2017 Fall

"Calculation and Applications Phase Equilibria"

Principles *of* Solidification

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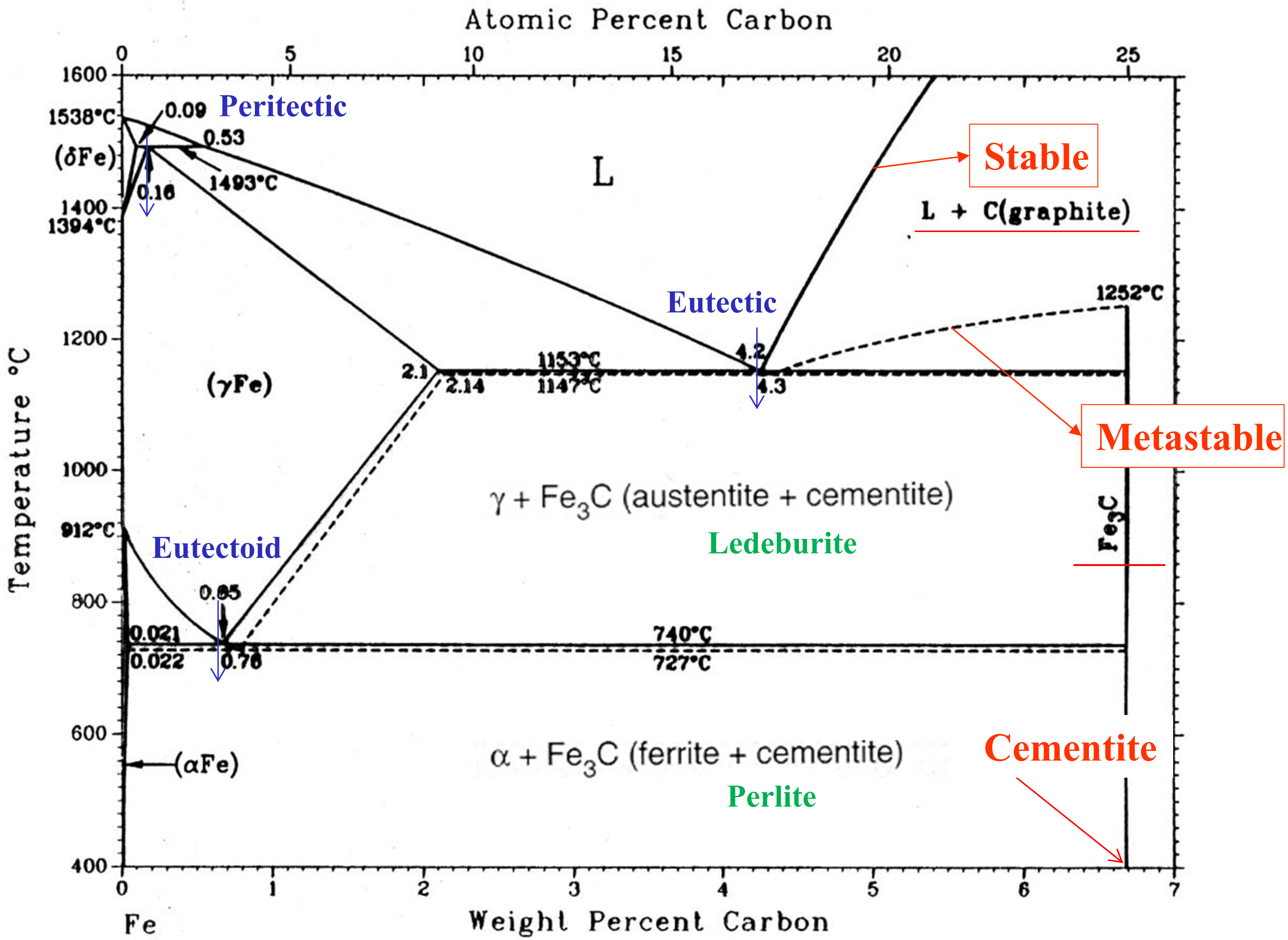
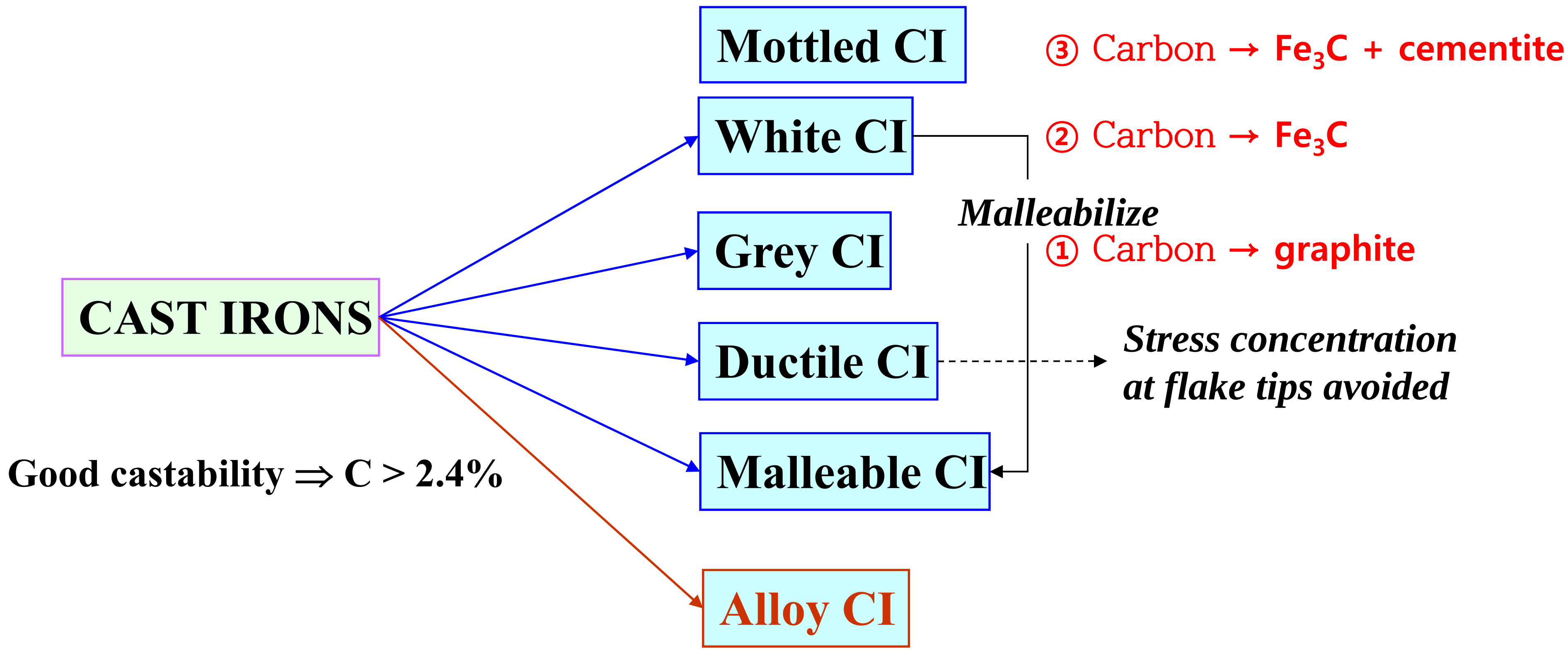
12) Cast Iron: Fe-C alloy (1.7 ≦ c ≦ 4.5%)



