20. Two-Beam Coupling and Phase Conjugation in Photorefractive Media

Introduction

$$P_i^{\mathrm{NL}}(\mathbf{r}) = 6\chi_{ijk\ell}^{(3)} A_{1j}(\mathbf{r}) A_{2k}(\mathbf{r}) A_{3\ell}^{*}(\mathbf{r}) e^{i[(\omega_1+\omega_2-\omega_3)t-(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3)\cdot\mathbf{r}]} + \text{c.c.}$$

$$\Delta n \propto A_1 A_3^{*} e^{i[(\omega_1-\omega_3)t-(\mathbf{k}_1-\mathbf{k}_3)\cdot\mathbf{r}]} + \text{c.c.}$$

$$A_4 \propto \Delta n A_2 \propto A_1 A_2 A_3^{*} \exp\{i[(\omega_1+\omega_2-\omega_3)t-(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3)\cdot\mathbf{r}]\}$$

Two-Wave Coupling in a Fixed Grating (I)

$$\mathbf{E}(\mathbf{r}) = \left[\frac{1}{2}\mathbf{A}_1(r)e^{-i\mathbf{k}_1\cdot\mathbf{r}} + \frac{1}{2}\mathbf{A}_2(r)e^{-i\mathbf{k}_2\cdot\mathbf{r}}\right]e^{i\omega t} + \text{c.c}$$

$$n(\mathbf{r}) = n_0 + n_1\cos(\mathbf{K}\cdot\mathbf{r} + \phi)$$

$$\frac{1}{2}\left(-2ik_1\frac{dA_1}{dz} - k_1^2 A_1\right)e^{-i\mathbf{k}_1\cdot\mathbf{r}} + \text{c.c} + \frac{1}{2}\left(-2ik_2\frac{dA_2}{dz} - k_2^2 A_2\right)e^{-i\mathbf{k}_2\cdot\mathbf{r}} + \text{c.c}$$

$$+\omega^2\mu\varepsilon_o\left[n_0^2 + \left(n_0 n_1 e^{-i\phi}e^{-i\mathbf{K}\cdot\mathbf{r}} + \text{c.c.}\right)\right]\left[\frac{A_1}{2}e^{-i\mathbf{k}_1\cdot\mathbf{r}} + \frac{A_2}{2}e^{-i\mathbf{k}_2\cdot\mathbf{r}}\right] = 0$$

$$\mathbf{k}_2 - \mathbf{k}_1 = \mathbf{K}$$ Bragg condition

Two-Wave Coupling in a Fixed Grating (II)

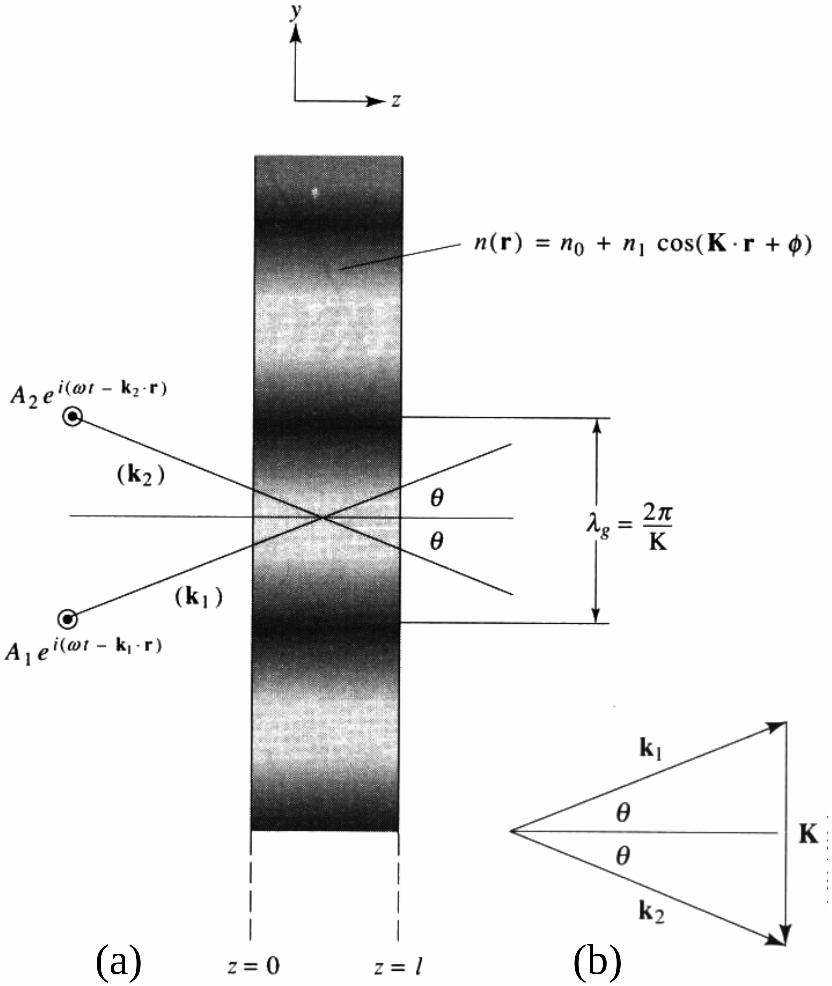
$$\cos\theta \frac{dA_1}{dz} = -\frac{\alpha}{2} A_1 - i\frac{\pi n_1}{\lambda_0} e^{i\phi} A_2 e^{i(\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{K})\cdot\mathbf{r}}$$

