

19. Phase Conjugation Optics



Phase Conjugation and Time Reversal

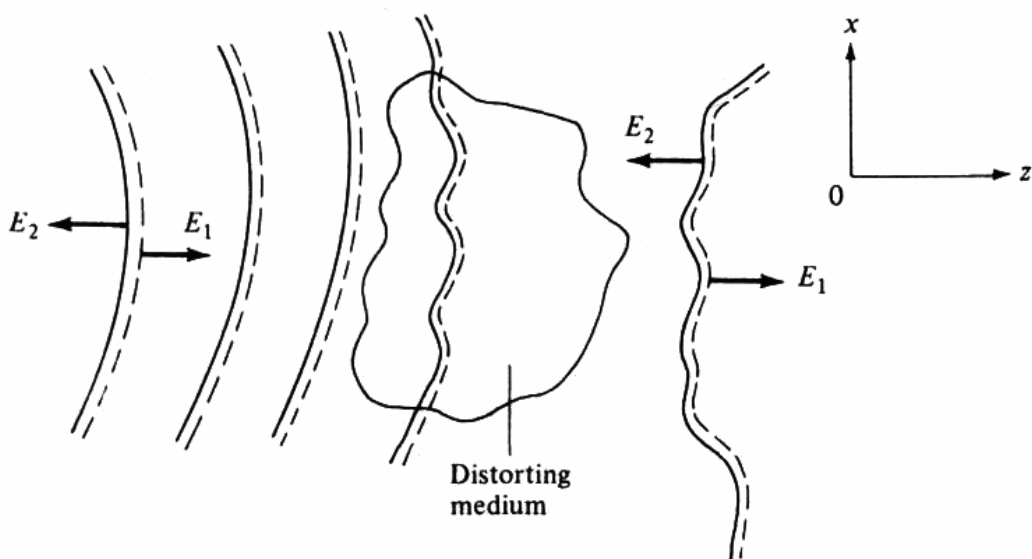
$$E_1(\mathbf{r}, t) = \text{Re}[\psi(\mathbf{r})e^{i(\omega t - kz)}]$$

$$E_2(\mathbf{r}, t) = \text{Re}[\psi^*(\mathbf{r})e^{i(\omega t + kz)}]$$

$$E_2(\mathbf{r}, t) = \text{Re}[\psi(\mathbf{r})e^{i[\omega(-t) - kz]}]$$



Distortion Correction Theorem



$$E_1 = \psi_1(\mathbf{r})e^{i(\omega t - kz)}$$

$$\nabla^2 E_1 + \omega^2 \mu \varepsilon(\mathbf{r}) E_1 = 0$$

$$\nabla^2 \psi_1 + [\omega^2 \mu \varepsilon(\mathbf{r}) - k^2] \psi_1 - 2ik \frac{\partial \psi_1}{\partial z} = 0$$

$$\nabla^2 \psi_1^* + [\omega^2 \mu \varepsilon^*(\mathbf{r}) - k^2] \psi_1^* + 2ik \frac{\partial \psi_1^*}{\partial z} = 0$$

$$E_2 = \psi_2(\mathbf{r})e^{i(\omega t + kz)}$$

$$\nabla^2 \psi_2 + [\omega^2 \mu \varepsilon(\mathbf{r}) - k^2] \psi_2 + 2ik \frac{\partial \psi_2}{\partial z} = 0$$

Generation of Phase Conjugate Waves

$$\mathbf{E}_1(\mathbf{r}, t) = \frac{1}{2} \mathbf{A}_1(r) e^{i(\omega_1 t - \mathbf{k}_1 \cdot \mathbf{r})} + \text{c.c.}$$

$$\mathbf{E}_2(\mathbf{r}, t) = \frac{1}{2} \mathbf{A}_2(r) e^{i(\omega_2 t - \mathbf{k}_2 \cdot \mathbf{r})} + \text{c.c.}$$

$$\mathbf{E}_3(\mathbf{r}, t) = \frac{1}{2} \mathbf{A}_3(r) e^{i(\omega_3 t - \mathbf{k}_3 \cdot \mathbf{r})} + \text{c.c.}$$

$$P_i^{(\text{NL})}(\mathbf{r}, t) = 6 \chi_{ijkl}^{(3)} A_{1j} A_{2k} A_{3l}^* e^{i[(\omega_1 + \omega_2 - \omega_3)t - (\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3) \cdot \mathbf{r}]} + \text{c.c.}$$



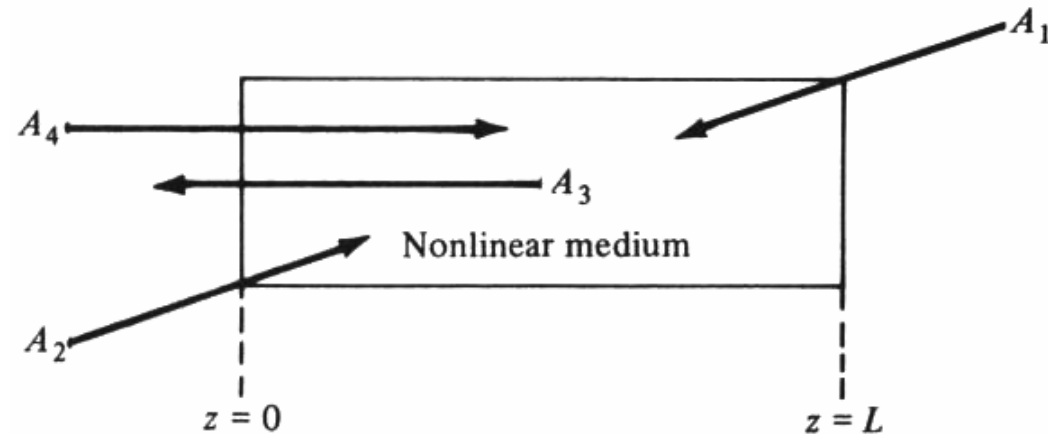
Coupled-Mode Formulation of Phase Conjugate Optics (I)

