi}^T \underset{\sim}{c} \underset{\sim}{\Phi},\quad \underset{\sim}{K} = \underset{\sim}{\Phi}^T \underset{\sim}{k} \underset{\sim}{\Phi}$$

$$\underline{P}(t) = \underset{\sim}{\Phi}^T \underline{p}(t)$$

$\underset{\sim}{M}, \underset{\sim}{K}$: 대각행렬

$\underset{\sim}{C}$: 고전적 감쇠 → 대각행렬
비 〃 → 비대각행렬

N DOF 방정식을 푸는 것보다 일반적으로 경제적이다.

Δt의 설정

- B은 모드 (계산에 포함되는, 1, 2, ---, J 모드) 에 대해서 정확한해을 보장해야 함
- $\Delta t / T_J \le 0.1$ (일반적으로)
- $\Delta t / T_J \le 0.1$: 선형가속도법이 중심차분법의 경우 안정조건 만족

15.2.1. 중심차분법

$$\dot{q}_i = \frac{q_{i+1} - q_{i-1}}{2\Delta t}$$

$$\ddot{q}_i = \frac{q_{i+1} - 2q_i + q_{i-1}}{(\Delta t)^2}$$

$$\underset{\sim}{M} \frac{q_{i+1} - 2q_i + q_{i-1}}{(\Delta t)^2} + \underset{\sim}{C} \frac{q_{i+1} - q_{i-1}}{2\Delta t} + \underset{\sim}{K} q_i = \underset{\sim}{P}_i$$

나머지 계산절차는 5.3절과 동일하다.

(표. 15.2.1 수록)

15.2.2. 뉴마크 방법

$$\dot{q}_{i+1} = \dot{q}_i + [(1-\gamma)\Delta t]\ddot{q}_i + (\gamma\Delta t)\ddot{q}_{i+1}$$

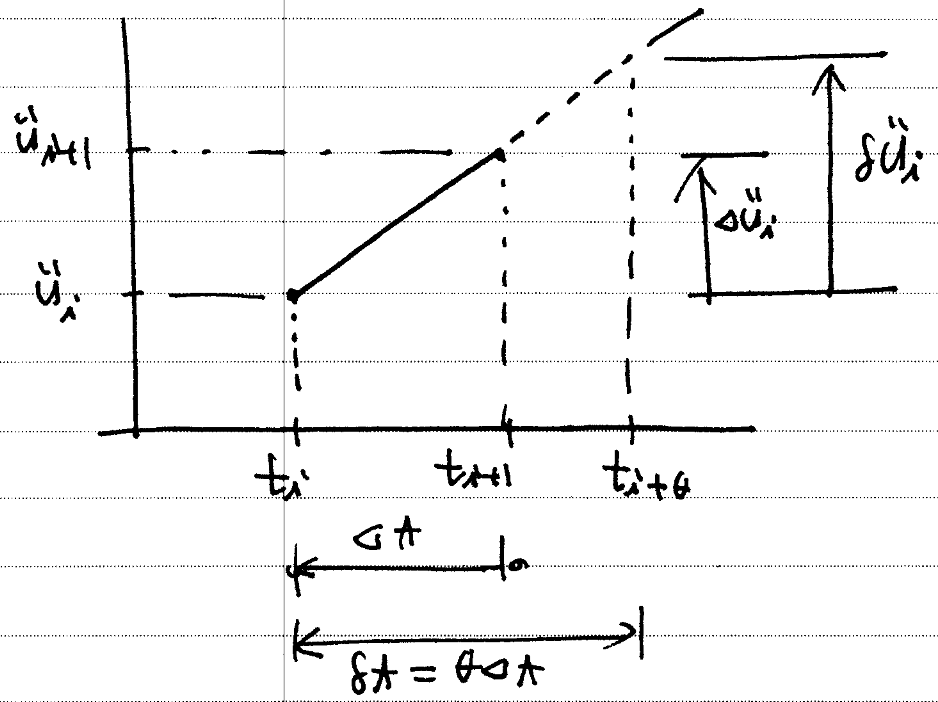
$$q_{i+1} = q_i + \Delta t\,\dot{q}_i + [(0.5-\beta)(\Delta t)^2]\ddot{q}_i + [\beta(\Delta t)^2]\ddot{q}_{i+1}$$