$$\cos\theta \frac{dA_2}{dz} = -\frac{\alpha}{2} A_2 - i\frac{\pi n_1}{\lambda_0} e^{-i\phi} A_1 e^{-i(\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{K})\cdot\mathbf{r}}$$

$$A_j \equiv \sqrt{I_j}\exp\left(-j\phi_j\right)$$

$$\cos\theta \frac{dI_1}{dz} = -\alpha I_1 + \frac{2\pi n_1}{\lambda_0}\sqrt{I_1 I_2}\sin\left(\phi_1 - \phi_2 + \phi\right)$$

$$\cos\theta \frac{dI_2}{dz} = -\alpha I_2 - \frac{2\pi n_1}{\lambda_0}\sqrt{I_1 I_2}\sin\left(\phi_1 - \phi_2 + \phi\right)$$



Two-Wave Coupling in a Fixed Grating (III)

For $\psi \equiv \phi_1 - \phi_2 + \phi = -\pi/2,$

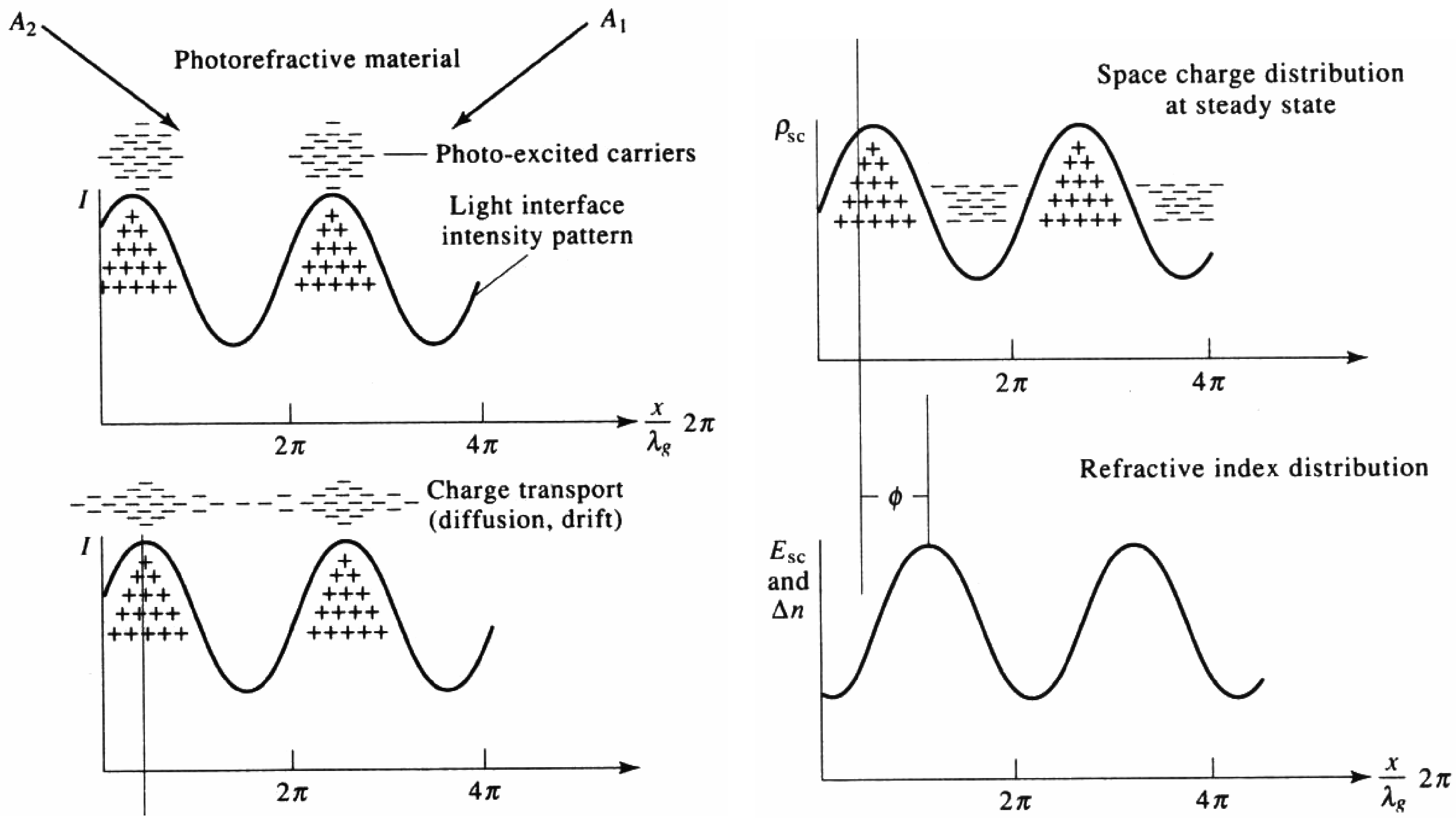
$$I_1(z) = I_1(0)e^{-\alpha z}\cos^2\left(\frac{\pi n_1 z}{\lambda_0 \cos\theta}\right)$$