$$\mathbf{E}_1(\mathbf{r}, t) = \frac{1}{2} \mathbf{A}'_1(r_1) e^{i(\omega t - \mathbf{k}_1 \cdot \mathbf{r})} + \text{c.c.}$$

$$\mathbf{E}_2(\mathbf{r}, t) = \frac{1}{2} \mathbf{A}'_2(r_2) e^{i(\omega t - \mathbf{k}_2 \cdot \mathbf{r})} + \text{c.c.}$$

$$\mathbf{E}_3(\mathbf{r}, t) = \frac{1}{2} \mathbf{A}'_3(z) e^{i(\omega t + kz)} + \text{c.c.}$$

$$\mathbf{E}_4(\mathbf{r}, t) = \frac{1}{2} \mathbf{A}'_4(z) e^{i(\omega t - kz)} + \text{c.c.}$$



Coupled-Mode Formulation of Phase Conjugate Optics (II)

$$\mathbf{k}_1 + \mathbf{k}_2 = 0$$

$$\nabla^2 \mathbf{E} - \mu \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu \frac{\partial^2 \mathbf{P}^{(NL)}}{\partial t^2}$$

$$\nabla^2 \mathbf{E}_4 = \frac{1}{2} \left[-k^2 \mathbf{A}'_4 - 2ik \frac{d\mathbf{A}'_4}{dz} + \frac{d^2 \mathbf{A}'_4}{dz^2} \right] e^{i(\omega t - kz)} + \text{c.c.}$$



Coupled-Mode Formulation of Phase Conjugate Optics (III)

$$\frac{dA_3}{dz} = i\kappa^* A_4^* \quad \frac{dA_4^*}{dz} = i\kappa A_3$$

$$A_3(z) = \frac{\cos|\kappa|z}{\cos|\kappa|L} A_3(L) + i \frac{\kappa^* \sin|\kappa|(z-L)}{|\kappa| \cos|\kappa|L} A_4^*(0)$$

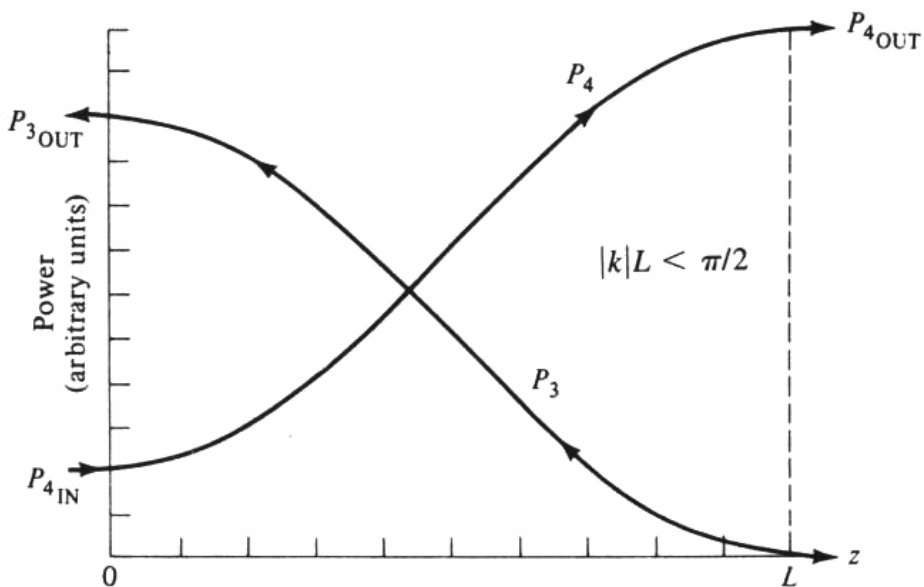
$$A_4(z) = -i \frac{|\kappa| \sin|\kappa|z}{\kappa \cos|\kappa|L} A_3^*(L) + \frac{\cos|\kappa|(z-L)}{\cos|\kappa|L} A_4(0)$$

$$\text{For } A_3(L) = 0, \quad A_3(0) = -i \left(\frac{\kappa^*}{|\kappa|} \tan|\kappa|L \right) A_4^*(0)$$

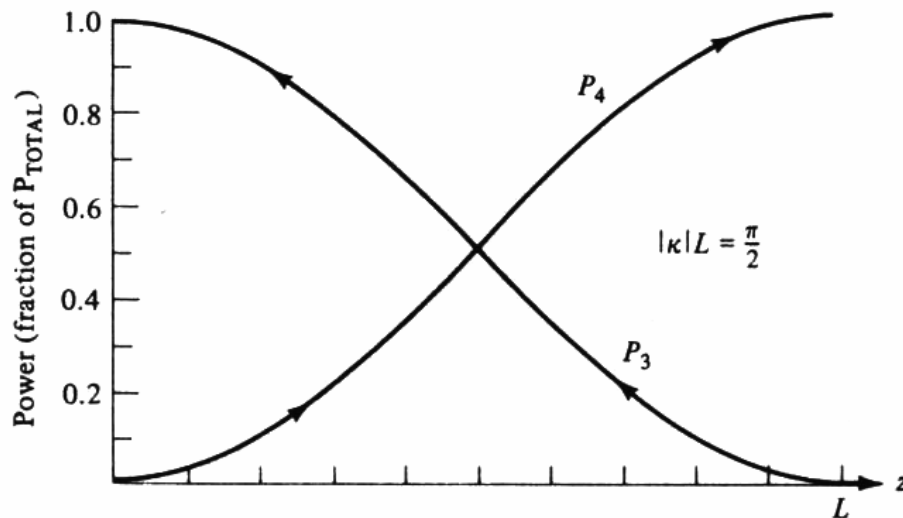
$$A_4(L) = \frac{A_4(0)}{\cos|\kappa|L}$$



