Ch.9 Compressible flow 압축성유동

high speed → density change → compressible

- liquid → almost incompressible ⇐ $Ma = \dfrac{V}{a} \ll 1$
- gas → compressible
  - ↳ gas dynamics

a ← speed of sound

9.1 Introduction

Incompressible flow : $\underline{V}$ (3), p (1) ($\rho = const$) → 4 unknowns ⟺ 4 eqs.

p ← mechanical press. ∇p

cont. eq (1)

N-S eqs. (3)

compressible flow : $\underline{V}$ (3), p (1), ρ (1), T (1) ← temperature → 6 unknowns

p ← thermodynamic press.

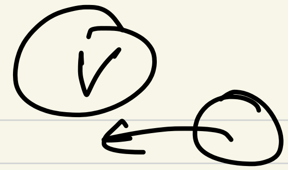
cont. eq (1)

N-S eqs (3)

6 eqs.

Change in ρ → change in T ⇑ energy eq. (1)

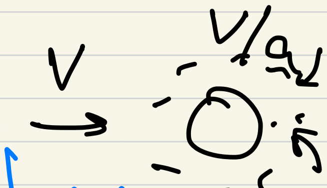
Change in ρ → change in P ⇐ state eq. $p = \rho R T$ (1)



- $\boxed{Ma = V/a}$ Mach number

ME

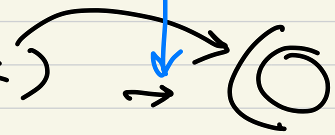
$\underline{Ma < 0.3}$: incomp. flow ($\rho \doteq$ const)



AA

$0.3 < Ma < 0.8$: subsonic flow (no shock)

$0.8 < Ma < 1.2$: transonic " (shock)



$1.2 < Ma < 3.0$: supersonic " (no subsonic)

$Ma > 3.0$: hypersonic "

- specific-heat ratio, $k = c_p / c_v$ $k = 1.4$ for air

perfect gas, $P = \rho R T$, $R = c_p - c_v = \text{const}$

$$c_p = \frac{k}{k-1} R$$

For real gas, c_p, c_v, k moderately vary with T

$du = c_v\,dT \qquad Tds = du + p\,d\left(\frac{1}{\rho}\right)$

$dh = c_p\,dT \qquad = dh - \frac{1}{\rho}dp = c_p\,dT - \frac{1}{\rho}dp$

$\Rightarrow ds = \frac{c_p}{T}dT - \frac{1}{\rho T}dp = \frac{c_p}{T}dT - \frac{R}{p}dp$

$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} = c_v \ln\frac{T_2}{T_1} - R\ln\frac{\rho_2}{\rho_1}$

* Isentropic process : $ds = 0 \rightarrow \boxed{\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{\rho_2}{\rho_1}\right)^{k}}$

