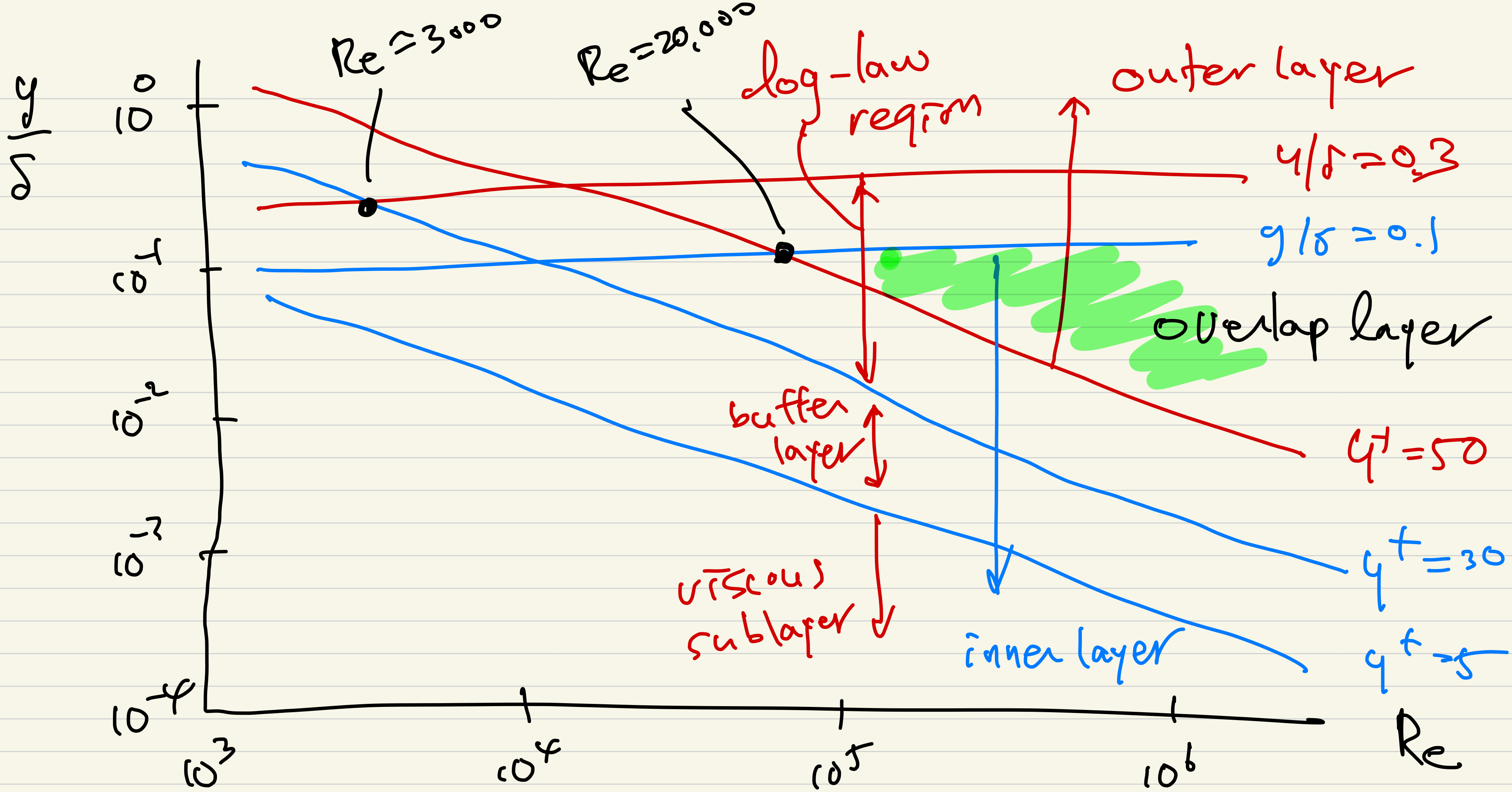
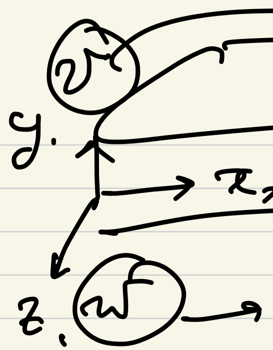