Coupled-Mode Formulation of Phase Conjugate Optics (IV)



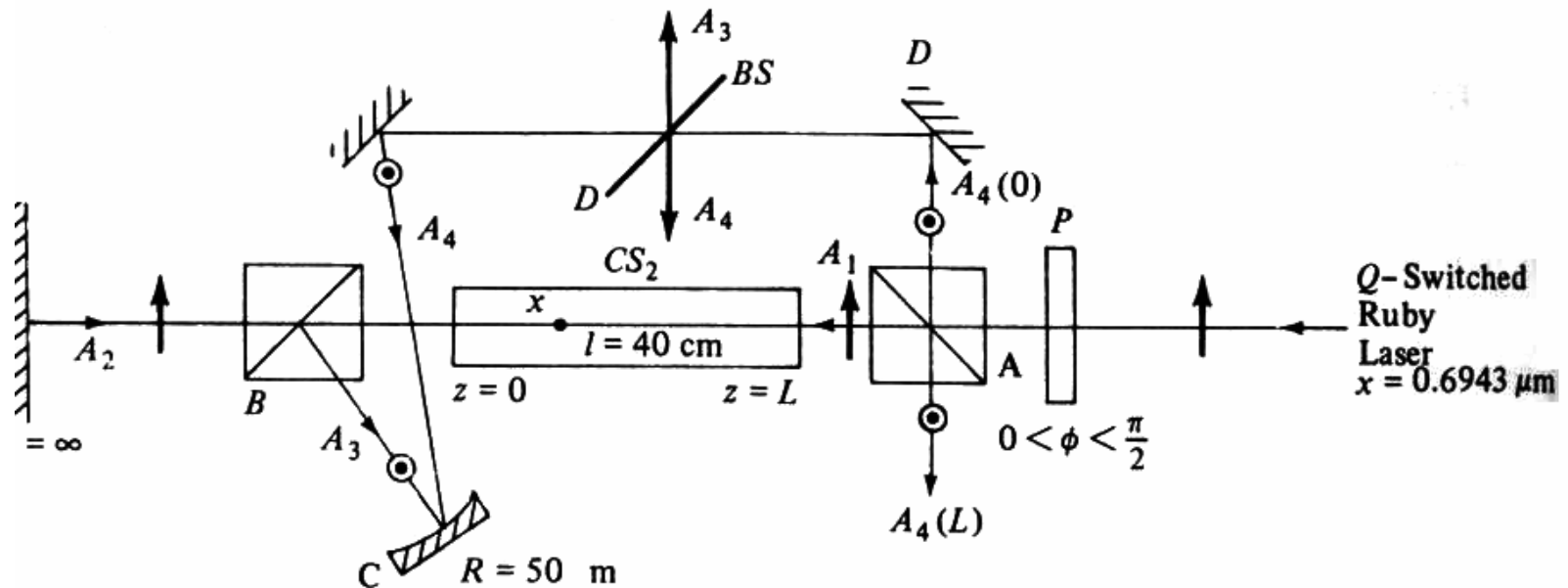
The intensity distribution inside the interaction region corresponding to the amplifier case $\pi/4 < |\kappa|L < \pi/2$