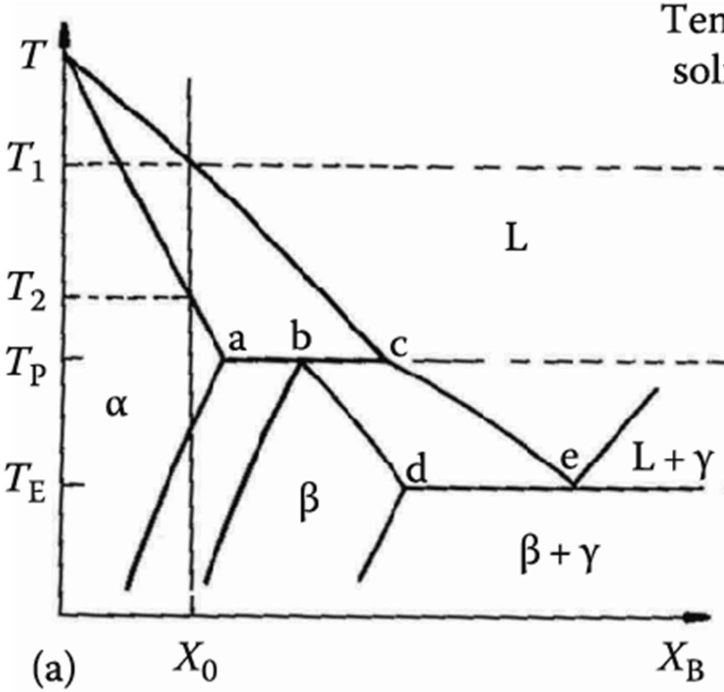
* Two eutectic system: Fe-graphite (stable) & Fe-Fe_3C (metastable)

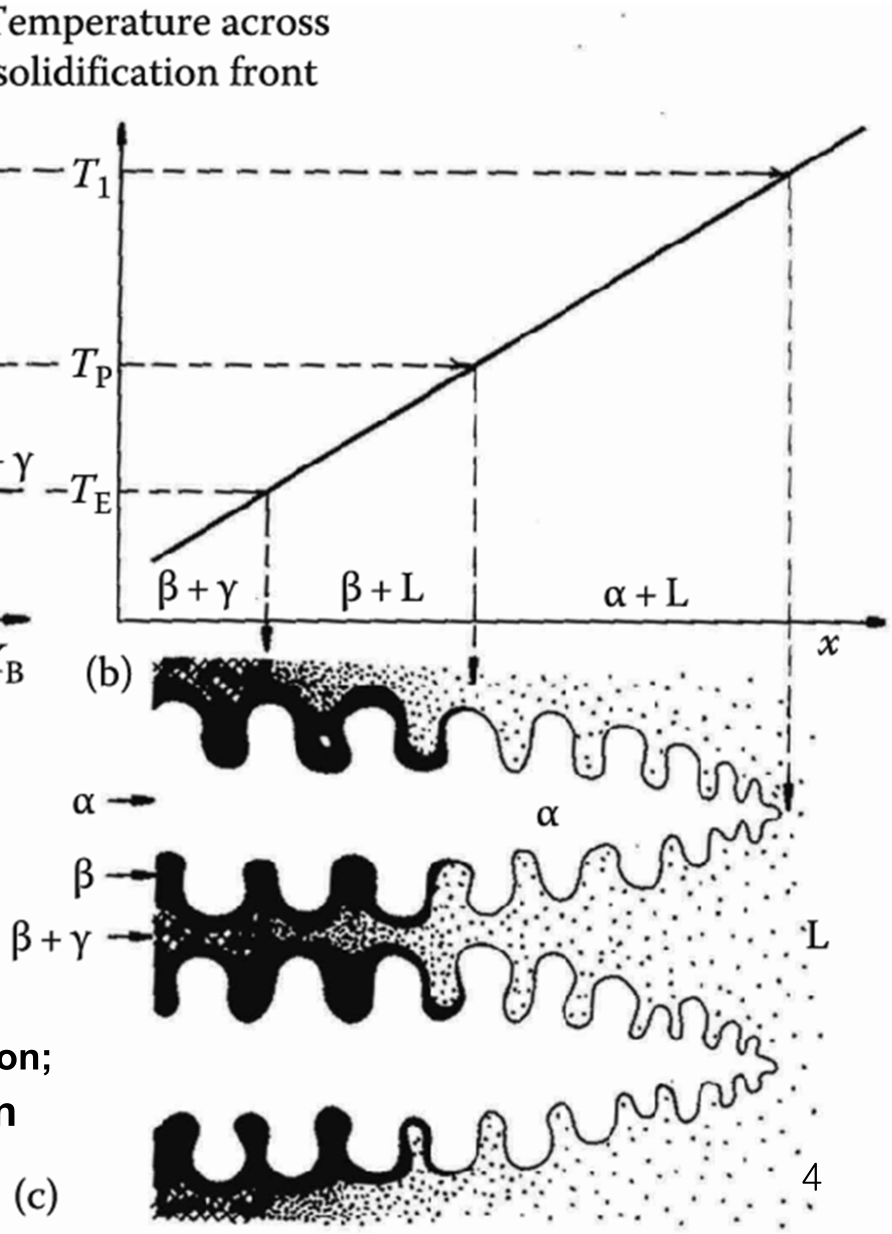
: If there is no other additive element, the Fe-graphite system is stable & Fe-Fe_3C (cementite) eutectic is formed by rapid cooling of liquid phase

* Classification of Cast Iron is possible depending on the type of Carbon.