$$I_2(z) = I_1(0)e^{-\alpha z}\sin^2\left(\frac{\pi n_1 z}{\lambda_0 \cos\theta}\right)$$

Diffraction efficiency

$$\eta = \frac{I_2(\ell)}{I_1(0)} = \exp\left(-\frac{\alpha\ell}{\cos\theta}\right)\sin^2\left(\frac{\pi n_1 \ell}{\lambda_0 \cos\theta}\right)$$

Photorefractive Effect



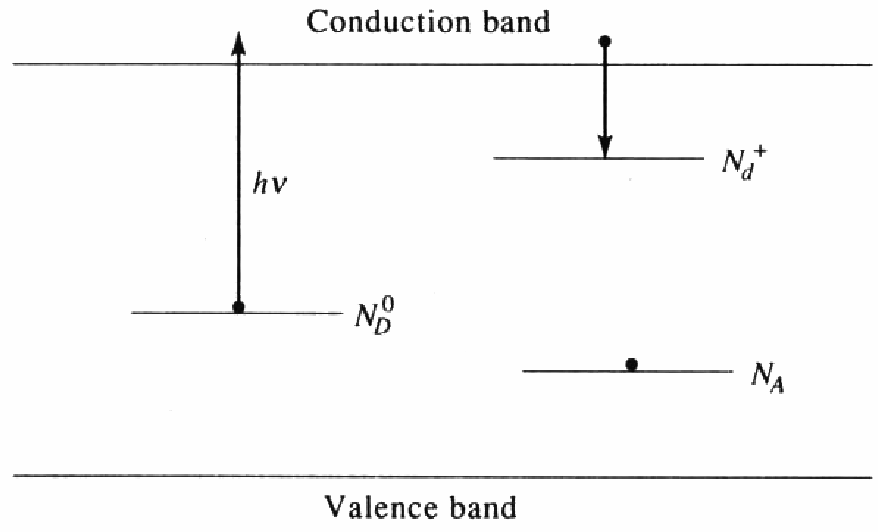
Two-Beam Coupling (I)

$$\frac{\partial N_D^+(x,t)}{\partial t} = \left(\frac{\alpha_D}{h\nu}\right) I(x)\left(N_D - N_D^+\right) - \gamma_D n_e N_D^+$$

$$J_x = \mu e n_e E_x + eD\frac{\partial n_e}{\partial x} = \mu e n_e E_x + k_B T \mu \frac{\partial n_e}{\partial x}$$

$$\frac{\partial J_x}{\partial x} = -e\frac{\partial}{\partial t}\left(N_D^+ - n_e\right)$$

$$\frac{\partial E_x}{\partial x} = \rho / \varepsilon = \frac{e\left(N_D^+ - n_e - N_A\right)}{\varepsilon}$$



Two-Beam Coupling (II)

$$N_D^+(x,t) = D_0 + \left[D_1 e^{-iKx} + \text{c.c.}\right]$$

$$n_e(x,t) = n_{e0} + \left[n_{e1} e^{-iKx} + \text{c.c.}\right]$$

$$E_x(x,t) = E_0 + \left[E_1^{sc} e^{-iKx} + \text{c.c.}\right]$$

$$\mathbf{E}_1(\mathbf{r},t) = \hat{\mathbf{e}}_1 A_1(\mathbf{r}) e^{i(\omega_1 t - \mathbf{k}_1 \cdot \mathbf{r})} + \text{c.c.}$$

$$\mathbf{E}_2(\mathbf{r},t) = \hat{\mathbf{e}}_2 A_2(\mathbf{r}) e^{i(\omega_2 t - \mathbf{k}_2 \cdot \mathbf{r})} + \text{c.c.}$$