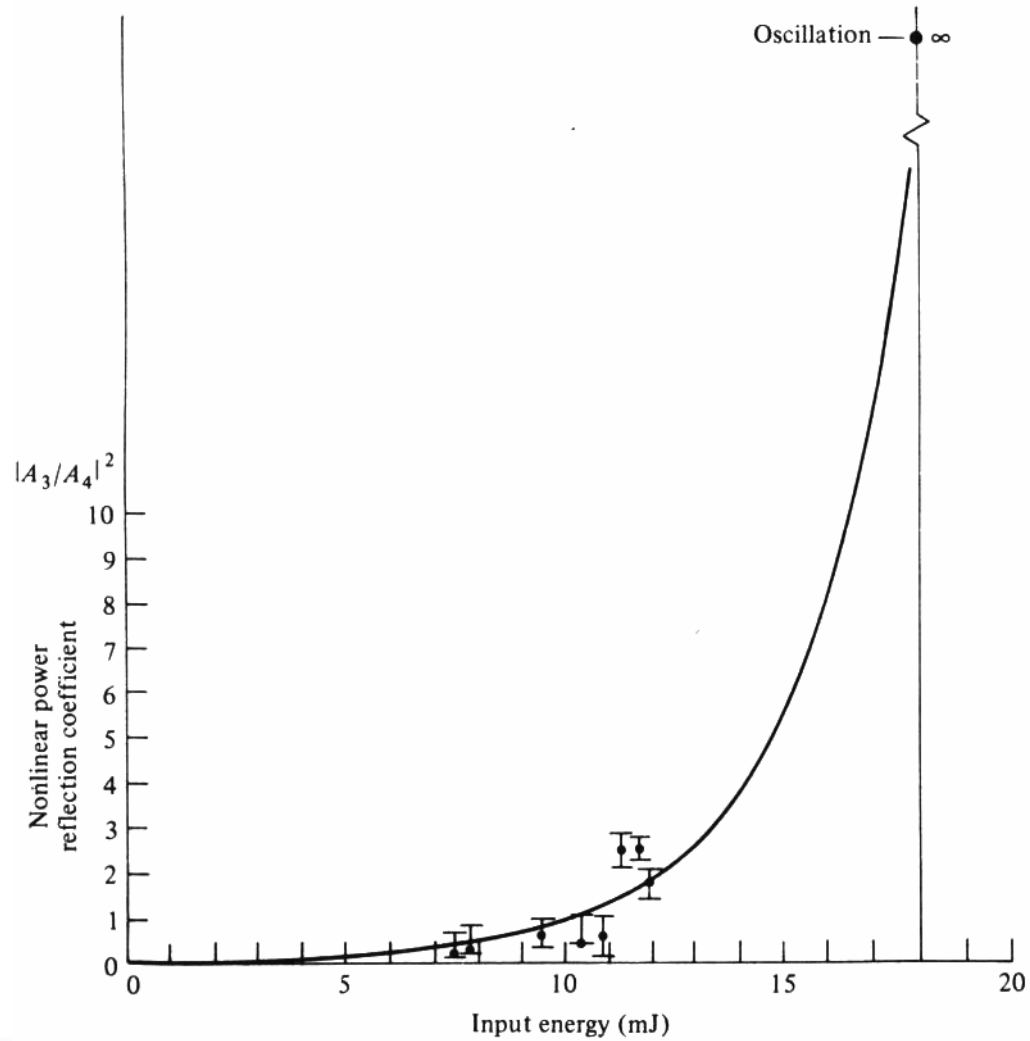
The intensity distribution inside the interaction region when the oscillation condition $|\kappa|L = \pi/2$ is satisfied.



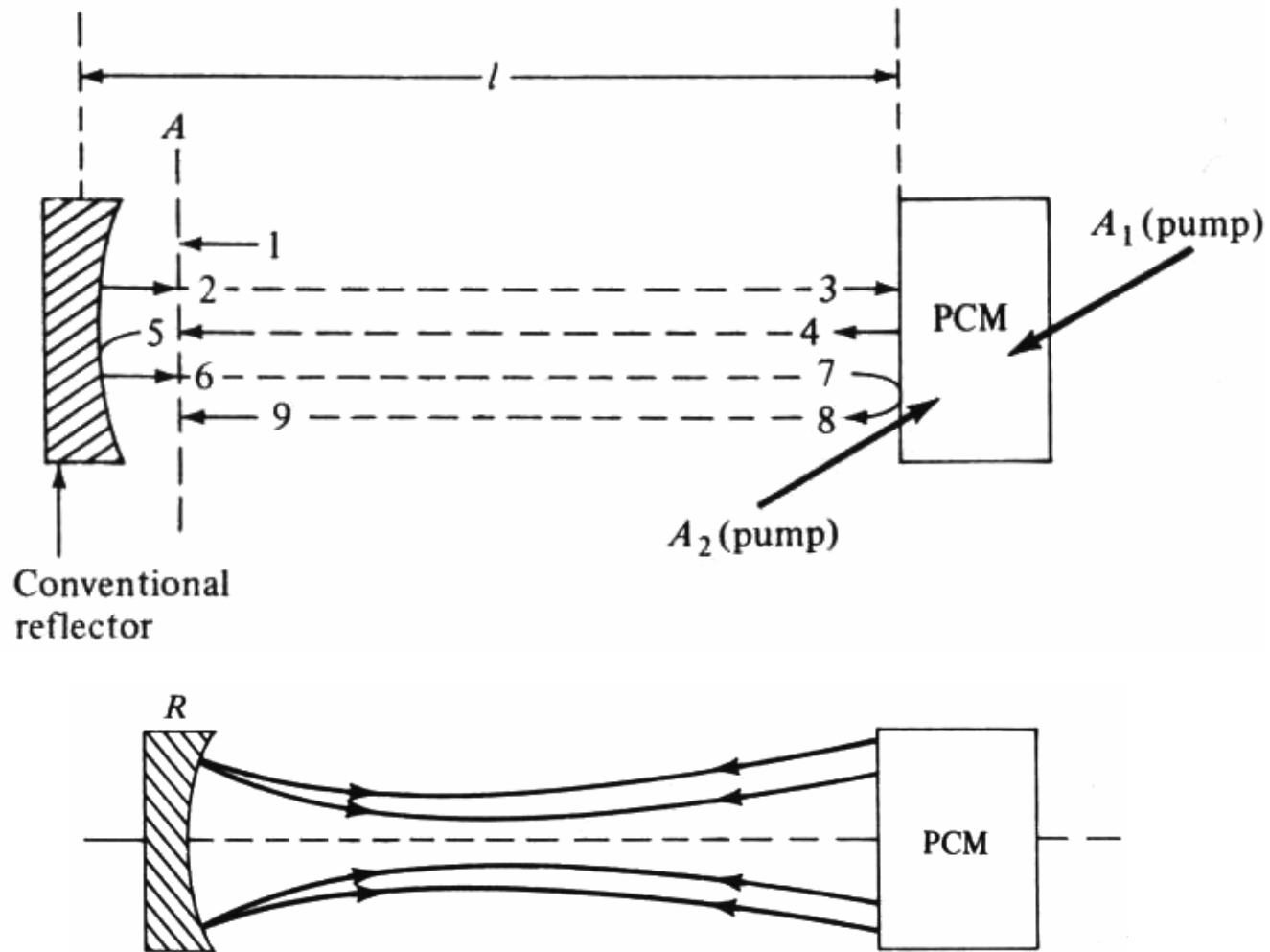
Experimental Example (I)



