2020 Spring

Advanced Solidification

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6) Degenerate eutectic structure

Pure eutectic (lamellar type) ~ a very wide range of solidification rate

→ structure degenerate at very slow rates of solidification (less than 1cm/hr)

*** Degenerate structure:**
resemble the beginning of the spheroidization process that occurs during prolonged annealing

→ But, the degenerate structure is formed during, and not after, solidification.

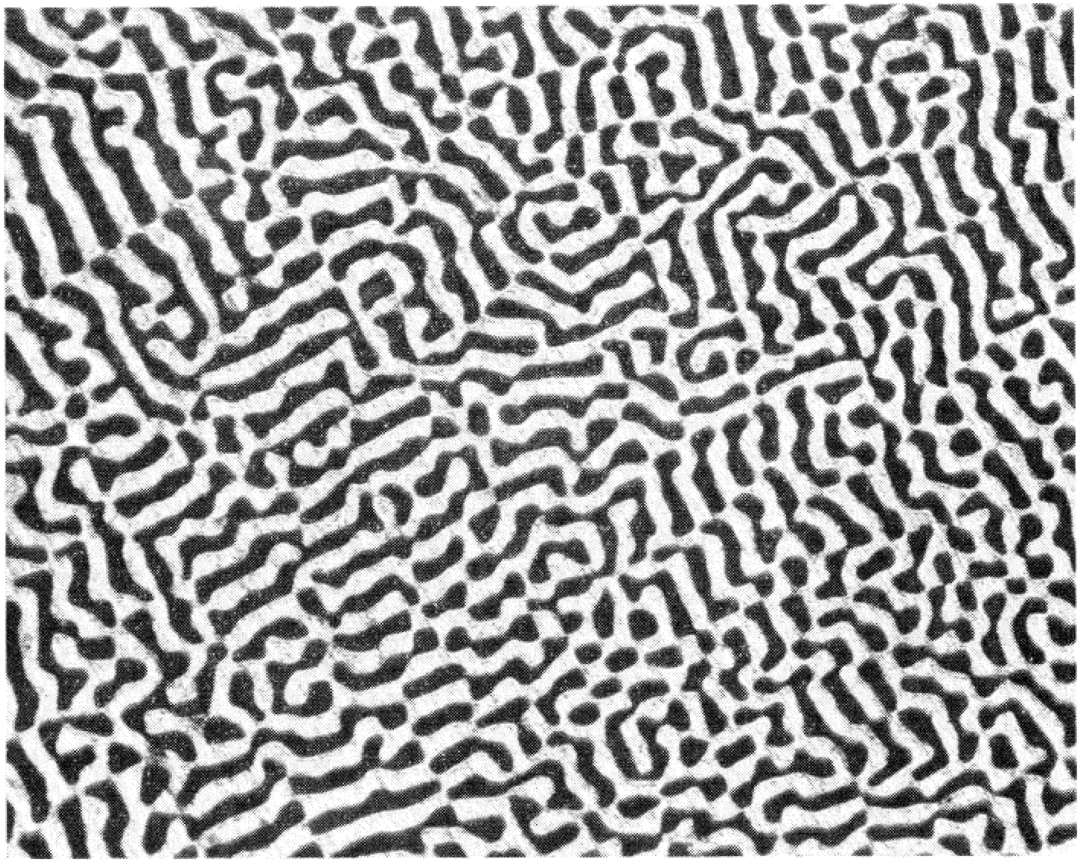


Fig. 6.20. Degenerate eutectic structure in CuAl2-Al eutectic at 0.8 cm/hr (X500).

7) Modification of Eutectics

Two degenerate forms of the lamellar structure *by impurities*

→ (a) Colony structure and (b) Rod structure

(a) Colony structure

: a cellular structure superimposed on the lamellar eutectic structure

* An impurity or an excess of one constituent, would diffuse much farther ahead of the interface than would be required for transverse interlamellar diffusion

→ The long range diffusion sets up constitutional supercooling → Cell formation and the resulting transverse diffusion of the impurity

→ if purity of the eutectic were sufficiently high, the colony structure are eliminated (regular lamellar structure is produced)

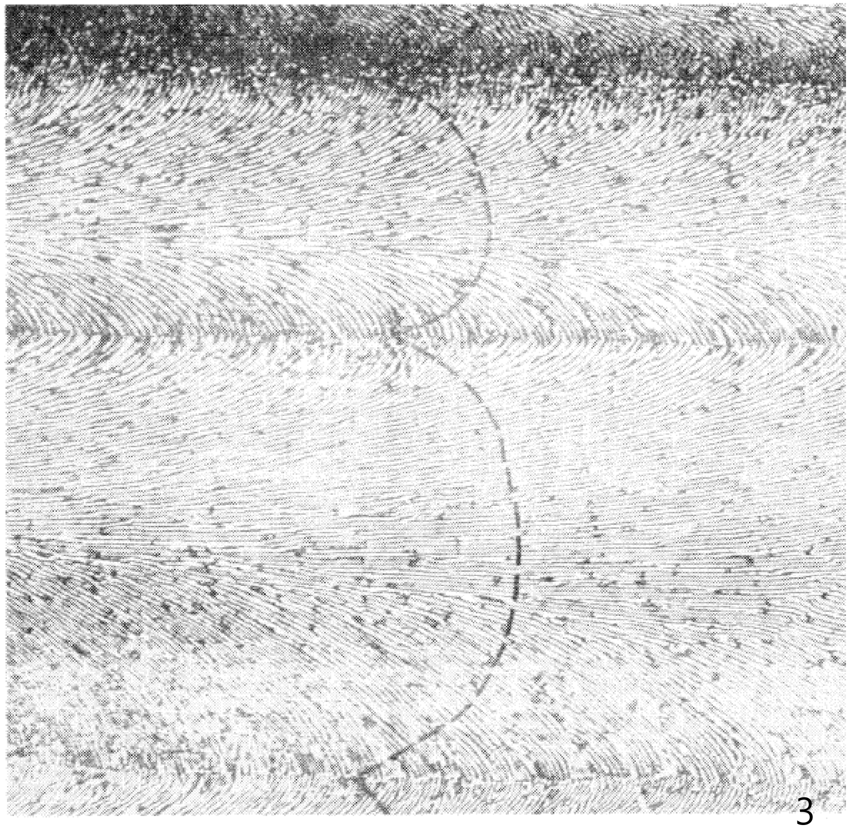


Fig. 6.21. Longitudinal section of impure $CuAl_2$–Al eutectic alloy. Broken line indicates shape of interface during growth.

(b) Rod structure

: Impurity has <u>sufficiently different distribution coefficients</u> for the two solid phases

*** <u>When the two distribution coefficient are very different</u>, the lamellae of one phase should grow into the liquid ahead of the other, and the lamellae of the lagging phase then break up into very small cells, separated by the other phase.**

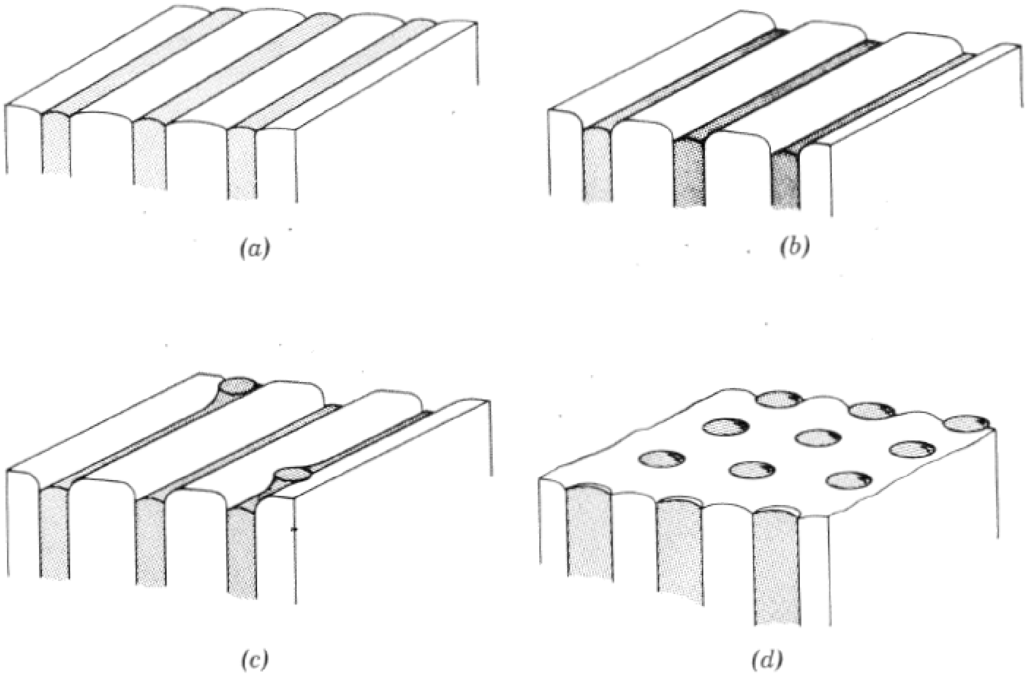


Fig. 6.22. Origin of “rod-type” eutectic structure (schematic).

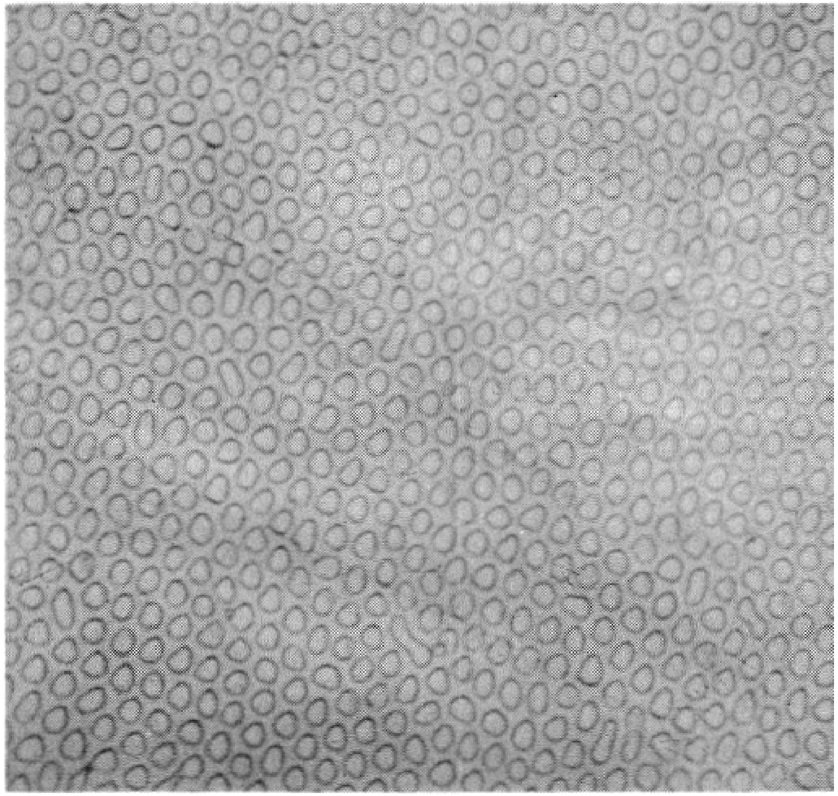


Fig. 6.23. Cross section of “rod-type” eutectic structure.

(C) Intermediate structure: Middle= lamellar structure/ edge = rod-type colony

: This is caused by an <u>impurity</u> which when present at a low concentration, has nearly equal distribution coefficient for the two solid phases, but which has <u>increasingly differing distribution coefficients as its concentration increases</u>.

*** Middle part of Cell**

: relatively low concentration of impurity & similar distribution coefficients

→ Lamellar structure

*** Edge of cell (near wall)**

: relatively high concentration of impurity & increasing differing distribution coefficients

→ Rod-type structure

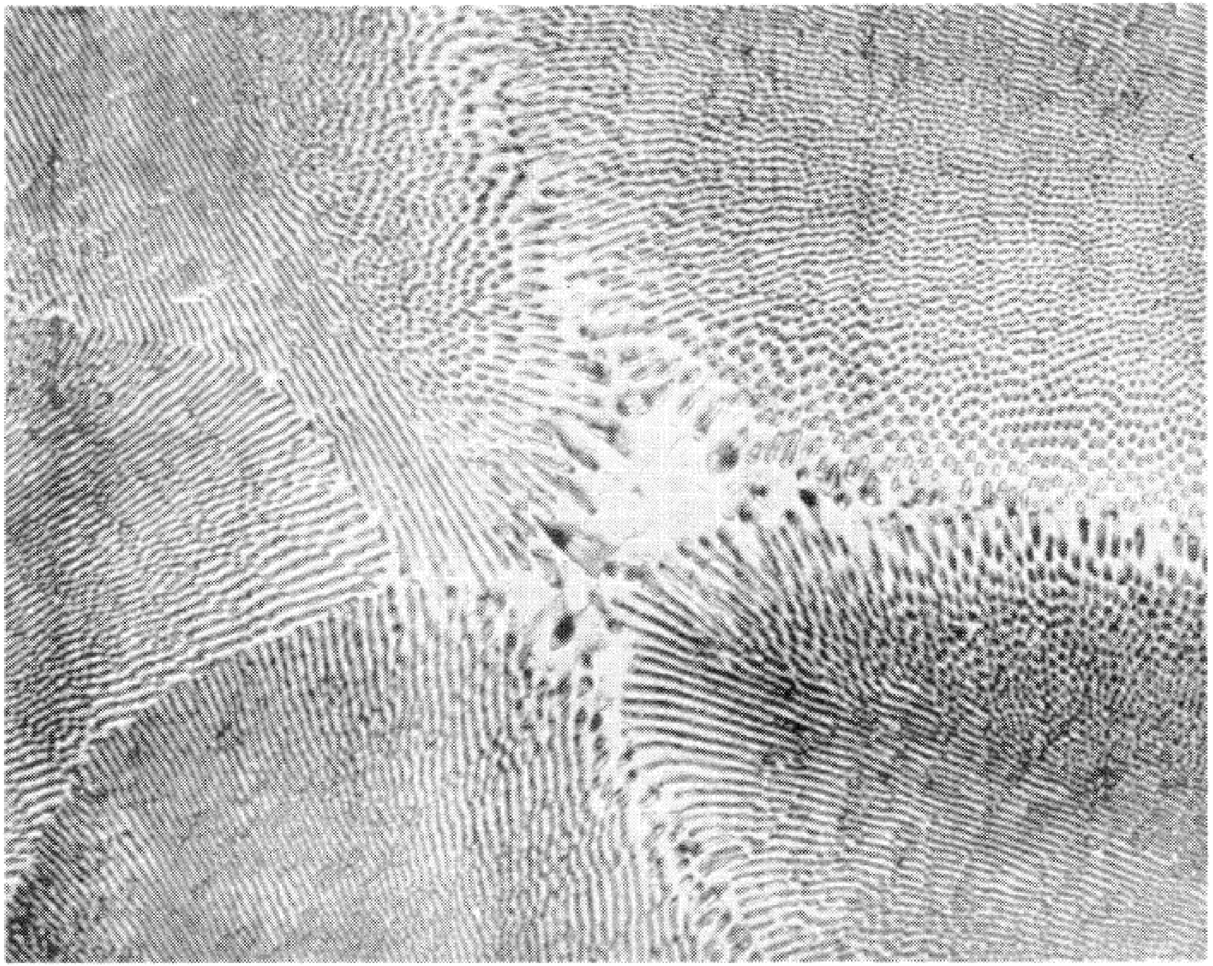


Fig. 6.24. "Mixed lamellar and rod structure" (Pb-Cd eutectic alloy with 0.1% Sn)

(d) Discontinuous eutectic structure

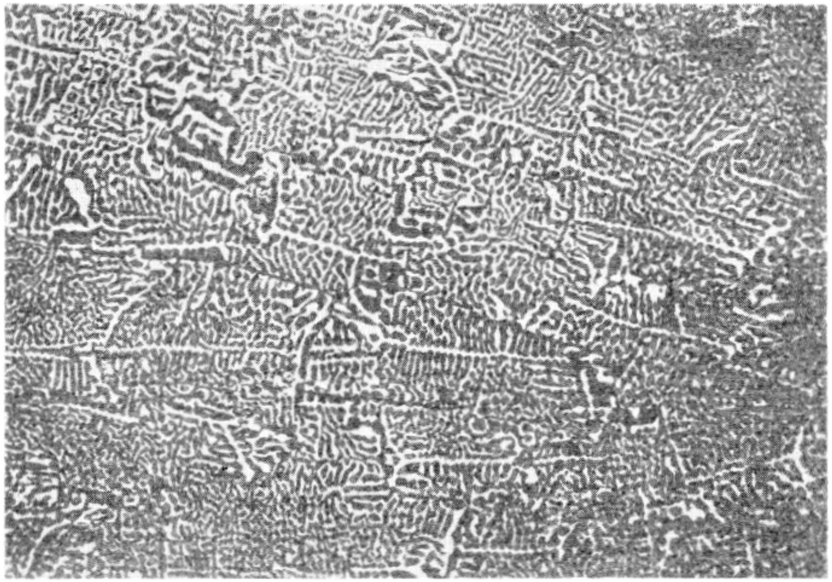
In lamellar type & degenerate form, each phase grows continuously
→ does not required repeated nucleation.

"Discontinuous eutectic" : required renucleate repeatedly due to "strong anisotropy" of growth characteristics of one of the phases

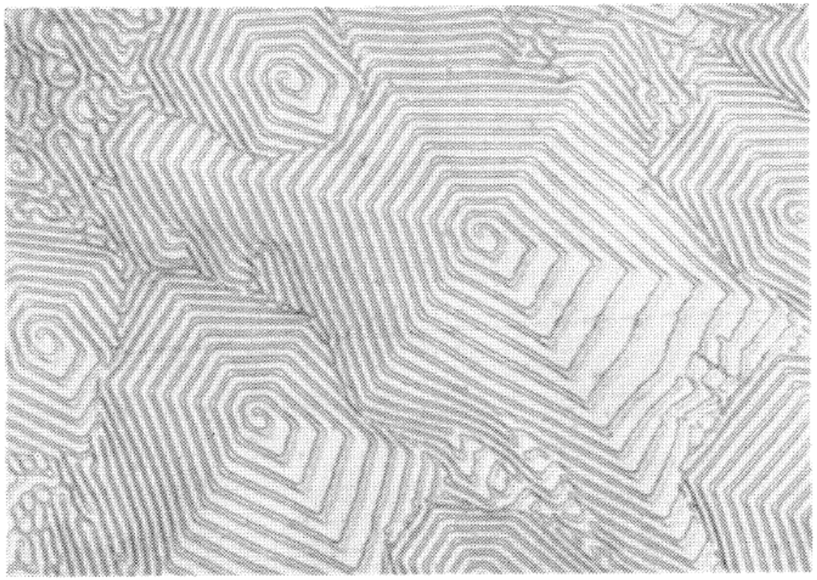
a) Case I: both phases renucleate repeatedly due to the termination of growth of crystals

b) Case II: "Spiral type의 discontinuous eutectic"

: one or both of the phases → anisotropic in growth rate



a) Fig. 6.25. "Chinese script" structure in Bi-Sn eutectic alloy



b) Fig. 6.28. Spiral eutectic structure in Zn-Mg alloy.

10) Divorced eutectic

- The primary phase continues to solidify past the eutectic point (along the line EA) of Fig. 6.31 until either the whole of the liquid has solidified or the other phase nucleated and forms a layer, which is some times dendritic, separating the two layers of the primary phase.

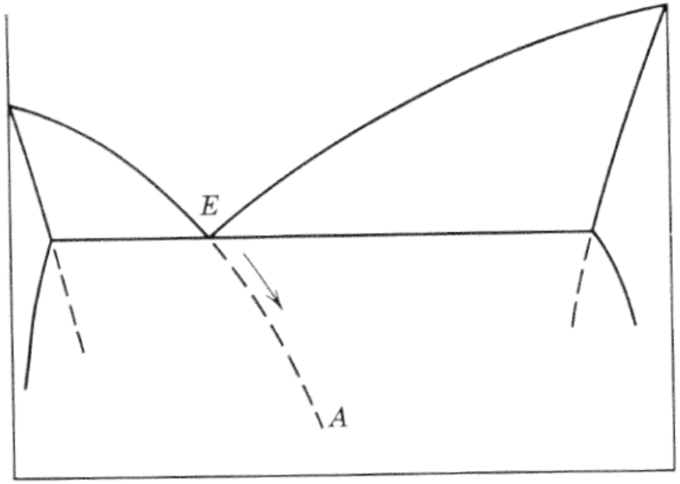
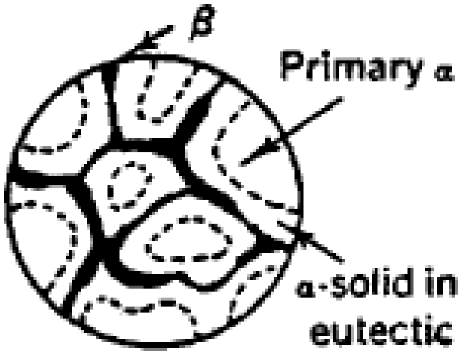


Fig. 6.31. Supercooling of eutectic in the absence of the second phase.

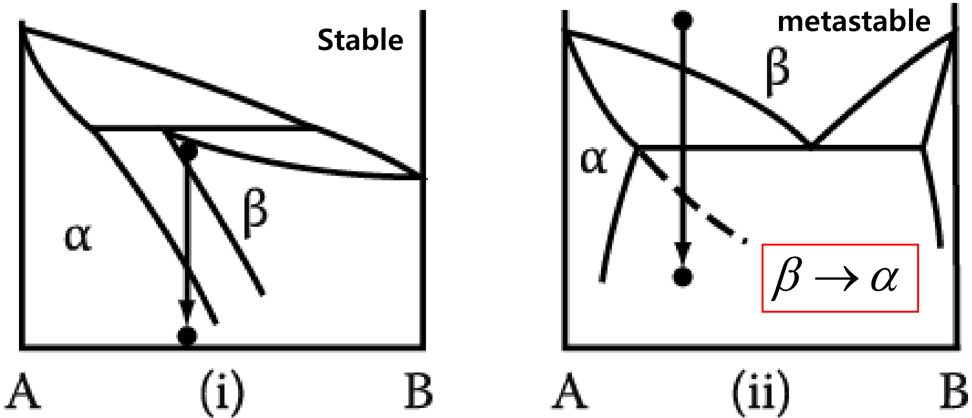
- One of the phases requires considerable supercooling for nucleation.



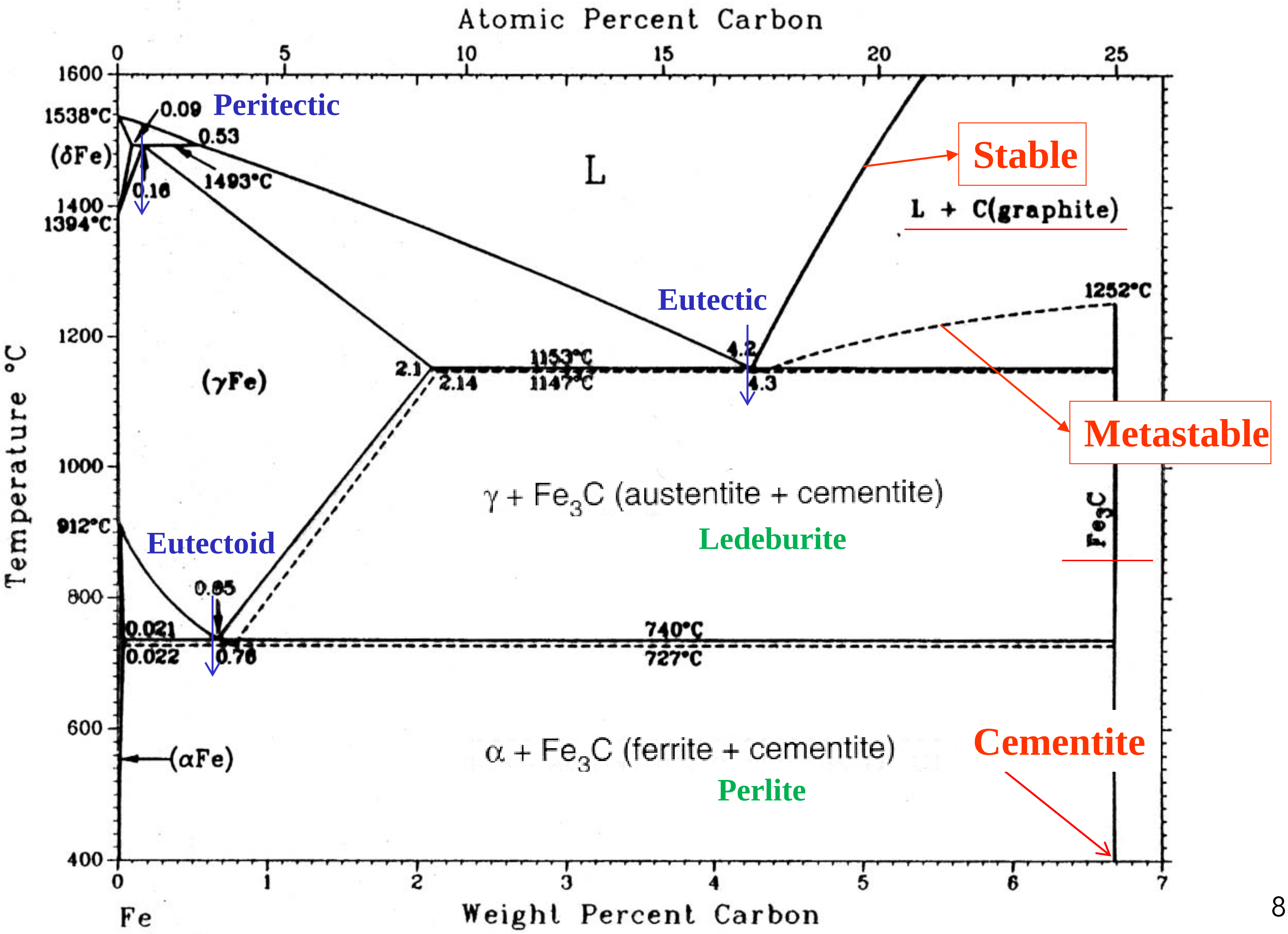
- "Divorced eutectic" is used to denote eutectic structures in which one phase is <u>either absent or present in massive form.</u>

- **Massive Transformation**

: The original phase decomposes into one or more new phases which have the same composition as the parent phase, but different crystal structures.



12) Cast Iron: Fe-C alloy (1.7 ≦ c ≦ 4.5%)



* Fe-Fe$_3$C eutectic temp $\overset{6^\circ C}{<}$ Fe-graphite eutectic temp.