$\frac{y}{\delta}$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^3
10^4
10^5
10^6
Re
$Re \simeq 3000$
$Re = 20,000$
log-law region
outer layer
$y/\delta = 0.3$
$y/\delta = 0.1$
overlap layer
$y^+ = 50$
buffer layer
$y^+ = 30$
viscous sublayer
inner layer
$y^+ = 5$

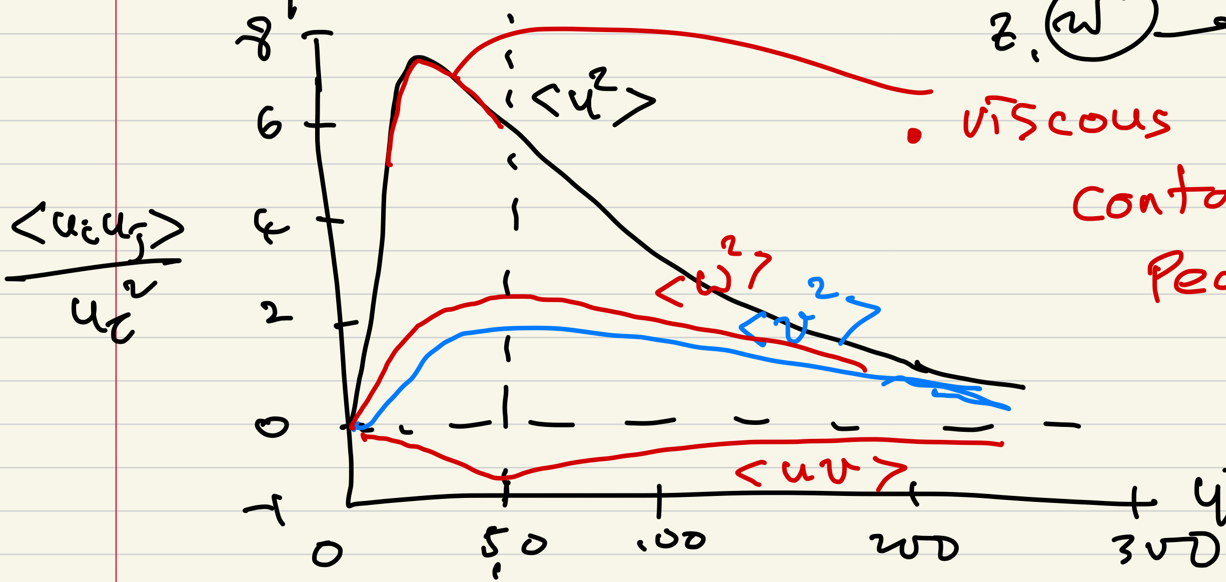
Turbulence statistics



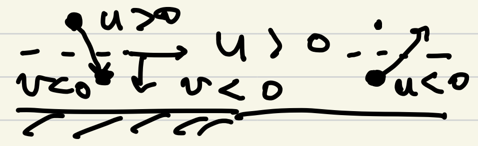
wall-normal vel.