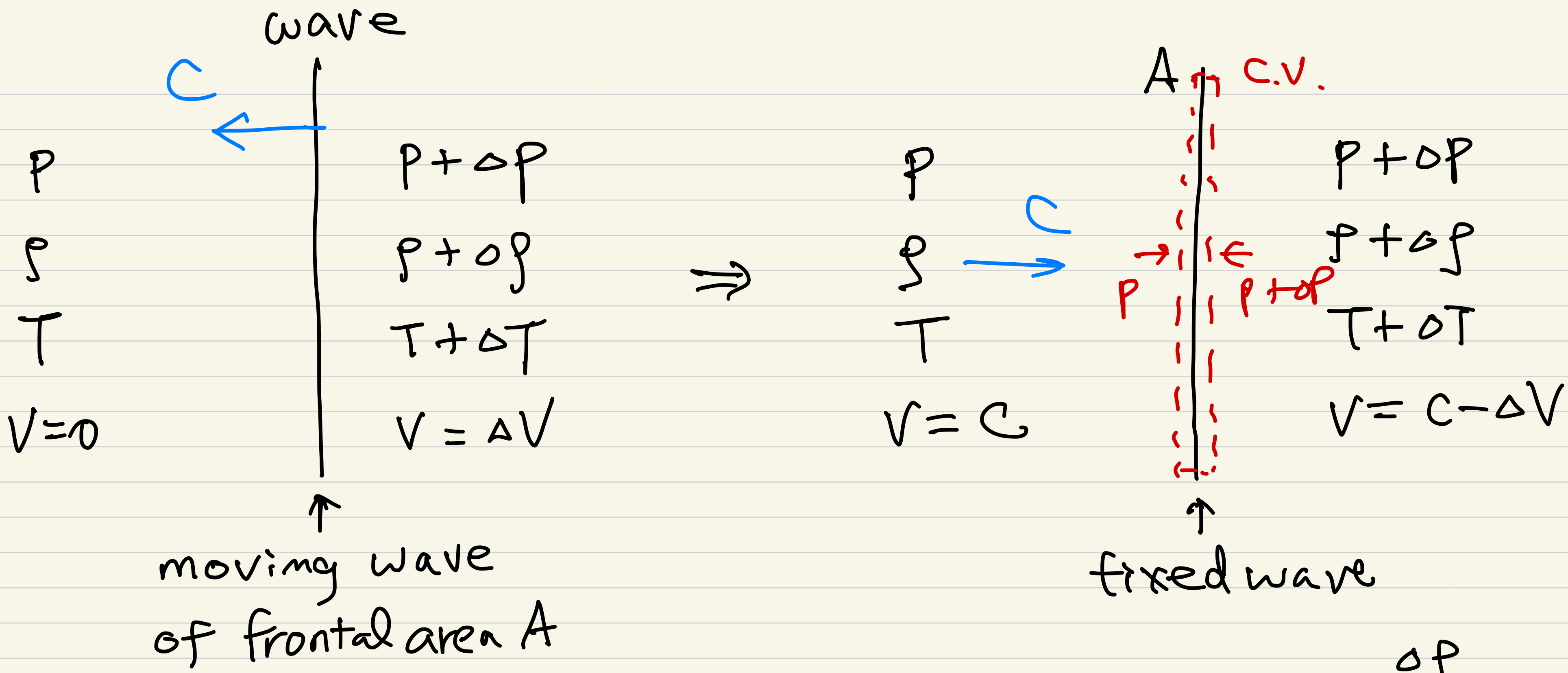
$\Delta s = 0$

reversible

adiabatic

9.2 Speed of sound

↳ a pressure pulse of infinitesimal strength



cont : $\rho c A = (\rho+\Delta\rho)(c-\Delta V)A \Rightarrow \Delta V = c\dfrac{\Delta\rho}{\rho+\Delta\rho}$ — ①

if $\Delta\rho = O(\epsilon)$,

$\Delta V = O(\epsilon)$.

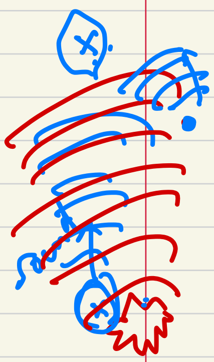
mtm : $\Sigma F = PA - (P+\Delta P)A$

$= -\rho c^2 A + (\rho+\Delta\rho)(c-\Delta V)^2 A$

$\Rightarrow \Delta P = \rho c \Delta V$ — ②

if $\Delta V = O(\epsilon)$,

$\Delta P = O(\epsilon)$

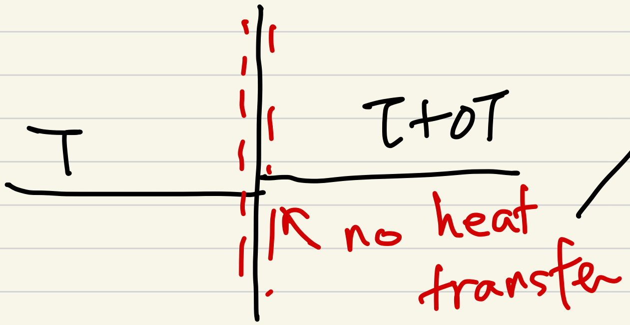


① → ②, $\boxed{c^2 = \frac{\Delta p}{\Delta \rho}\left(1+\frac{\Delta \rho}{\rho}\right)}$ $\qquad p = p(\rho, T)$

$$\left.\begin{array}{l}\Delta\rho \to 0 \\ \Delta p \to 0\end{array}\right] \Rightarrow a^2 = \left.\frac{\partial p}{\partial \rho}\right|_{?}$$: speed of sound

Powerful explosion ($\Delta\rho = O(1)$) waves move much faster than sound wave ($\Delta\rho = O(\epsilon)$)

Newton (1686) obtained 'a' using the assumption of isothermal process. → 20% error ∴ wrong!



