Chapter 18. Revenue Management with Capacity Controls

Matching supply to demand when supply is fixed

- Examples of **fixed supply**:
  - Travel industries (fixed number of seats, rooms, cars, etc).
  - Advertising time (limited number of time slots).
  - Telecommunications bandwidth.
  - Size of the MBA program.
  - Doctor’s availability for appointments.

- Revenue management is a solution:
  - If adjusting supply is impossible – **adjust the demand**!
  - Segment customers into high willingness to pay and low willingness to pay.
  - Limit the number of tickets sold at a low price, i.e., control the average price by changing the mix of customers.

Revenue management and margin arithmetic

$$Profit = R \times M - F = Net\ profit\ \% \times R$$

$$R = Revenue$$
$$M = Gross\ margin\ as\ a\ percentage\ of\ revenue$$
$$F = Fixed\ costs$$
$$Net\ profit\ \% = Net\ profit\ as\ a\ percentage\ of\ revenue$$ (1~10%)

$$\%\ change\ in\ profit = \frac{[(1+Revenue\ increase) \times R \times M - F] - [R \times M - F]}{R \times M - F} \times 100\%$$

$$= \frac{Revenue\ increase \times R \times M}{R \times M - F} \times 100\%$$

$$= \frac{Revenue\ increase \times M}{Net\ profit\%}$$

Revenue management and margin arithmetic

- Small changes in revenue can have a big impact on profit, especially for high gross margin and low net profit % industries:

Percentage change in profit for different gross margins, revenue increases and net profits as a percentage of revenue.

Net profit % = 2%

Gross margin	Revenue increase 1%	2%	5%	8%
100%	50%	100%	250%	400%
90%	45%	90%	225%	360%
75%	38%	75%	188%	300%
50%	25%	50%	125%	200%
25%	13%	25%	63%	100%
15%	8%	15%	38%	60%

Net profit % = 6%

Gross margin	Revenue increase 1%	2%	5%	8%
100%	17%	33%	83%	133%
90%	15%	30%	75%	120%
75%	13%	25%	63%	100%
50%	8%	17%	42%	67%
25%	4%	8%	21%	33%
15%	3%	5%	13%	20%

Environments suitable for revenue management

- The same unit of capacity (e.g., airline seat) can be used to deliver services to different customer segments (e.g., business and leisure customers) at different prices.
- High gross margins (so that the variable cost of additional sales is low).
- Perishable capacity (it cannot be stored) and limited capacity (all possible customers cannot always be served).
- Capacity is sold in advance of demand.
- There is an opportunity to segment customers (so that different prices can be charged) and different segments are willing to pay different prices.
- It is not illegal or morally irresponsible to discriminate among customers.

Revenue Management:

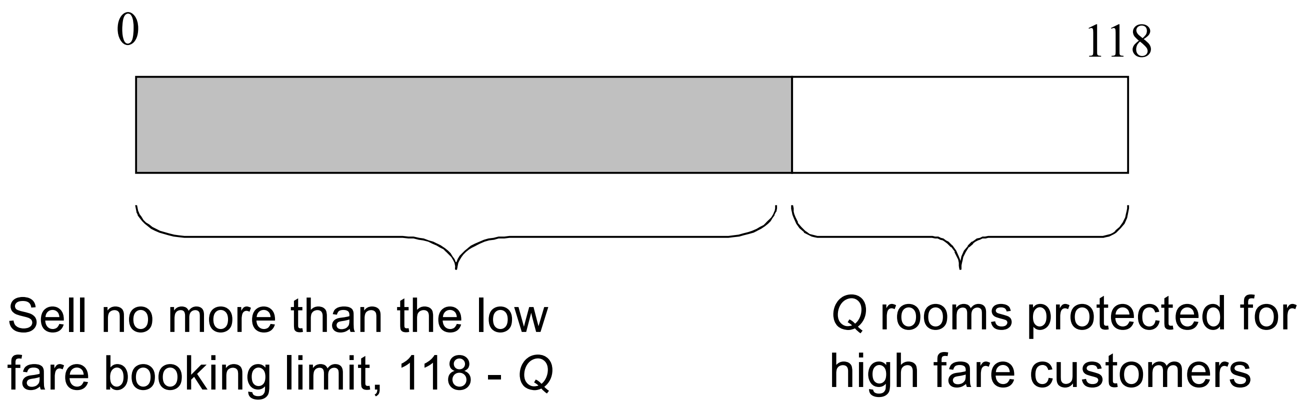
Booking limits and protection levels

Practical problem

- The Park Hyatt Philadelphia at the Bellevue.
- 118 King/Queen rooms.
- Hyatt offers a r_L= \$159 (low fare) discount fare for a mid-week stay targeting leisure travelers.
- Regular fare is r_H= \$225 (high fare) targeting business travelers.
- Demand for low fare rooms is abundant.
- Let D be *uncertain* demand for high fare rooms.
  - Suppose D has Poisson distribution with mean 27.3.
- Assume most of the high fare (business) demand occurs only within a few days of the actual stay.
- Objective:
  - Maximize expected revenues by controlling the number of low fare rooms you sell.

Yield management decisions

- The ***booking limit*** is the number of rooms you are willing to sell in a fare class or lower.
- The ***protection level*** is the number of rooms you reserve for a fare class or higher.
- Let *Q* be the protection level for the high fare class.
- *Q* is in effect while you sell low fare tickets.
- Since there are only two fare classes, the booking limit on the low fare class is 118 – *Q*:
  - You will sell no more than 118-*Q* low fare rooms because you are protecting (or reserving) *Q* rooms for high fare passengers.



The connection to the newsvendor

- A single decision is made before uncertain demand is realized.

- There is an over-protecting cost:
  - If $D < Q$ then you protected too many rooms (you over protected) ...
  - … so some rooms are empty which could have been sold to a low fare traveler.

- There is an under-protecting cost:
  - If $D > Q$ then you protected too few rooms (you under protected) …
  - … so some rooms could have been sold at the high fare instead of the low fare.

- Choose Q to balance the over-protecting and under-protecting costs.

Optimal protection level

- **Over-protecting cost:**
  - If $D < Q$ we protected too many rooms and earn nothing on $Q - D$ rooms.
  - We could have sold those empty rooms at the low fare, so $C_o = r_L$.
- **Under-protecting cost:**
  - If $D > Q$ we protected too few rooms.
  - $D - Q$ rooms could have been sold at the high fare but were sold instead at the low fare, so $C_u = r_H - r_L$
- Optimal high fare protection level:

$$F(Q^*) = \frac{C_u}{C_o + C_u} = \frac{r_H - r_L}{r_H}$$

- Optimal low fare booking limit = 118 – Q^*

- Choosing the optimal high fare protection level is a Newsvendor problem with properly chosen underage and overage costs.

Hyatt example

- Critical ratio:

$$\frac{C_u}{C_o + C_u} = \frac{r_h - r_l}{r_h} = \frac{225 - 159}{225} = \frac{66}{225} = 0.2933$$

- Poisson distribution with mean 27.3:

Q	$F(Q)$	Q	$F(Q)$	Q	$F(Q)$
10	0.0001	20	0.0920	30	0.7365
11	0.0004	21	0.1314	31	0.7927
12	0.0009	22	0.1802	32	0.8406
13	0.0019	23	0.2381	33	0.8803
14	0.0039	24	0.3040	34	0.9121
15	0.0077	25	0.3760	35	0.9370
16	0.0140	26	0.4516	36	0.9558
17	0.0242	27	0.5280	37	0.9697
18	0.0396	28	0.6025	38	0.9797
19	0.0618	29	0.6726	39	0.9867

- Answer: 24 rooms should be *protected* for high fare travelers. Similarly, *a **booking limit*** of *118-24 = 94* rooms should be applied to low fare reservations.

Related calculations

- How many high-fare travelers will be refused a reservation?
  - *Expected lost sales* = 4.10.
  - (Look this up in a Poisson Loss function table with mean 27.3)

- How many high-fare travelers will be accommodated?
  - *Expected high-fare sales* = *Expected high-fare demand* - *Lost sales* = 27.3 - 4.1 = 23.2

- How many rooms will remain empty?
  - *Expected left over inventory* = *Q* - *Expected sales* = 24 - 23.2 = 0.8.