* If solidification proceeds at interface temperature above the cementite eutectic temperature, **Graphite eutectic formation**

→ Gray cast Iron

* If the solidification proceed **below Cementite eutectic temperature** due to lower the liquidus temperature through fast quenching and a suitable nucleation agent to form an over-solute layer,

→ White cast Iron

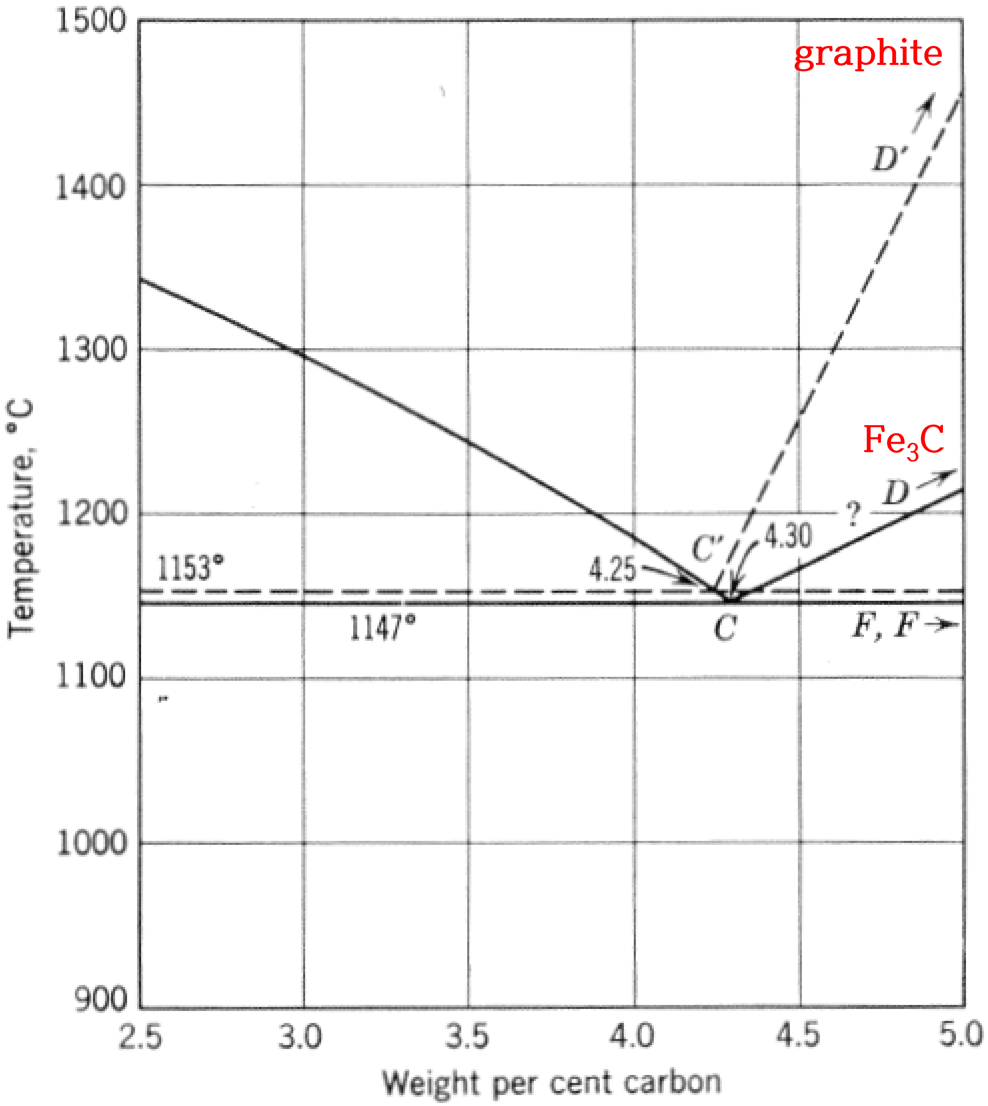
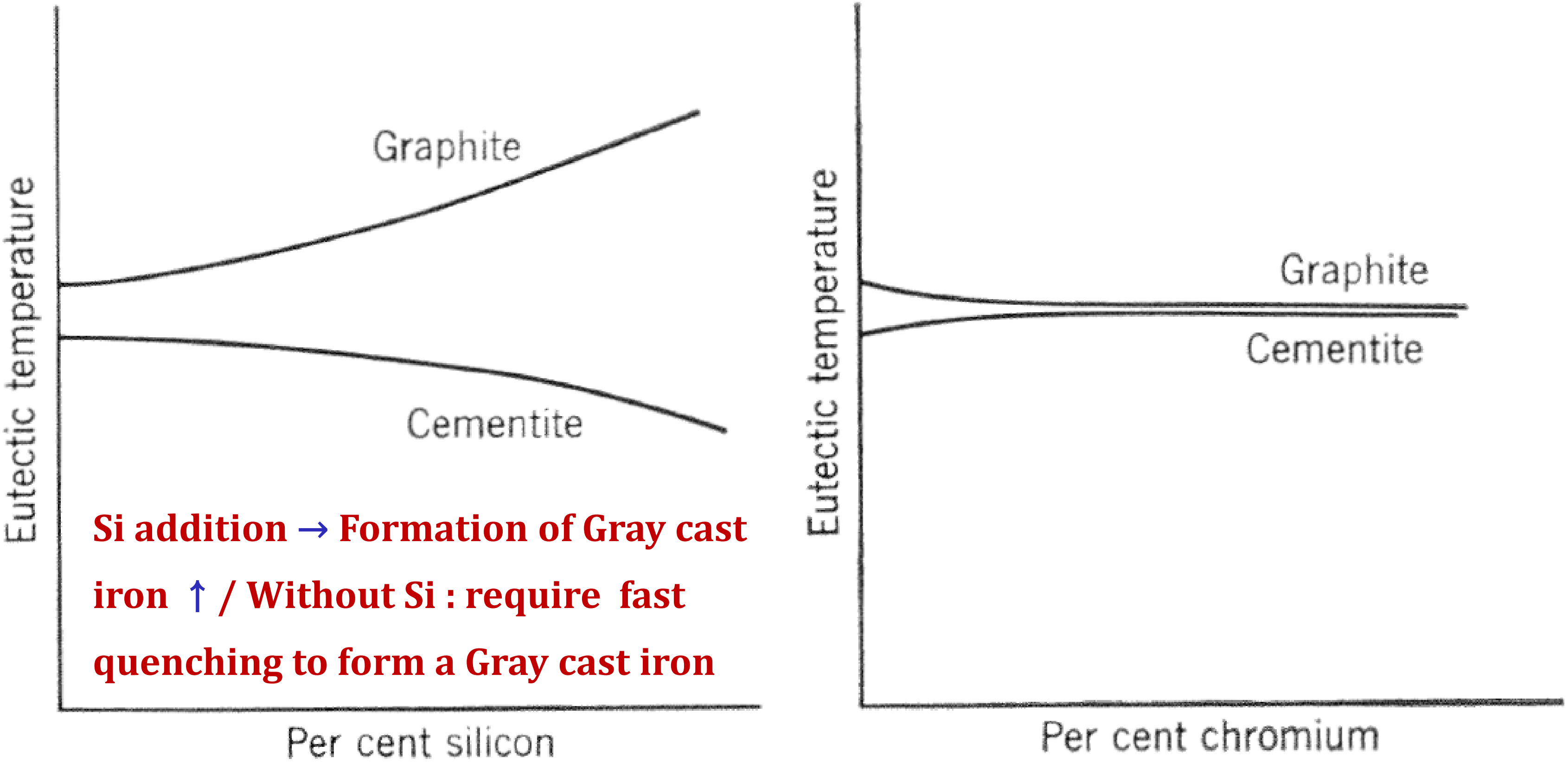


Fig. 6.35. Eutectic region of the iron carbon system.

* Addition effects of other elements

① Si → $T_E^{graphite}$ ↑ / $T_E^{Cementite}$ ↓

② Cr: decreasing the temperature rage where graphite is formed



Si addition → Formation of Gray cast iron ↑ / Without Si : require fast quenching to form a Gray cast iron

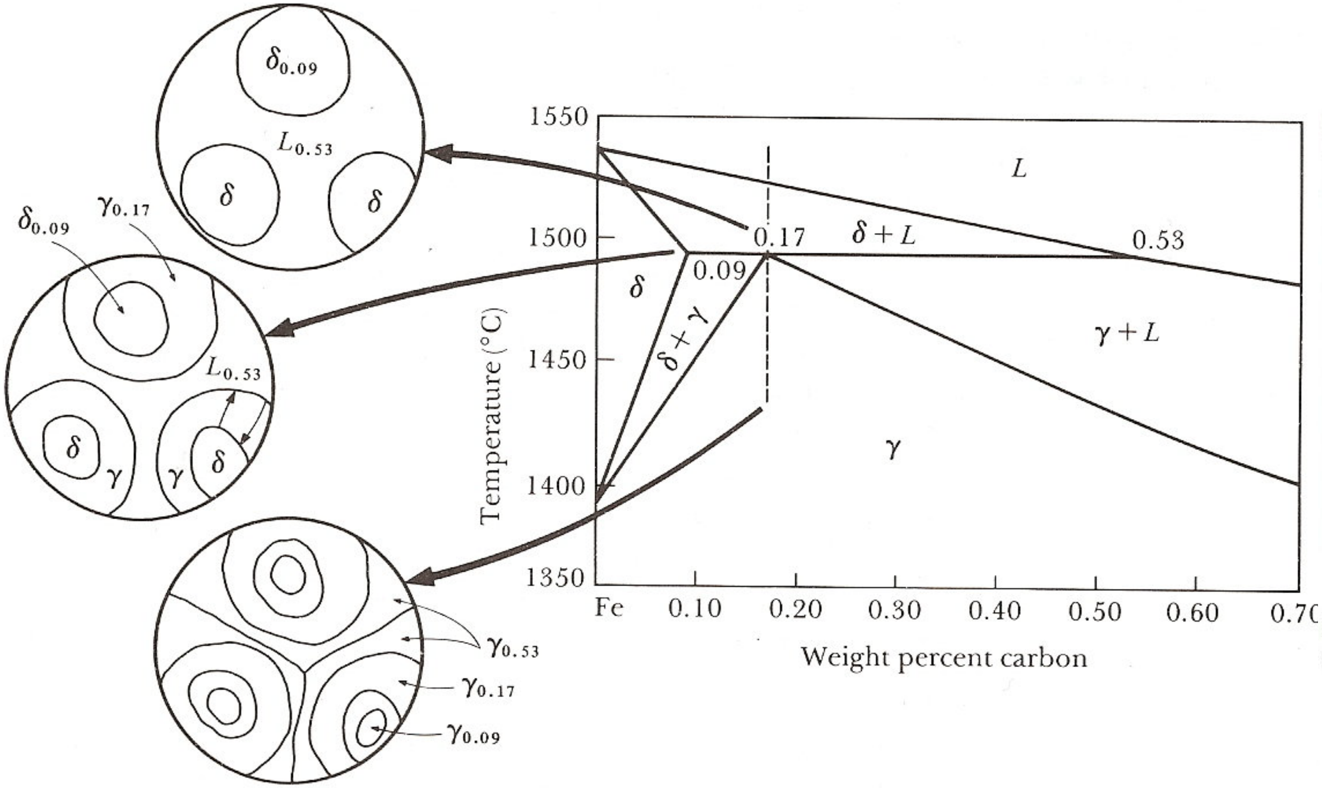
Fig. 6.36. Effect of third component on the eutectic temperatures (schematic). (a) Silicon type, (b) chromium type.

13) Peritectic Solidification

: Occurs when two liquidus lines intersect with a slope of the same direction

*** Solidification and microstructure that develop as a result of the peritectic reaction**

→ Unlike eutectic, <u>peritectic does not grow into lamellar structure</u>.



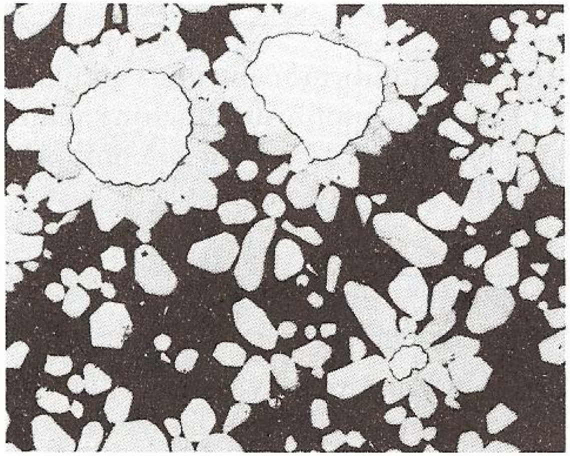
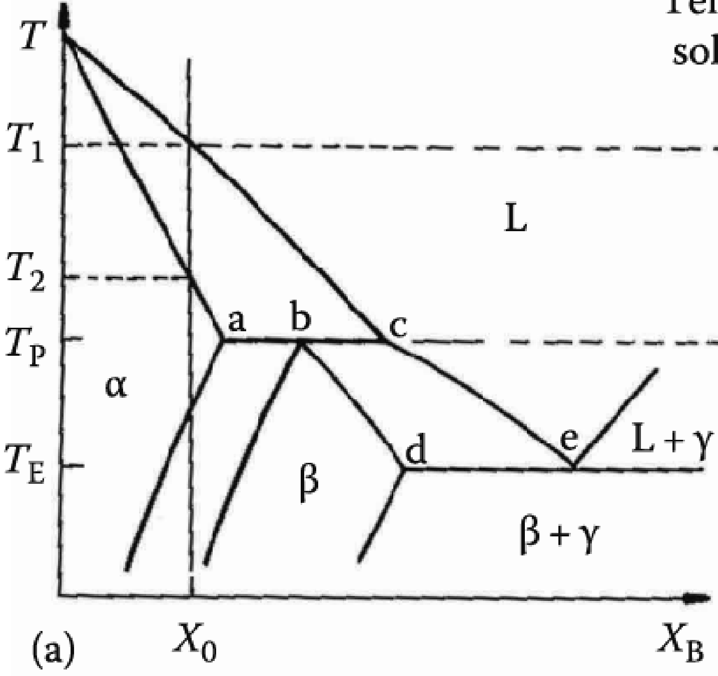
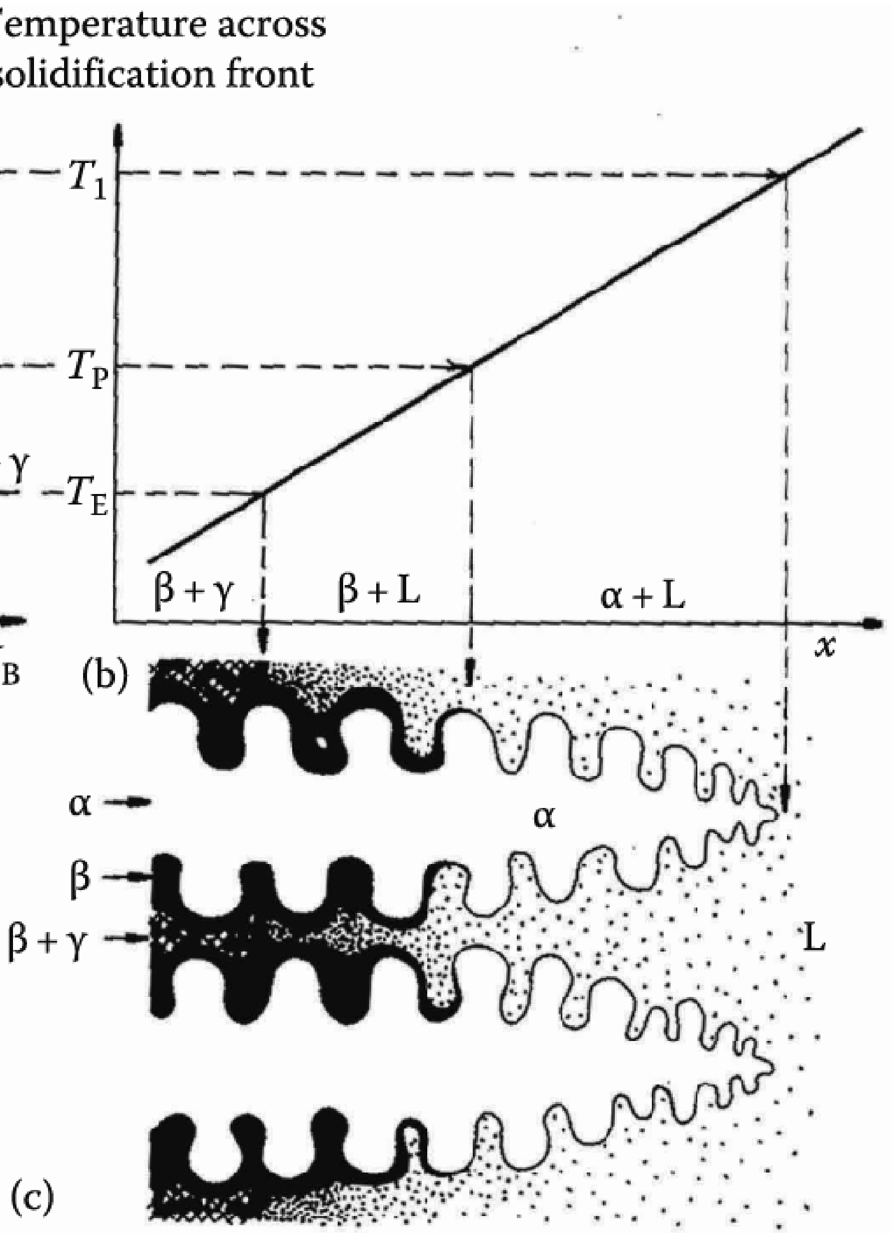


FIGURE 10-24 The peritectic reaction in a Cd-10% Cu alloy begins when rounded

*** L + α → β is <u>a very slow reaction</u> except for the initial state, because liquid and α are separated by β**




Temperature across solidification front

- **L + α → β , <u>difficult to complete</u>.**

- **α dendrites first form at T_1;**
  Liquid reaches the composition 'c';
  β forms as the result of the peritectic reaction;
  α coring is isolated from further reaction
  finally β + γ eutectic forms.

6.4. Solidification in the presence of a solid phase

- If liquid metals contain particles of solid in suspension; their distribution in the resulting solid influence dislocation content (page 58) or directly the mechanical properties. → relevant to consider the interaction btw an advancing S-L interface and solid particles in the liquid.

- Three factors that may influence the final location of a particle

(1) If "density" of particle is different from that of liquid: particle ~ float or sink

- Particle behavior dominated by its buoyancy (positive or negative)

: depends on density difference and the size and shape of the particle

(2) Second factor = "Fluid motion"_ generated as the liquid enters the mold

large enough to maintain in suspension particles that would sink or float in a stationary liquid

: persist for a considerable time before it gives way to convection caused by thermal and composition gradient.

(3) Third factor = "Interface speed" : Although there may be some vertical separation due to flotation or sedimentation, and some radial separation resulting from centrifugal forces, the smaller particles may remain suspended with a nearly random distribution.

→ ∴ The final distribution in the solid depends on whether a particle is "trapped" in situ by the advancing S-L interface or whether it is "pushed ahead" as the interface moves forward.

* Solidification of a liquid in a porous solid

: Little attention has been paid to the solidification of a liquid metal that is contained in interconnected channels in a porous solid that is chemically inert to the solidifying liquid.

(ex) Nonmetallic system: **Freezing of water in Soil → Induce "frost heaving load"**

Causes serious damage to highways and other structure

- These forces arise not because water expands on freezing, but because a water layer persists between ice and solid particles. As ice is formed, more water is drawn into the region of contact to replace what has frozen. This water in turn starts to freeze, causing more water to be "sucked" in, and forcing the existing ice away from the soil particle.

→ **Preference, energetically, for the existence of a liquid layer btw the two solids**

→ *A liquid metal contained in a porous matrix may have a similar surface E relationship*, in which case very large forces could be exerted, tending to disrupt the matrix.

7. Macroscopic Heat Flow and Fluid Flow

7.2. Fluid Flow

* The ability of a molten metal to flow =

(1) poured from a container in which it was melted into a mold in which it is to solidify.

: effect of the macroscopic geometry of the casting (Chapter 7)

(2) Relative motion of different parts of the liquid can occur while it is solidifying.

: its implications in relation to the structure of the solidified metal (Chapter 8)

1) Viscosity of liquid metal

liquid metal : Flow rate depends on the force = shear rate is proportional to the shear stress

ex) Flow rate of a liquid through a tube depends on the pressure difference btw the ends of the tube (ΔP), on its length (l) and on the radius of the tube (r).

The quantity flowing per unit time, Q

$$\boxed{Q = \frac{\pi r^4}{8\mu} \cdot \frac{P_1 - P_2}{l}}$$

μ = viscosity

→ The formula given above applies only in cases in which the flow is of the "stream-line" or laminar type, which occurs at relatively slow rates of flow.

Fragility

- **Fragility** ~ ability of the liquid to withstand changes in medium range order with temp.
  ~ extensively use to figure out liquid dynamics and glass properties
  corresponding to "frozen" liquid state

< Classification of glass >

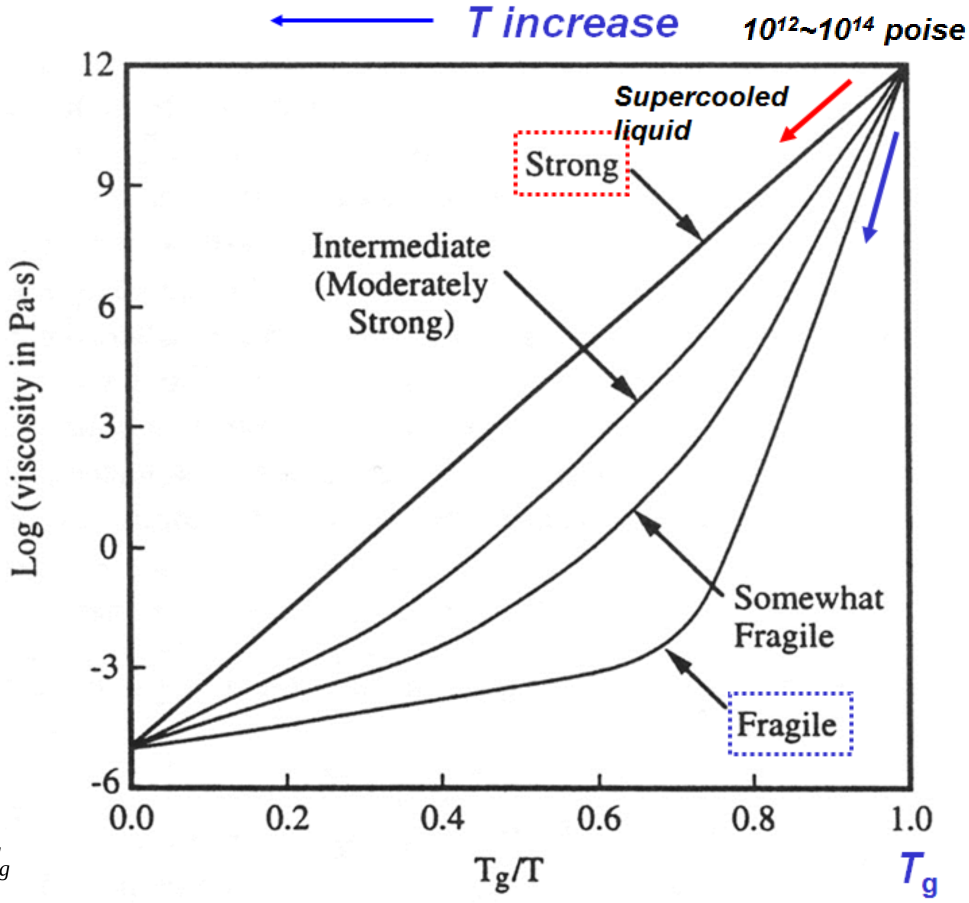
Strong network glass : Arrhenius behavior

$$\eta = \eta_0 \exp[\frac{E_a}{RT}]$$

↕

Fragile network glass : Vogel-Fulcher relation

$$\eta = \eta_0 \exp[\frac{B}{T - T_0}]$$



< Quantification of Fragility >

$$m = \left.\frac{d \log \eta(T)}{d(T_{g,n}/T)}\right|_{T=T_{g,n}} = \left.\frac{d \log \tau(T)}{d(T_g/T)}\right|_{T=T_g}$$

Slope of the logarithm of viscosity, η (or structural relaxation time, τ) at T_g

Mold Filling

Bernouli's Equation:

$$p/w + Z + q^2/2g = \text{constant}$$

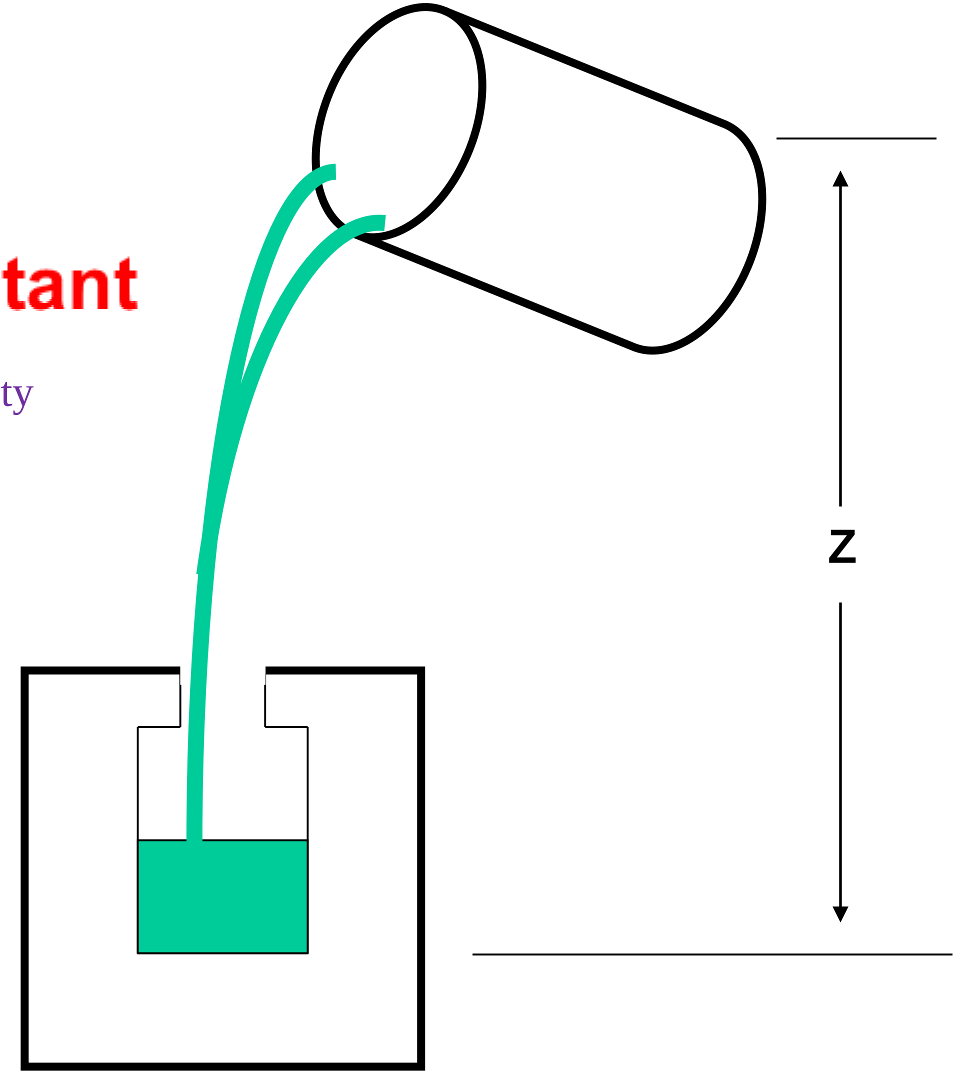
(p= pressure w = specific weight q = velocity
g = gravity z = elevation)

Reynold's Number:

$$\mathrm{Re} = \gamma v l / \mu$$

γ = density , v = velocity,
μ = viscosity, l = linear dimension

- **Short filling times**
- **Potential Turbulence**



*Bernoulli theorem: Applicable for dynamic behavior of fluid_Fluid Mechanics

By assuming that fluid motion is governed only by pressure and gravity forces, applying Newton's second law, *F* = *ma*, leads us to the Bernoulli Equation.

For a flowing liquid,

$$p/w + Z + q^2/2g = \text{constant} \text{ along a streamline}$$

The pressure due
to head of liquid

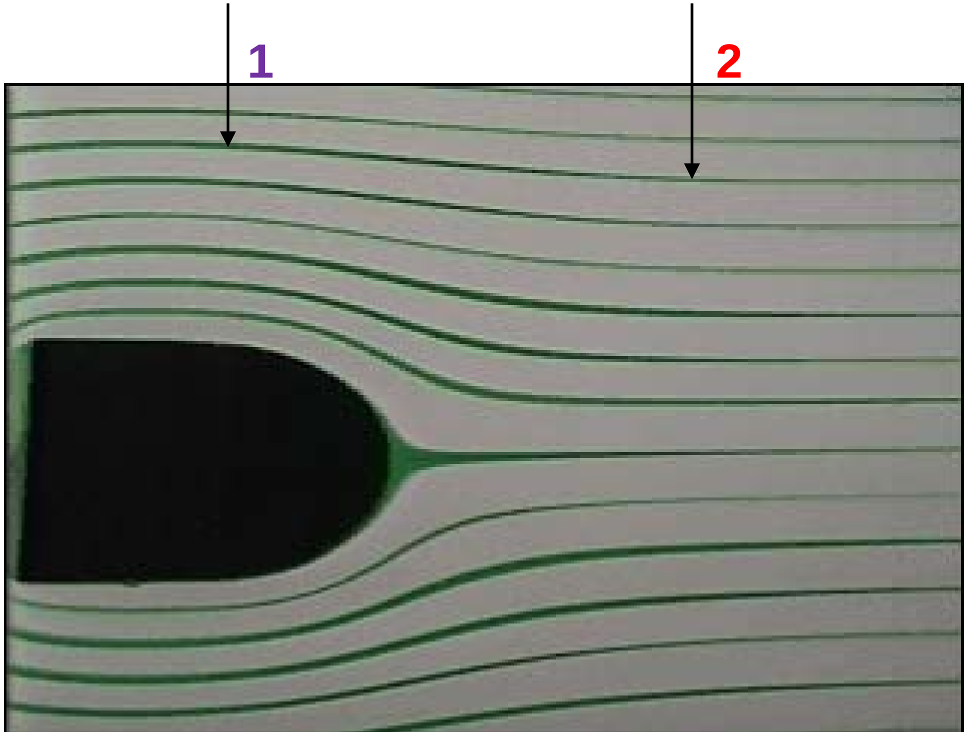
(*p*= pressure *w* = specific weight *q* = velocity *g* = gravity *z* = elevation)

In a steady flow, the sum of all forms of energy in a fluid along a streamline is same at all points on that streamline: "principle of conservation of energy"

A streamline is the path of one particle of water. Therefore, at any two points along a streamline, the Bernoulli equation can be applied and, using a set of engineering assumptions, unknown flows and pressures can easily be solved for.