Two-Beam Coupling (III)

$$I(x) \equiv \tfrac{1}{2}\left\langle \left(\mathbf{E}_1 + \mathbf{E}_2\right)\cdot\left(\mathbf{E}_1 + \mathbf{E}_2\right)\right\rangle_{\text{space}-\text{time}}$$

$$= \left|A_1\right|^2 + \left|A_2\right|^2 + \hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_2 \left(A_1 A_2^* e^{i\left[(\omega_1 - \omega_2)t - (\mathbf{k}_1 - \mathbf{k}_2)\cdot\mathbf{r}\right]} + \text{c.c.}\right)$$

$$\mathbf{K} = \mathbf{k}_1 - \mathbf{k}_2 = \hat{\mathbf{e}}_x \left|\mathbf{k}_1 - \mathbf{k}_2\right|$$

$$I_0 \equiv \left|A_1\right|^2 + \left|A_2\right|^2$$

$$\Omega = \omega_2 - \omega_1$$

$$I_1 \equiv \hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_2 A_1 A_2^*$$

Two-Beam Coupling (IV)

$$E_1^{\mathrm{sc}} = -i\frac{I_1}{I_0}\frac{E_N(E_0 + iE_D)\left(e^{i\Omega t} - e^{-t/\tau}\right)}{\left[E_0 - \Omega t_0\left(E_D + E_\mu\right)\right] + i\left(E_N + E_D + \Omega t_0 E_0\right)}$$

$$\Omega \equiv \omega_2 - \omega_1 \qquad E_N = \frac{eN_A}{\varepsilon K} \qquad E_\mu = \frac{\gamma_D N_A}{\mu K}$$

$E_0 =$ externally applied field,

$$E_D = \frac{k_B T K}{e}$$

$$t_0 = \frac{N_A h\nu}{\alpha_D N_D I_0}$$

$$\tau = t_0 \frac{E_0 + i\left(E_D + E_\mu\right)}{E_0 + i\left(E_N + E_D\right)}$$

Two-Beam Coupling (V)