adiabatic process

$\Delta T = O(\epsilon)$ → reversible process

⇒ isentropic process

$\Delta s = 0$: $\frac{p}{\rho^k} = \text{const} \rightarrow \ln p - k \ln \rho = \text{const.}$

$$\rightarrow \frac{dp}{p} - k\frac{d\rho}{\rho} = 0$$

$$a^2 = \left.\frac{\partial p}{\partial \rho}\right|_s$$

$$\rightarrow \quad \left.\frac{\partial p}{\partial \rho}\right|_s = k\frac{p}{\rho} = kRT = a^2$$

$$\Rightarrow \quad \boxed{a = \sqrt{kRT}}$$

@ 1 atm, 15°C, air $a = 340$ m/s

water 1490 m/s

steel 5060 m/s

) bulk modulus

Ma = 0.3 water $V = 0.3 \times 1500 = 450$ m/s

9.3 Adiabatic and isentropic steady flow

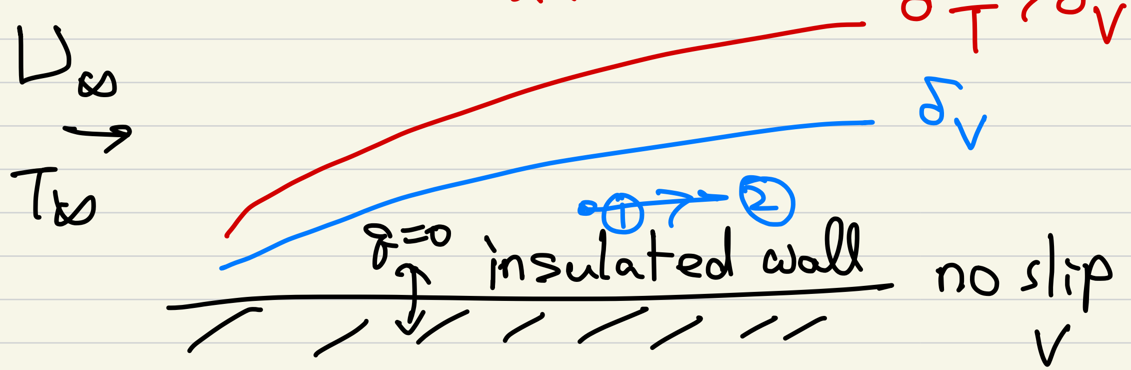
$$Pr = \frac{\nu}{\alpha} < 1 \qquad \text{air } Pr = 0.7 \text{ or } 0.71$$

$$\hookrightarrow \boxed{\nu < \alpha}$$

$$\rho\frac{\partial V}{\partial t} = \cdots + \mu\nabla^2 V$$

$$\rho c_p \frac{\partial T}{\partial t} = \cdots + \alpha\nabla^2 T$$

$\alpha > \nu$



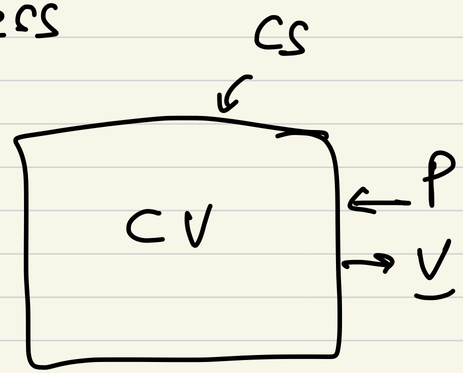
$T_{fluid} = T_{solid}$ ⇐ no jump condition for T energy conservation $B = E$, $\beta = dE/dm = e$

$$\left(\begin{aligned} e &= e_{internal} + e_{kinetic} + e_{potential} + \cdots \\ &= \hat{u} + \tfrac{1}{2}V^2 + gz + \cdots \end{aligned} \right)$$

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt} = \frac{d}{dt}\int_{CV} e\rho \, d\forall + \int_{CS} e\rho(\underline{V}\cdot\underline{n})\,dA$$

$$\dot{W} = \frac{dW}{dt} = \dot{W}_{shaft} + \dot{W}_{press} + \dot{W}_{viscous\ stress}$$

$$= \dot{W}_s + \dot{W}_p + \dot{W}_v$$



$$\dot{W}_p = \int_{CS} p(\underline{V}\cdot\underline{n})\,dA$$

$$\dot{W}_v = -\int_{CS} \underline{\tau}\cdot\underline{V}\,dA$$ $\underline{\tau}$: stress vector on dA

$$\rightarrow \dot{Q} - \dot{W}_s - \dot{W}_v = \frac{\partial}{\partial t}\int_{CV} e\rho\,d\forall + \int_{CS} \left(e + \frac{p}{\rho}\right)\rho(\underline{V}\cdot\underline{n})\,dA$$