(a) At any two points on a streamline:

$$p_1/w + Z_1 + q_1^2/2g = p_2/w + Z_2 + q_2^2/2g$$



(b) If the fluid velocity, q, of the liquid increases, the pressure of the liquid decreases due to the effect of the passing tube. → ∴ In the case of liquid metals flowing through a complicated mold, the pressure decreases due to the influence of air bubbles entering the liquid phase from the mold wall and flowing together. These air bubbles cause internal void formation in casting.

Mold Filling

Bernouli's Equation (incompressible flow):

$$p/w + Z + q^2/2g = \text{constant}$$

(p= pressure w = specific weight q = velocity
g = gravity z = elevation)

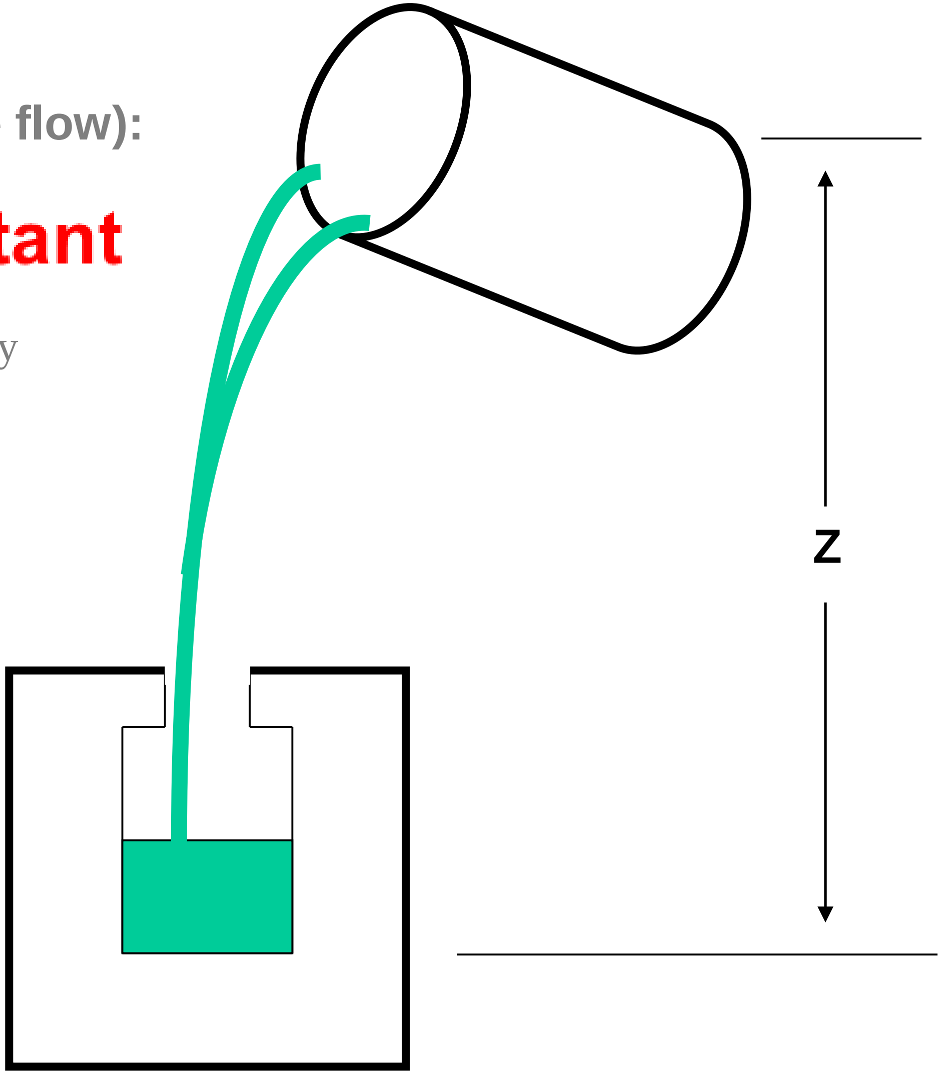
principle of conservation of energy

Reynold's Number:

$$\text{Re} = \gamma v l / \mu$$

γ = density , v = velocity,
μ = viscosity, l = linear dimension

- **Short filling times**
- **Potential Turbulence**



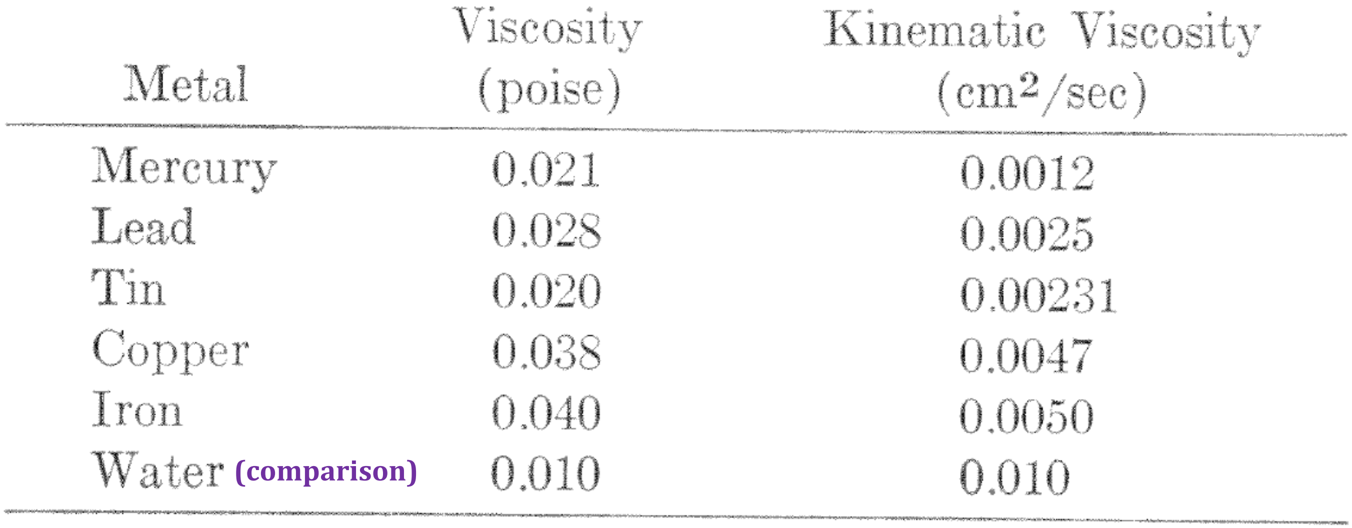
→ To compare "rates of flow" in this case,

Reynolds' number = $\gamma v l / \mu$

γ = density , v = velocity,
μ = viscosity, l = linear dimension

* If the value of Reynolds' number is high (>1400) for a tube leading out of a containing vessel, the flow becomes turbulent and Q drops below the value that would be calculated from the above formula. → Derive the Kinematic viscosity, μ/γ from the above equation : Used for calculation of flow rate when pressure difference is caused by flowing liquid → For solidification it is considered more important.

Table 7.1 Values of viscosity and kinematic viscosity of some liquid metals at T_m



Metal	Viscosity (poise)	Kinematic Viscosity (cm^2/sec)
Mercury	0.021	0.0012
Lead	0.028	0.0025
Tin	0.020	0.00231
Copper	0.038	0.0047
Iron	0.040	0.0050
Water (comparison)	0.010	0.010

→ Liquid metals, when they are completely liquid, flow rather more easily than water, and that their viscosity is seldom, if ever, a limiting factor in the process of filling a mold, even through a rather narrow channel.

* Fluidity: The ability of being fluid or free-flowing_distinguished from viscosity

Measurement of Fluidity

: Maximum length melt can reach

(a) Fluidity Spiral

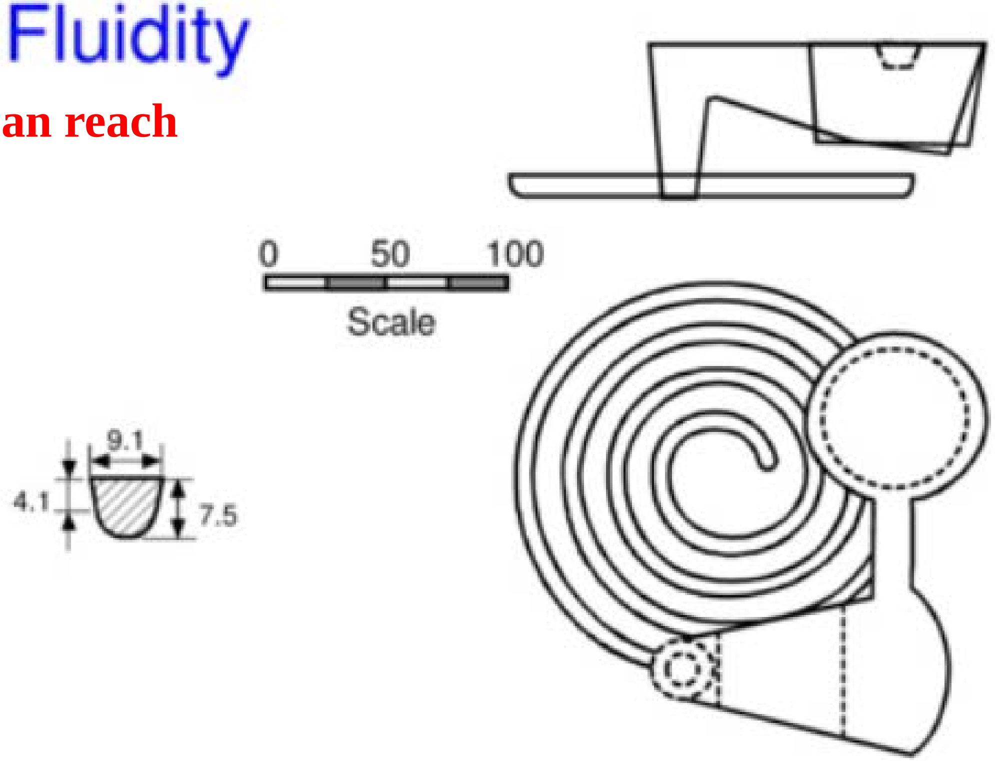
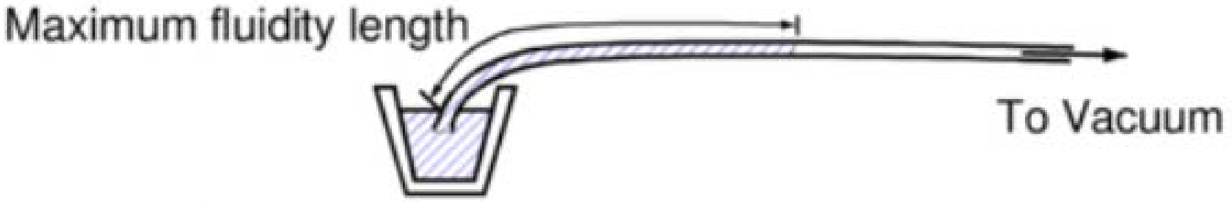
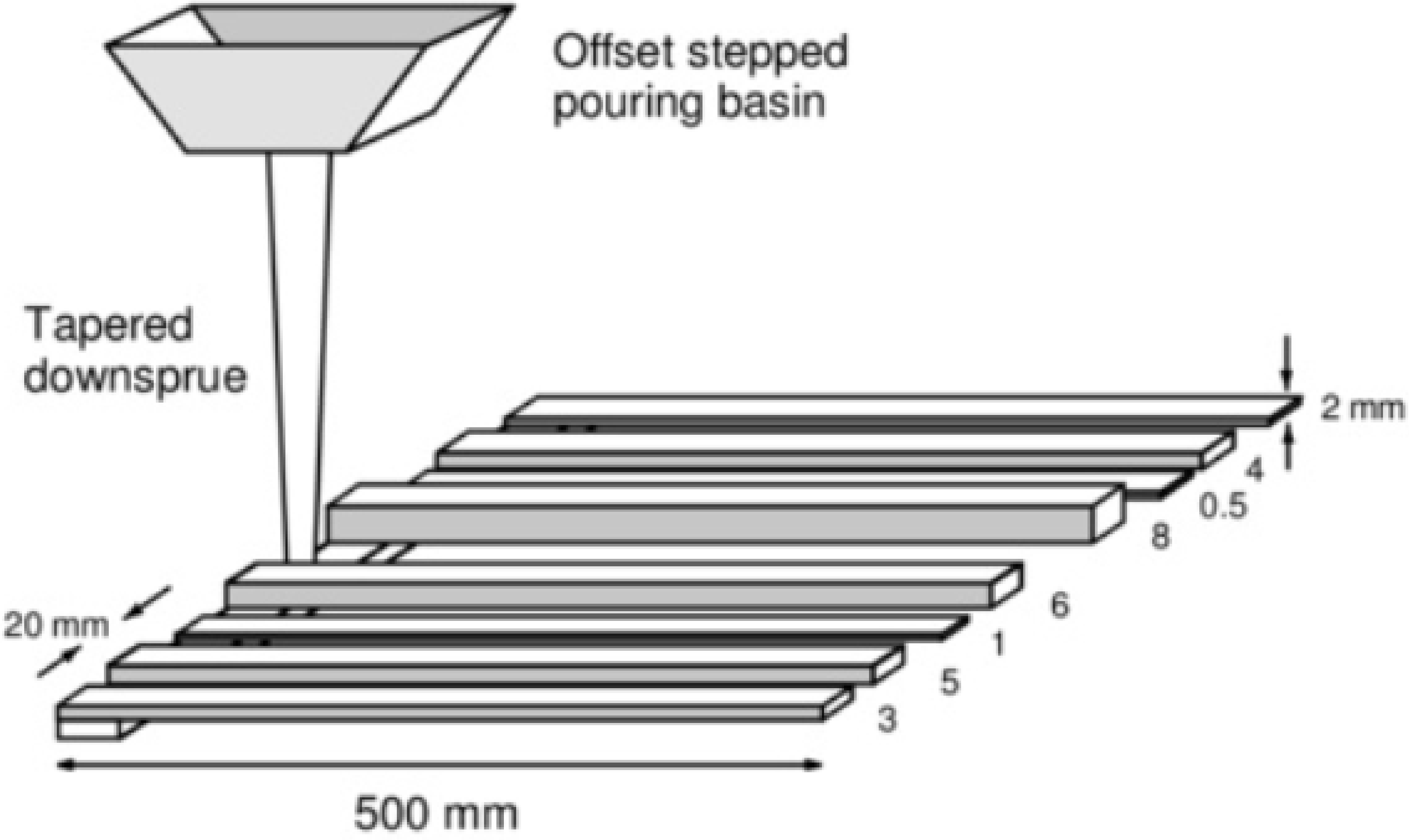


Fig. 7.1. Mold for fluidity test.

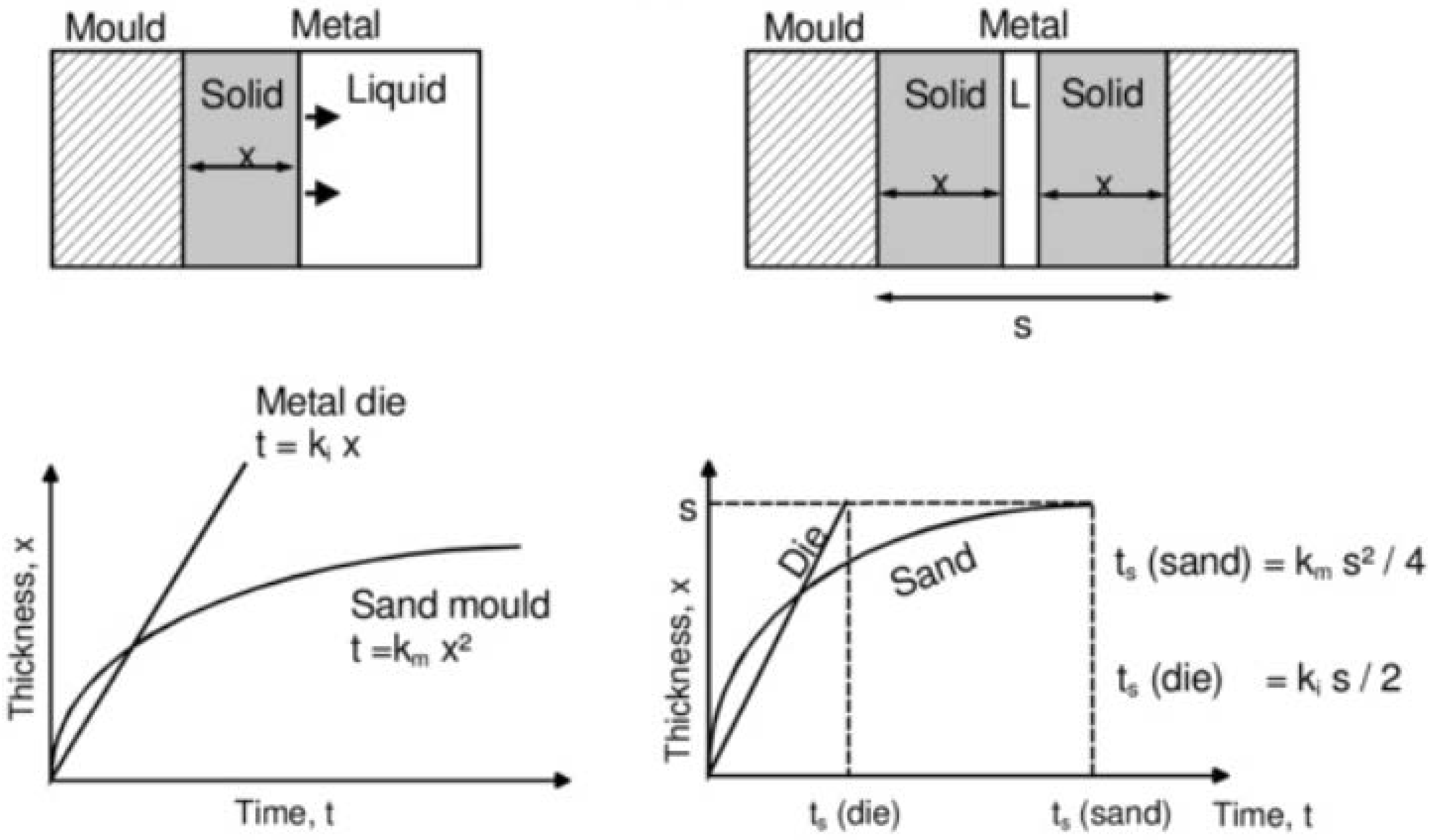
(b) Laboratory Test



New Design of Fluidity Test piece

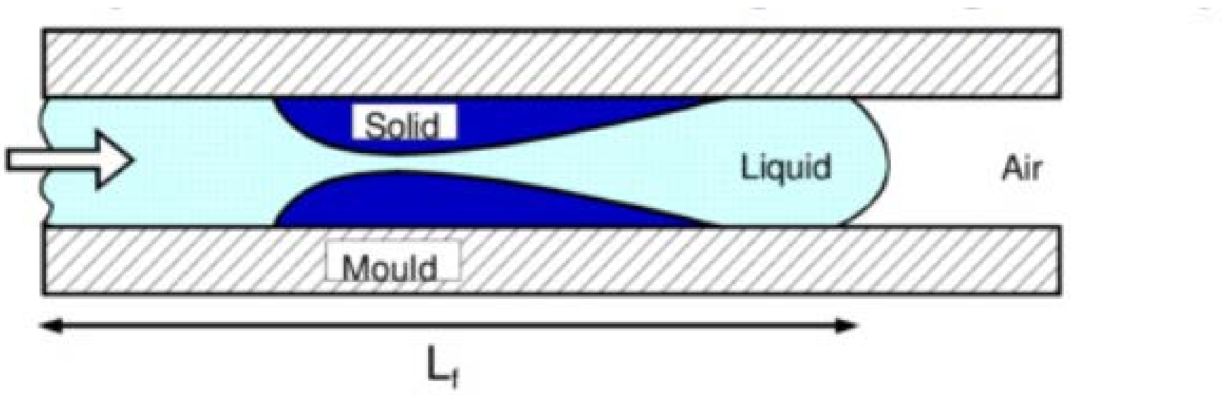


Solidification Rate



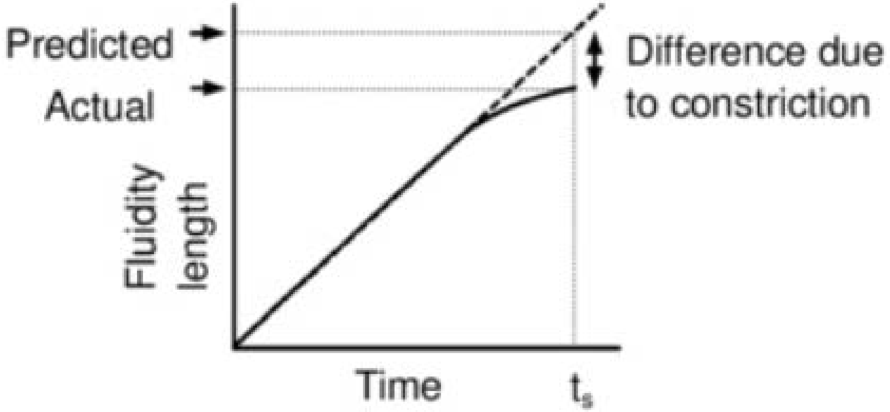
① Effect of composition

Fluidity of short Freezing Range Alloys

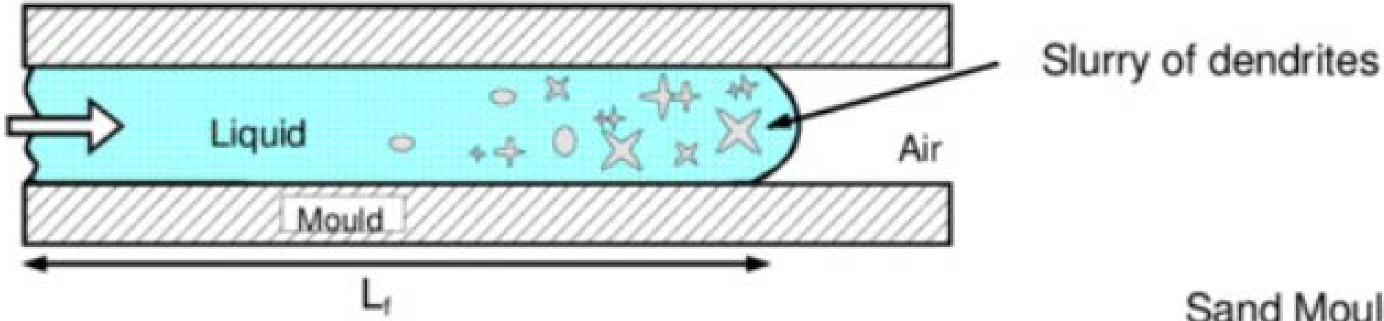


Fluidity Distance $L_f = V\,t_s$ where V = flow velocity
t_s = solidification time

	Sand Mould	Metal Die
t_s	$k_m\,s^2 / 4$	$k_i\,s / 2$
L_f	$V\,k_m\,s^2 / 4$	$V\,k_i\,s / 2$



Fluidity of Long Freezing Range Alloys

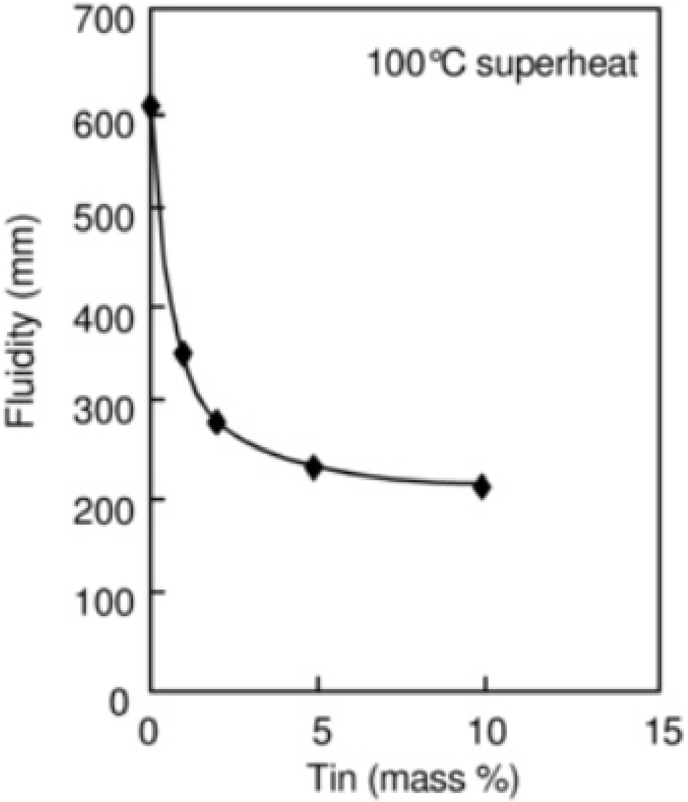


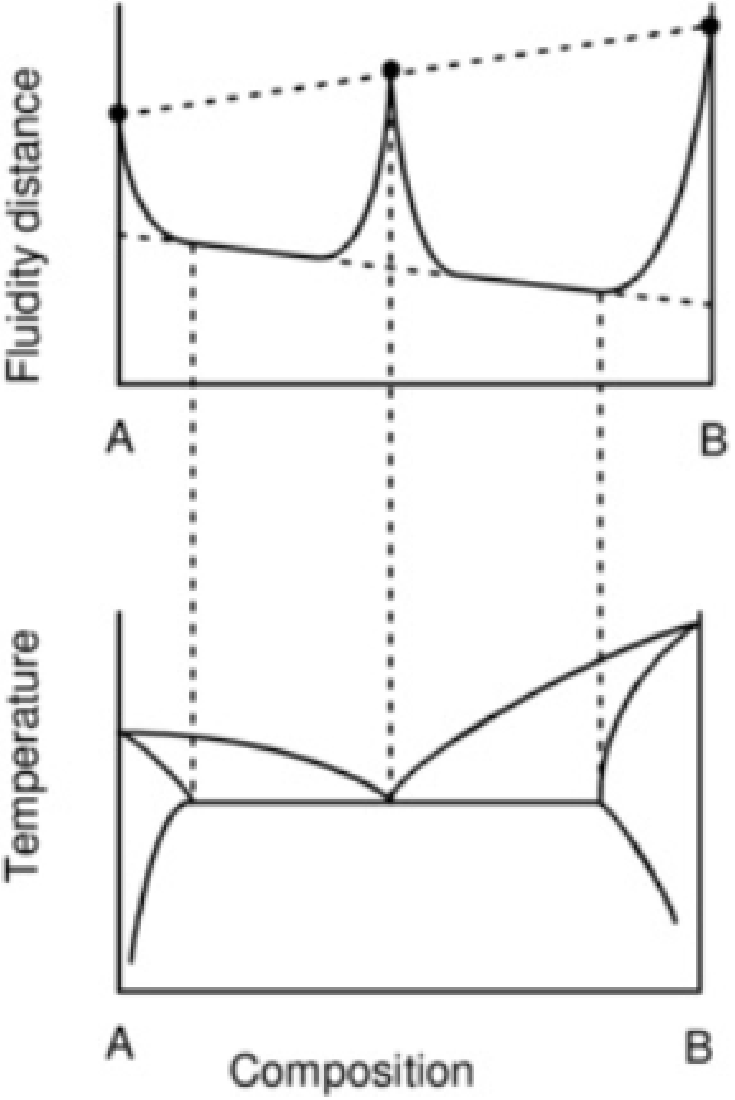
Flow stops when 25 - 50% solid is present,
i.e. when x = S/8 to S/4