나머지 계산 절차는 5.4 절과 동일

- ✓ 평균가속도법 ($\gamma = \frac{1}{2}$, $\beta = 1/4$)
  - ○ 무조건적으로 안정
- ✓ 선형가속도법 ($\gamma = \frac{1}{2}$, $\beta = \frac{1}{6}$)
  - ○ 조건적으로 안정 $\Delta t \le 0.55 T_J$
  - ○ 안정 조건 내에서는 평균가속도법보다 정확
- ✓ (표 15.2.2) 수록
- ✓ 그림 E 15.17 수록

15.3.2. Wilson 방법

- ✓ 조건적으로 안정한 선형가속도법을 무조건적으로 안정하도록 수정하였음.
- ✓ $\delta t = \theta\Delta t$ 에 걸쳐 가속도의 선형 변화
- ○ 정확도와 안정성은 θ값에 의존함



$$\delta\dot{u}_i = (\delta t)\ddot{u}_i + \frac{\delta t}{2}\delta\ddot{u}_i \qquad (5a)$$

$$\delta u_i = (\delta t)\dot{u}_i + \frac{(\delta t)^2}{2}\ddot{u}_i + \frac{(\delta t)^2}{6}\delta\ddot{u}_i \qquad (5b)$$

(5b)로부터

$$\delta\ddot{u}_i = \frac{6}{(\delta t)^2}\delta u_i - \frac{6}{\delta t}\dot{u}_i - 3\ddot{u}_i \qquad (6)$$

(6) → (5a)

$$\delta\dot{u}_i = \frac{3}{\delta t}\delta u_i - 3\dot{u}_i - \frac{\delta t}{2}\ddot{u}_i \qquad (7)$$

$$m\delta\ddot{u}_i + c\,\delta\dot{u}_i + k\,\delta u_i = \delta p_i \qquad (8)$$

$$\delta p_i = \theta(\Delta p_i)$$

$$m\left(\frac{6}{(\delta t)^2}\delta u_i-\frac{6}{\delta t}\dot{u}_i-3\ddot{u}_i\right)$$

$$+c\left(\frac{3}{\delta t}\delta u_i-3\dot{u}_i-\frac{\delta t}{2}\ddot{u}_i\right)$$

$$+k\,\delta u_i=\delta p_i$$

$$\hat{k}_i\,\delta u_i=\delta\hat{p}_i$$

$$\hat{k}_i=k_i+\frac{3}{\theta\Delta t}c+\frac{6}{(\theta\Delta t)^2}m$$

$$\delta\hat{p}_i=\theta(\Delta p_i)+\left(\frac{6}{\theta\Delta t}m+3c\right)\dot{u}_i$$

$$+\left(3m+\frac{\theta\Delta t}{2}c\right)\ddot{u}_i$$

$$\Delta\ddot{u}_i=\frac{1}{\theta}\delta\ddot{u}_i$$

- $\theta=1.0$: 선형가속도법
- $\theta\geq 1.37$: 무조건 안정
- $\theta=1.42$: 최적의 정확성

(그림 E15.3) 수록

Frequency Domain Method

적용방법 #1

- ✓ 고전적 감쇠 시스템 (선형)
- ✓ 무감쇠 시스템의 모드
- ✓ 비연계 방정식

$$M_n \ddot{q}_n + C_n \dot{q}_n + K_n q_n = P_n$$

$$\ddot{q}_n + 2\zeta_n \omega_n \dot{q}_n + \omega_n^2 q_n = P_n / M_n$$

- ✓ Transfer function (system function)

$$H_n(\omega) = \left(\frac{1}{M_n}\right)\left(\frac{1}{\omega_n^2 - \omega^2 + 2i\zeta_n \omega_n \omega}\right)$$