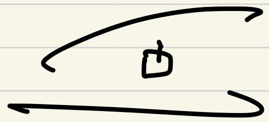
enthalpy

$$= \left(\hat{u} + \tfrac{1}{2}V^2 + gz\right) + \frac{p}{\rho} = h + \tfrac{1}{2}V^2 + gz$$

1D & steady : ① $\rightarrow$ ②

$$\dot Q - \dot W_s - \dot W_v = -\dot m \left(h_1 + \tfrac{1}{2}V_1^2 + g z_1\right) + \dot m \left(h_2 + \tfrac{1}{2}V_2^2 + g z_2\right)$$

$$\rightarrow h_1 + \tfrac{1}{2}V_1^2 + g z_1 = h_2 + \tfrac{1}{2}V_2^2 + g z_2 - \underbrace{q}_{=\dot Q/\dot m} + \dot W_s/\dot m + \dot W_v/\dot m$$



inside the bdry layer : $q \neq 0$, $\dot W_v \neq 0$

outside " " " : $q = \dot W_v = 0$

$$\hookrightarrow h_1 + \tfrac{1}{2}V_1^2 + g z_1 = h_2 + \tfrac{1}{2}V_2^2 + g z_2$$

neglect $g(z_2 - z_1) \simeq 0$

$$\rightarrow h_1 + \tfrac{1}{2}V_1^2 = h_2 + \tfrac{1}{2}V_2^2 = \text{const} = \boxed{h_0 = h + \tfrac{1}{2}V^2}$$

↑ stagnation enthalpy

h_0 varies inside the thermal boundary layer, but its average value is the same because of energy conservation.

For perfect gas, $h = c_p T \rightarrow c_p T_0 = c_p T + \frac{1}{2} V^2$

T_0 ↑ stagnation temperature

$$V_{max} = (2 c_p T_0)^{\frac{1}{2}} = \left(2 \frac{kR}{k-1} T_0\right)^{\frac{1}{2}}$$

$$c_p T_0 = c_p T + \frac{1}{2} V^2$$

$$c_p T = \frac{kR}{k-1} T = \frac{a^2}{k-1} \qquad \left(a = \sqrt{kRT}\right)$$

$$\rightarrow \frac{T_0}{T} = 1 + \frac{V^2}{2 c_p T} = 1 + \frac{(k-1) V^2}{2a^2} \qquad Ma = \frac{V}{a}$$

$$\rightarrow \boxed{\frac{T_0}{T} = 1 + \frac{k-1}{2} Ma^2}$$ adiabatic

given T & Ma $\rightarrow T_0$

$$\boxed{\frac{a_0}{a} = \left(\frac{T_0}{T}\right)^{\frac{1}{2}} = \left(1 + \frac{k-1}{2} Ma^2\right)^{\frac{1}{2}}}$$ adiabatic

If isentropic process,

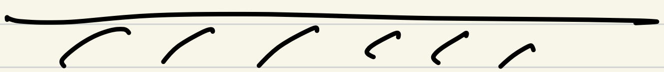
$$\boxed{\begin{aligned}\frac{p_0}{p} &= \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}} = \left(1+\frac{k-1}{2}Ma^2\right)^{\frac{k}{k-1}} \\ \frac{\rho_0}{\rho} &= \left(\frac{T_0}{T}\right)^{\frac{1}{k-1}} = \left(1+\frac{k-1}{2}Ma^2\right)^{\frac{1}{k-1}}\end{aligned}}$$

p_0 : stag. press

ρ_0 : " density

$1 \bullet \longrightarrow \bullet\, 2$ irrev. adiabatic $\Rightarrow$ $T_{0_1} = T_{0_2} \rightarrow h_{0_1} = h_{0_2}$

$a_{0_1} = a_{0_2}$



$$p_1 + \frac{1}{2}\rho_1 V_1^2 = p_{0_1} \neq p_{0_2} = p_2 + \frac{1}{2}\rho_2 V_2^2$$

$$\rho_{0_1} \neq \rho_{0_2}$$

isentropic $\Rightarrow$ $T_{0_1} = T_{0_2}$, $a_{0_1} = a_{0_2}$, $p_{0_1} = p_{0_2}$, $\rho_{0_1} = \rho_{0_2}$.

- $h + \frac{1}{2}V^2 = \text{const}$ (adiabatic)

$\rightarrow dh + VdV = 0$

$T\,ds = dh - \frac{1}{\rho}dp = 0 \rightarrow dh = \frac{1}{\rho}dp$

isentropic

$$\boxed{\frac{dp}{\rho} + VdV = 0}$$

Bernoulli eq

$\boxed{\text{isentropic process.}}$