Related calculations

- What is the expected revenue?
  - $225 x *Exp. sales* + $159 x *Booking limit* = $20,166

Note: without yield management the **worst case scenario** (all rooms are sold at the discounted price) generates revenue $159 x 118 = $18,762

What is the **maximum expected revenue?**

$225 x *27.3* + $159 x (118-27.3) = $20,564

$18,762 < $20,166 < $20,564

(78% of potential revenue improvement was captured!)

Example

- A BigJet flight from Philadelphia to Boston has 60 seats. The high fare is $400 and the low fare is $100. There is ample demand for the low fare class and they buy well in advance before high fare customers. Demand for the high fare is Poisson with mean 10.

- *To choose a protection level…*
  - *What is Co?*
  - *What is Cu?*
- *What is the optimal protection level?*
- *With the optimal protection level …*
  - *How many high fare seats can then expect to sell?*
  - *What is the probability of a full flight?*
- *What is the optimal booking limit?*

Data for a Poisson with mean 10

q	f(q)	F(q)	L(q)
0	0.000	0.000	10.0
1	0.000	0.000	9.0
2	0.002	0.003	8.0
3	0.008	0.010	7.0
4	0.019	0.029	6.0
5	0.038	0.067	5.0
6	0.063	0.130	4.1
7	0.090	0.220	3.2
8	0.113	0.333	2.5
9	0.125	0.458	1.8
10	0.125	0.583	1.3
11	0.114	0.697	0.8
12	0.095	0.792	0.5
13	0.073	0.864	0.3
14	0.052	0.917	0.2
15	0.035	0.951	0.1

Revenue Management:

Overbooking

Ugly reality: cancellations and no-shows

- Approximately 50% of reservations get cancelled at some point in time.
  - In many cases (car rentals, hotels, full fare airline passengers) there is no penalty for cancellations (which almost invites “no-shows”).

- Problem:
  - the company may fail to fill its capacity (e.g., seat, room or car) if the passenger cancels at the very last minute or does not show up.

- Solution: **Overbooking!**
  - accept/sell more reservations than capacity.

  Prohibiting overbooking would cost the world’s airlines $3billion annually due to no-shows

- Danger:
  - some customers may have to be denied service even though they have a confirmed reservation.

Hyatt's Problem

- The forecast for the number of customers that do not show up (X) is Poisson with mean 8.5.
- The net cost of denying a room to the customer with a confirmed reservation is $350 in ill-will and penalties.
- How many rooms (Y) should be overbooked (sold in excess of capacity)?
- Newsvendor setup:
  - Single decision when the number of no-shows is uncertain.
  - Underage cost if $X > Y$ (insufficient number of rooms overbooked).
  - Overage cost if $X < Y$ (too many rooms overbooked).

Overbooking solution

- Underage cost:
  - if $X > Y$ then we could have sold X-Y more rooms…
  - … to be conservative, we could have sold those rooms at the low fare, $C_u = r_L$.
  - Note: C_u does not depend on whether the fare is refundable or not.

- Overage cost:
  - if $X < Y$ then we bumped $Y - X$ customers …
  - … and incur an overage cost C_o = $350 on each bumped customer.

- Optimal overbooking level:

$$F(Y) = \frac{C_u}{C_o + C_u}.$$

- Critical ratio:

$$\frac{C_u}{C_u + C_o} = \frac{159}{350 + 159} = 0.3124$$

Optimal overbooking level

- Poisson distribution with mean 8.5

Q	$F(Q)$	Q	$F(Q)$
0	0.0002	10	0.7634
1	0.0019	11	0.8487
2	0.0093	12	0.9091
3	0.0301	13	0.9486
4	0.0744	14	0.9726
5	0.1496	15	0.9862
6	0.2562	16	0.9934
7	0.3856	17	0.9970
8	0.5231	18	0.9987
9	0.6530	19	0.9995

- Optimal number of overbooked rooms is Y=7.
- Hyatt should allow up to 118+7 reservations.
- Suppose 125 reservations are made. Then, there is about a $F(6)$=25.62% chance that the Hyatt will find itself turning down travelers with reservations.