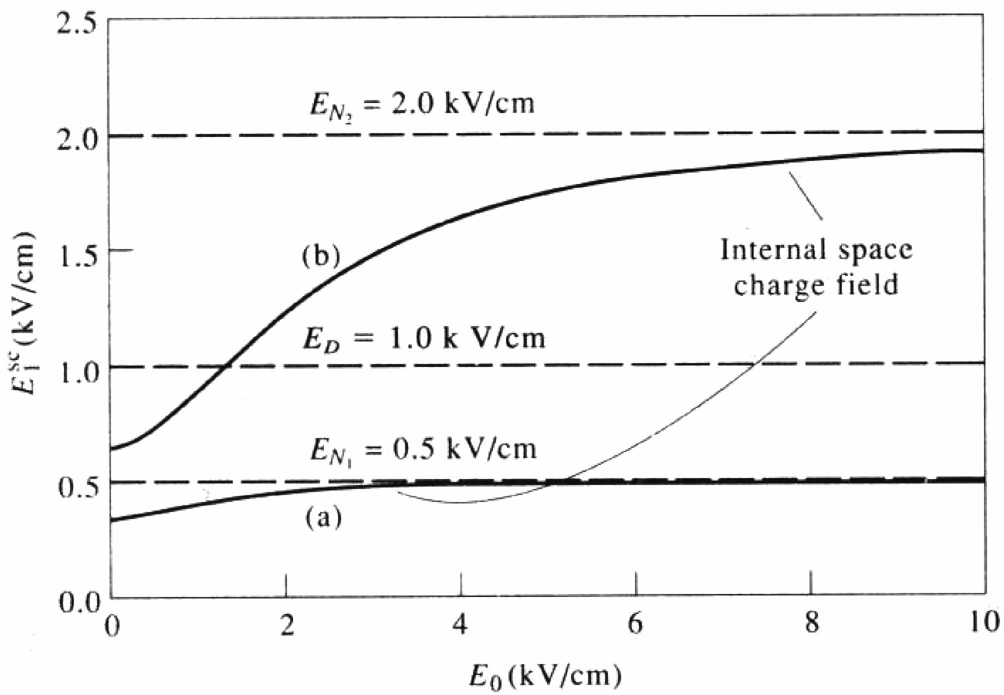
With $\Omega = 0$ and $E_0 = 0$

$$E_1^{\mathrm{sc}} \approx -i\frac{I_1}{I_0}\frac{E_D}{1+\dfrac{E_D}{E_N}}$$

$$\tau = t_0\frac{1+E_\mu/E_D}{1+E_N/E_D}$$

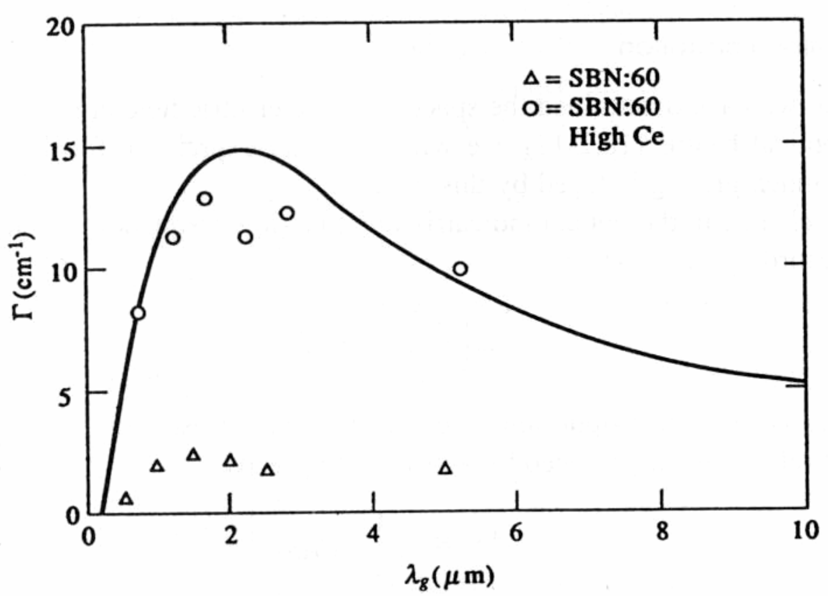
$$J_e = \mu e n_e E_1^{\mathrm{sc}} + k_B T\mu\frac{\partial n_e}{\partial x}$$

Two-Beam Coupling (VI)

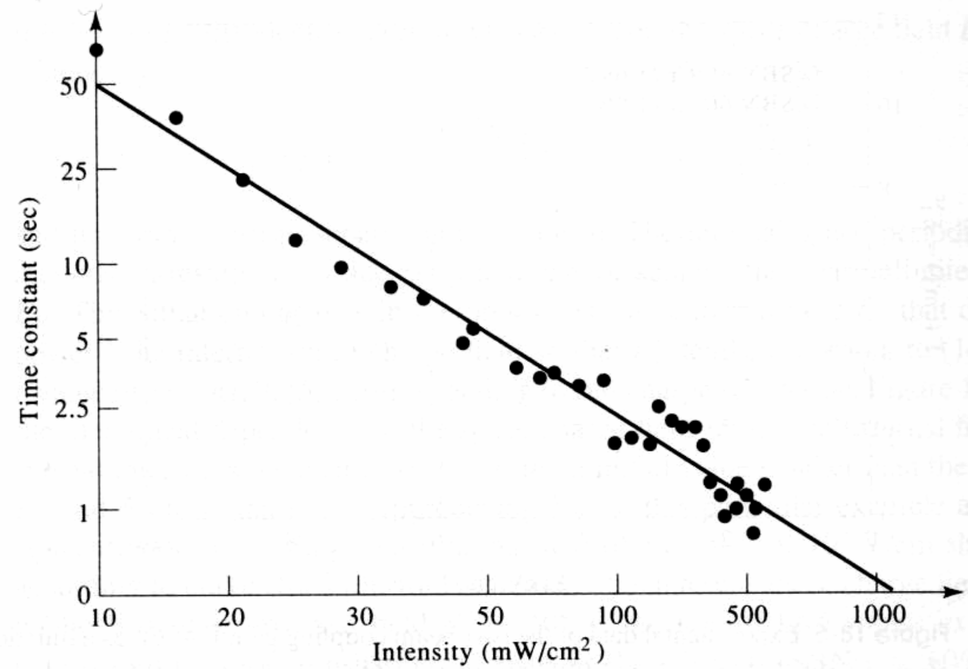


A theoretical plot of the amplitude E_1^{sc} of the spatially periodic internal electric field in photorefractive crystals with $\Omega = 0$, $t \to \infty$ as a function of the externally applied field E_0. The characteristic fields are: $E_D = 10^3$V/cm and (a) $E_N = 5 \times 10^2$V/ cm , (b) $E_N = 2 \times 10^3$V/cm .

Two-Beam Coupling (VII)



Two-beam coupling coefficient versus grating wavelength for E_0=0. The coupling coefficient Γ is proportional to the internal field E_1^{sc}.



Photorefractive response time of the $BaTiO_3$ crystal versus intensity for λ=0.605μm, λ_g=1.4 μm .