	Sand Mould	Metal Die
	$t = k_m x^2$	$t = k_i x$
25% solid	$t = k_m S^2 / 64$	$t = k_i S / 8$
50% solid	$t = k_m S^2 / 16$	$t = k_i S / 4$
Therefore	$L_f = V k_m S^2 / 64$ to $V k_m S^2 / 16$	$V k_i S / 8$ to $V k_i S^2 / 4$
Remember that for short freezing range alloys:	$L_f = V k_m S^2 / 4$	$V k_i S / 2$
Therefore $\frac{\text{Short freezing range}}{\text{long freezing range}}$:	4 - 16	2 - 4

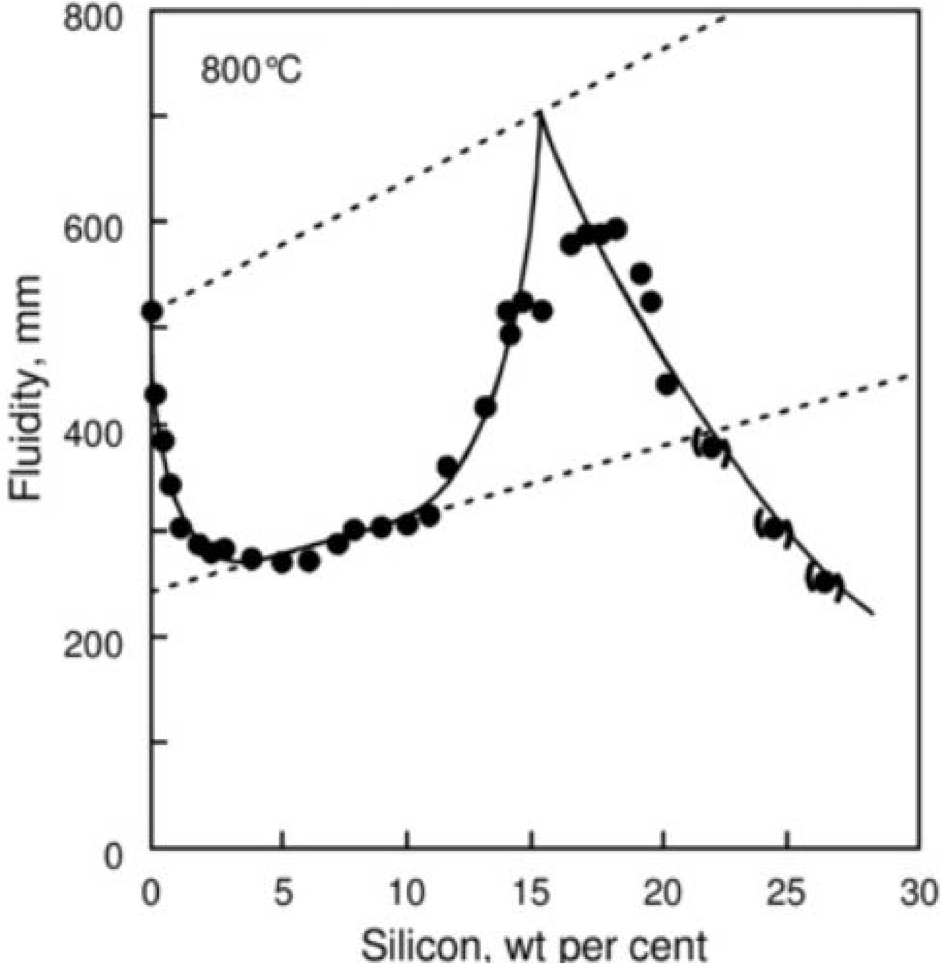
Mapping the Fluidity of Binary Alloys

Fluidity of Al-Sn alloys

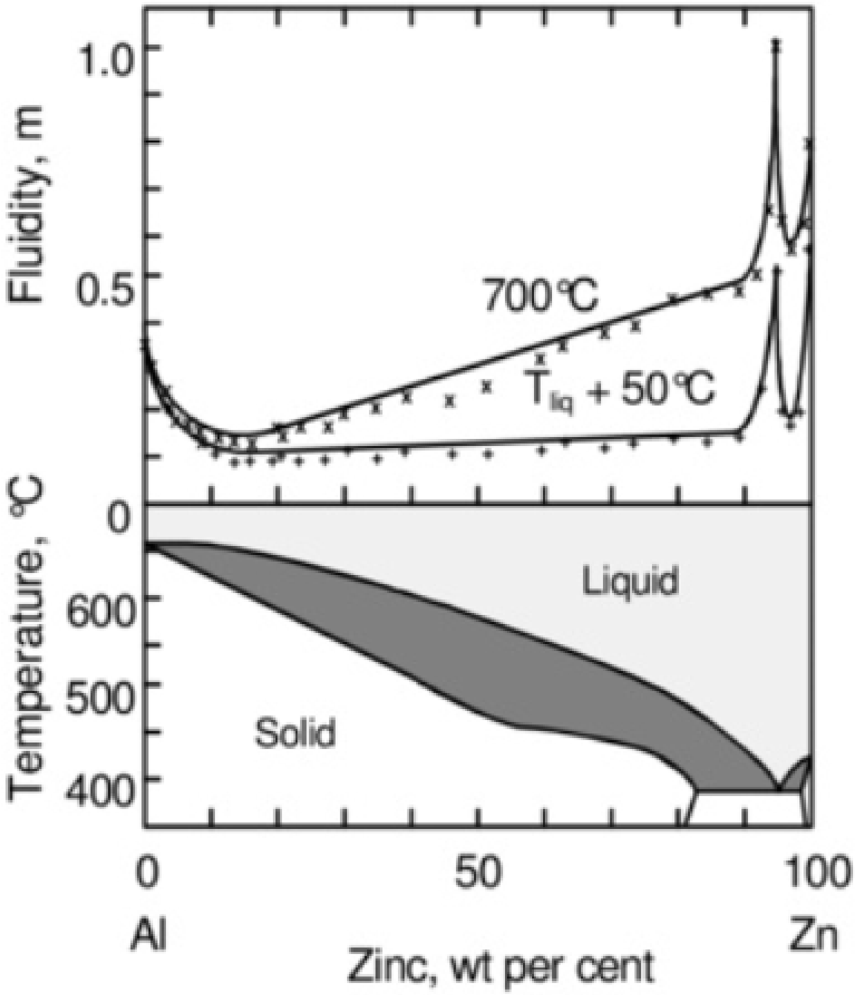




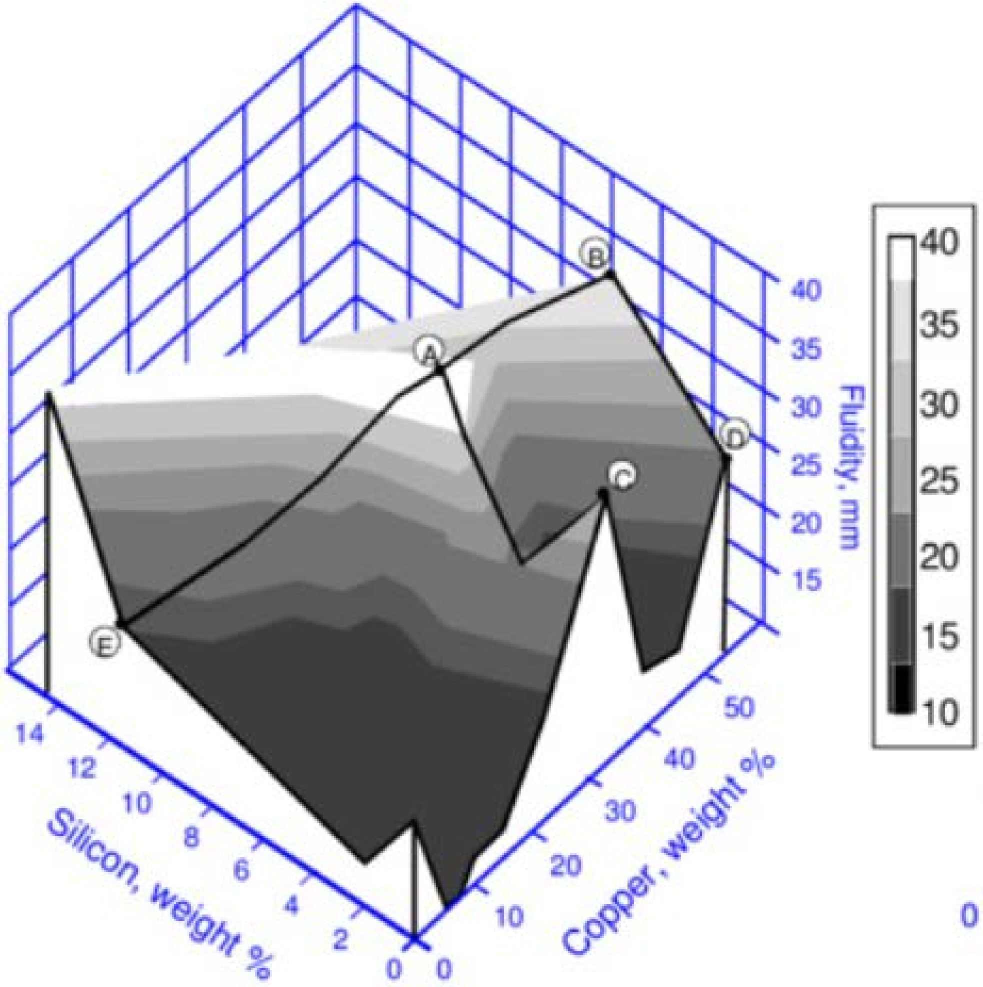
The Fluidity of Al-Si alloys



The Fluidity of Al-Zn alloys

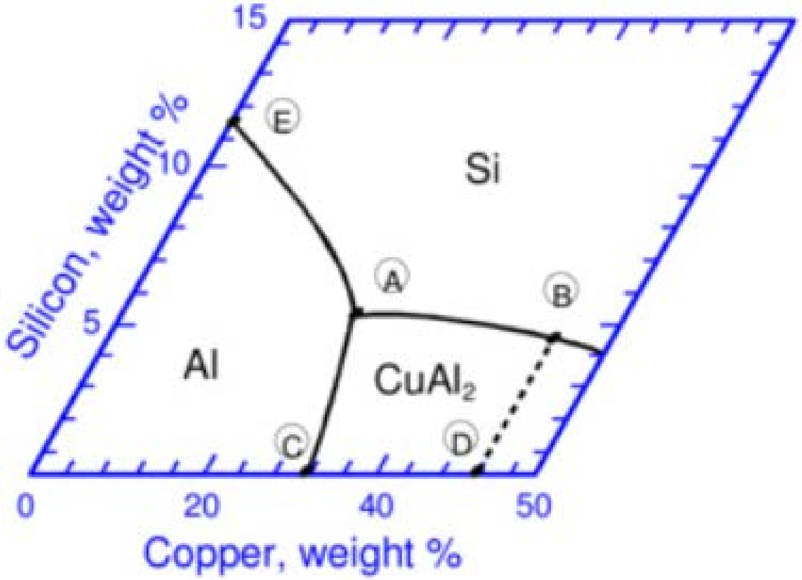


The Fluidity of Al-Cu-Si Alloys



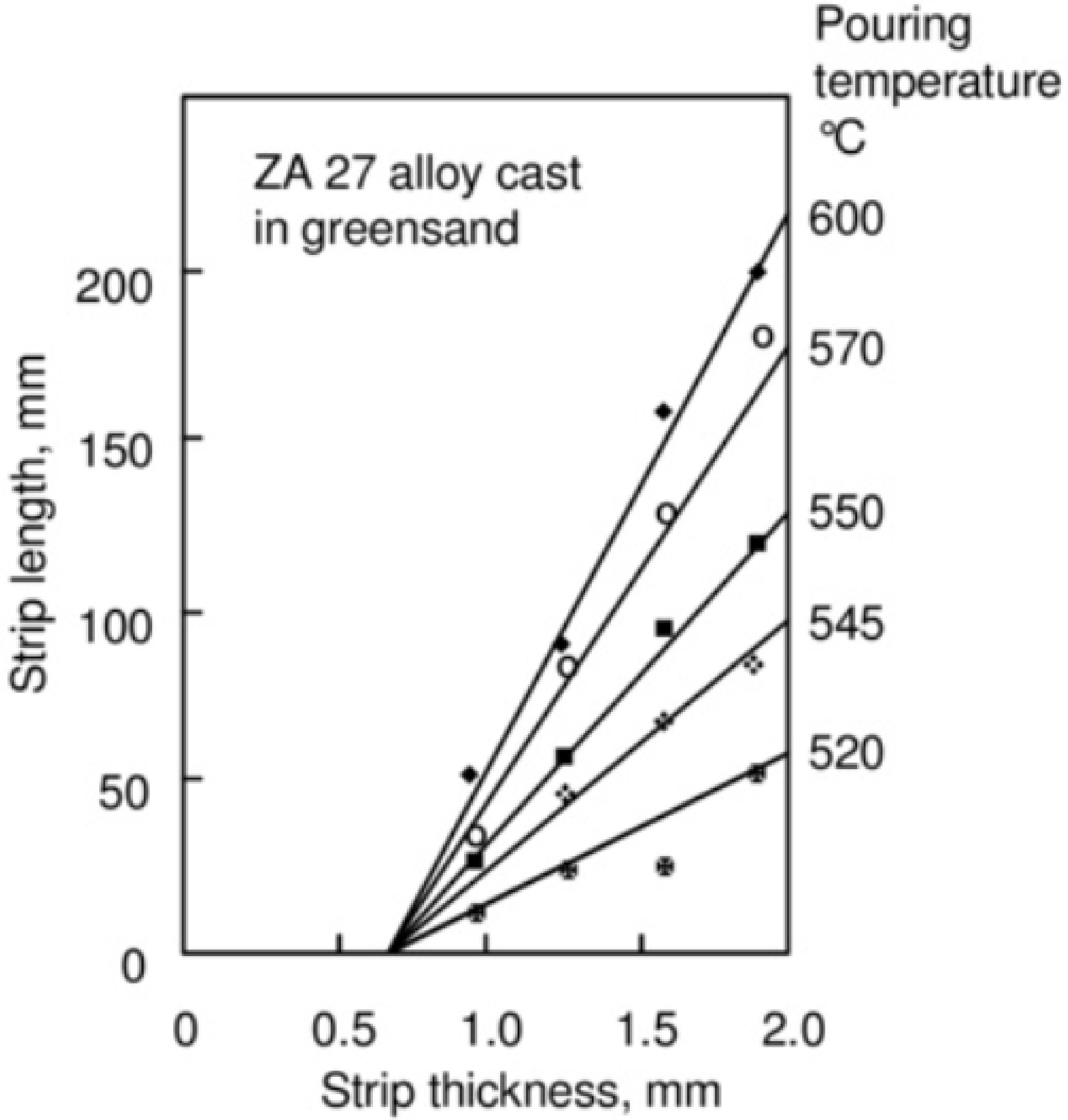
The liquidus surface of the Al-Cu-Si system

	Cu	Si	T°C	
Ⓐ	27.5	5.25	524	Ternary eutectic
Ⓑ	51	4.5	571	Ternary eutectic
Ⓒ	33	-	548	Binary eutectic
Ⓓ	51	-	591	$CuAl_2$
Ⓔ	-	11.7	577	Binary eutectic

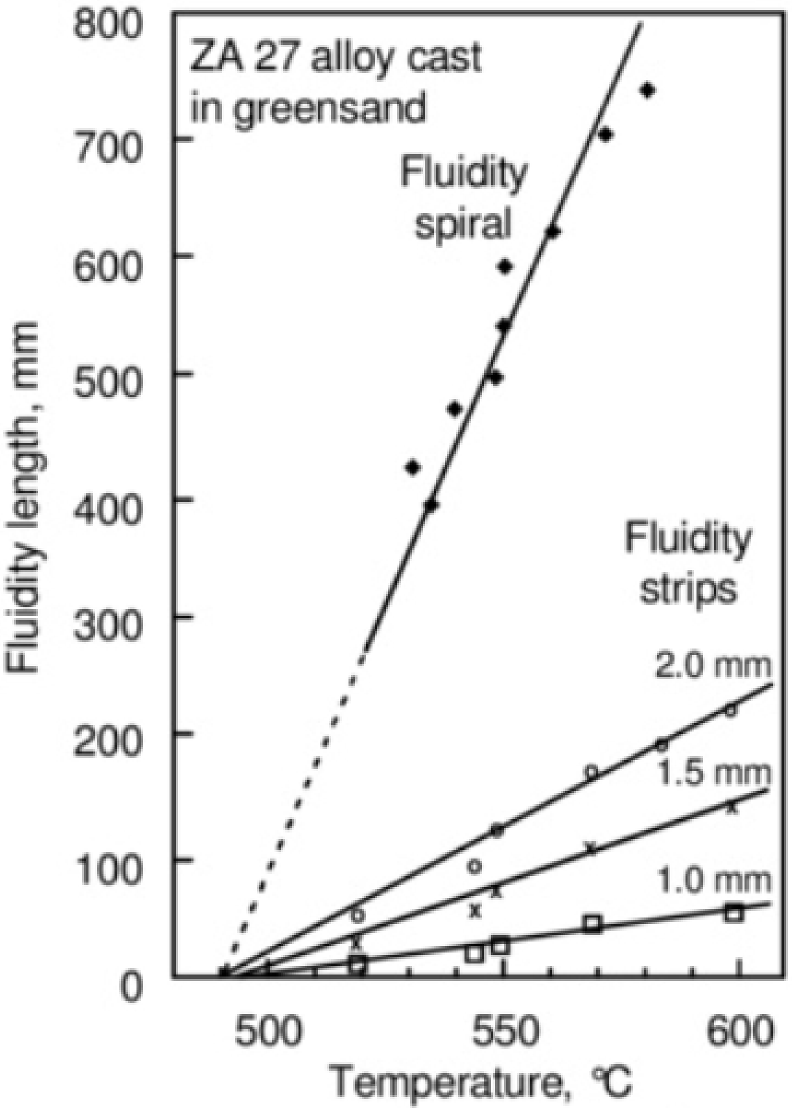


② **Effect of temperature**

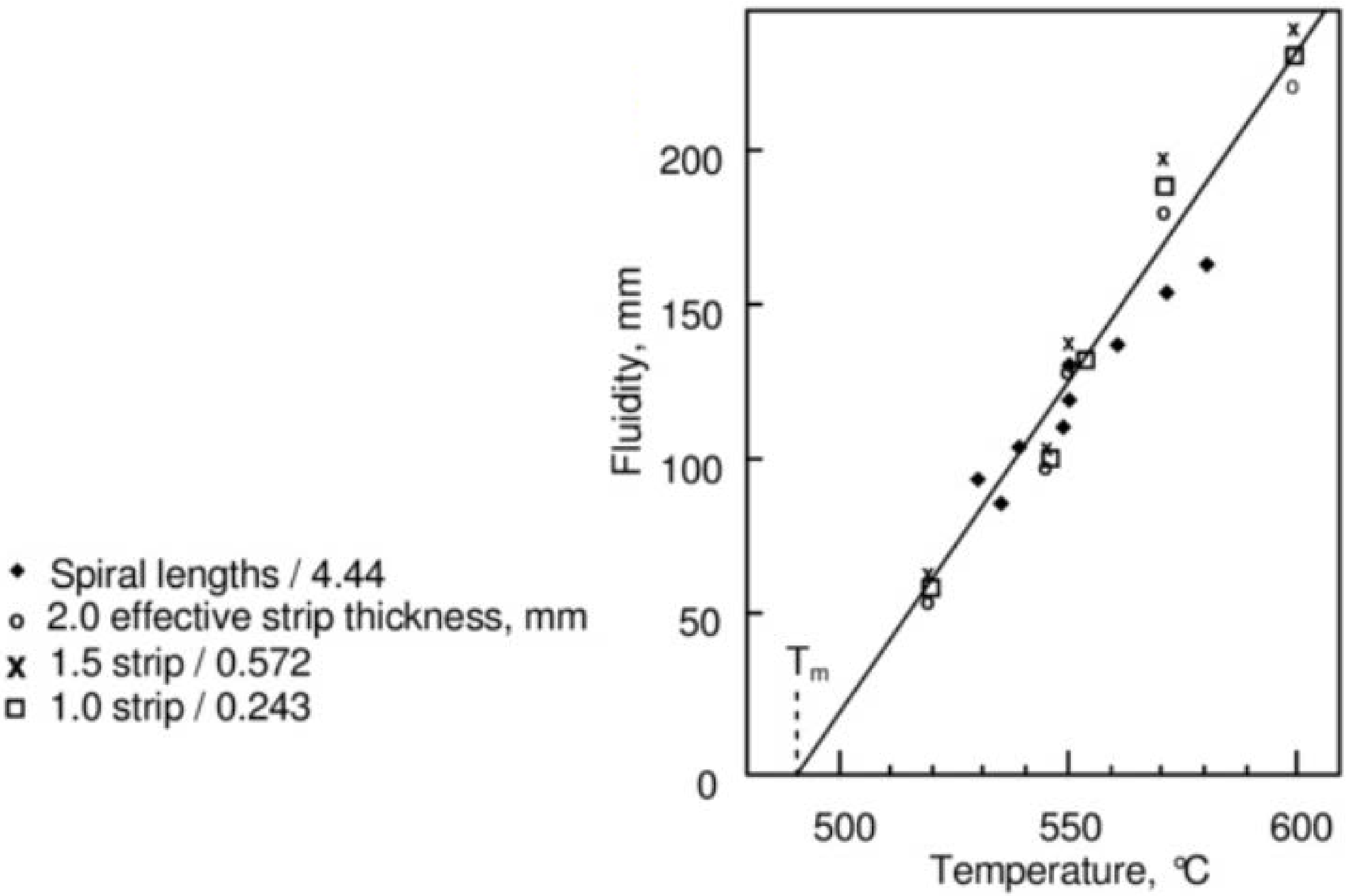
The Fluidity of ZA 27 Zinc-Aluminum Alloy