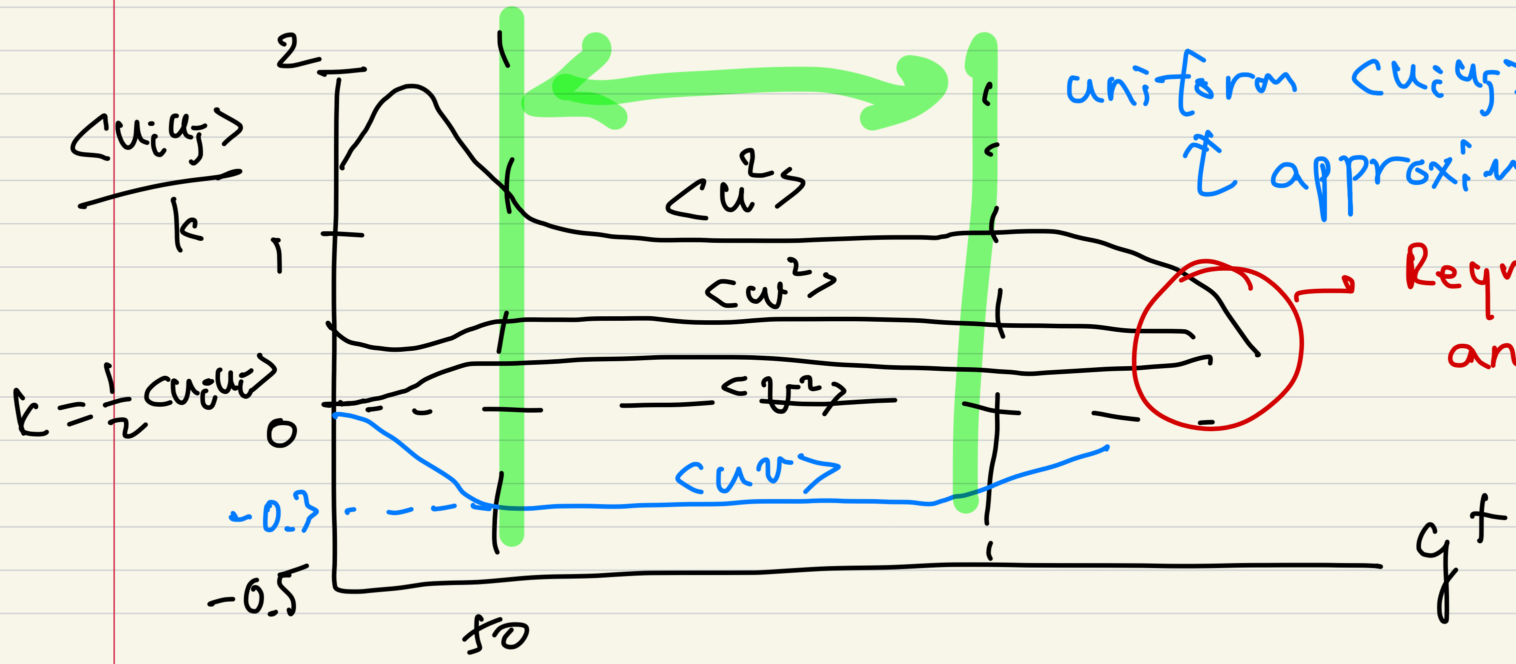
spanwise vel.

- Reynolds stresses



- viscous wall region ($y^+ < 50$) contains vigorous turbulent activity.
  Peak occurs at $y^+ < 20$.
  - $\langle w^2 \rangle > \langle v^2 \rangle$
- $-\langle uv \rangle > 0$



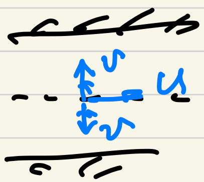


$k = \frac{1}{2}\langle u_i u_i \rangle$

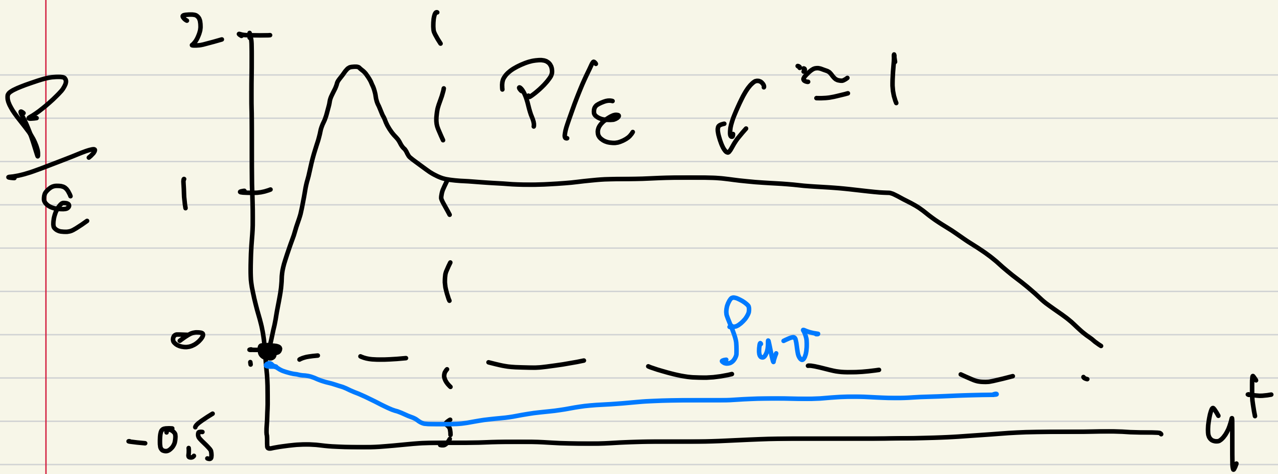
uniform $\langle u_i u_j \rangle / k$

↳ approximate self-similarity

Reynolds stresses are anisotropic even at the centerline.