Experimental Example (II)



Optical Resonator with Phase Conjugate Reflector (I)



Optical Resonator with Phase Conjugate Reflector (II)

$$\phi_2 = \phi_1 + \phi_R$$

$$\phi_3 = \phi_2 + \phi_l(m, n) = \phi_1 + \phi_R + \phi_l(m, n)$$

$$\phi_4 = -\phi_3 + \alpha = -(\phi_1 + \phi_R + \phi_l(m, n)) + \alpha$$

$$\phi_5 = \phi_4 + \phi_l(m, n) = -\phi_1 - \phi_R + \alpha$$

$$\phi_6 = \phi_5 + \phi_R = -\phi_1 + \alpha$$

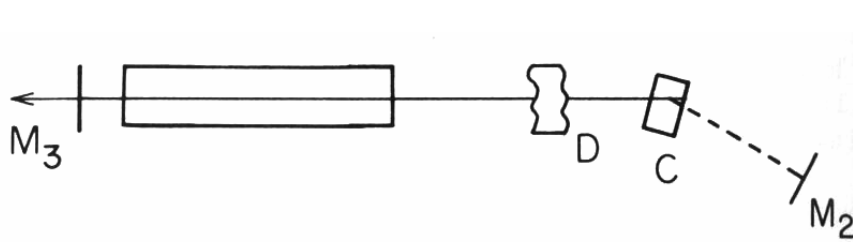
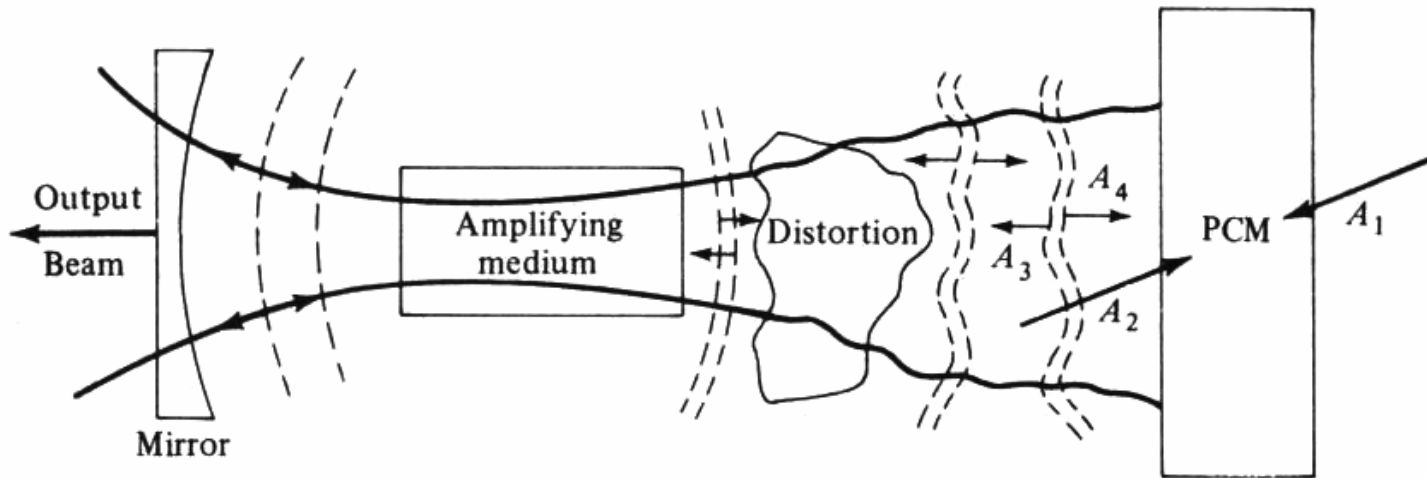
$$\phi_7 = \phi_6 + \phi_l(m, n) = -\phi_1 + \alpha + \phi_l(m, n)$$

$$\phi_8 = -\phi_7 + \alpha = \phi_1 - \phi_l(m, n)$$

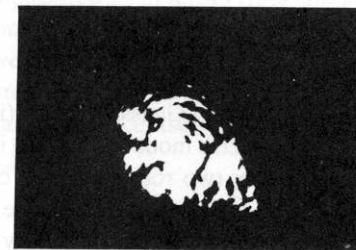
$$\phi_9 = \phi_8 + \phi_l(m, n) = \phi_1$$



Dynamic Distortion Correction

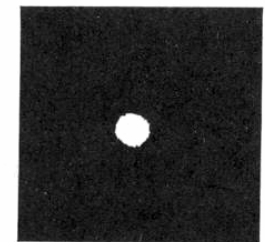


(a)