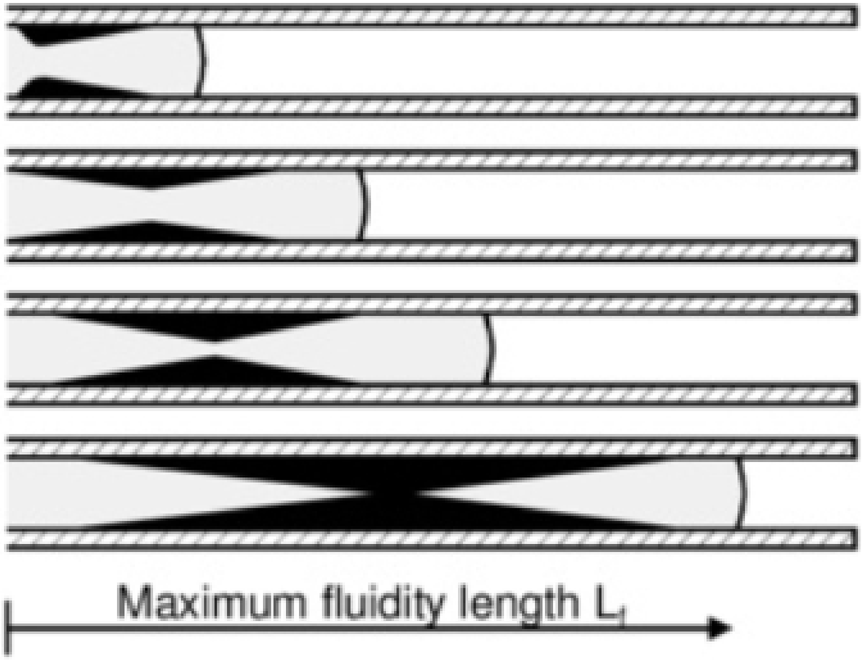
Comparison of Fluidity Measurements

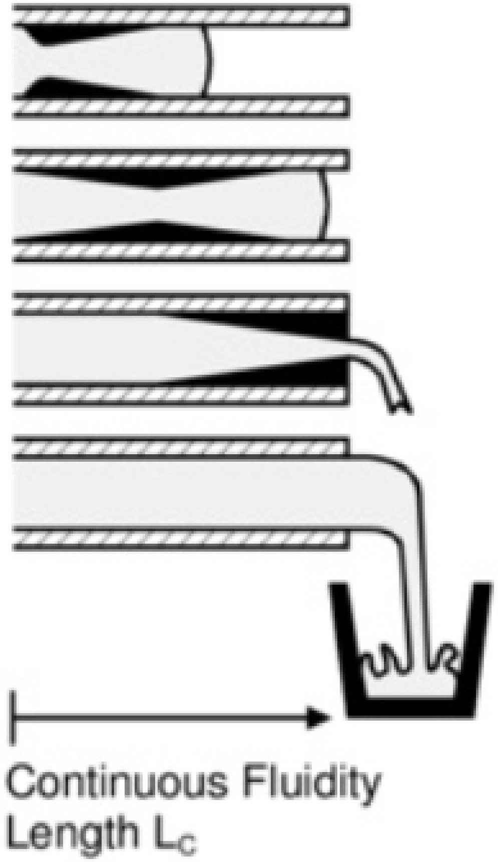


Rationalisation of Fluidity Measurement

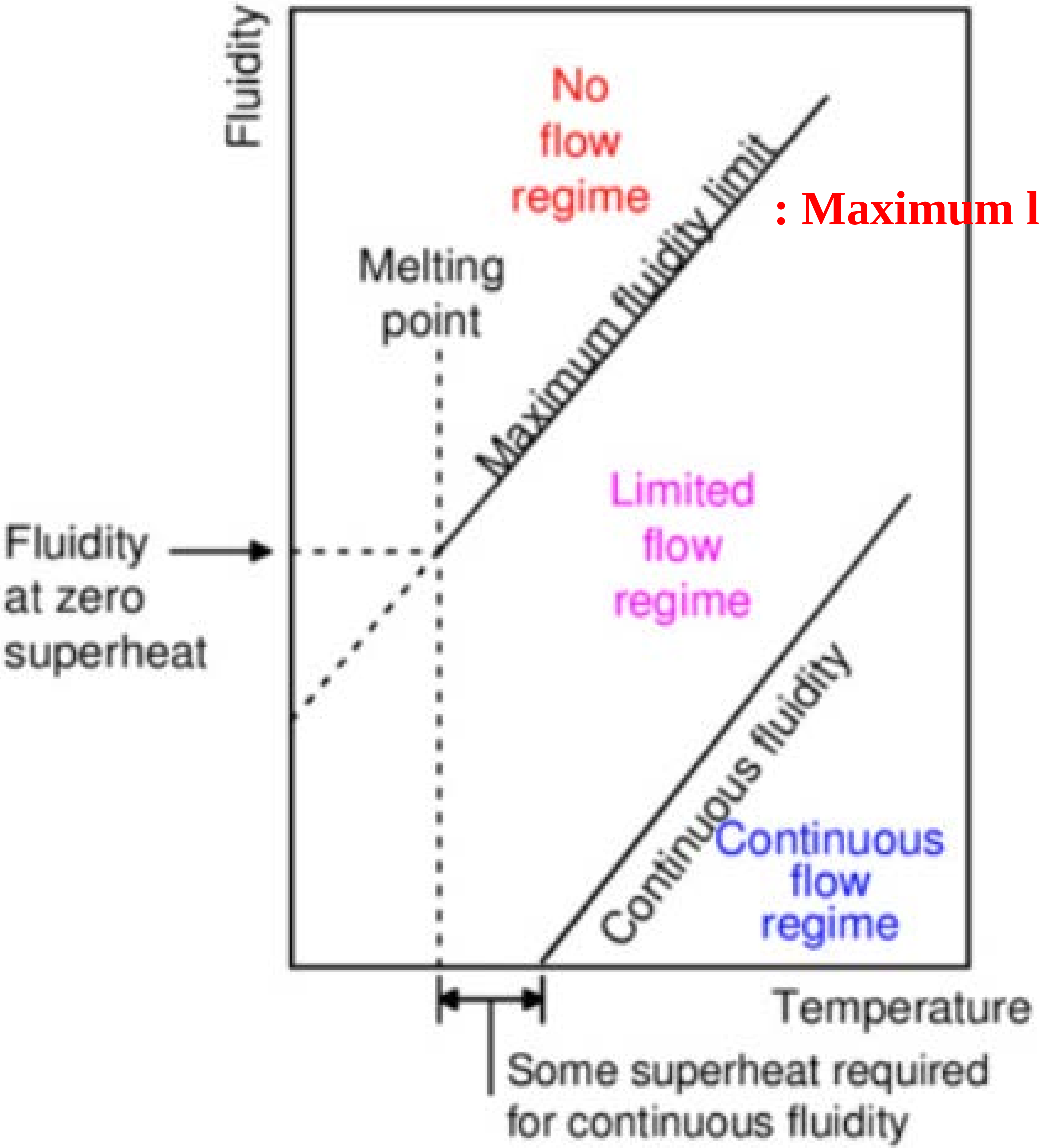


Continuous Fluidity

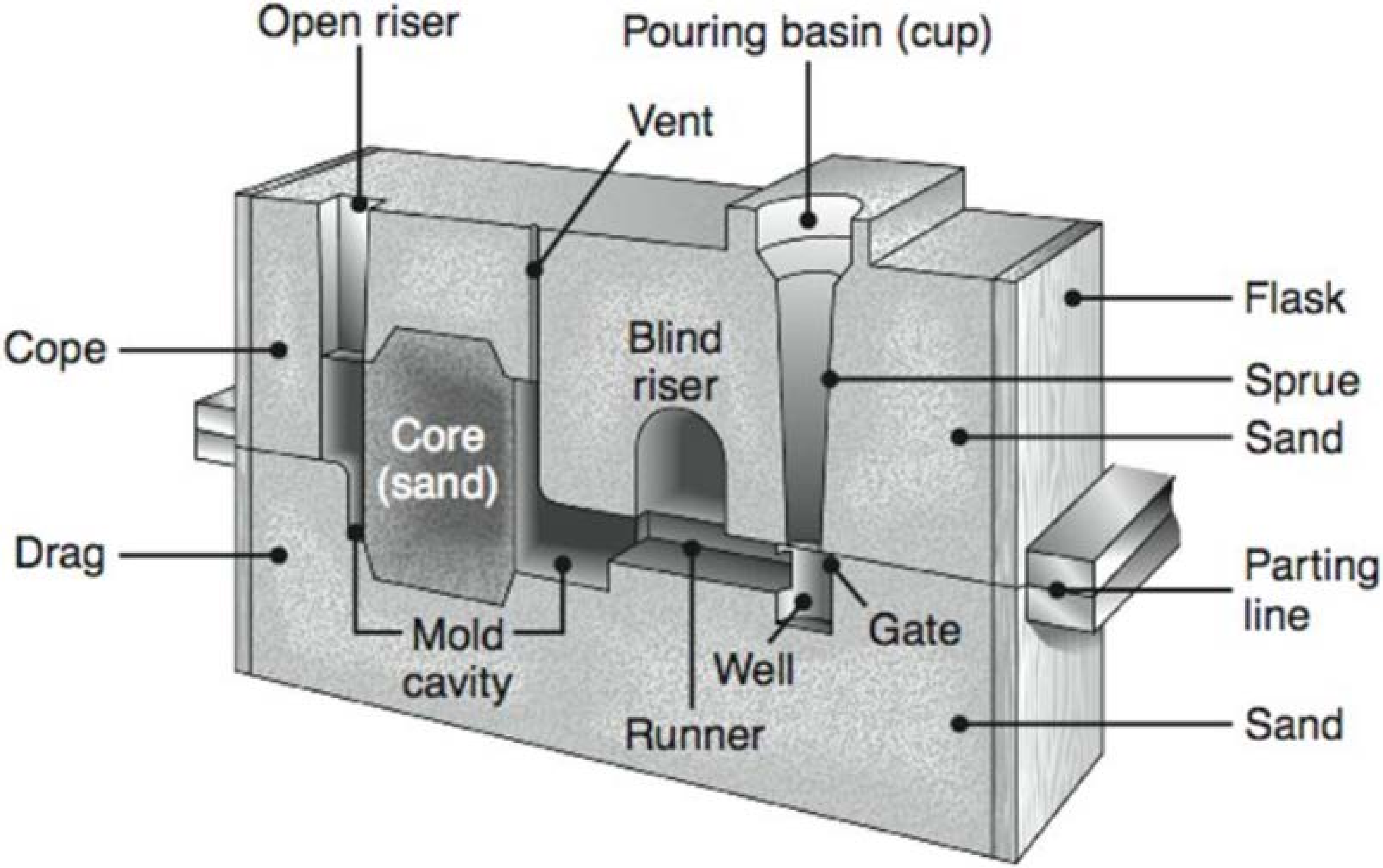




Regimes of continuous, partial and impossible flow

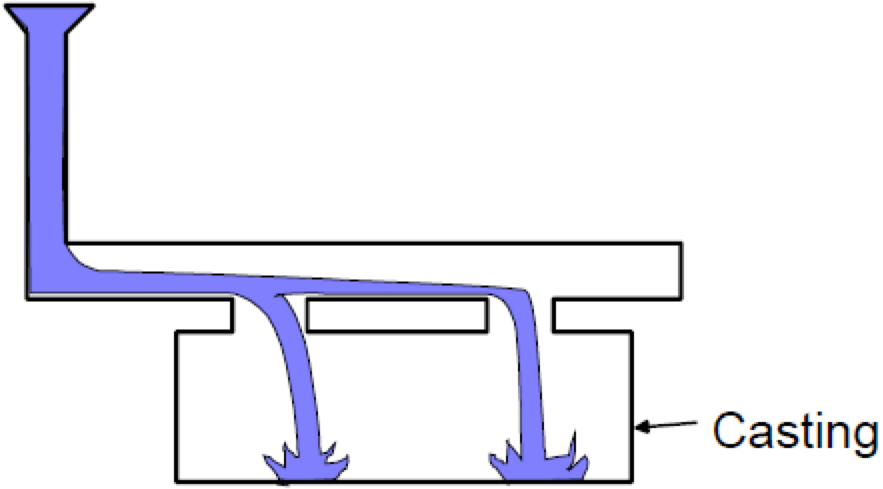


: Maximum length melt can reach

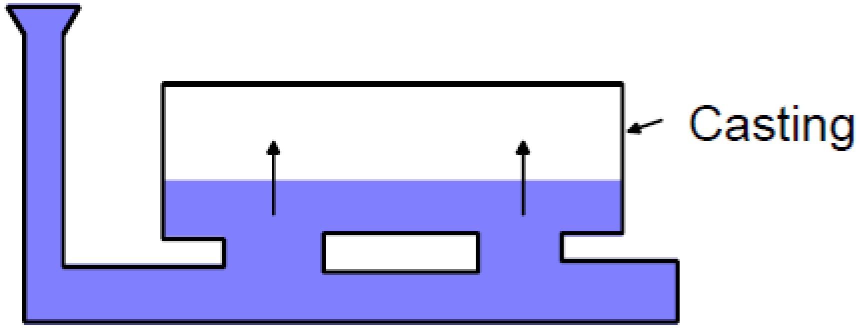


Fluid Flow : Molten metal → Pouring basin → Sprue → Runner → Cavity → Riser

Top versus Bottom Gating

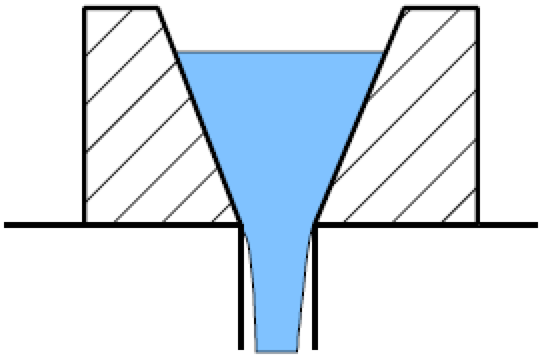


Top gating - causes turbulence

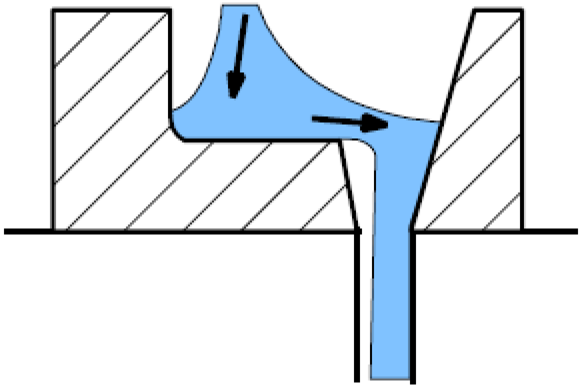


Bottom gating - prevents turbulence

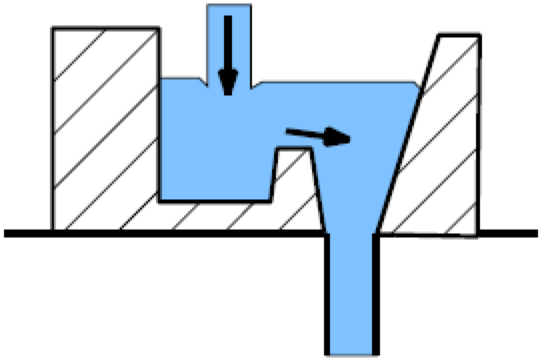
Good Design 1: Pouring Basin



BAD
conical basin



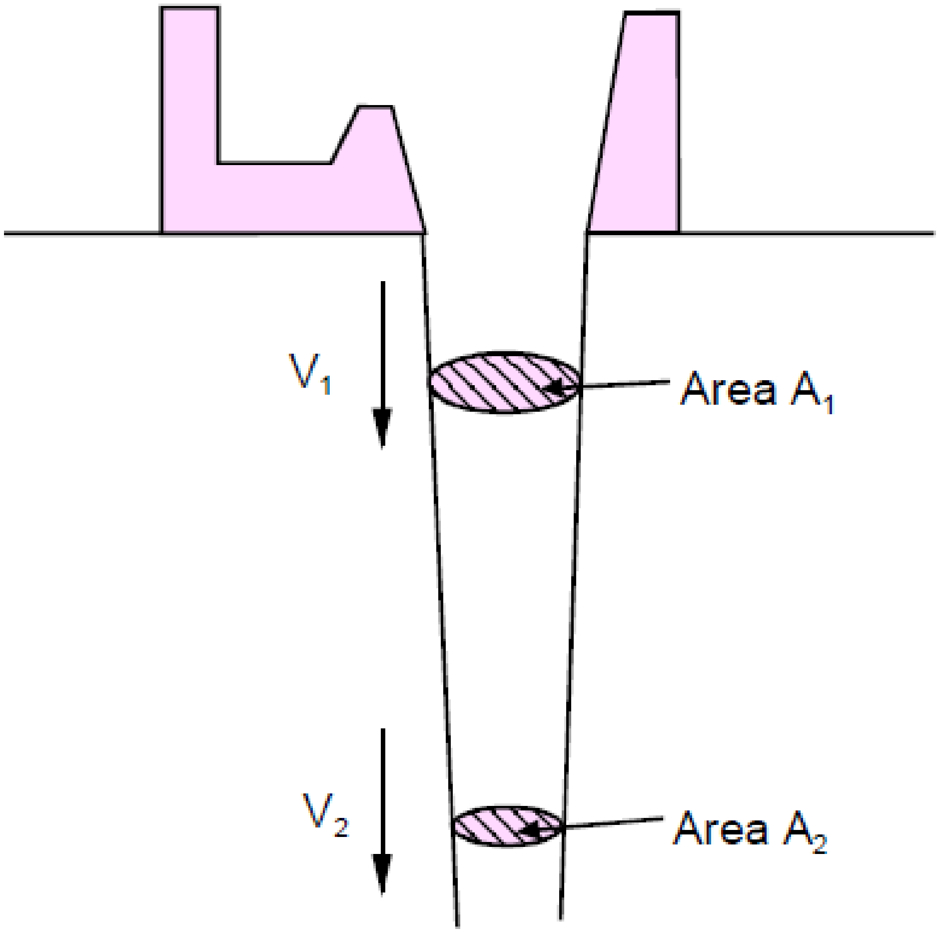
BETTER
offset basin



BEST
offset stepped basin

Good Design 2: Tapered Sprue

탕구



Metal accelerates from
V_1 to V_2 due to gravity.

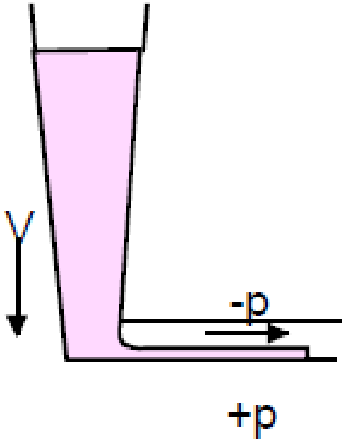
Sprue will remain full of metal
if the sprue is tapered so that

$$A_1 \cdot V_1 = A_2 \cdot V_2$$

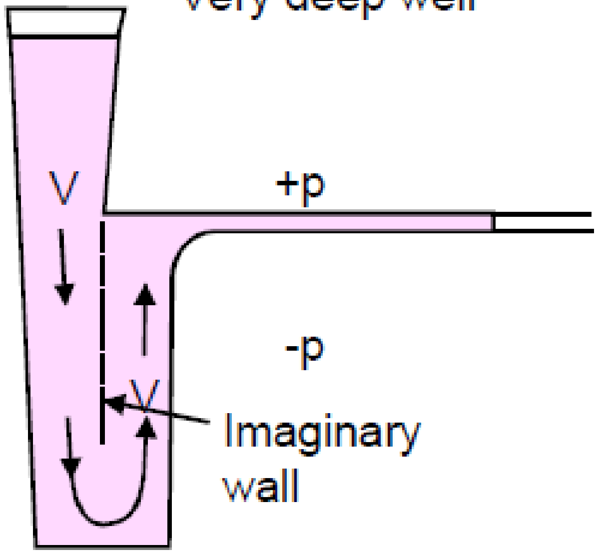
Good Design 3: Sprue Well

The sprue well helps to:
(i) Declarate metal
(ii) Stop first splash
(iii) Fill runner

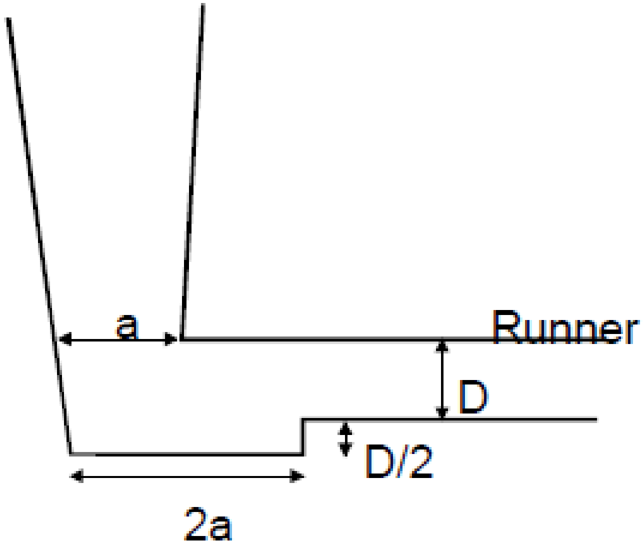
No well



Very deep well



Optimum well design



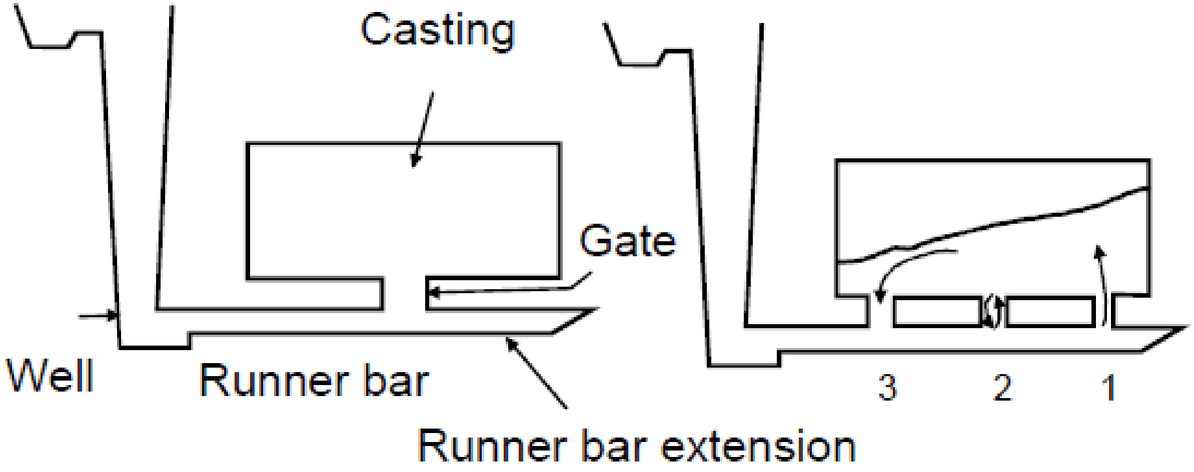
Good Design 4: Runner Bar and Gates

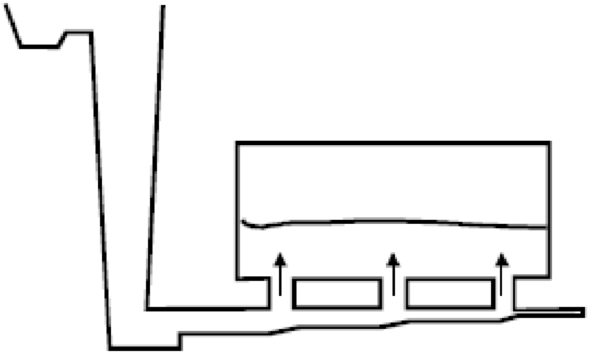
AIMS: (i) to distribute metal to lowest point(s) on a casting
(ii) to reduce metal velocity.

Casting with a single ingate

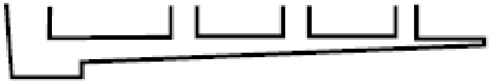
Uneven flow in multigated casting as a result of incorrect runner bar design

Uniform flow promoted by the use of stepped runner bar





Alternative tapered runner bar

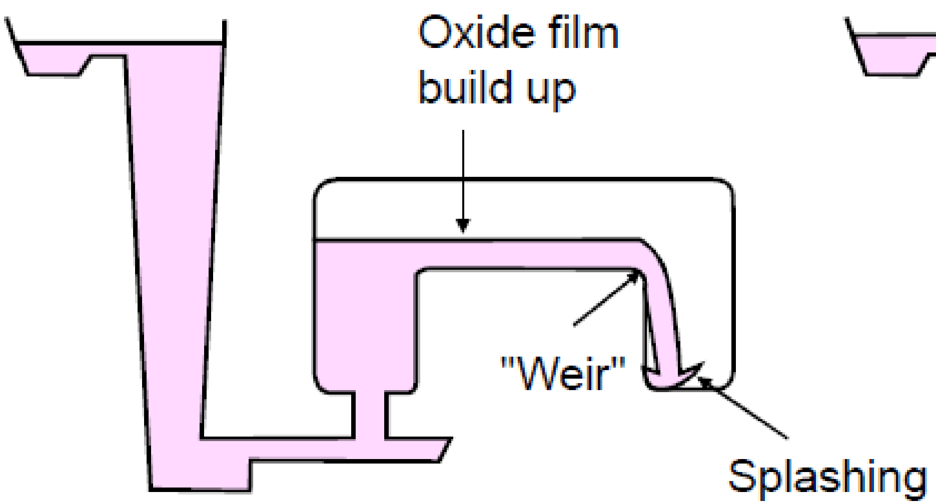


Good Design 4 (Continued): Runner Bar and Gates

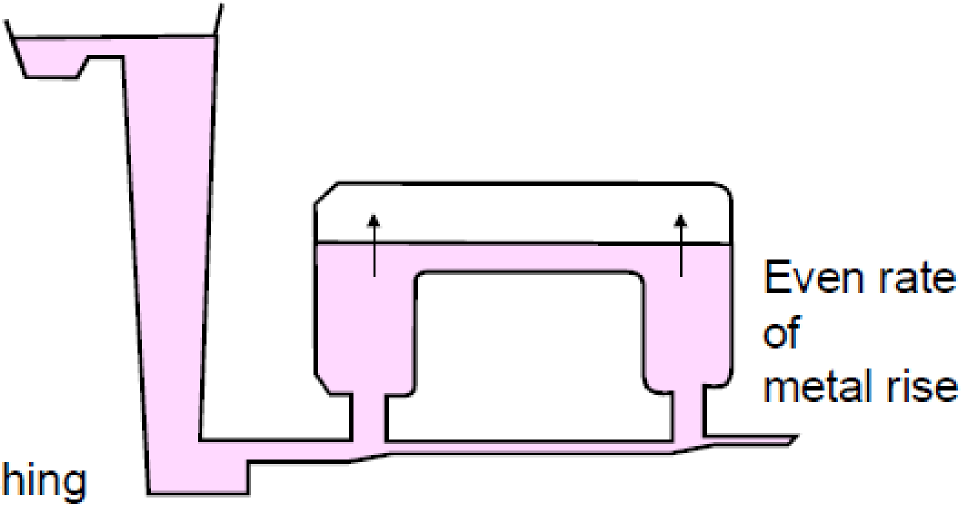
Waterfall effects must be avoided so that:

(a) splashing is prevented
(b) the critical velocity is not exceeded
(b) the metal meniscus is never stationary

Bad



Good



7.3. Heat Flow

Heat Balance Equation

$$K_S T'_S = K_L T'_L + vL_V$$

*** Solidification rate of solid/liquid interface ∝ $\Delta T = T_e - T$ (actual temp): To maintain ΔT, latent heat generated during solidification needs to be removed.**

→ the amount of solidification at a given time ∝ the amount of heat removed during that time, Q

* Heat transfer in the casting process

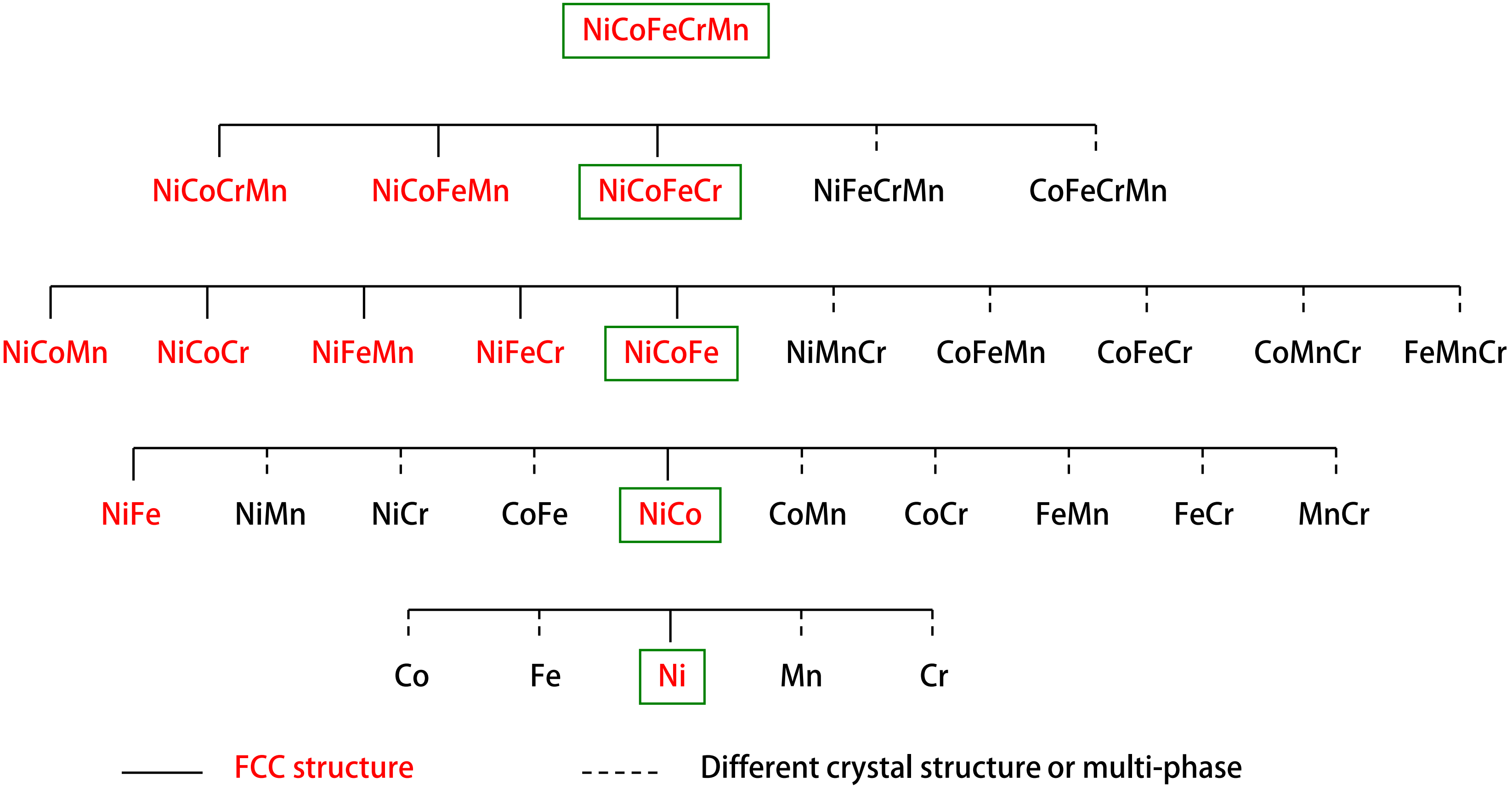
1) Thermal conductivity: Generally in pure metals and low alloys, TC decreases/ in high alloys TC increases when T increases/ TC of metal >> TC of ceramic.

2) Convection heat transfer: Convection occurs due to density difference by temperature difference in flowing of molten metal.

3) Radiative heat transfer: in high-temperature molten metal or mold surface contacting the atmosphere, radiation heat transfer in which heat energy moves in the form of electromagnetic waves should be considered.