→ $\langle uv \rangle |_{\text{centerline}} \to 0$



$$P = -\langle uv\rangle \frac{d\langle U\rangle}{dy}$$

$$= 0 \text{ @ wall } (\langle uv\rangle = 0)$$

$$= 0 \text{ @ centerline } (d\langle U\rangle/dy = 0)$$

Very close to the wall,

$$u(y) = \underbrace{\cancel{u\big|_{y=0}}}_{0} + y\left.\frac{\partial u}{\partial y}\right|_0 + \frac{1}{2}y^2\left.\frac{\partial^2 u}{\partial y^2}\right|_0 + \cdots \Rightarrow u \sim y$$

$$v(y) = \underbrace{\cancel{v\big|_{y=0}}}_{0} + y\underbrace{\left.\frac{\partial v}{\partial y}\right|_0}_{= -\left.\frac{\partial u}{\partial x}\right|_0 - \left.\frac{\partial w}{\partial z}\right|_0 = 0} + \frac{1}{2}y^2\left.\frac{\partial^2 v}{\partial y^2}\right|_0 + \cdots \Rightarrow v \sim y^2$$

$$w(y) = \underbrace{\cancel{w\big|_{y=0}}}_{0} + y\left.\frac{\partial w}{\partial y}\right|_0 + \frac{1}{2}y^2\left.\frac{\partial^2 w}{\partial y^2}\right|_0 + \cdots \Rightarrow w \sim y$$

$$\Rightarrow w \gg v$$
$$u \gg v$$

very near the wall ($v \simeq 0$)