5mm

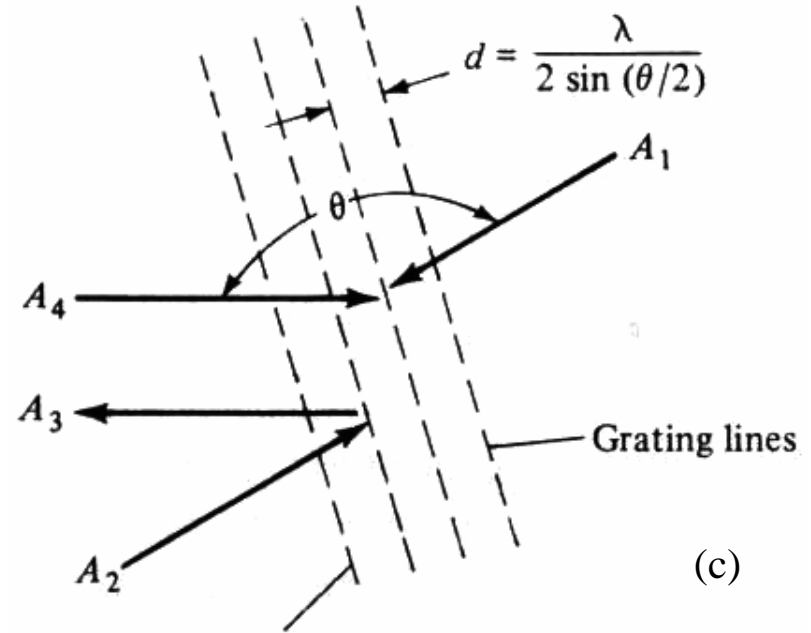
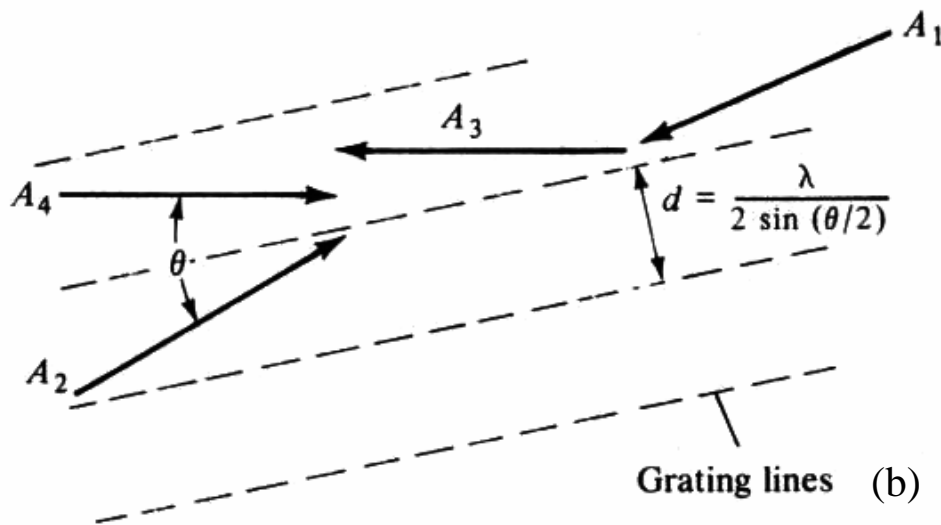
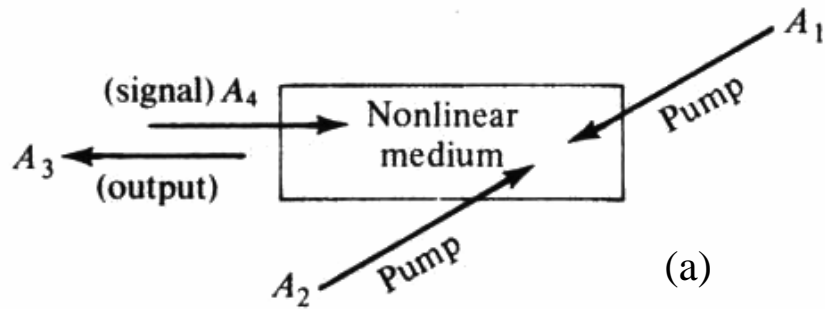
(b)