4) Phase transformation and latent heat: Release or absorption of latent heat occurs when there is a phase transformation/ in this case, exothermic or endothermic term should be added by heat energy conservation law.

manipulation of σ_y/κ ratio in FCC solid solutions



NiCoFeCrMn

NiCoCrMn NiCoFeMn NiCoFeCr NiFeCrMn CoFeCrMn

NiCoMn NiCoCr NiFeMn NiFeCr NiCoFe NiMnCr CoFeMn CoFeCr CoMnCr FeMnCr

NiFe NiMn NiCr CoFe NiCo CoMn CoCr FeMn FeCr MnCr

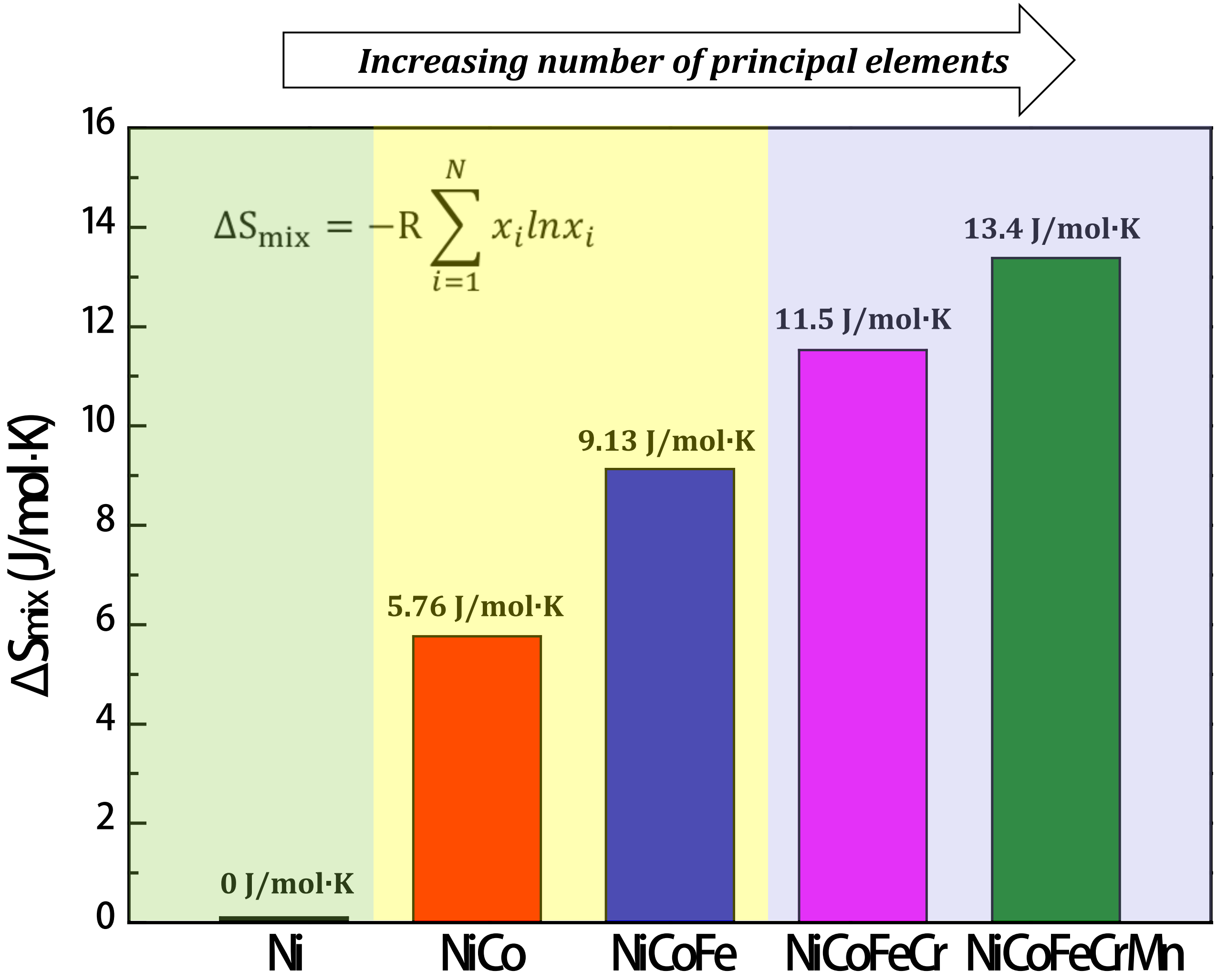
Co Fe Ni Mn Cr

——— FCC structure

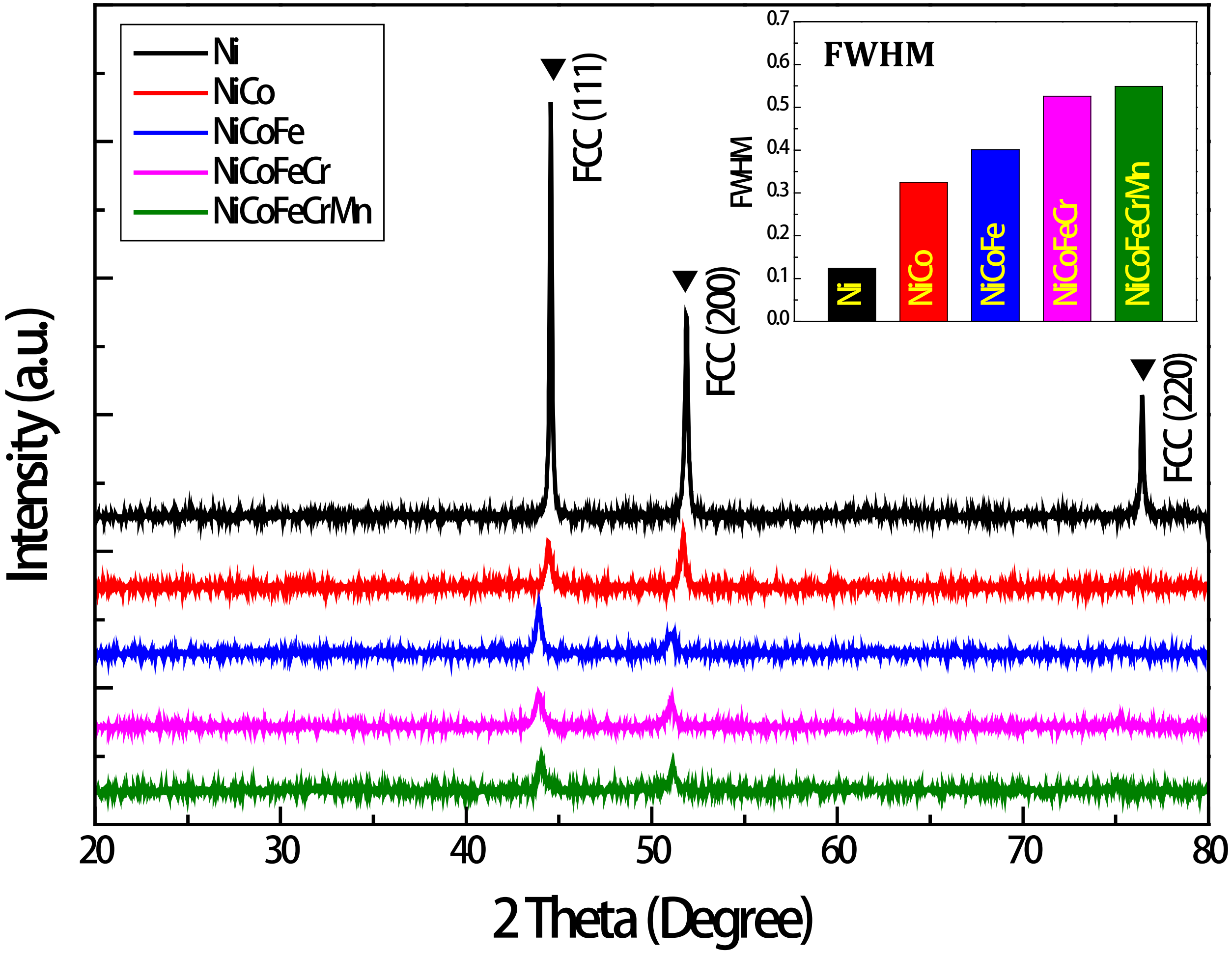
- - - - - Different crystal structure or multi-phase

→ Guideline for design of multi-principal elements alloys in cryogenic applications

Relationship btw # of principle elements (NPEs) & ΔS_{mix}



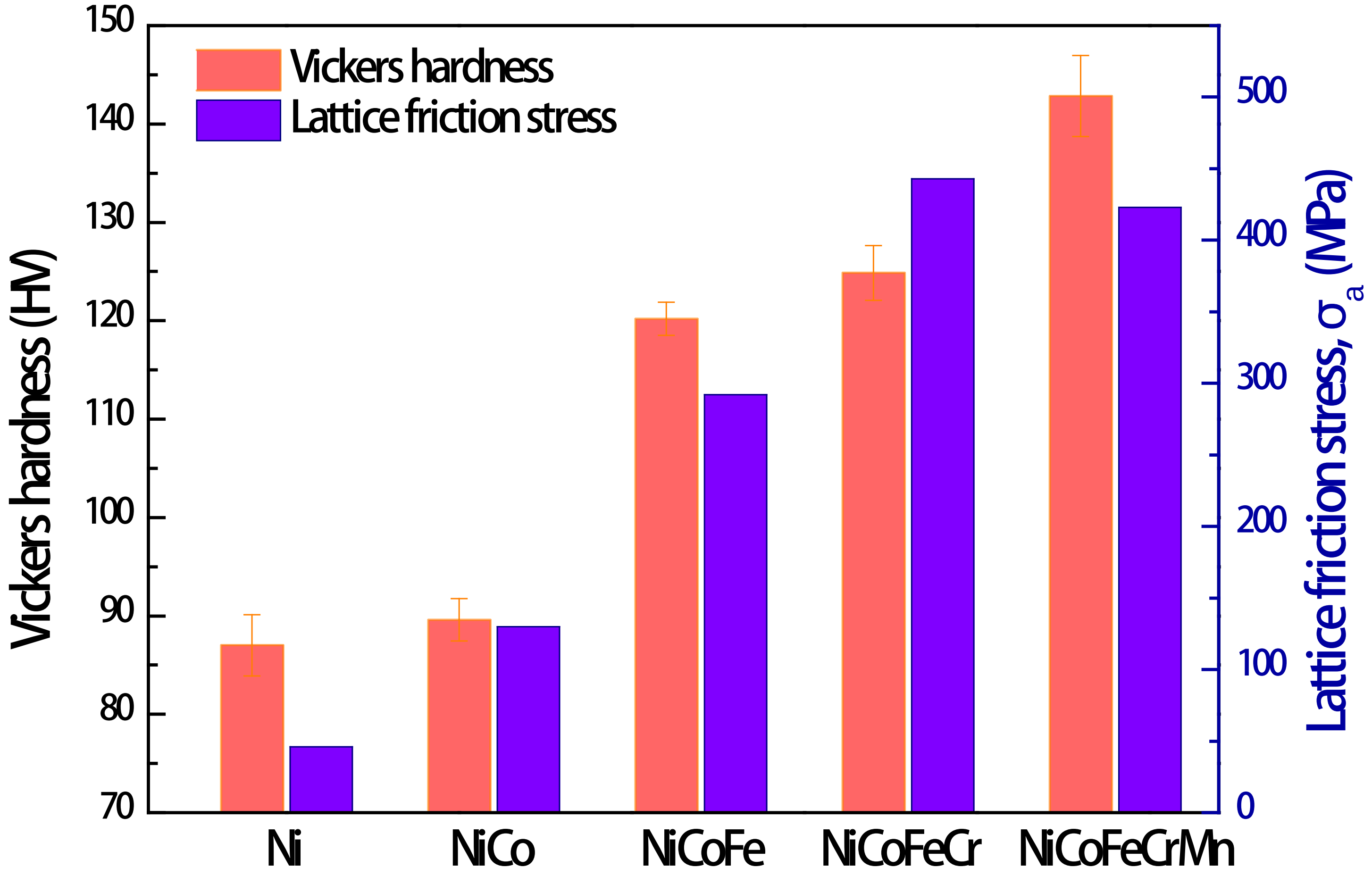
XRD results of the FCC solid solutions



Number of principal elements ↑ → Peak intensity ↓ & Full width at half maximum ↑

→ Increase in lattice distortion with increasing NPEs

Mechanical properties of the FCC solid solutions

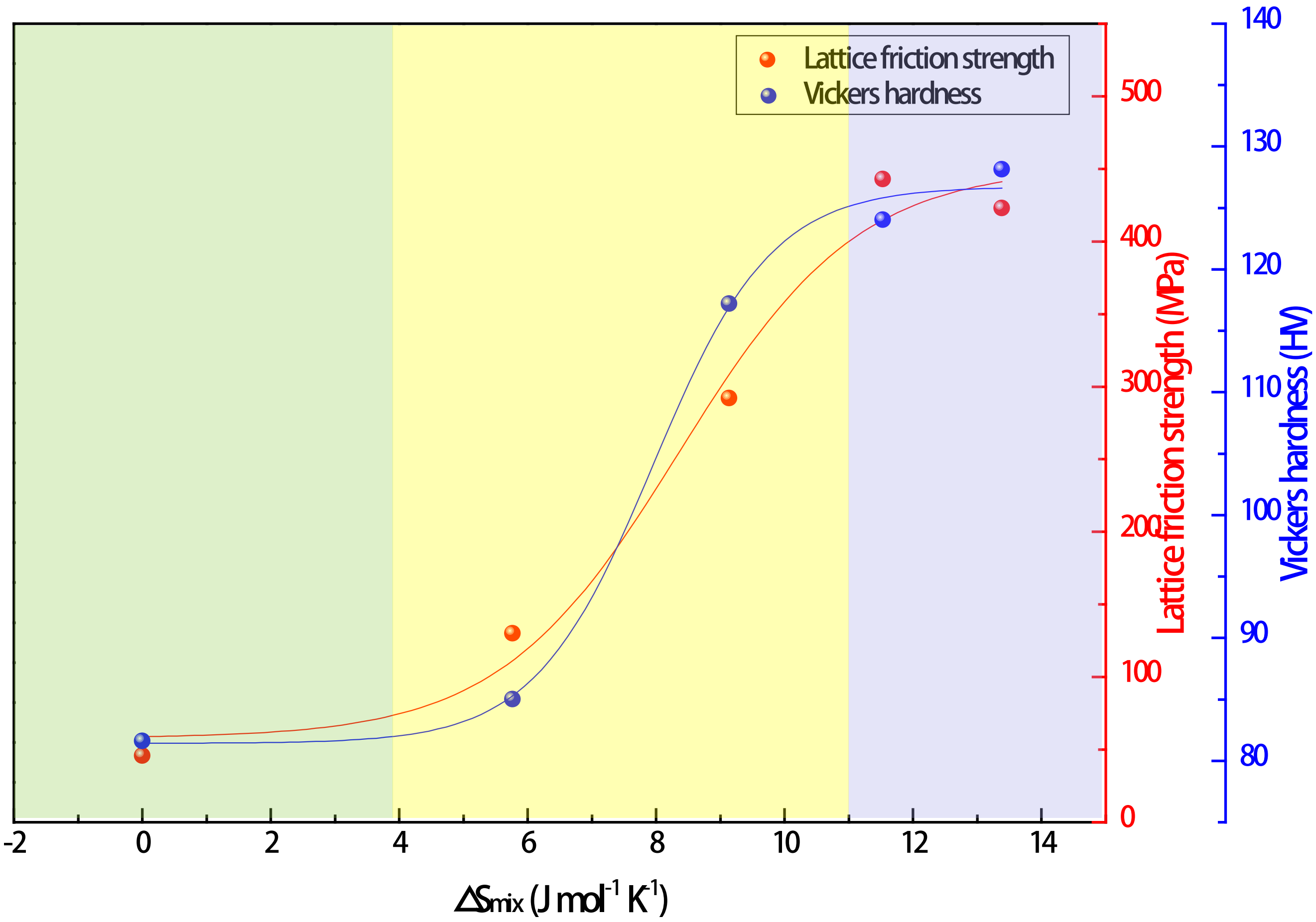


NPEs ↑ → Lattice distortion ↑ → Vickers hardness & Lattice friction stress ↑

: grain size and temperature-independent intrinsic lattice resistance to dislocation motions

Z.Wu et al. (2014)

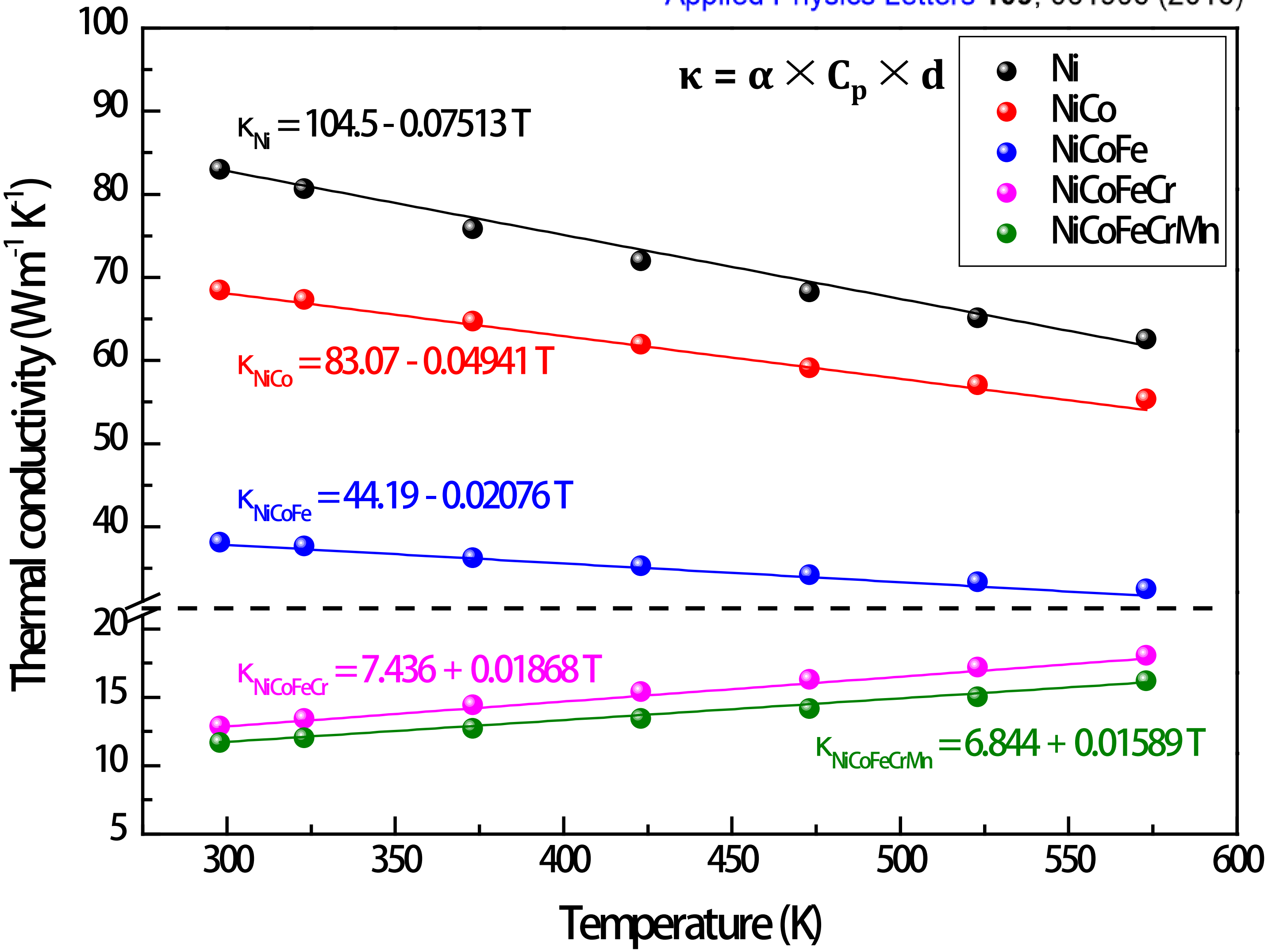
Mechanical properties of the FCC solid solutions



ΔS_{mix} (NPEs) ↑ → Lattice distortion & Compositional complexity ↑
→ Solid-solution hardening ↑ (HV & σ_a ↑)

Thermal conductivity (κ) of the FCC solid solutions

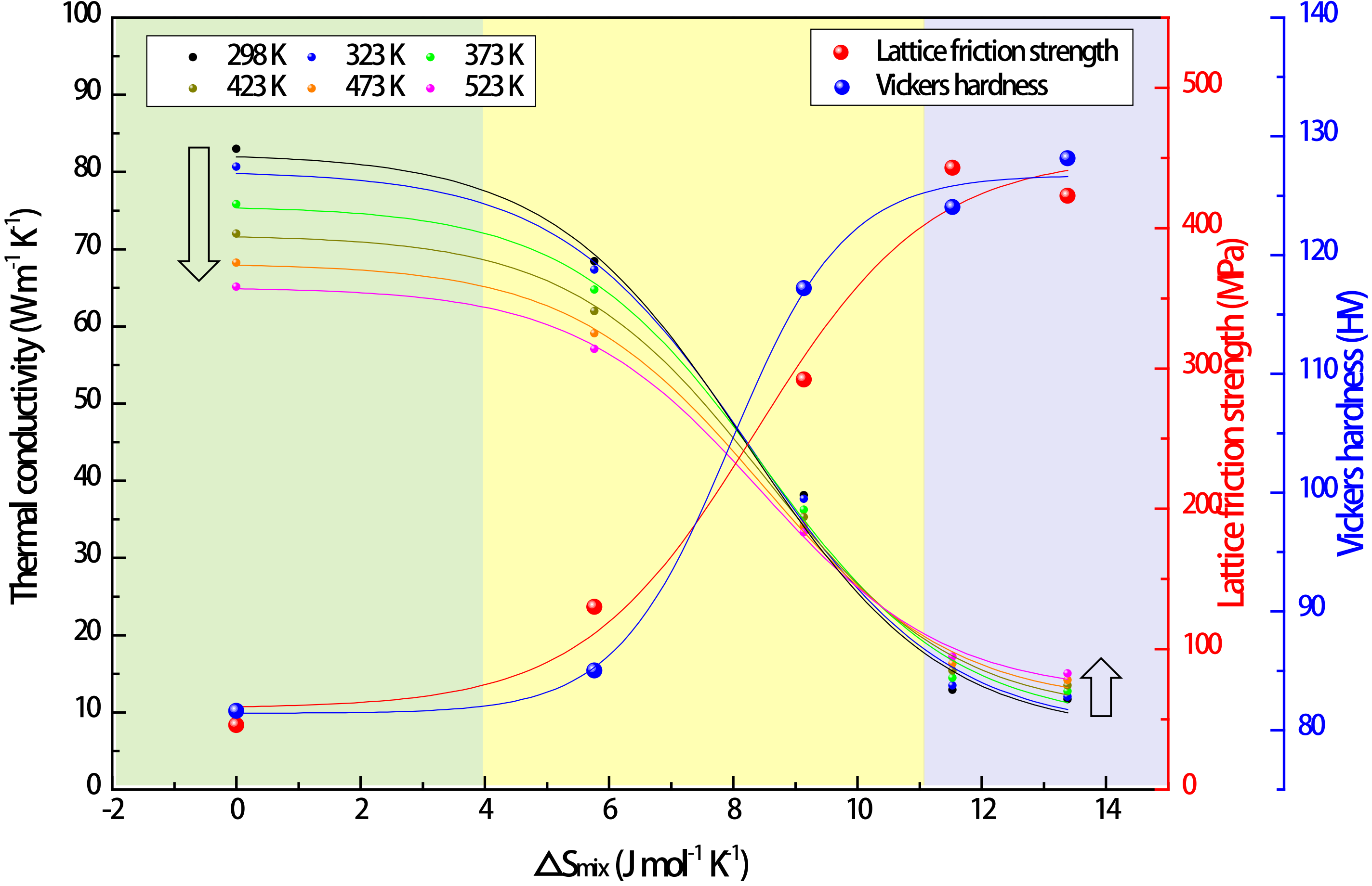
Applied Physics Letters **109**, 061906 (2016)



Ni, NiCo and NiCoFe: Negative trend / **NiCoFeCr and NiCoFeCrMn: Positive trend**

Reversed trends in κ and σ_y with increasing ΔS_{mix}

Applied Physics Letters **109**, 061906 (2016)



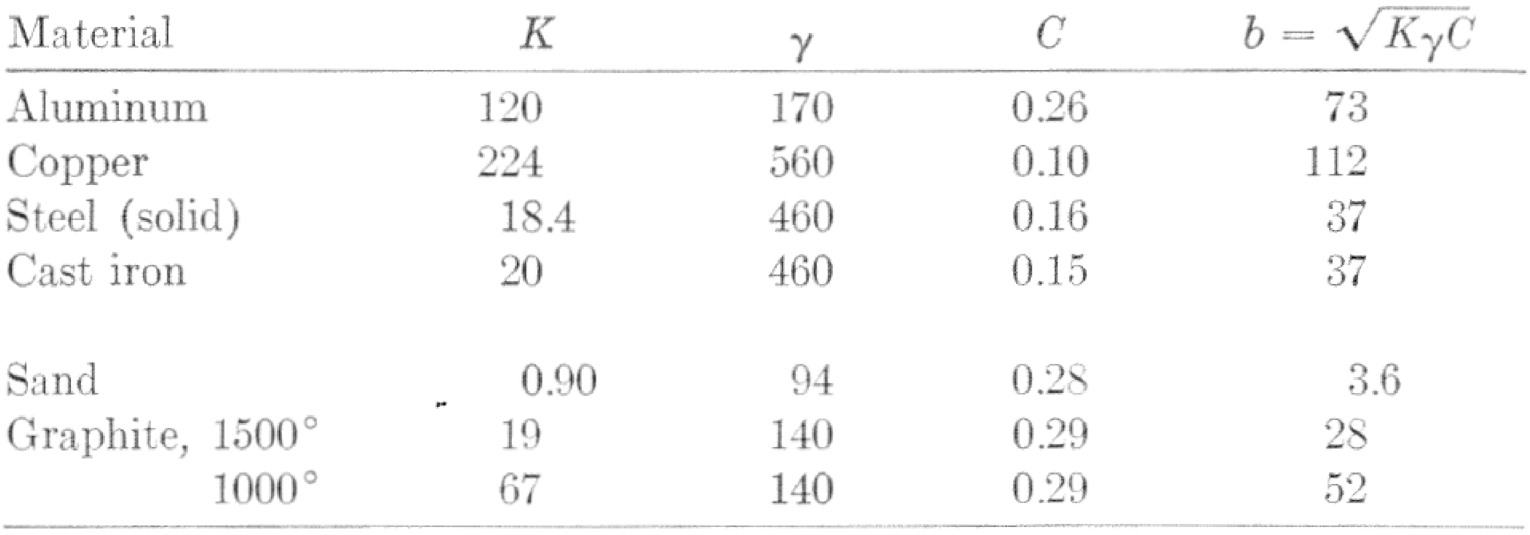
ΔS_{mix} (NPEs) ↑ → Lattice distortion & Compositional complexity ↑
→ Solid-solution hardening ↑ (HV & σ_a ↑) & Electron scattering ↑ (ρ ↑, κ ↓)

* Possible to classify of various solidification processes according to thermal property of mold (Related to heat release associated with solidification rate)

* Heat diffusivity **b =** $\sqrt{K\gamma C}$

K= thermal conductivity/ γ = density/ C= specific heat

Table 7.2 (Values in feet, pounds, °F units)



Material	K	γ	C	$b = \sqrt{K\gamma C}$
Aluminum	120	170	0.26	73
Copper	224	560	0.10	112
Steel (solid)	18.4	460	0.16	37
Cast iron	20	460	0.15	37
Sand	0.90	94	0.28	3.6
Graphite, 1500°	19	140	0.29	28
1000°	67	140	0.29	52

A. For metal molds with thermal conductivity similar to solidifying metal

* n = (b of mold metal)/ (b of solidifying metal)

: Solidification of steel in cast iron mold n = 1.12 ~ close to 1.

*** If the metal mold thickness is not larger than the section thickness to be cast,**

Initial solidification: control by heat flow into mold

final solidification: conduction through mold / Heat loss outside the mold

*** 3 different types of metal mold**

(1) Typical ingot mold: Heat release by radiation from outer surface and convection

(2) Metal mold cooled on surface relatively far from casting

→ - Most continuous casting is made by water cooling.
- Water cooling in the absence of mold has a similar effect.
- Arc melting also uses this type of mold.

(3) Huge metal mold compared to the solidification part

: Heat loss outside mold is not large until final solidification

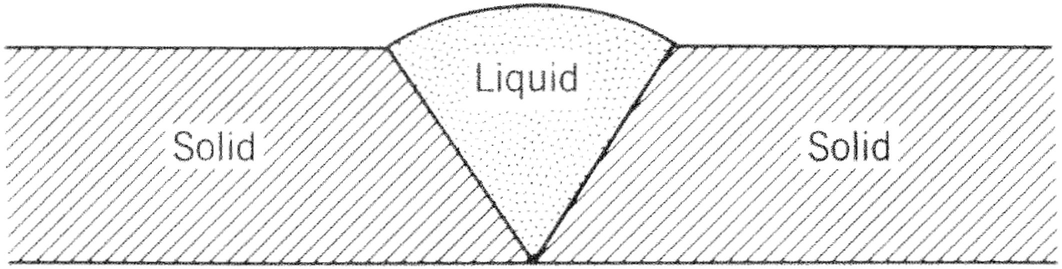


Fig. 7.2. cross section of butt weld (schematic).

B. Sand mold: Thermal conductivity is much smaller than solidifying metal

Ex) n = 0.13 → the dependence of the mold thickness is greatly reduced

(∵ heat loss from the mold surface is independent of thickness variation)

C. If the heat release is controlled by controlled heat supply control and heat sink to the liquid metal, as in the case of zone refining or single crystal growth: the process rate depends on the L-S temperature gradient and the interfacial velocity, v.

1. Solidification Rate

* The solidification rate *f*(t) is to obtain the increase in time of the solid layer in contact with the mold. Although in case of ①pure metal or eutectic without solidus-liquidus interval, it is possible to calculate the heat flow by solidifying while maintaining the planar interface, heat flow calculations and interpretation of results at the ②dendritic interface are complicated due to influence by various interface conditions → Solidification rate can be calculated by measuring the Temp.-time relationship in various parts of the solidifying metal.