Grating Formation

$$\Delta\left(\frac{1}{n^2}\right)_{ij} = r_{ijk}E_k$$

$$\Delta n \equiv -\frac{1}{2}n_0^3 r_{\text{eff}} E$$

$$n(x,t) \equiv n_0 + \frac{1}{2}\left[\frac{n_1 e^{-i\phi} I_1}{I_0} e^{i(\Omega t - Kx)} + \text{c.c.}\right]$$

With $\Omega = 0$,

$$n_1 e^{-i\phi} \simeq r_{\text{eff}} n_0^3 \frac{iE_N\left(E_0 + iE_D\right)}{E_0 + i\left(E_N + E_D\right)}$$

Refractive Two-Beam Coupling

$$\nabla^2\mathbf{E} + \omega^2\mu\varepsilon(\mathbf{r})\mathbf{E} = 0, \qquad \varepsilon(\mathbf{r}) = \varepsilon_0 n^2(\mathbf{r})$$

$$\nabla^2\mathbf{E} + \frac{\omega^2}{c^2}\left[n_0^2 + \frac{n_0 n_1 e^{-i\phi}\hat{\mathbf{e}}_1\cdot\hat{\mathbf{e}}_2 A_1 A_2^*}{I_0} e^{i(\Omega t - Kx)} + \text{c.c.}\right]\mathbf{E} = 0$$

$$\cos\theta_1 \frac{dA_1}{dz} = -\frac{\alpha}{2}A_1 - i\frac{\pi n_1}{\lambda_0}e^{-i\phi}\frac{|A_2|^2}{I_0}\hat{\mathbf{e}}_1\cdot\hat{\mathbf{e}}_2 A_1$$

$$\cos\theta_2 \frac{dA_2}{dz} = -\frac{\alpha}{2}A_2 - i\frac{\pi n_1^*}{\lambda_0}e^{+i\phi}\frac{|A_1|^2}{I_0}\hat{\mathbf{e}}_1\cdot\hat{\mathbf{e}}_2 A_2$$

$$\cos\theta_1 \frac{d\mathcal{J}_1}{dz} = -\alpha\mathcal{J}_1 - \frac{2\pi n_1}{\lambda_0}\hat{\mathbf{e}}_1\cdot\hat{\mathbf{e}}_2 \sin(\phi)\mathcal{J}_1\mathcal{J}_2/(\mathcal{J}_1 + \mathcal{J}_2)$$

$$\cos\theta_2 \frac{d\mathcal{J}_2}{dz} = -\alpha\mathcal{J}_2 + \frac{2\pi n_1}{\lambda_0}\hat{\mathbf{e}}_1\cdot\hat{\mathbf{e}}_2 \sin(\phi)\mathcal{J}_1\mathcal{J}_2/(\mathcal{J}_1 + \mathcal{J}_2)$$

Two-Beam Coupling - Symmetric Geometry

$$\mathcal{J}_1(r) = \mathcal{J}_1(O)e^{-\alpha r}\frac{\mathcal{J}_1(O)+\mathcal{J}_2(O)}{\mathcal{J}_1(O)+\mathcal{J}_2(O)e^{2\Gamma r}}$$

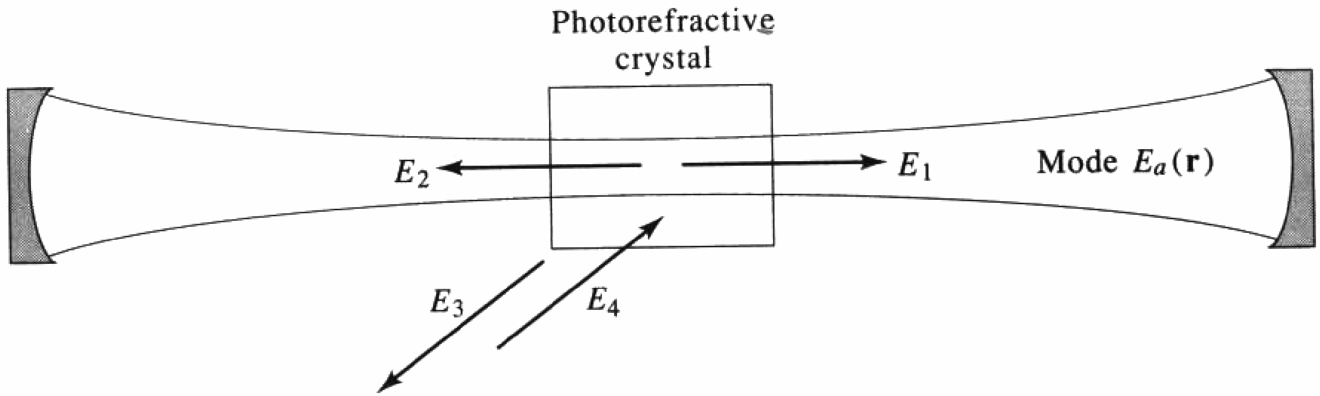
$$\mathcal{J}_2(r) = \mathcal{J}_2(O)e^{-\alpha r}\frac{\mathcal{J}_1(O)+\mathcal{J}_2(O)}{\mathcal{J}_1(O)e^{-2\Gamma r}+\mathcal{J}_2(O)}$$