∗ Peritectic reaction (L + α → β) is a very slow reaction except for the initial state, because liquid and α are separated by β





- **L + α → β , difficult to complete.**

- **α dendrites first form at T_1;**
  Liquid reaches the composition ‘c’;
  β forms as the result of the peritectic reaction;
  α coring is isolated from further reaction
  finally β + γ eutectic forms.

6.4. Solidification in the presence of a solid phase

- Three factors that may influence the final location of a particle

(1) If "density" of particle is different from that of liquid: particle ~ float or sink

- Particle behavior dominated by its buoyancy (positive or negative)

  : depends on density difference and the size and shape of the particle

(2) Second factor = "Fluid motion"_ generated as the liquid enters the mold

large enough to maintain in suspension particles that would sink or float in a stationary liquid

: persist for a considerable time before it gives way to convection caused by thermal and composition gradient.

(3) Third factor = "Interface speed" : Although there may be some vertical separation due to flotation or sedimentation, and some radial separation resulting from centrifugal forces, the smaller particles may remain suspended with a nearly random distribution.

→ ∴ The final distribution in the solid depends on whether a particle is "trapped" in situ by the advancing S-L interface or whether it is pushed ahead as the interface moves forward.

7. Macroscopic Heat Flow and Fluid Flow

7.2. Fluid Flow

* The ability of a molten metal to flow =

(1) poured from a container in which it was melted into a mold in which it is to solidify.

: effect of the macroscopic geometry of the casting (Chapter 7)

(2) Relative motion of different parts of the liquid can occur while it is solidifying.

: its implications in relation to the structure of the solidified metal (Chapter 8)

1) Viscosity of liquid metal

liquid metal : "Flow rate depends on the force" = "shear rate is proportional to the shear stress"

→ To compare "rates of flow",

Reynolds' number = $\gamma v l / \mu$

γ = density , v = velocity,
μ = viscosity, l = linear dimension

* **If the value of Reynolds' number is <u>high</u> (>1400) for a tube leading out of a containing vessel, <u>the flow becomes turbulent</u> and Q drops below the value that would be calculated from the above formula.** → Derive the "**Kinematic viscosity, μ/γ**" from the above equation : Used for calculation of flow rate when pressure difference is caused by flowing liquid → **For solidification it is considered more important.**

Fragility

- **Fragility ~ ability of the liquid to withstand changes in medium range order with temp.**
  ~ extensively use to figure out liquid dynamics and glass properties corresponding to "frozen" liquid state

< Classification of glass >

Strong network glass **: Arrhenius behavior**

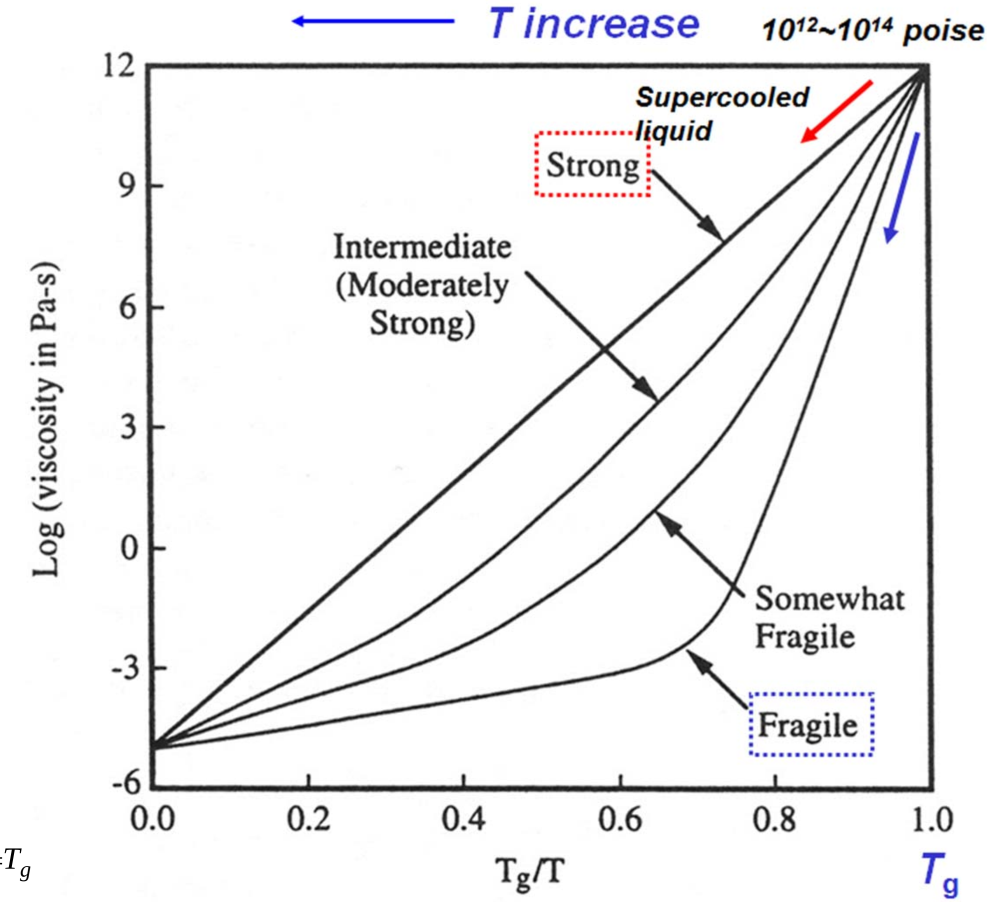
$$\eta = \eta_0 \exp[\frac{E_a}{RT}]$$

Fragile network glass **: Vogel-Fulcher relation**

$$\eta = \eta_0 \exp[\frac{B}{T - T_0}]$$

< Quantification of Fragility >

$$m = \left.\frac{d \log \eta(T)}{d(T_{g,n}/T)}\right|_{T=T_{g,n}} = \left.\frac{d \log \tau(T)}{d(T_g/T)}\right|_{T=T_g}$$



Slope of the logarithm of viscosity, η (or structural relaxation time, τ) at T_g

*Bernoulli theorem: Applicable for dynamic behavior of fluid_Fluid Mechanics

By assuming that fluid motion is governed only by pressure and gravity forces, applying Newton's second law, *F* = *ma*, leads us to the Bernoulli Equation.