Assumption: mold and metal: semi-infinite / initial liquid temperature T_m / liquid: pure metal & solid-liquid interface temperature~ constant / metal: constant temperature with mold interface

① For heat conduction in one direction in the mold (i.e., perpendicular to the planar mold wall), Temp, θ of an element of volume at t

Thermal conductivity

$$\frac{\partial\theta}{\partial t} = \frac{K}{\gamma C}\frac{\partial^2\theta}{\partial x^2}$$

density Heat capacity

② ΔT at a specific location in time t :

initial temp. at the surface, θ_0 → instantly raised to θ_i at t=0 → change to θ_m at t=t_1

$$\theta_m = \theta_0 + (\theta_i - \theta_0)\,\mathrm{erfc}\left(\frac{X}{2\sqrt{\alpha t_1}}\right)$$

where, $\alpha = \dfrac{K}{\gamma C}$

③ Heat removed by casting at any time t_1 :

$$\frac{\partial Q}{\partial t} = -K\left[\frac{\partial\theta}{\partial x}\right]_{x=0} = \frac{K(\theta_i - \theta_0)}{\sqrt{\pi\alpha t}} = 0.564\frac{K(\theta_i - \theta_0)}{\sqrt{\alpha t}} = 0.564b\frac{(\theta_i - \theta_0)}{\sqrt{t}}$$

By differentiation of the efrc equation

④ **Total heat conducted into the mold Q up to time t** : (where, b = $\sqrt{K\gamma C}$

$$Q = b(\theta_i - \theta_0)\int_0^t \frac{0.564}{\sqrt{t}} = 1.128b(\theta_i - \theta_0)\sqrt{t}$$

⑤ Thickness of the solidified layer, D:

$$\boxed{D = q\sqrt{t}}$$

where, q = solidification constant

$$1.128\ [b(\theta_i - \theta_0)/L\gamma'];$$

L: latent heat/ density of solidified metal: γ'

⑥ Reflection of Superheat condition: more complicated problem

Total heat to be extracted from solidifying metal = $W[L + S(\theta_c - \theta_f)]$

where W = weight of casting
θ_c = initial temperature of the liquid
θ_f = final solidification temperature

→ This amount of heat must be conducted into the mold during the time t taken for solidification.

<u>If the area of the surface of contact of mold and metal is A</u>, then

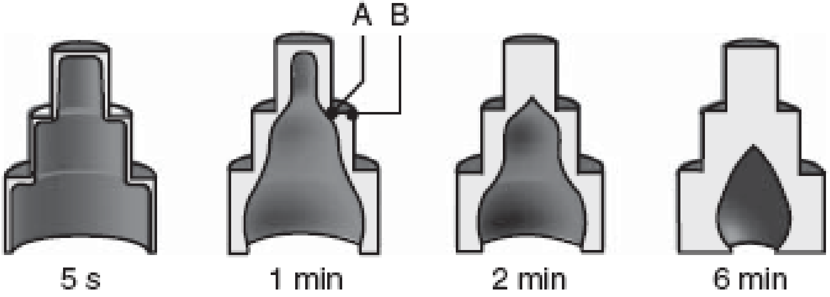
$$\sqrt{t} = \frac{W[L + S(\theta_c - \theta_f)]}{1.128A\sqrt{K\gamma C}\,(\theta_i - \theta_0)}$$

from which

$$\boxed{t = \left(\frac{V}{A}\right)^2 \times \text{constant}}$$

(Chvorinov's rule)

Solidification Time



- Total solidification time T_{TS} = time required for casting to solidify after pouring
- T_{TS} depends on size and shape of casting by relationship known as *Chvorinov's Rule*

$$T_{TS} = C_m \left(\frac{V}{A} \right)^n$$

where T_{TS} = total solidification time; V = volume of the casting; A = surface area of casting; n = exponent with typical value = 2; and C_m is *mold constant.*

$$T_{TS} = C_m \left(\frac{V}{A} \right)^n$$

$$\sqrt{t} = \frac{W[L + S(\theta_c - \theta_f)]}{1.128 A \sqrt{K \gamma C}\,(\theta_i - \theta_0)}$$

Mold Constant in Chvorinov's Rule

- Mold constant C_m depends on:
  - Mold material
  - Thermal properties of casting metal
  - Pouring temperature relative to melting point

- Value of C_m for a given casting operation can be based on experimental data from previous operations carried out using same mold material, metal, and pouring temperature, even though the shape of the part may be quite different.

Solidification times for various shapes

Three metal pieces being cast have the same volume but different shapes: On is a sphere, one a cube, and the other a cylinder with its height equal to its diameter.
Which piece will solidify the fastest, and which one the slowest? Assume that n = 2

Solution The volume of the piece is taken as unity. Thus from Eq.

$$\text{Solidification time} \propto \frac{1}{(\text{Surface area})^2}$$

The respective surface areas are as follows:

Sphere:

$$V = \left(\frac{4}{3}\right)\pi r^3,\ r = \left(\frac{3}{4\pi}\right)^{1/3}$$

$$A = 4\pi r^2 = 4\pi\left(\frac{3}{4\pi}\right)^{2/3} = 4.84$$

Cube:

$$V = a^3,\ a = 1,\ \text{and}\ A = 6a^2 = 6$$

Cylinder:

$$V = \pi r^2 h = 2\pi r^3,\ r = \left(\frac{1}{2\pi}\right)^{1/3}$$

$$A = 2\pi r^2 + 2\pi r h = 6\pi r^2 = 6\pi\left(\frac{1}{2\pi}\right)^{2/3} = 5.4$$

The respective solidification times are therefore

$$t_{\text{sphere}} = 0.043C,\ t_{\text{cube}} = 0.028C,\ t_{\text{cylinder}} = 0.033C$$

Hence, the cube-shaped piece will solidify the fastest, and the spherical piece will solidify the slowest.

What Chvorinov's Rule Tells Us

- Casting with a higher volume-to-surface area ratio cools and solidifies more slowly than one with a lower ratio
  - To feed molten metal to the main cavity, T_{TS} for riser must be greater than T_{TS} for main casting
- Since mold constants of riser and casting will be equal, design the riser to have a larger volume-to-area ratio so that the main casting solidifies first
  - This minimizes the effects of shrinkage & waste metal.

*** In the case of sand mold,**

the superheat of the liquid phase is uniformly reflected throughout and <u>delays the start of solidification</u> until the liquid reaches the liquidus temp.

*** Thickness of the solidified layer D in sand mold casting**

$$D = q\sqrt{t} - c$$

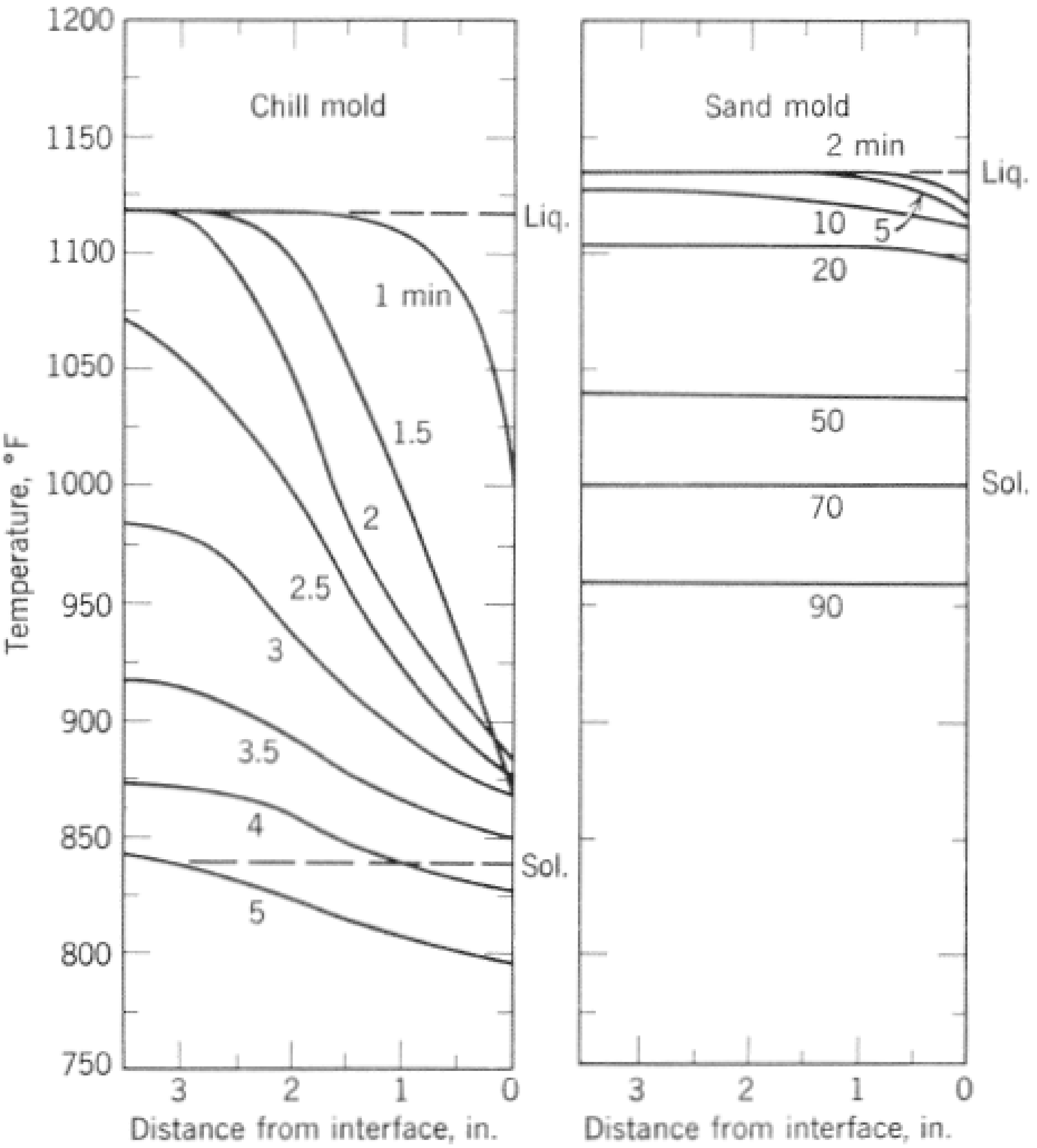


Fig. 7.3. Variation of temperature during solidification of Al 5% Mg alloy in a 7-inch square mold. (a) Metal mold, (b) sand mold.

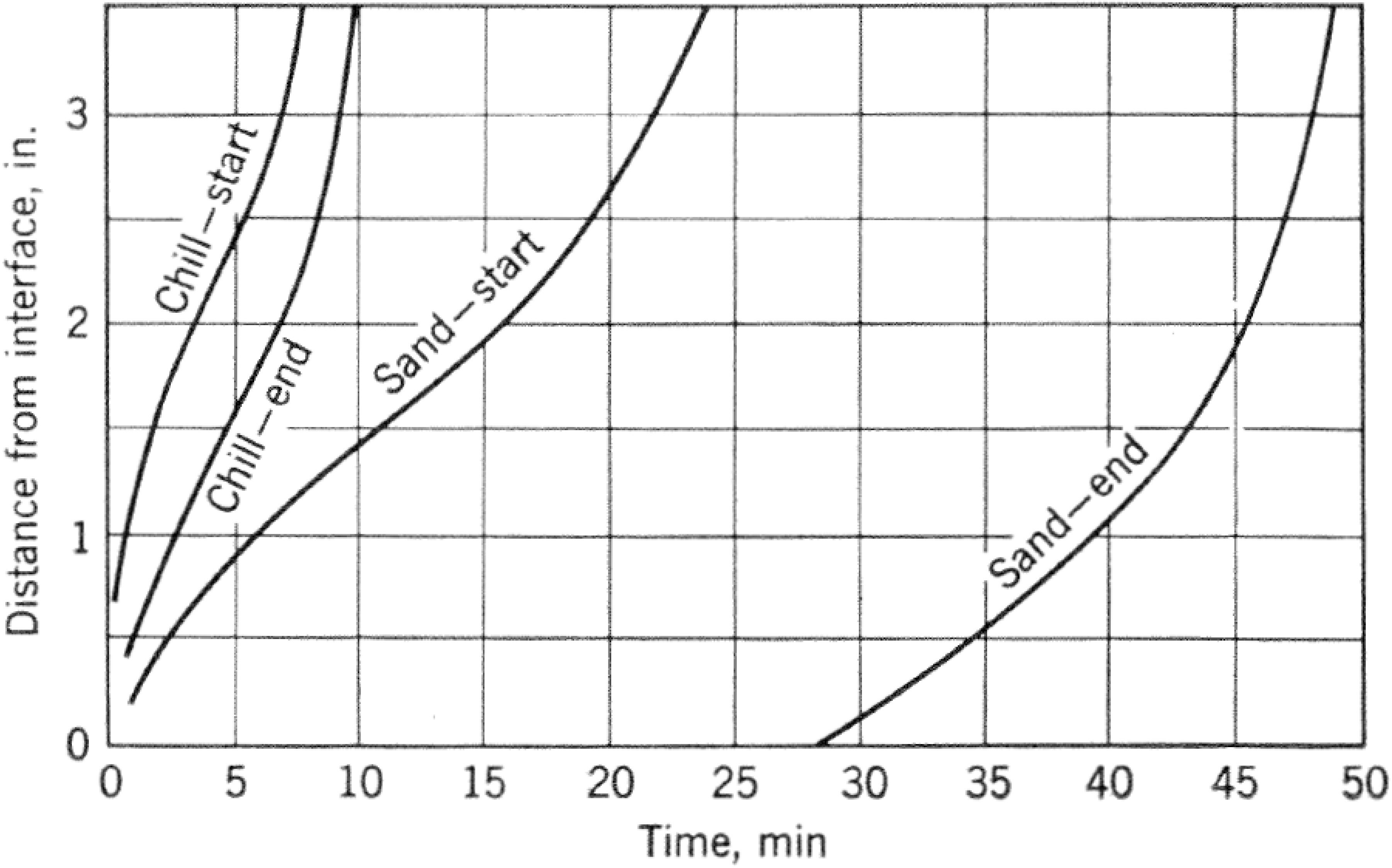
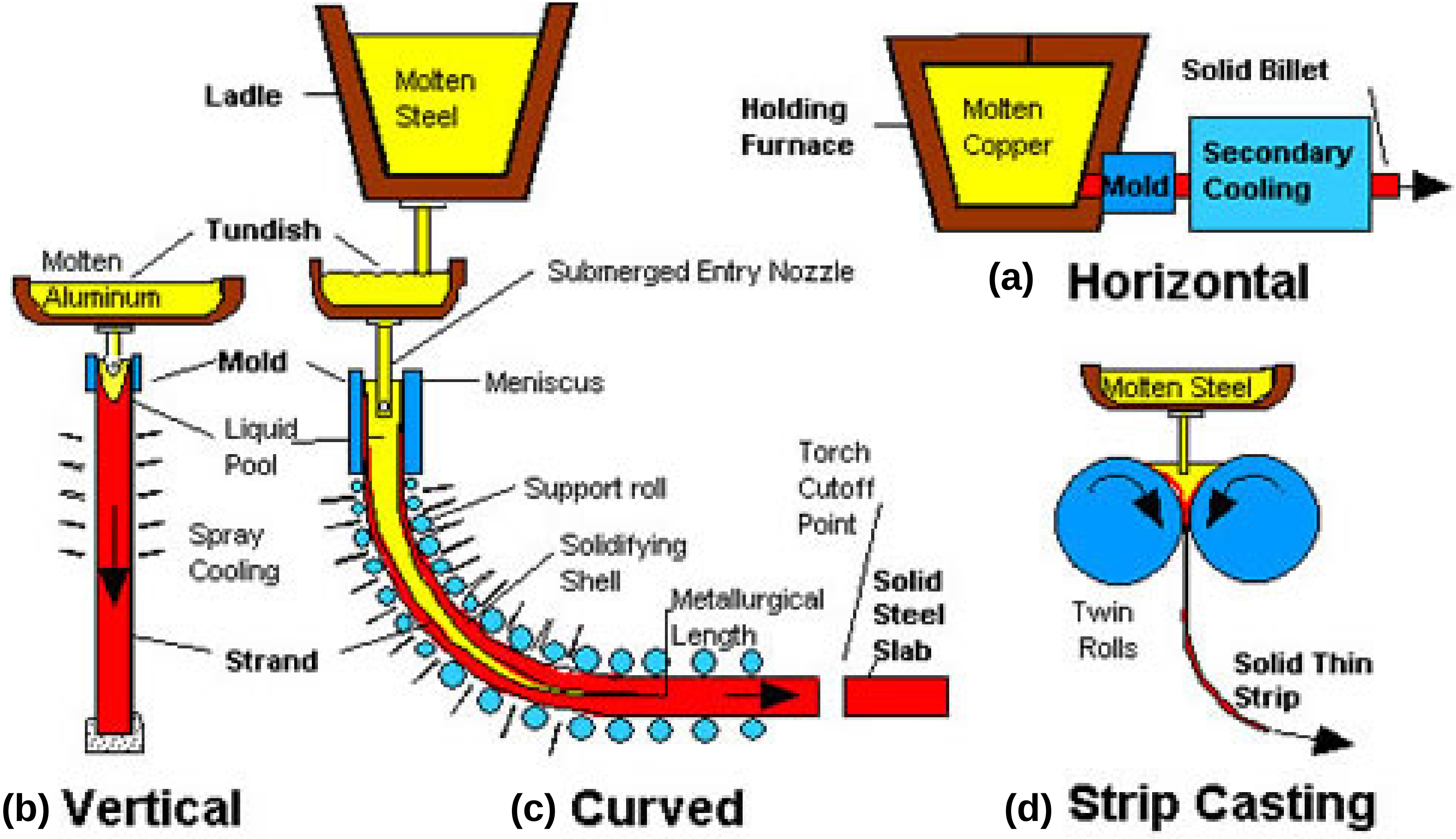


Fig. 7.4. Movement of liquidus and solidus tempeeratures during solidification of a 0.6% carbon steel.

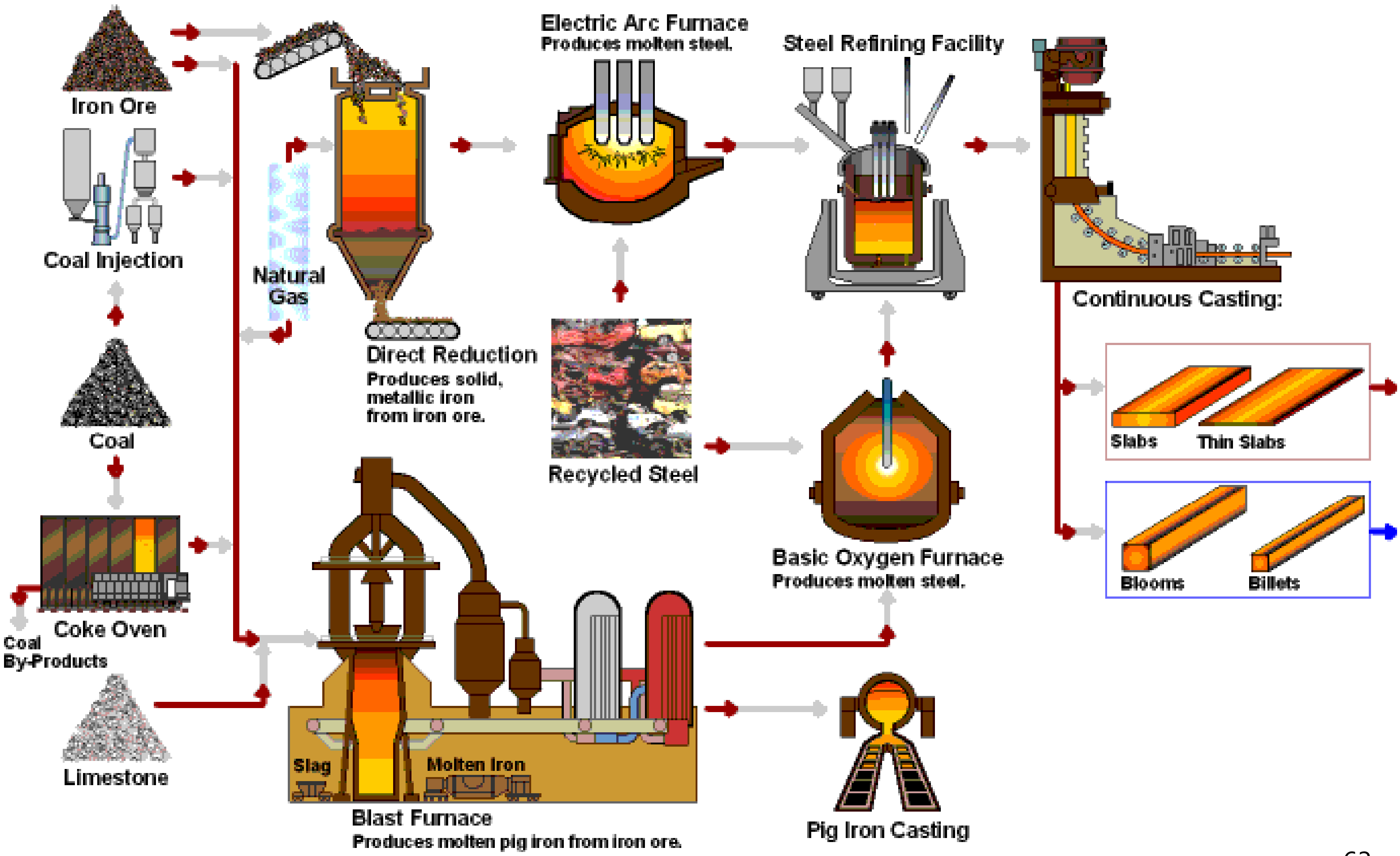
2. continuous casting: a number of dynamic industrial process

The molten metal is poured continuously into a water-cooled mold from which the solidified metal is continuously withdrawn in plate or rod form. (solid-liquid interface)

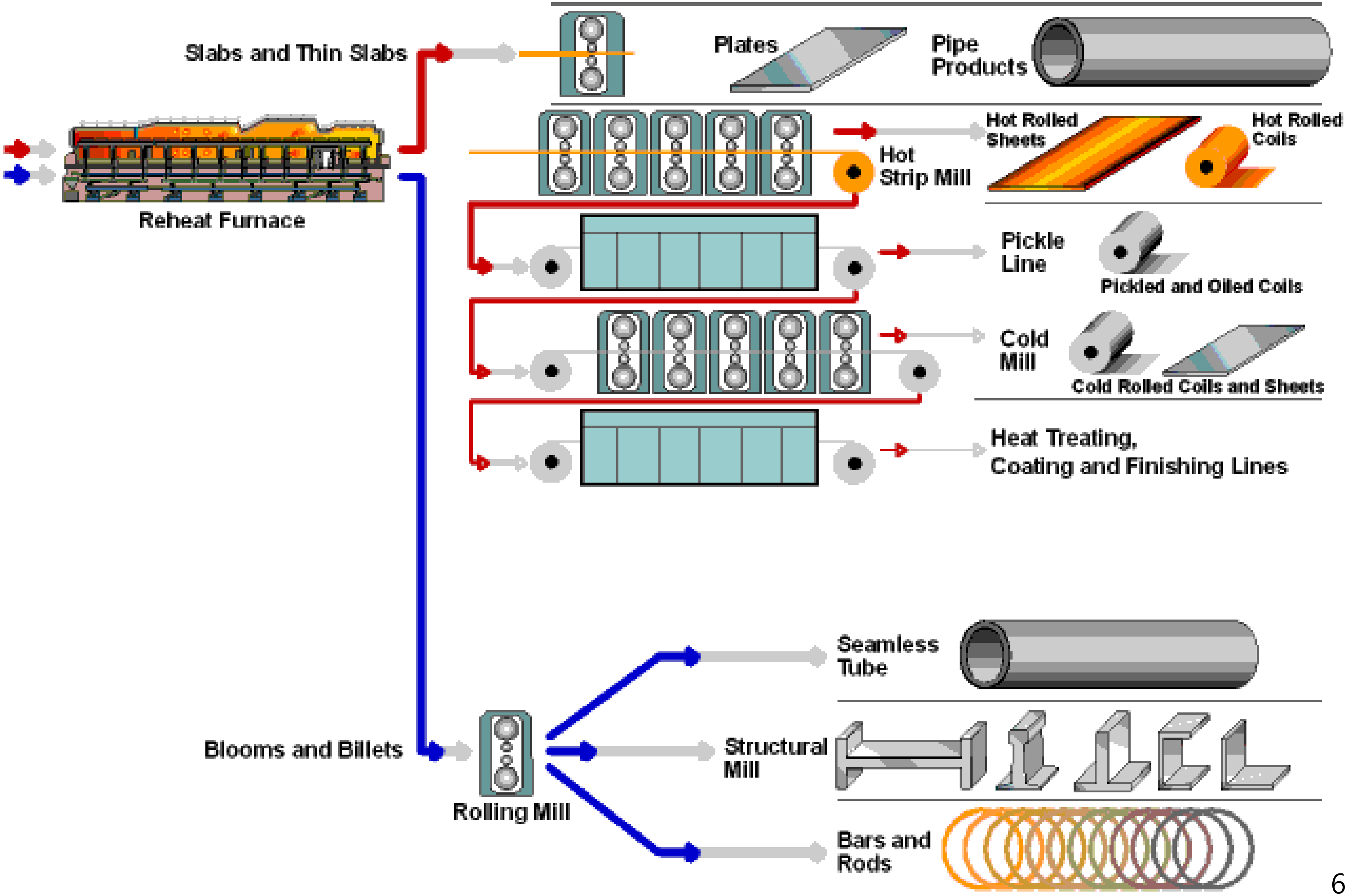


continuous casting

continuous casting



4.4.3 continuous casting

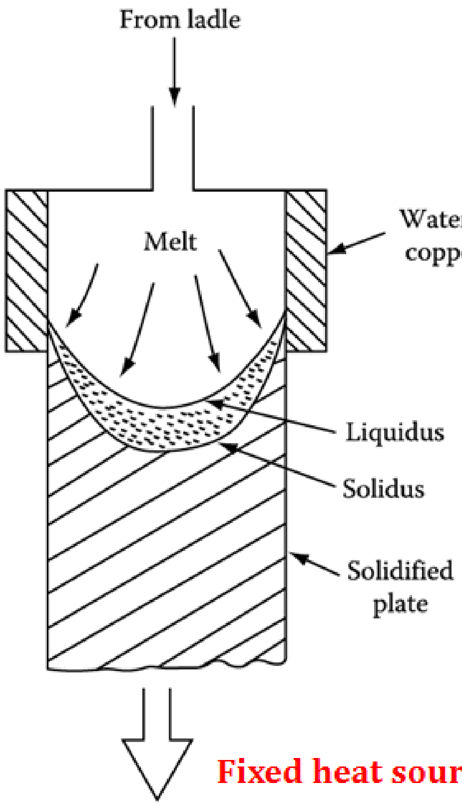


continuous casting: a number of dynamic industrial process

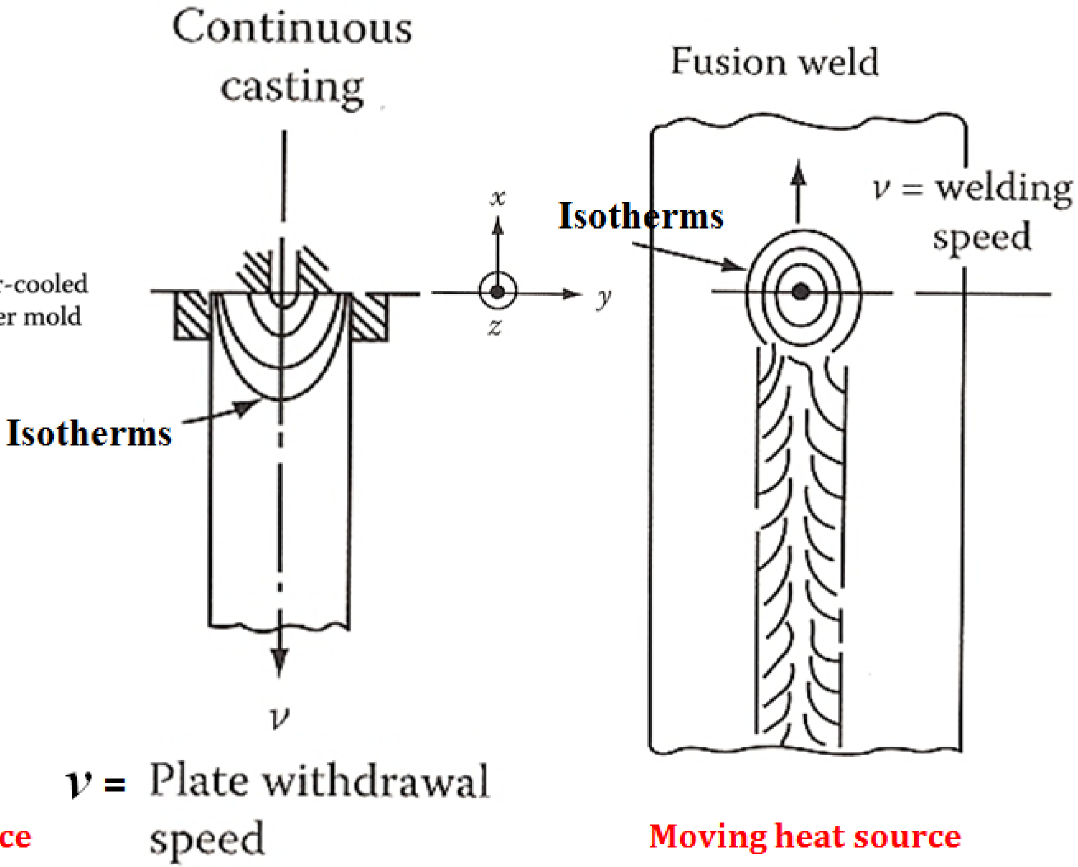
: large mass (economic advantage)/ high speed (property good)

→ Process speed: related to latent heat removal & metal flow during solidification.