→ u, w two component flow. very near the wall

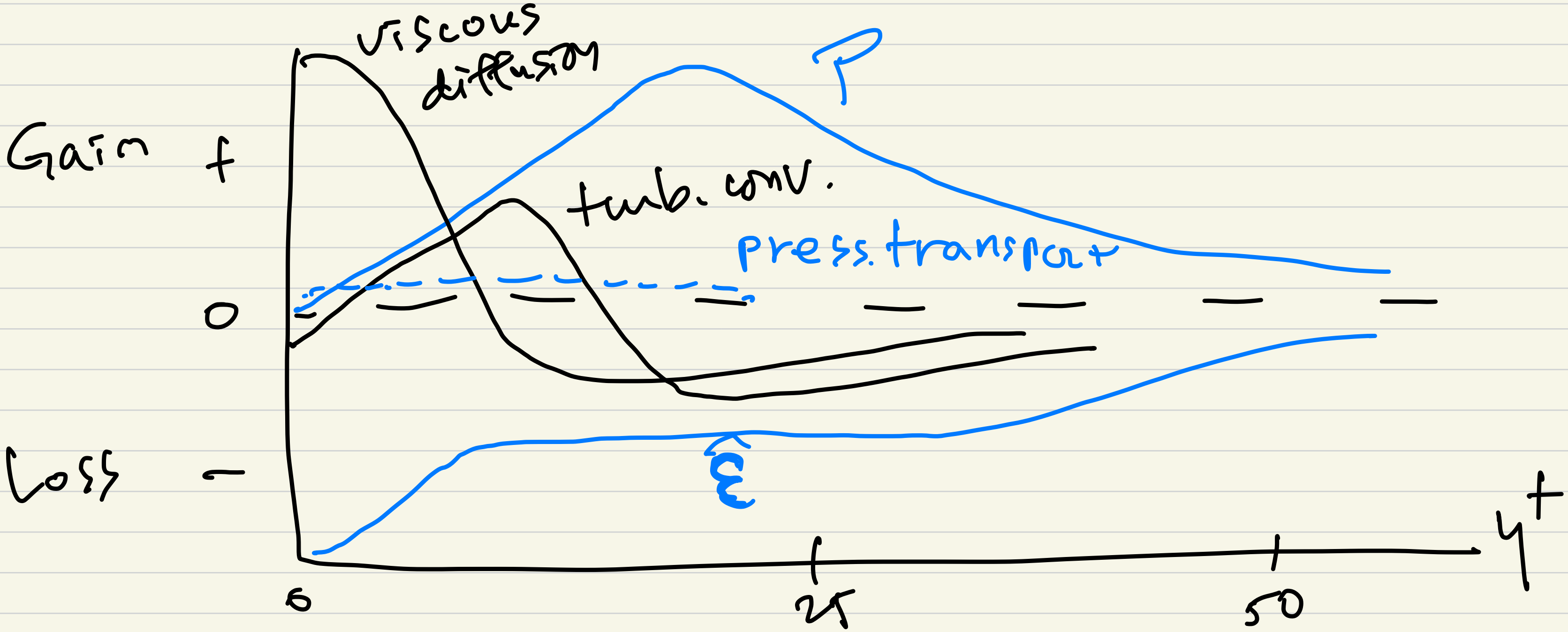
or wall-parallel flow

$$\Rightarrow \langle u^2 \rangle \sim y^2, \quad \langle v^2 \rangle \sim y^4, \quad \langle w^2 \rangle \sim y^2$$

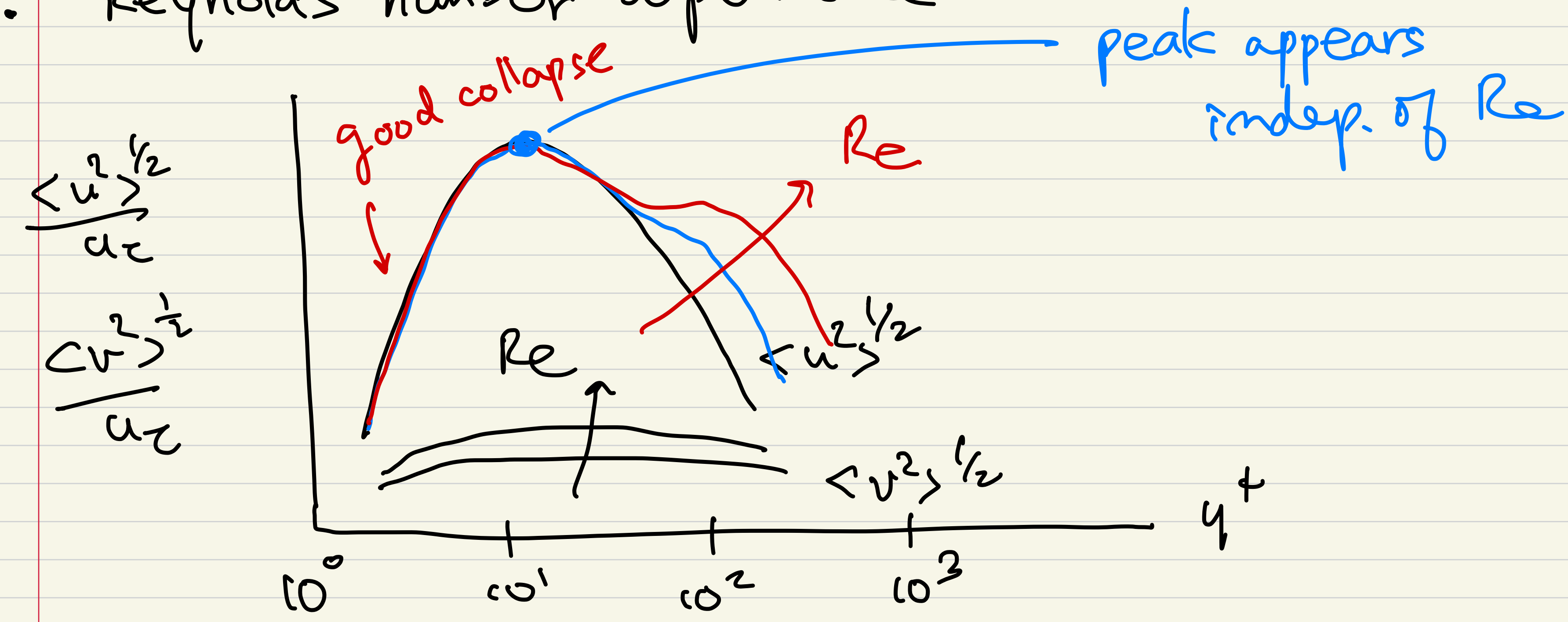
$$\langle uv \rangle \sim y^3$$

- Balance eq. for k (Budget analysis)

$$\frac{dk}{dt} + \cdots \rightarrow 0 = \underbrace{P}_{\text{production}} - \underbrace{\hat{\varepsilon}}_{\text{pseudo-dissipation}} + \underbrace{\nu \frac{d^2 k}{dy^2}}_{\text{viscous diffusion}} - \underbrace{\frac{d}{dy}\left\langle \frac{1}{2} v u_j u_j \right\rangle}_{\text{turb. convection}} - \underbrace{\frac{1}{\rho}\frac{d}{dy}\langle v p' \rangle}_{\text{press. transport}}$$