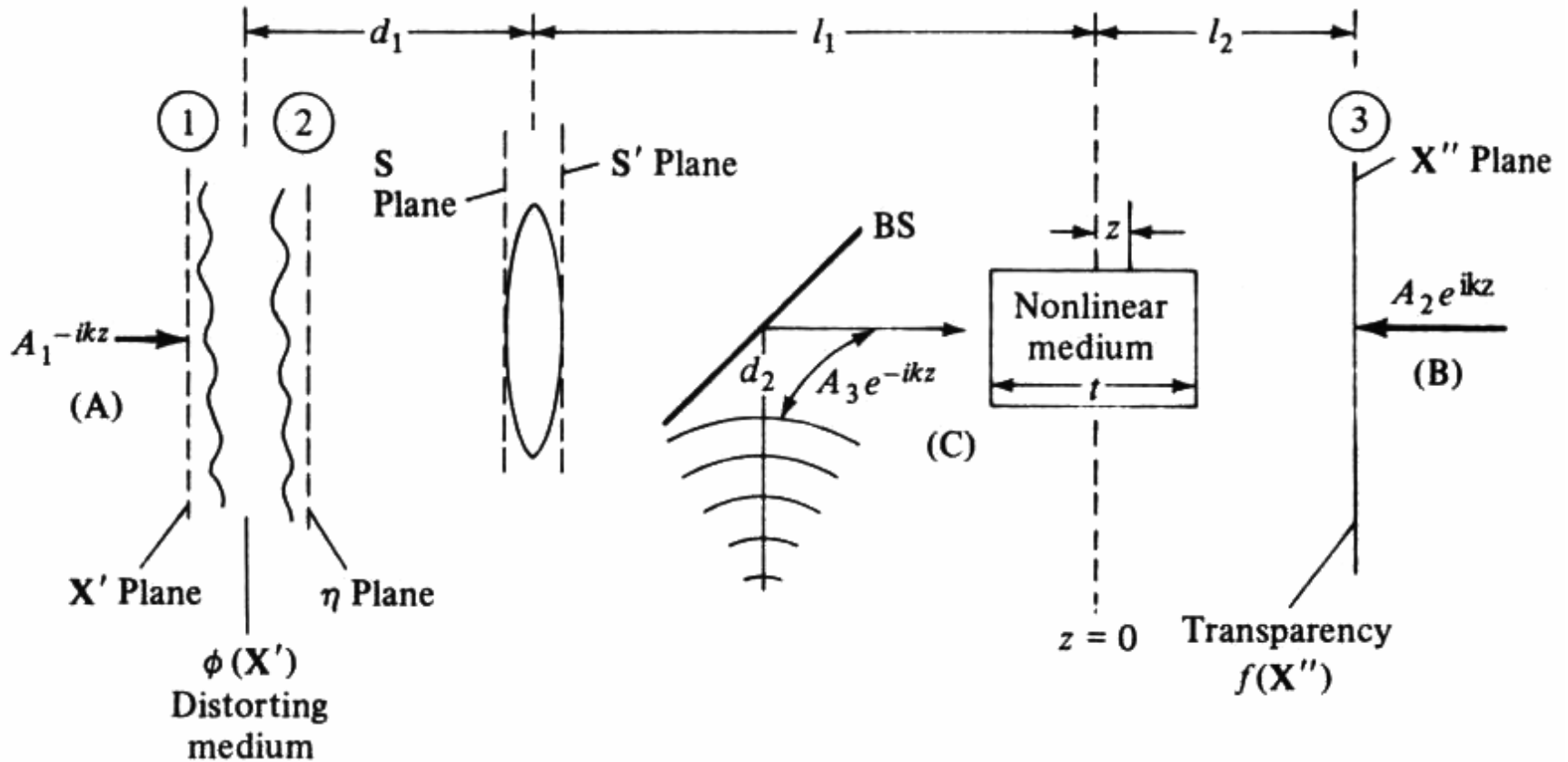
(c)



Holographic Analogs of Phase Conjugate Optics



Imaging through a Distorted Medium (I)



Imaging through a Distorted Medium (II)

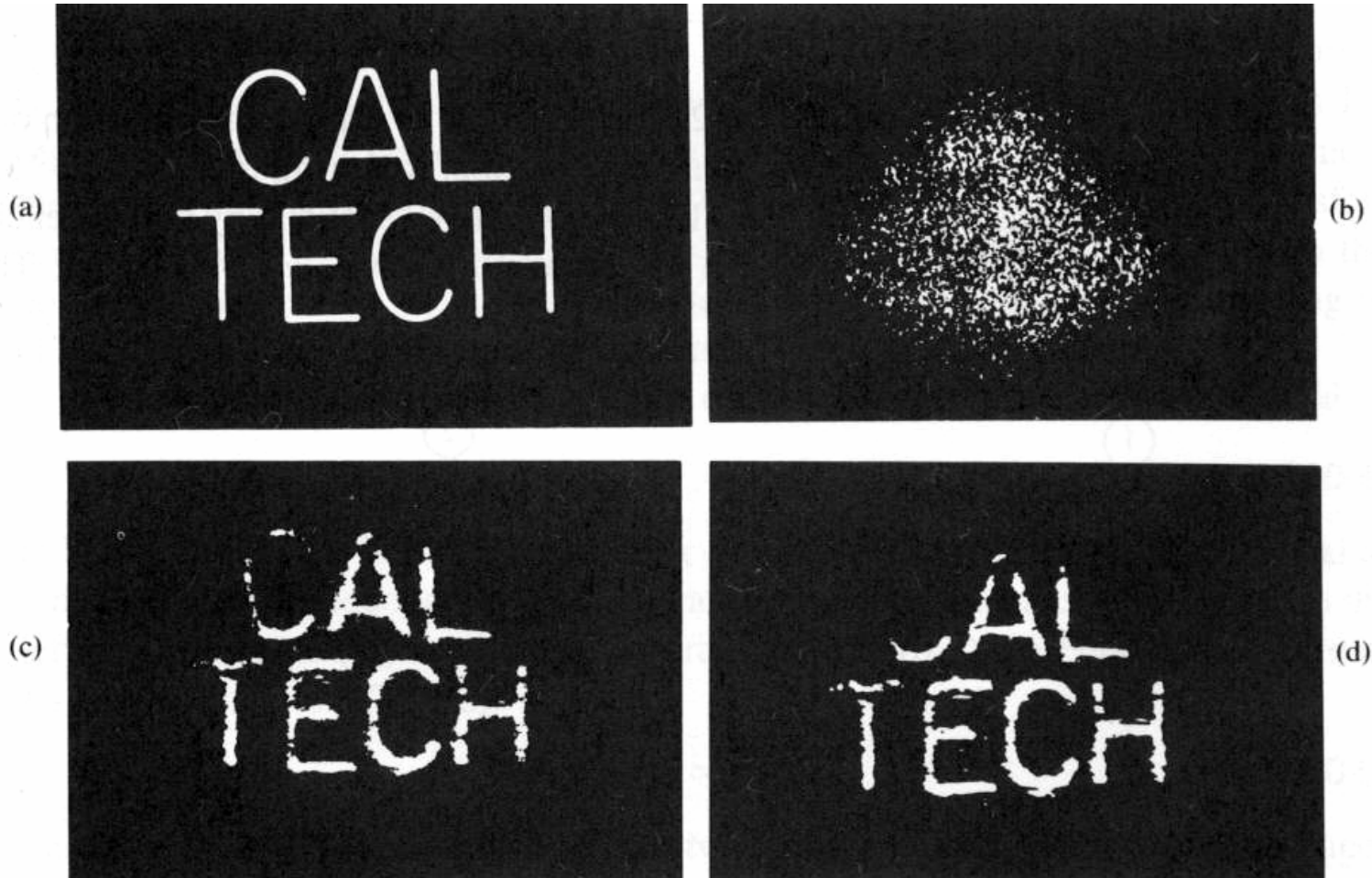


Image Processing by Four-Wave Mixing (I)