① Dynamic process: importance of isotherm distribution



Fixed heat source



v = Plate withdrawal speed

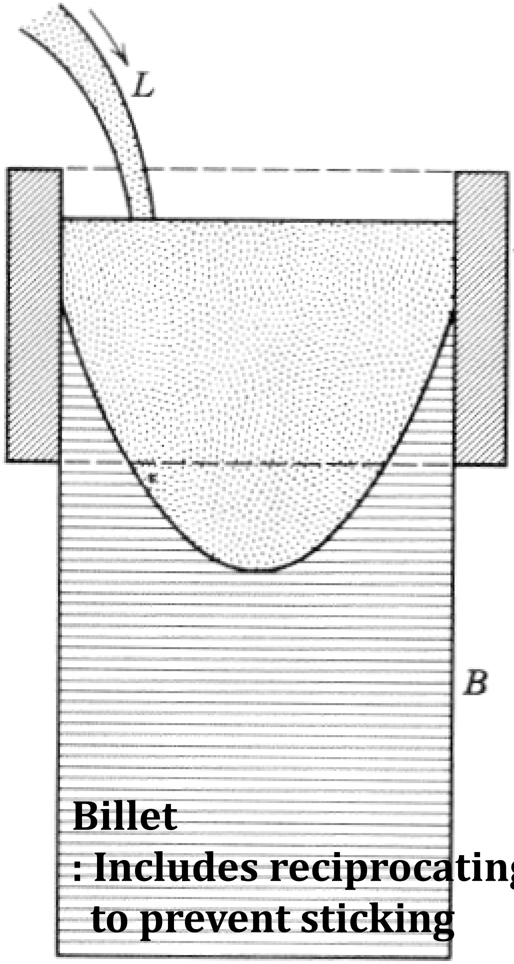
Moving heat source

Fig. 4.44 Schematic illustration of a continuous casting process

Fig. 4.45 Illustrating the essential equivalence of isotherms about the heat sources in fusion welding and continuous casting

continuous casting: a number of dynamic industrial process

② the temperature gradient is maintained in a steady state→ related to constant shape of the interface and the solidification rate (<u>here, the solidification rate is changed by not to time but to position from the surface</u>)



Mold
: Square or circular section

a) cooling by water spray or water jet/

b) Extra cooling for the metal extracting from the mold

Billet
: Includes reciprocating motion to prevent sticking

Fig. 7.5. Continuous casting (schematic).

* solidification rate varies depending on the position as shown in the following figure

(∵ Since solidification occurs in a direction perpendicular to the solidification interface)

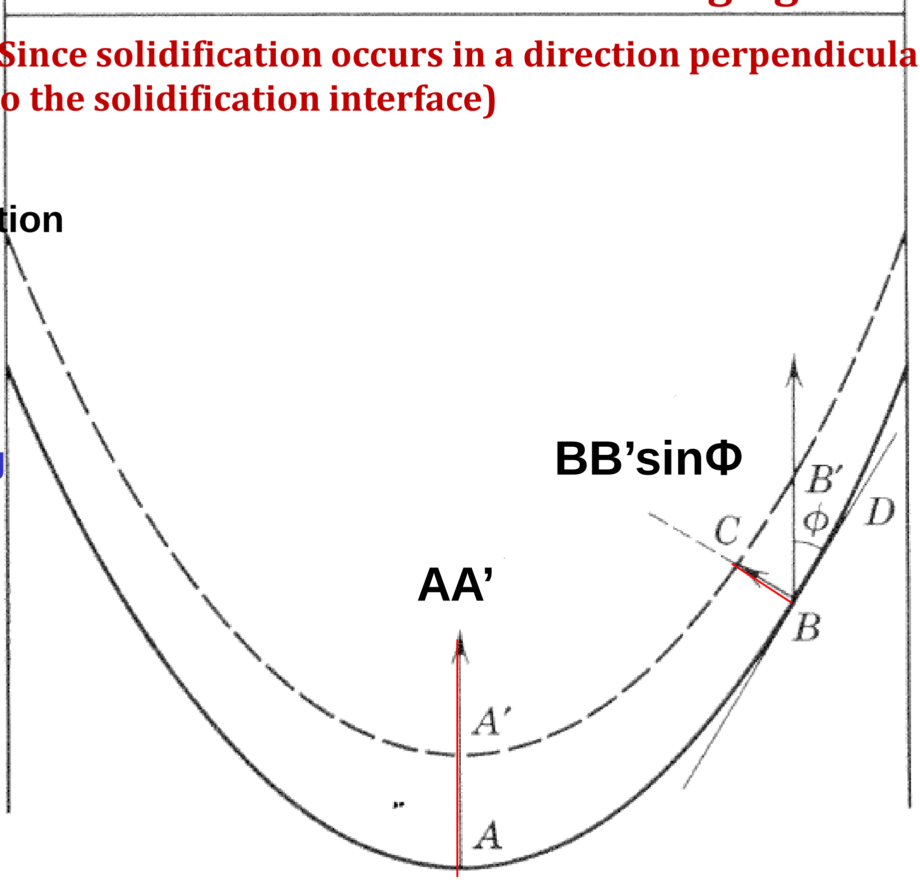


Fig. 7.6. Interface shape and rate of solidification in continuous casting.

* To obtain the interface shape of Fig. 7.6,

in the case of max. emission of latent heat at A/ min. emission of latent heat at D

→ For this, efficient cooling of the billet beyond the mold is necessary.

* To obtain solid-liquid interface shape of Fig. 7.7,

At the billet center, the solidification rate becomes minimum.

→ In this case. a large tendency of segregation and porosity by shrinkage in the center line where solidification ends

→ additional cooling of the lower part of the mold is required

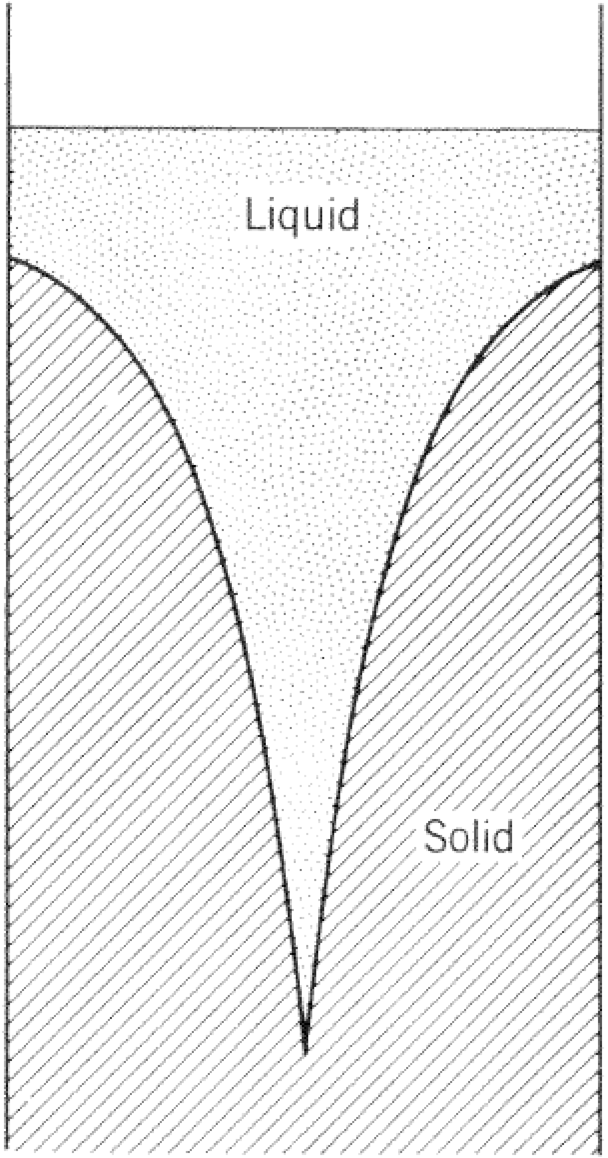


Fig. 7.7. Alternative interface shape in continuous casting

* Quantitative aspects of continuous casting

① depth of liquid core h_c:

$$\boxed{h_c = \frac{I\gamma R^2 v_c}{4K(\theta_f - \theta_s)}}$$

where I = latent heat plus heat extracted, for unit mass, during fall of temperature from θ_f to $(\theta_f + \theta_s)/2$

γ = density

R = radius of billet (assumed cylindrical)

v_c = rate of withdrawal of the billet

K = thermal conductivity of the solid metal

θ_f = melting point

θ_s = surface temperature (assumed uniform)

(1) Depth of core for billets of equal diameter ∝ casting rate (v_c)

(2) Depth of Liquid core ∝ (billet dia.)^2

(3) For billets of a given diameter and a given alloy composition, v_l = impossible to exceed in practice

limiting rate of solidification v_l

$$v_l = \frac{4K(\theta_f - \theta_s)}{I\gamma R}$$

→ The existence of this limit follows from the consideration that as casting velocity ↑ → length of liquid core ↑ → changing angle Φ → a much greater normal freezing rate is required to give the required longitudinal rates.

→ In practice, a very long liquid core produces segregation and porosity difficulties of the kind mentioned in connection with interfaces of reverse curvature

*** Figure 7.8 shows the validity of the calculation of the liquid core depth.**

casting rate ↑ → depth of liquid core (hc) ↑

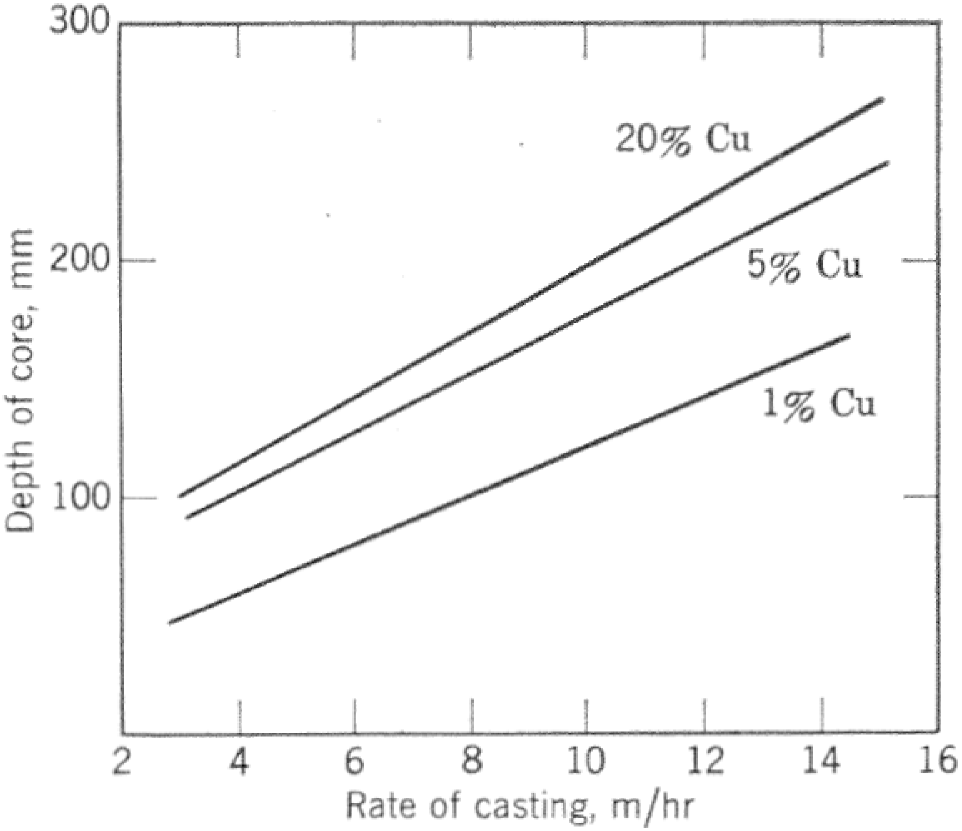


Fig. 7.8. Relationship between depth of liquid core and rate of casting.

TC Find temp. Surface temp.

$$f(h) = \frac{K(\theta_f - \theta_s)}{L + \frac{1}{2}\gamma_c(\theta_f - \theta_s)}$$

Latent heat density

is inversely proportional to the pool depth (*hc*) for a given combination of casting speed and billet size

$$h_c = \frac{I\gamma R^2 v_c}{4K(\theta_f - \theta_s)}$$

*** Table 7.3 shows *f* (*h*) of various alloys.**

Metal	K	L	K/γ_c	$f(h)$
Aluminum	0.53	93	0.82	0.72
Copper	0.92	50	0.98	1.02
Brass	0.28	37	0.33	0.36
Mild steel	0.11	78	0.09	0.10

→ *f*(*h*) steel < *f*(*h*) Al → pool depth (*hc*) of steel >> pool depth (*hc*) of Al

①A very long liquid pool increases the danger of the liquid breaking out through the solid skin (Strong solid skin is required to withstand the hydrostatic pressure induced by the liquid head at the liquid interface/ more important when <Speed ↑>)
limiting speed or cross section for continuous casting of steel is much less than for other metals due to low *f*(*h*)

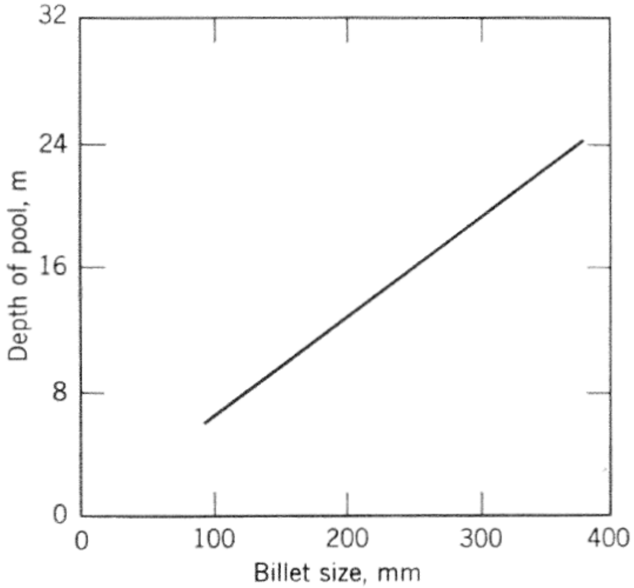


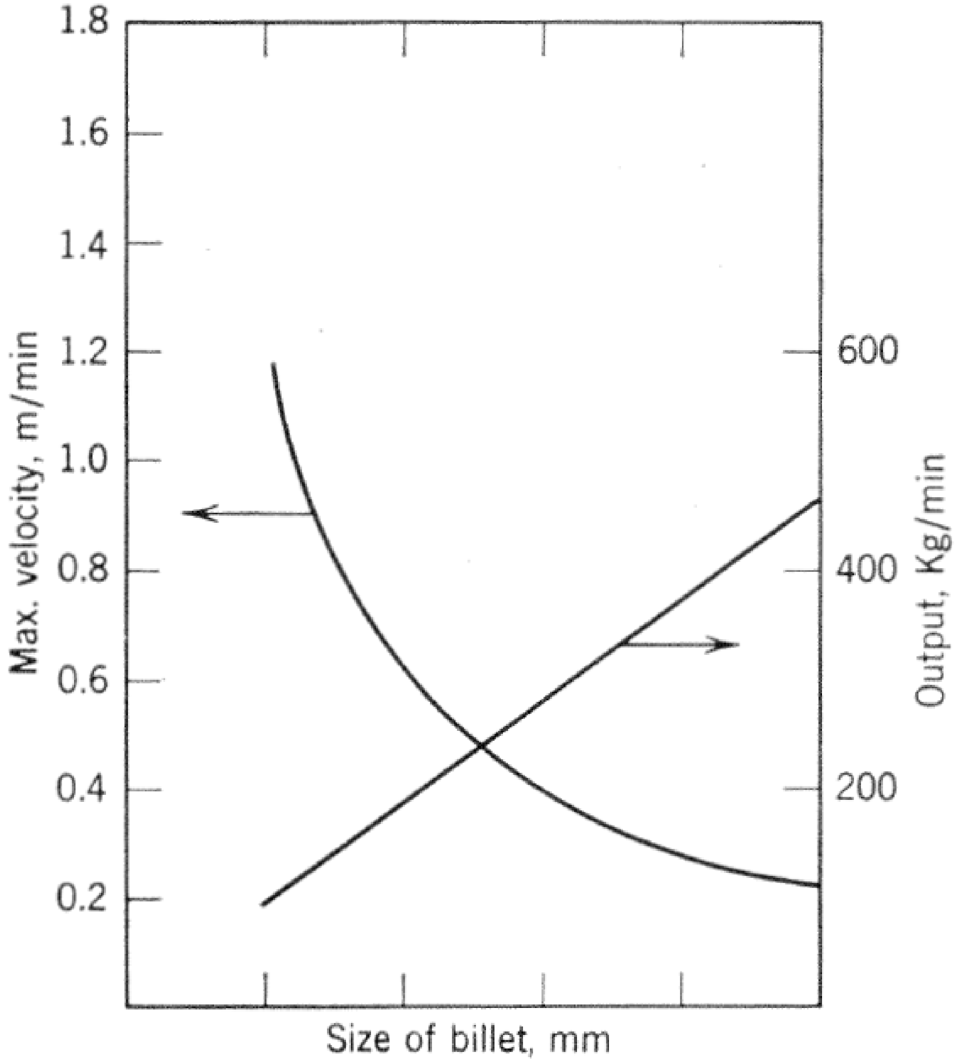
Fig. 7.9. Pool depth for continuously cast steel billets as a function of billet size.

②Billet size ↑ → liquid pool depth ↑

→ The billet can not be cut until the liquid area is completely solidified

→ a very long device is needed for high speed casting

③ **By theoretical considerations a) Increasing the cross section size of the billet is obtained by slower solidification / b) Increasing output is obtained by increasing cross section size.**



7.10. Maximum casting speed and output per unit time as a function of bullet size.

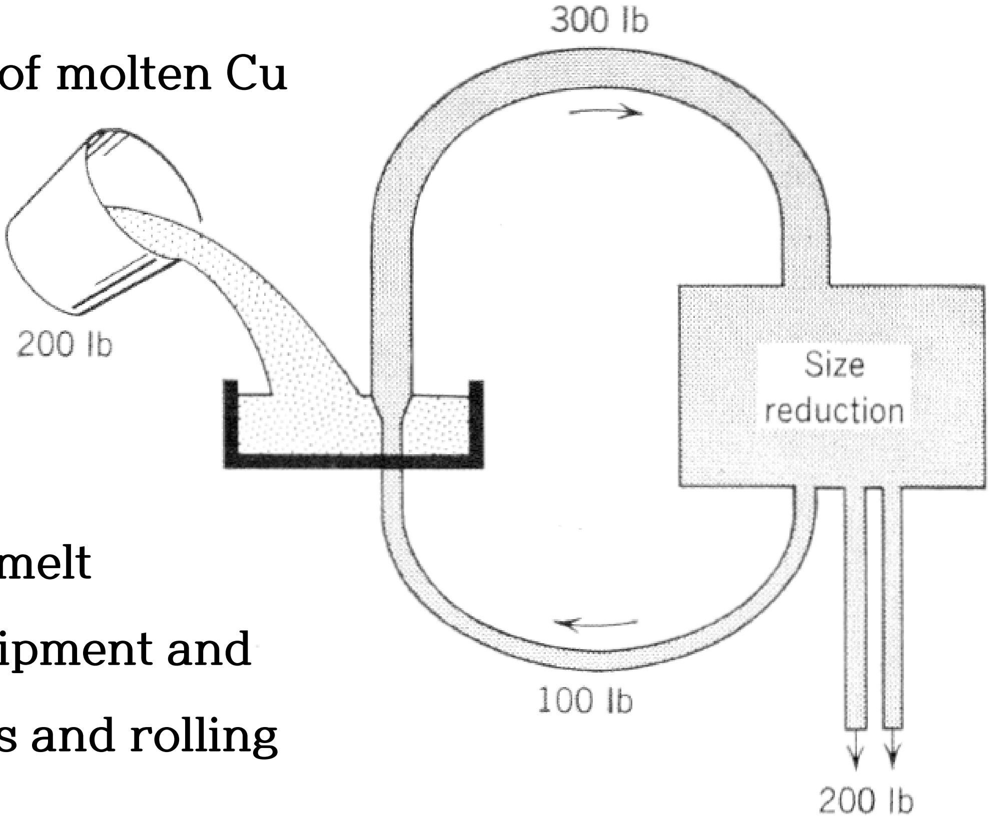
3. Dip forming: 2nd type of continuous casting

: the amount and shape of solidified metal is controlled by a heat sink rather than by a mold/ <u>developed for fabrication of Cu rod</u>: Cu rod (cold when it enters) → heated up by the bath & some of which solidifies on the rod → increase of thickness

100 lb rod → pass through a bath of molten Cu =300 lb rod → 1/3 of it is used for further dip forming (100 lb rod)/ 2/3 200lb rod_ready for further processing

① Direct production of rod from melt

② high speed, eliminating the equipment and time required for pouring ingots and rolling them into rod



7.11. Principle of dip forming.

Dip Casting

- Dip a part or mold into liquid resin
- Resin can be it’s own part (gloves) or it can be a portion of a larger part

* Calculation of thermal factor for Dip forming

① **thickness of added metal ~ *f* (time in bath melt, speed of bar, temperature, thermal property, geometry of the system)**

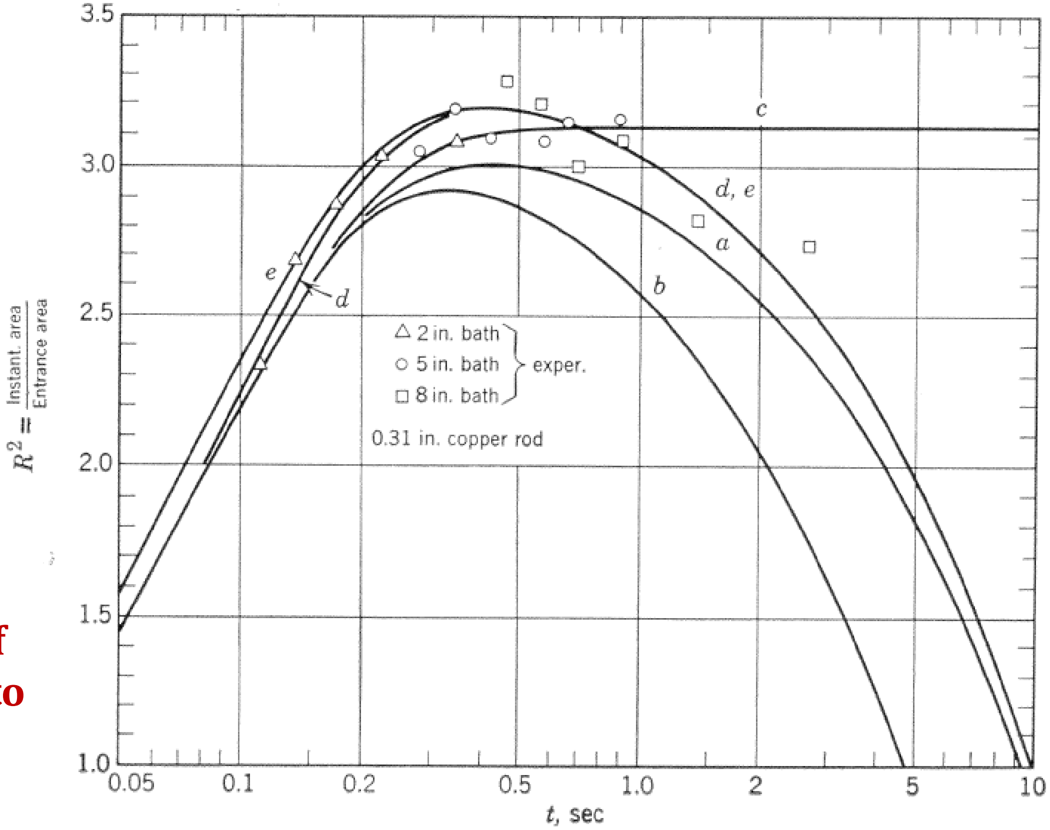
② **amount of added metal ~ if depth of the bath is sufficient, increase to max. and then decreases**

Fig. 7.12 shows some experimental points (for speeds ranging from 15 ft/min to 90ft/min & bath temp. of about and theoretical curves using different values of the effective temp. of the bath and the rod.

→ Calculation and experimental : good similarity

But calculations are made on the assumption of a smooth interface. This condition would be satisfied in the case of pure Cu, but when the process is applied to an alloy, a dendritic or cellular-dendritic interface is to be expected.

∴ Max. amount solidified would be somewhat greater for an alloy



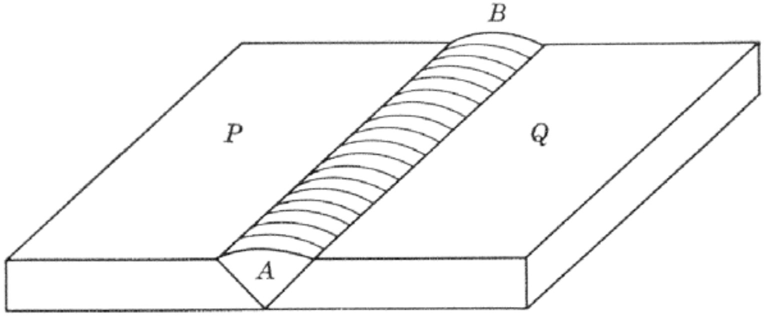
7.12. Comparison of experimental and theoretical results of dip forming of copper.

7.4. Thermal stresses in a solidifying Body

: Effect of macroscopic heat flow → Temperature gradient in the solidified metal
during solidification → different contraction of the solid metal
→ thermal stresses within the solid metal ↑

① Extreme case : welded joint

: Unless the plates are pre-heated, the "bead" contracts far more, after it is solid, than do this plates, and high stresses is in a welded joint btw two metal plates.
→ Plastic deformation may relieve the stresses, but cracking often occurs instead.

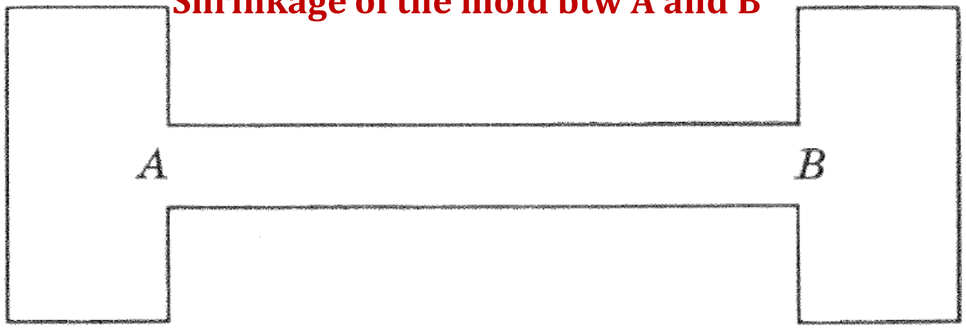


7.13. Origin of stresses in a weld.

② stress arise as a result of contraction of solidified metal

: if alloy solidifies over a long temp. range, the contraction of the first part to solidify may tear the metal apart where there is still some liquid, resulting in extreme weakness, although the casting may appear to be sound.

→ "hot tearing"



Shrinkage of the mold btw A and B

Fig. 7.14. Origin of contraction stresses in a casting.