- Reynolds number dependence



⊙ Length scales and mixing length

- In the log-law region $\quad u^+ = \frac{1}{\kappa}\ln y^+ + B$

① $S = \frac{d\langle U\rangle}{dy} = \frac{u_\tau}{\kappa y}$ or $\frac{du^+}{dy^+} = \frac{1}{\kappa y^+}$

② $P/\varepsilon \simeq 1$

③ $-\langle uv\rangle/k \simeq 0.3$

④ $\frac{\mathcal{S}k}{\varepsilon} = \frac{k}{|\langle uv\rangle|} \cdot \frac{\mathcal{P}}{\varepsilon}$ $\leftarrow \mathcal{P} = -\langle uv\rangle \frac{\partial \langle U\rangle}{\partial y} = -\langle uv\rangle \mathcal{S}$

$\doteq \frac{1}{0.3} \doteq 3$

- turbulent length scale

$$L \equiv k^{3/2}/\varepsilon = \frac{k^{\frac{3}{2}}}{\varepsilon} \cdot \frac{\mathcal{P}}{\mathcal{P}} = \frac{k^{\frac{3}{2}}}{\varepsilon} \cdot \frac{\mathcal{P}}{-\langle uv\rangle \mathcal{S}} \quad \mathcal{S} = \frac{u_\tau}{\kappa y}$$

$$= \kappa y \underbrace{\frac{|\langle uv\rangle|^{\frac{1}{2}}}{u_\tau}}_{\text{const}} \cdot \underbrace{\frac{\mathcal{P}}{\varepsilon}}_{\text{const}} \cdot \underbrace{\left|\frac{\langle uv\rangle}{k}\right|^{-\frac{3}{2}}}_{\text{const}} = C_L y \sim y$$

$(C_L \simeq 2.5)$

- $\mathcal{S} \sim \frac{1}{y}$, $\mathcal{P} \sim \frac{1}{y}$, $\varepsilon \sim \frac{1}{y}$, $L \sim y$, $\tau = k/\varepsilon \sim y$

$(\because |\langle uv\rangle| \sim \text{const})$ $(\because \mathcal{P}/\varepsilon \sim 1)$

from DNS at moderate Reynolds number, there is no overlap region, and the shear stress changes appreciably over the log-law region $\rightarrow$ DNS currently does not show the behavior of L.

$$\frac{\partial \langle u\rangle}{\partial t} + \underbrace{\boxed{\frac{\partial}{\partial x}}\langle uu\rangle}_{=0} + \frac{\partial}{\partial y}\langle uv\rangle + \underbrace{\frac{\partial}{\partial z}\langle uw\rangle}_{=0} = \cdots$$

- mixing length

$$\boxed{-\langle uv\rangle = \nu_T \frac{d\langle U\rangle}{dy}}$$

ν_T : turbulent viscosity $[VL]$

$$\nu_T = u^* l_m$$

l_m : mixing length

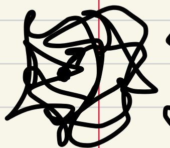
$$u^* = \sqrt{-\langle uv\rangle}$$

u^* : char. vel. scale

$$\Rightarrow u^{*2} = u^* l_m \frac{d\langle U\rangle}{dy} \rightarrow u^* = l_m \frac{d\langle U\rangle}{dy}$$

At high Re, $-\langle uv\rangle \sim u_\tau^2$ in the overlap region

$$\frac{d\langle U\rangle}{dy} = \frac{u_\tau}{\kappa y} = \frac{u^*}{l_m} = \frac{\sqrt{-\langle uv\rangle}}{l_m} = \frac{u_\tau}{l_m}$$



$$\Rightarrow \boxed{l_m = \kappa y} \sim y \quad \text{in overlap region}$$

$$\rightarrow \boxed{\nu_T = u^* l_m = l_m^2 \frac{d\langle U\rangle}{dy}}$$

Prandtl's mixing-length hypothesis.

what about other regions? (§7.3)