For a flowing liquid,

$$p/w + Z + q^2/2g = \text{constant} \text{ along a streamline}$$

The pressure due to head of liquid

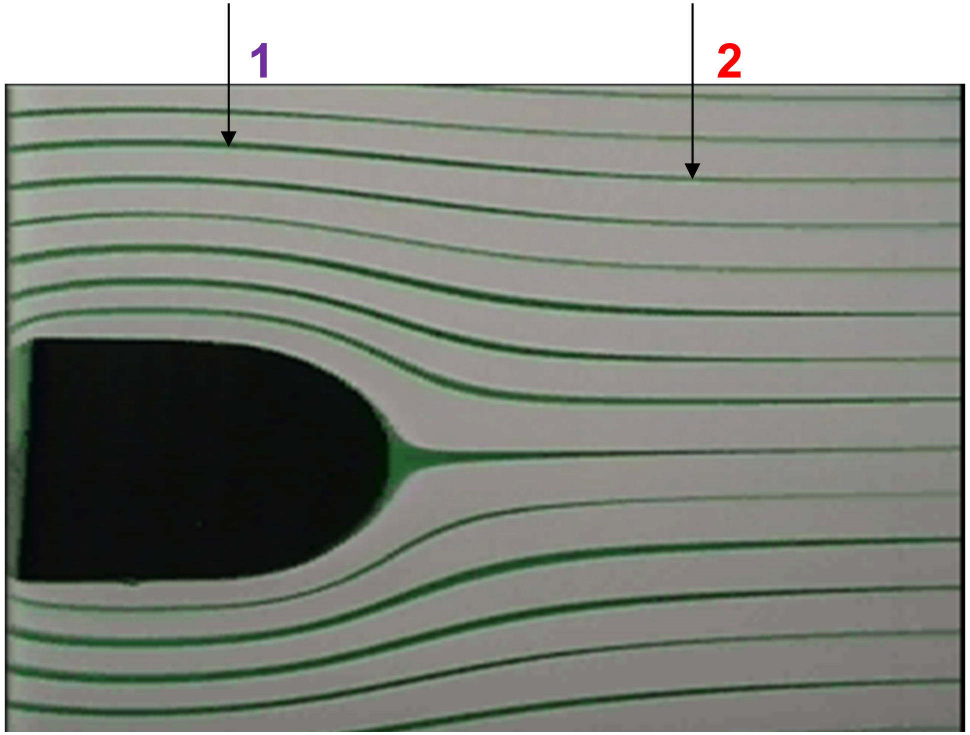
(p= pressure w = specific weight q = velocity g = gravity z = elevation)

In a steady flow, the sum of all forms of energy in a fluid along a streamline is same at all points on that streamline: "principle of conservation of energy"

A streamline is the path of one particle of water. Therefore, at any two points along a streamline, the Bernoulli equation can be applied and, using a set of engineering assumptions, unknown flows and pressures can easily be solved for.

(a) At any two points on a streamline:

$$p_1/w + Z_1 + q_1^2/2g = p_2/w + Z_2 + q_2^2/2g$$



(b) If the fluid velocity, q, of the liquid increases, the pressure of the liquid decreases due to the effect of the passing tube. → ∴ In the case of liquid metals flowing through a complicated mold, the pressure decreases due to the influence of air bubbles entering the liquid phase from the mold wall and flowing together. These air bubbles cause internal void formation in casting.