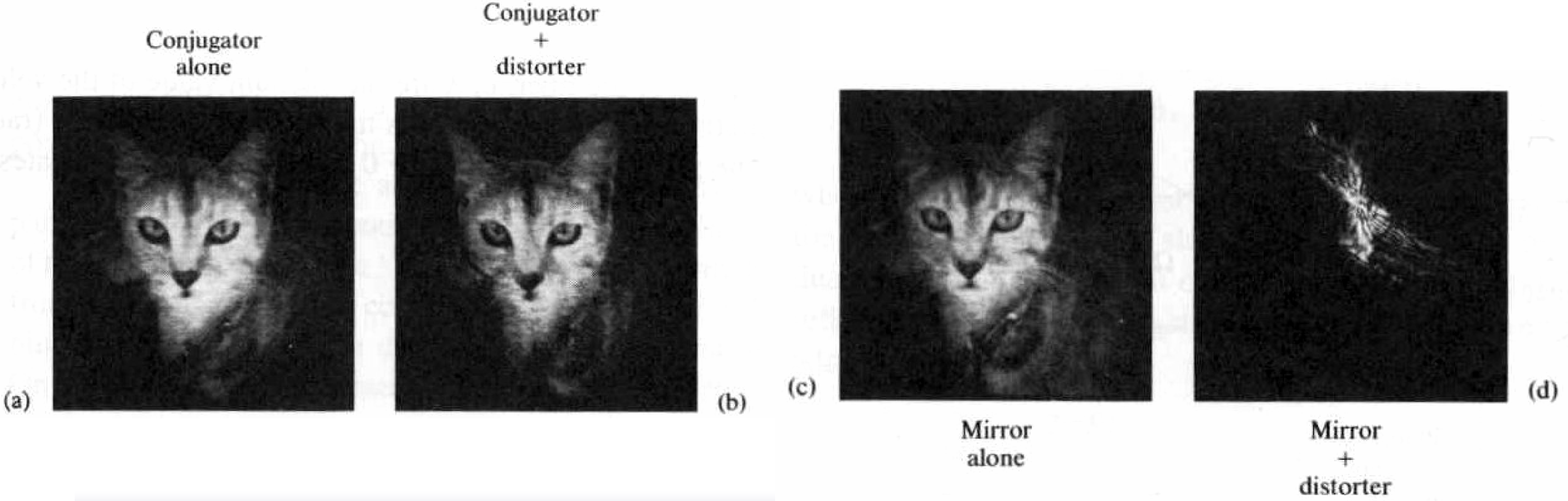
$$2\Gamma = \frac{2\pi n_1}{\lambda}\hat{\mathbf{e}}_1\cdot\hat{\mathbf{e}}_2\sin\phi$$

In the case of $\mathcal{J}_2(O) << \mathcal{J}_1(O)e^{-2\Gamma r}$,

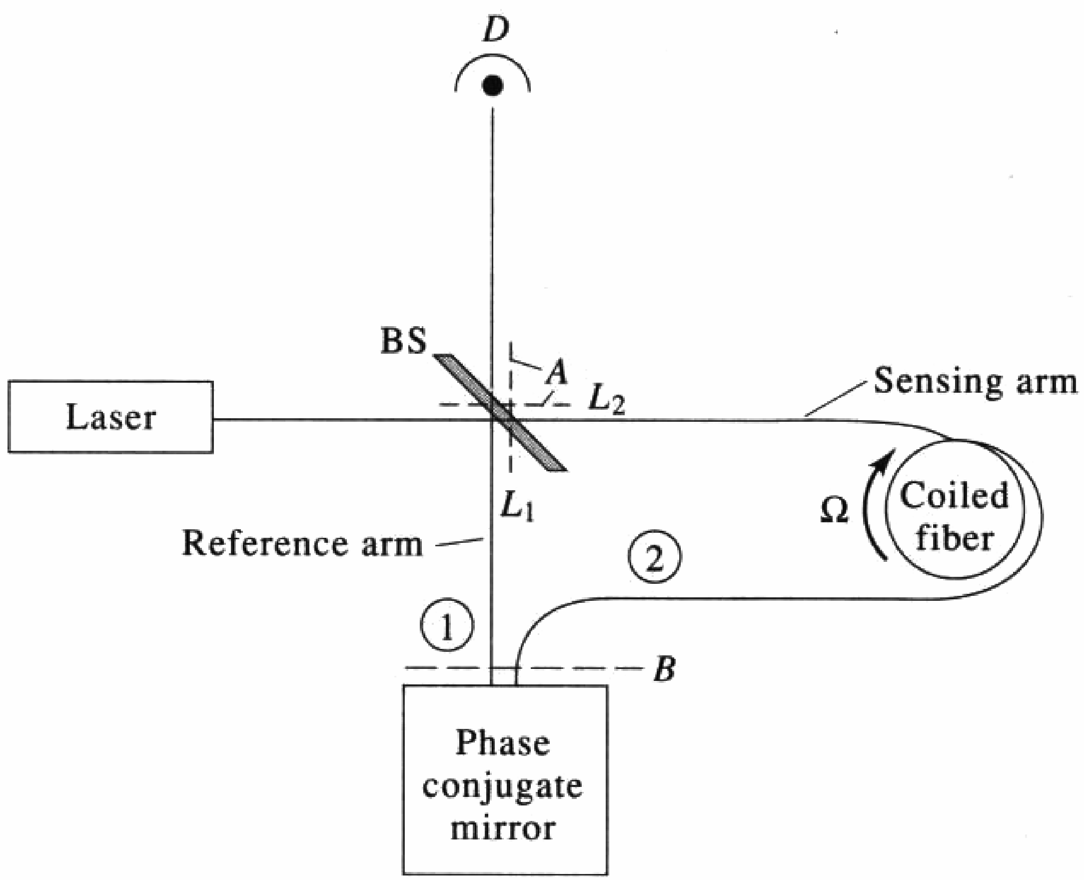
$$\mathcal{J}_2(r) = \mathcal{J}_2(O)e^{(2\Gamma-\alpha)r}$$