$$E_3 \propto A_2 \tilde{u}_1 \tilde{u}_4^*$$

$$u_3(x, y) \propto u_1 \star u_4$$

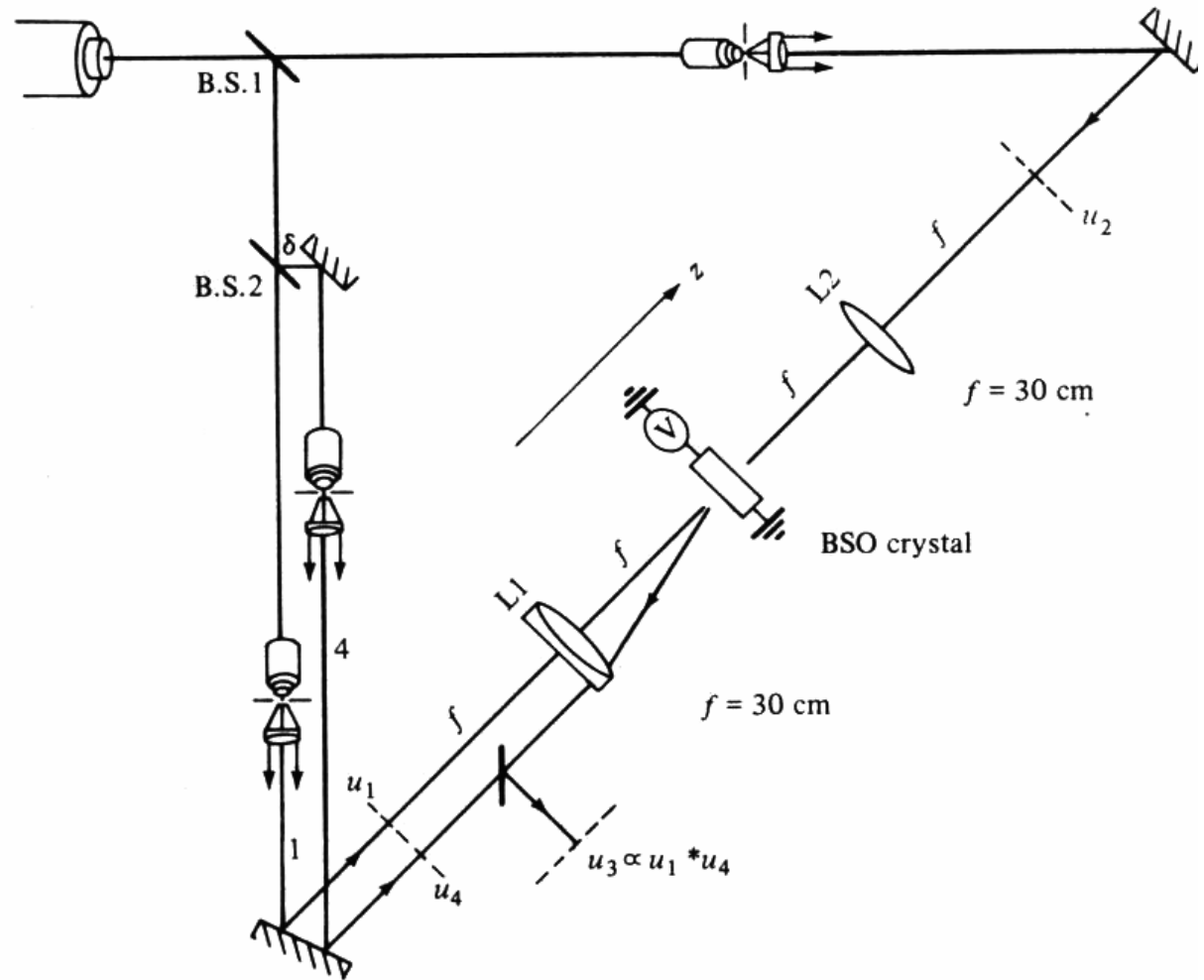

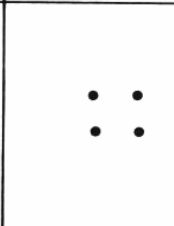
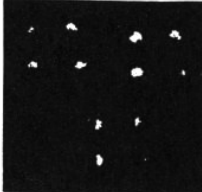


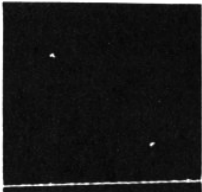
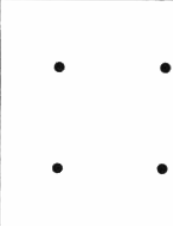


Image Processing by Four-Wave Mixing (II)

	U_1	U_2	U_4	U_3
(a)		DELTA FUNCTION		
(b)		DELTA FUNCTION	E	
(c)	C	DELTA FUNCTION	CAL TECH	
(d)	C		DELTA FUNCTION	