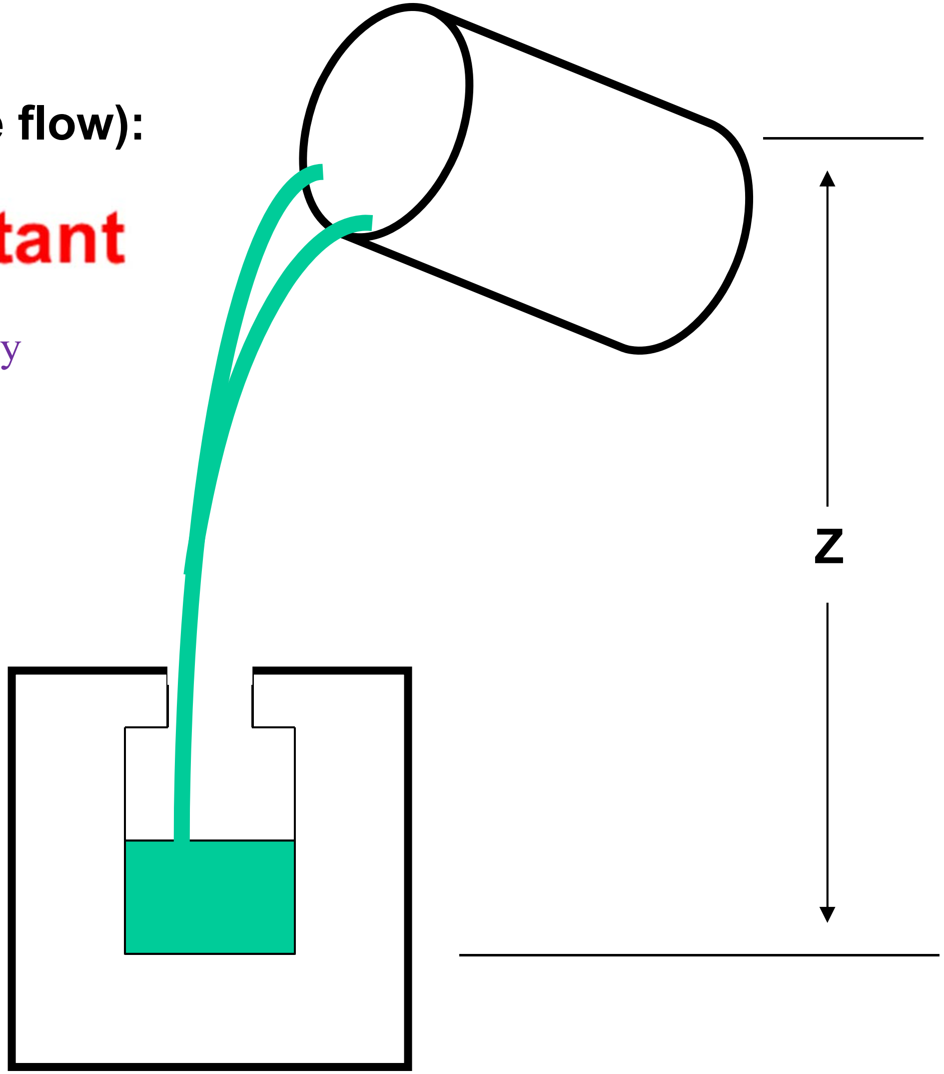
Mold Filling

Bernouli's Equation (incompressible flow):

$$p/w + Z + q^2/2g = \text{constant}$$

(p= pressure w = specific weight q = velocity
g = gravity z = elevation)

principle of conservation of energy



Reynold's Number:

$$\mathrm{Re} = \gamma v l / \mu$$

γ = density , v = velocity,
μ = viscosity, l = linear dimension

- **Short filling times**
- **Potential Turbulence**

* Fluidity: **The ability of being fluid or free-flowing_distinguished from viscosity**

Measurement of Fluidity

: Maximum length melt can reach

(a) Fluidity Spiral

Spiral section enlarged

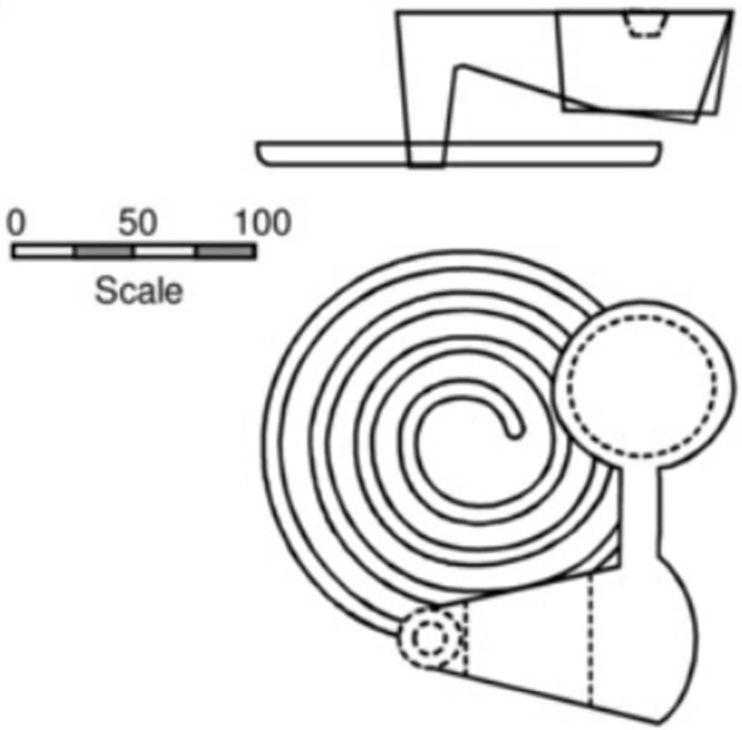
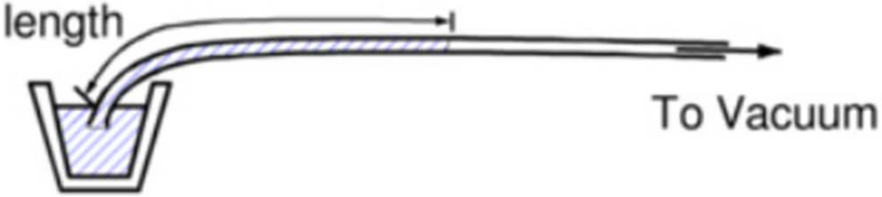


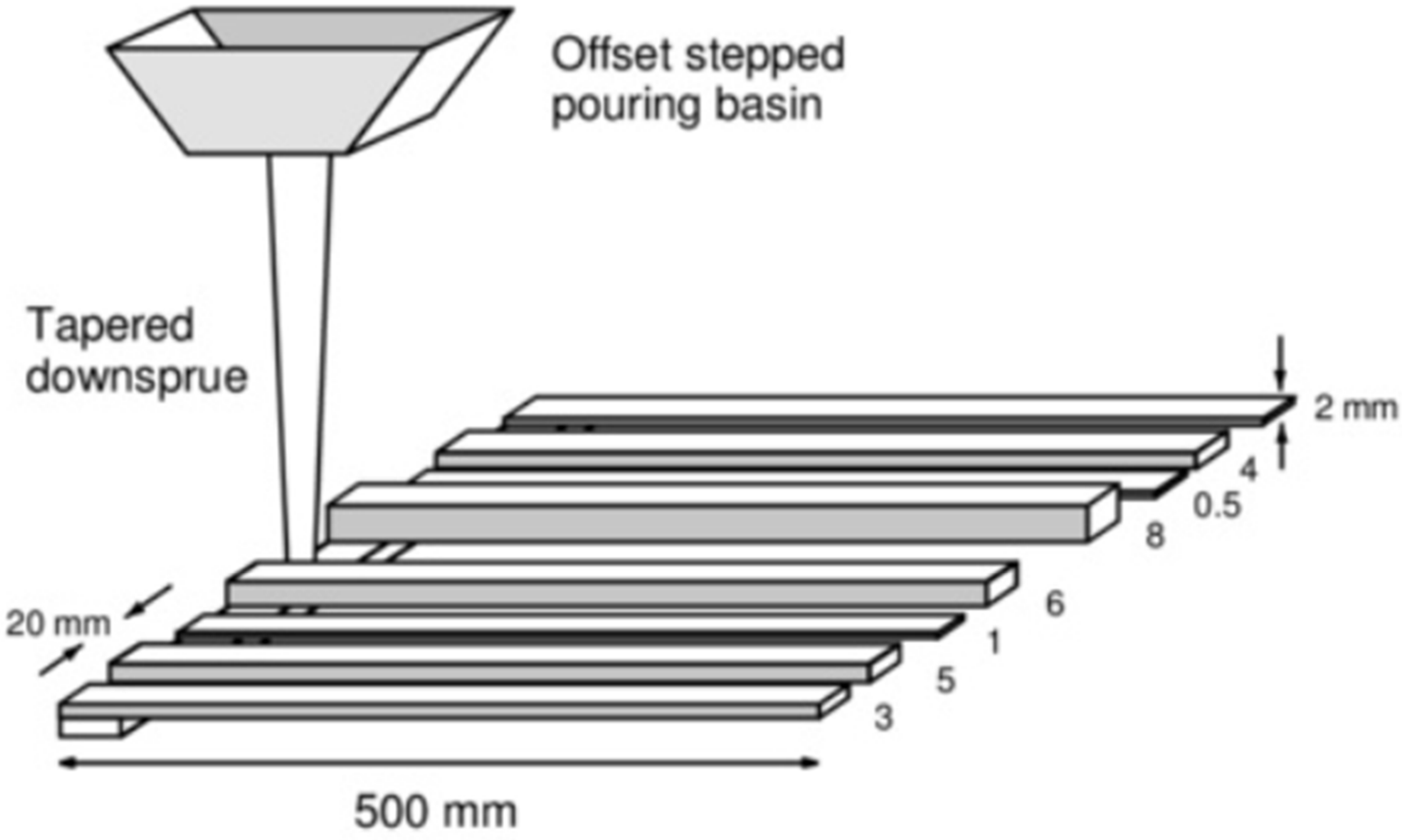
Fig. 7.1. Mold for fluidity test.