Photorefractive Self-Pumped Phase Conjugation

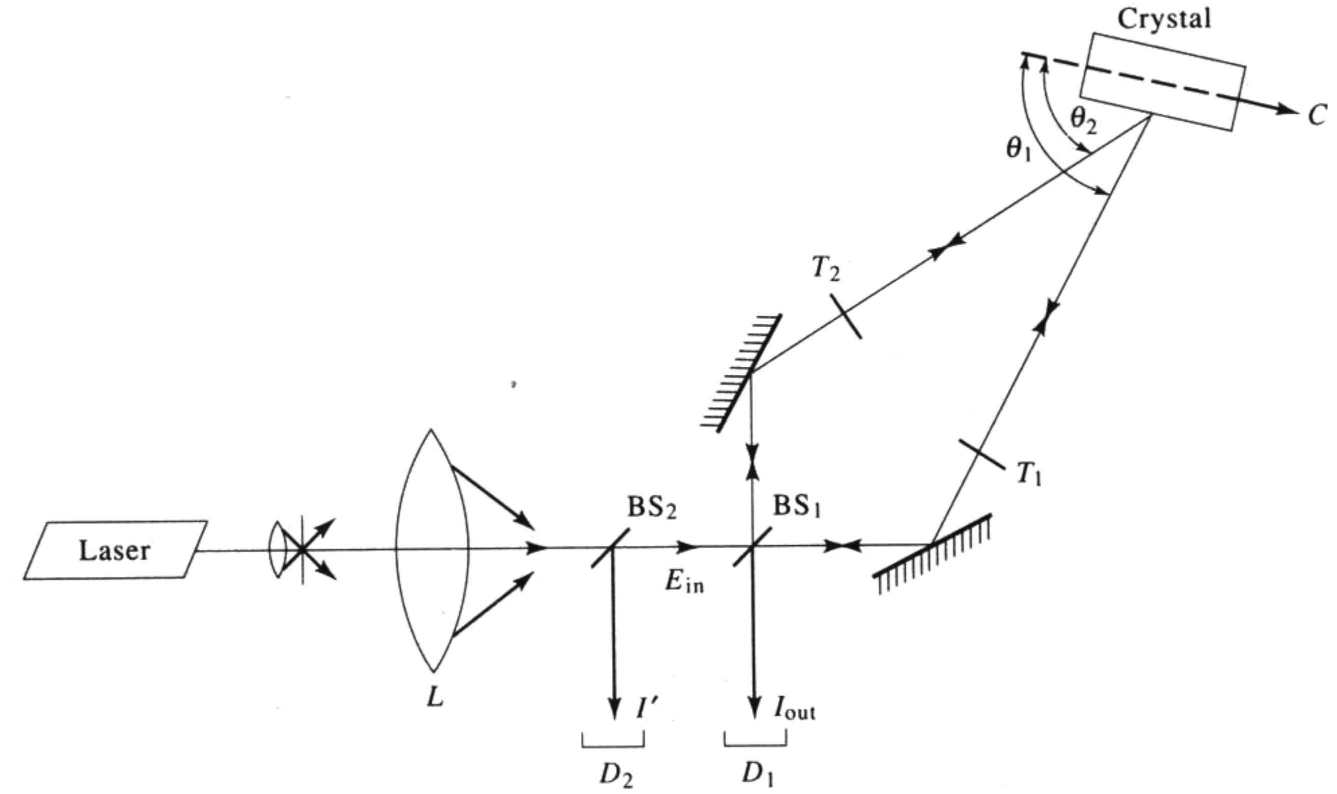




Application - Rotating Sensing



Application - Logic Operations (I)



Application - Logic Operations (II)