- ✓ Fourier transform of $P_n \Rightarrow \hat{P}_n$

$$\hat{P}_n = \mathcal{F}[P_n(t)]$$

- ✓. Fourier Transform of 변 $q_n(t)$

$$Q_n(\omega) = H_n(\omega)\hat{P}_n(\omega) = \mathcal{F}[q_n]$$

- ✓. Inverse transform

$$\underline{y}(t) = \sum_{n=1}^{N} \mathcal{F}^{-1}[Q_n(\omega)]\underline{\phi}_n$$

적용방법 #2 (Direct Solution in Frequency Domain)

✓. 선형 MDOF System

$$\underset{\sim}{m}\ddot{\underset{\sim}{u}} + \underset{\sim}{c}\dot{\underset{\sim}{u}} + \underset{\sim}{k}\underset{\sim}{u} = \underset{\sim}{p}$$

✓ Fourier transform

$$-\omega^2 \underset{\sim}{m}\hat{u} + i\omega\underset{\sim}{c}\hat{u} + \underset{\sim}{k}\hat{u} = \hat{p}(\omega)$$

$$\underbrace{(\underset{\sim}{k} - \omega^2\underset{\sim}{m} + i\omega\underset{\sim}{c})}_{\text{Dynamic stiffness matrix}}\hat{u} = \hat{p}(\omega)$$

✓ Transfer function matrix
(System function)

$$\underset{\sim}{H}(\omega) = (\underset{\sim}{k} - \omega^2 m + i\omega c)^{-1}$$

$$\hat{u} = \underset{\sim}{H}(\omega)\,\hat{\underset{\sim}{p}}(\omega)$$

✓ Inverse transform

$$\underset{\sim}{u}(t) = \mathcal{F}^{-1}[\hat{u}] = \mathcal{F}^{-1}[H(\omega)\hat{p}(\omega)]$$

적용방법 #3

$$\underset{\sim}{m}\,\ddot{\underline{u}} + \underset{\sim}{c}\,\dot{\underline{u}} + \underset{\sim}{k}\,\underline{u} = \underline{p} \qquad (1)$$

무감쇠 시스템의 모드: $\phi_1, \phi_2, \cdots, \phi_N$

$$\underline{u}(t) \cong \sum_{n=1}^{J} \phi_n q_n(t) = \underset{(N\times J)}{\Phi}\;\underset{(J\times 1)}{\underline{q}(t)} \qquad (2)$$

↑

저차 J개의 모드로 근사

$$\underset{\sim}{M}\,\ddot{\underline{q}} + \underset{\sim}{C}\,\dot{\underline{q}} + \underset{\sim}{K}\,\underline{q} = \underline{P} \qquad (3)$$

$$\underset{\sim}{M} = \Phi^T \underset{\sim}{m}\,\Phi,\quad \underset{\sim}{C} = \Phi^T \underset{\sim}{c}\,\Phi,\quad \underset{\sim}{K} = \Phi^T \underset{\sim}{k}\,\Phi$$

$$P(t) = \Phi^T p(t)$$

식(3)에 Fourier Transform 적용

$$(\underset{\sim}{K} - \omega^2 \underset{\sim}{M} + i\omega \underset{\sim}{C})\,\hat{\underline{q}} = \hat{\underline{P}}$$

$$\hat{\underline{q}} = \underbrace{(\underset{\sim}{K} - \omega^2 M + i\omega C)^{-1}}_{H(\omega)\ \text{or}\ \hat{h}(\omega)}\,\hat{\underline{P}}$$

$$\underline{q} = \mathcal{F}^{-1}(\hat{\underline{q}}) = \mathcal{F}^{-1}(H(\omega)\hat{P}(\omega))$$