(b) Laboratory Test

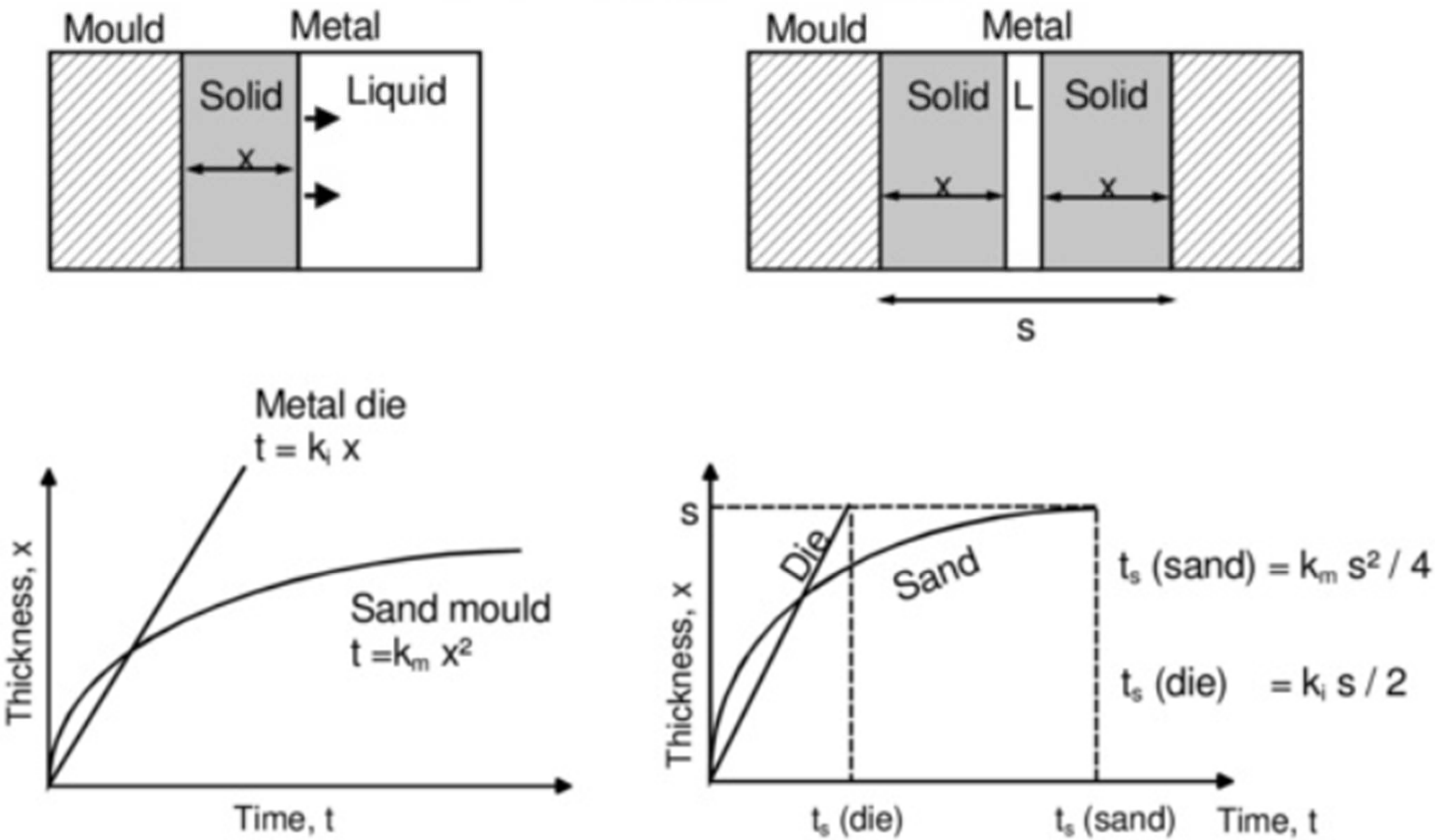
Maximum fluidity length



New Design of Fluidity Test piece

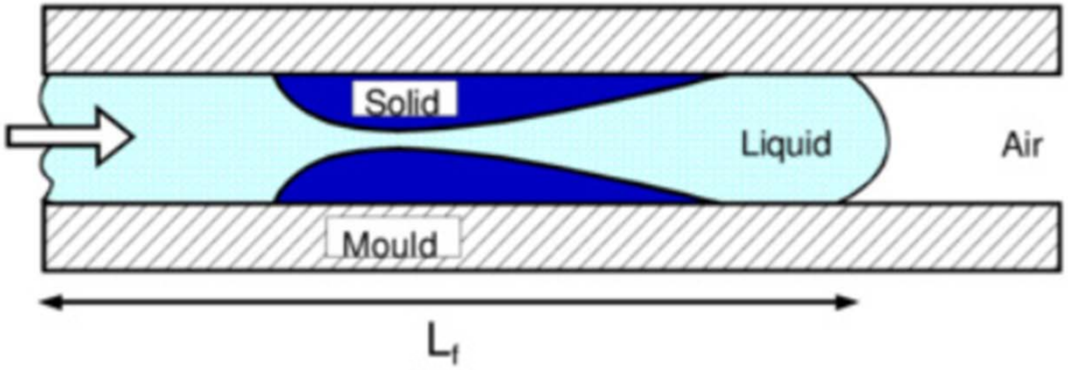


Solidification Rate



① Effect of composition

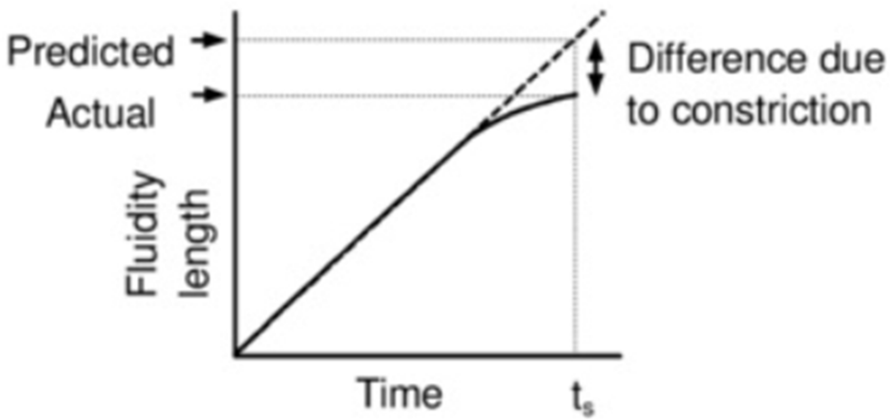
Fluidity of short Freezing Range Alloys



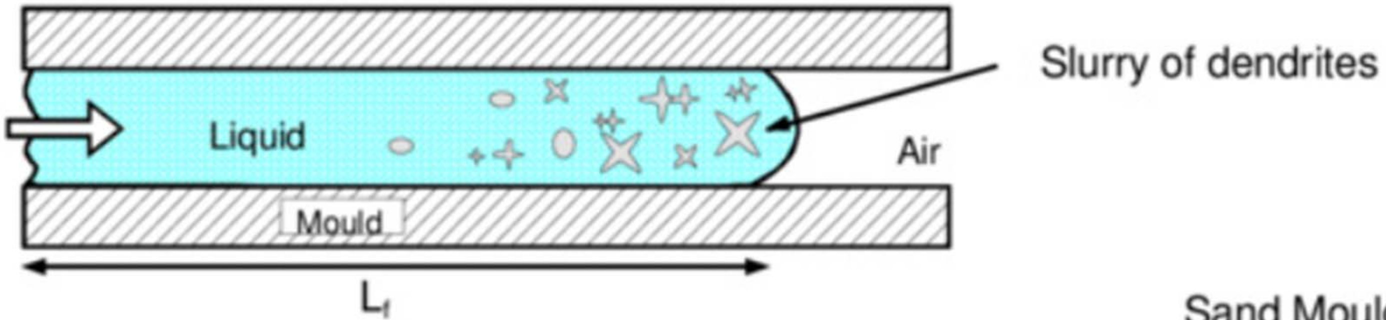