Revenue Management:

Complications

Revenue management challenges …

- Demand forecasting.
  - Wealth of information from reservation systems but there is seasonality, special events, changing fares and truncation of demand data.
- Dynamic decisions.

  →Refer to **Moon and Kang (1999, Journal of the OR Society)**
- Variable capacity:
  - Different aircrafts, ability to move rental cars around.
- Group reservations.
- How to construct good “fences” to differentiate among customers?
  - Non-refundability.
  - One-way vs round-trip tickets.
  - Non-stop vs. connecting flights.
  - Saturday-night stay requirement.
- Multi-leg passengers/multi-day reservations for cars and hotels:
  - Not all customers using a given piece of capacity (a room for one night) are equally valuable.

A solution to the multi-leg customer: buckets

Fare class	O'Hare to JFK	O'Hare to Heathrow
Y	$724	$1,610
M	$475	$829
Q	$275	$525

- With segment control there are only three booking limits for the O'Hare-JFK leg, one for each fare class.
- But an O'Hare-Heathrow customer may be more valuable, so you could have six booking limits, one for each fare-itinerary combination.
- But that leads to many booking limits, so group similar fare-itineraries into buckets:

Bucket	Itinerary	Fare class
0	O'Hare to Heathrow	Y
1	O'Hare to Heathrow	M
	O'Hare to JFK	Y
2	O'Hare to Heathrow	Q
	O'Hare to JFK	M
3	O'Hare to JFK	Q

Another solution to multi-legs: bid prices

Fare class	O'Hare to JFK	O'Hare to Heathrow
Y	$724	$1,610
M	$475	$829
Q	$275	$525

- Assign a bid price to each segment:

	O'Hare to JFK	JFK to Heathrow
Bid price	$290	$170

- A fare is accepted if it exceeds the sum of the bid prices on the segments it uses:
  - For example, an O'Hare-JFK fare is accepted if it exceeds $290
  - A O'Hare-Heathrow fare is accepted if it exceeds $290+$170 = $460

- The trick is to choose good bid-prices.→ **sophisticated optimization**

오버부킹된 유나이티드항공, 승객 무력 하차 파문

[헤럴드미디어] 2017.04.11 07:08:09 가+ 가-

[헤럴드경제=이슈섹션] 정상적으로 탑승한 승객을 무력으로 하차시킨 미국 유나이티드 항공에 대한 비난이 쇄도하고 있다.

11일(한국시간) 미국 언론에 따르면 전날 저녁 시카고 오헤어 국제공항을 출발해 켄터키 주 루이빌로 향할 예정이었던 유나이티드 항공 3411편에서 한 남자 승객이 공항 경찰 등 당국자에 의해 강제로 끌려 나오는 일이 발생했다.

머니투데이

한국인 3명 두고 이륙한 델타항공, 피해자 찾아

박가영 기자 2019.11.27. 09:00

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오버부킹(overbooking·초과 예약)을 이유로 한국인 3명을 태우지 않고 이륙한 델타항공이 뒤늦게 고개를 숙였다. 델타 측은 피해자들을 직접 찾아가 사과하며 재발 방지를 약속했다.

27일 피해 한국인들에 따르면 전날인 26일 델타항공 한국 대표가 이들을 찾아가 사과했다. 델타 측은 피해자들이 항공기에 탑승하지 못해 발생한 비용 전액 등을 보상하겠다고 밝혔다.

한국인 3명은 지난 5일 오후 3시55분(현지시간) 뉴욕 JFK 공항을 출발해 시애틀 타코마 국제공항으로 향하는 DL2699편에 탑승하려 했다. 그러나 델타항공이 예정 시간보다 이른 3시 37분쯤 문을 닫고 이륙하면서 탑승하지 못했다. 이들 자리에는 예비 예약자가 탑승한 상태였다.

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