Fluidity Distance $L_f = V\, t_s$ where V = flow velocity

t_s = solidification time

	Sand Mould	Metal Die
t_s	$k_m\, s^2 / 4$	$k_i\, s / 2$
L_f	$V\, k_m\, s^2 / 4$	$V\, k_i\, s / 2$



Fluidity of Long Freezing Range Alloys



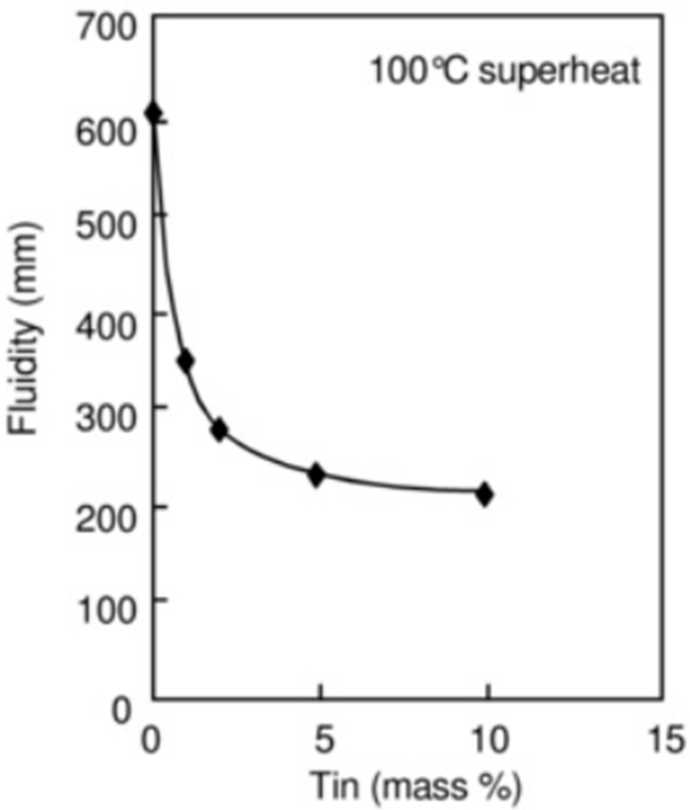
Flow stops when 25 - 50% solid is present,
i.e. when $x = S/8$ to $S/4$

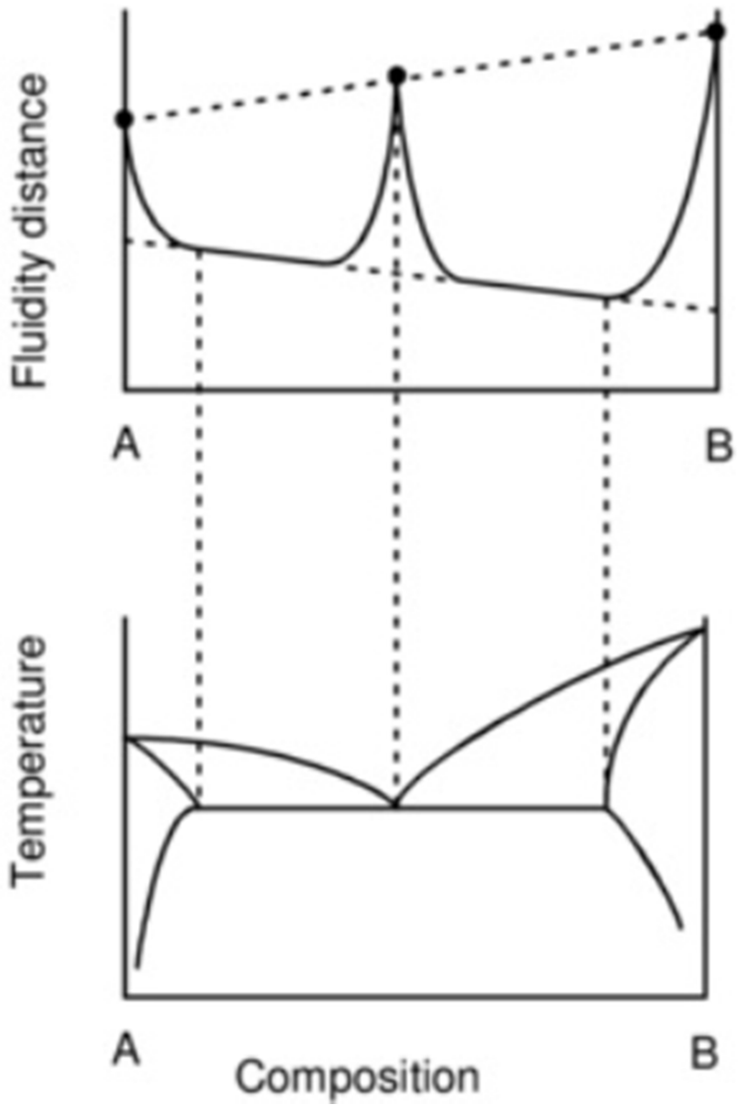
	Sand Mould	Metal Die
	$t = k_m x^2$	$t = k_i x$
25% solid	$t = k_m S^2 / 64$	$t = k_i S / 8$
50% solid	$t = k_m S^2 / 16$	$t = k_i S / 4$
Therefore	$L_f = V k_m S^2 / 64$ to $V k_m S^2 / 16$	$V k_i S / 8$ to $V k_i S^2 / 4$
Remember that for short freezing range alloys:	$L_f = V k_m S^2 / 4$	$V k_i S / 2$

Therefore $\frac{\text{Short freezing range}}{\text{long freezing range}}$: 4 - 16   2 - 4

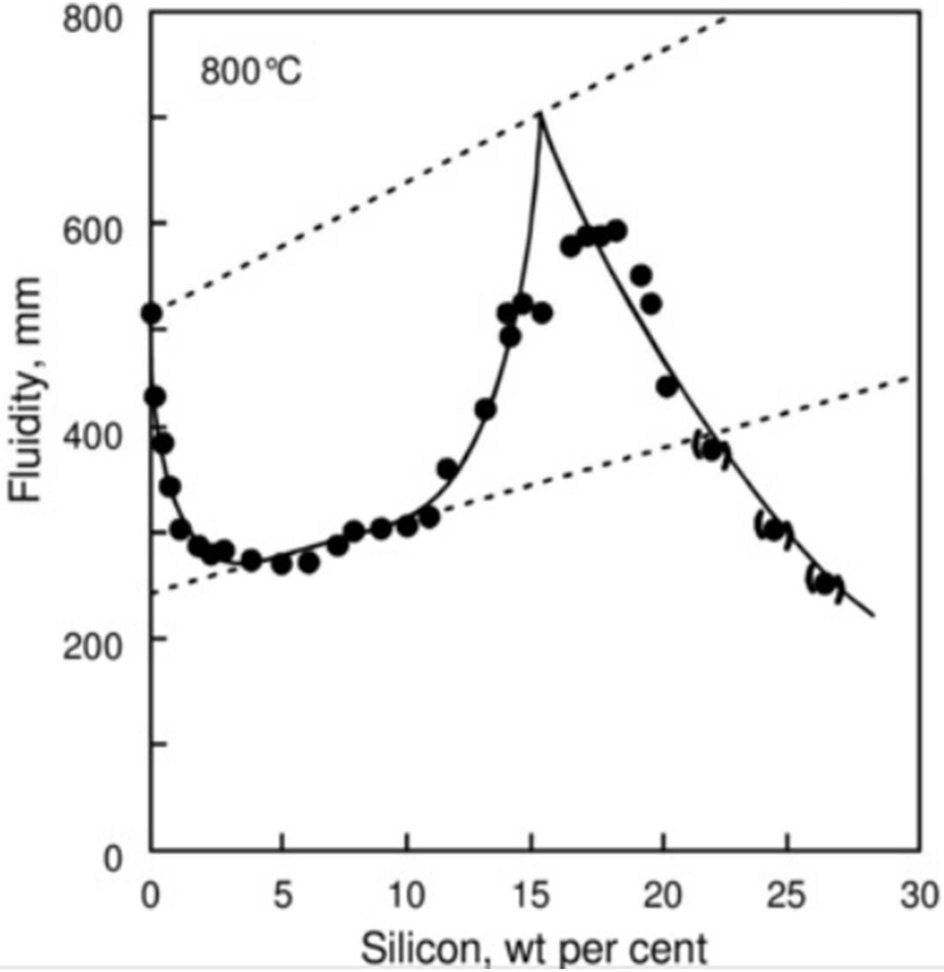
Mapping the Fluidity of Binary Alloys

Fluidity of Al-Sn alloys

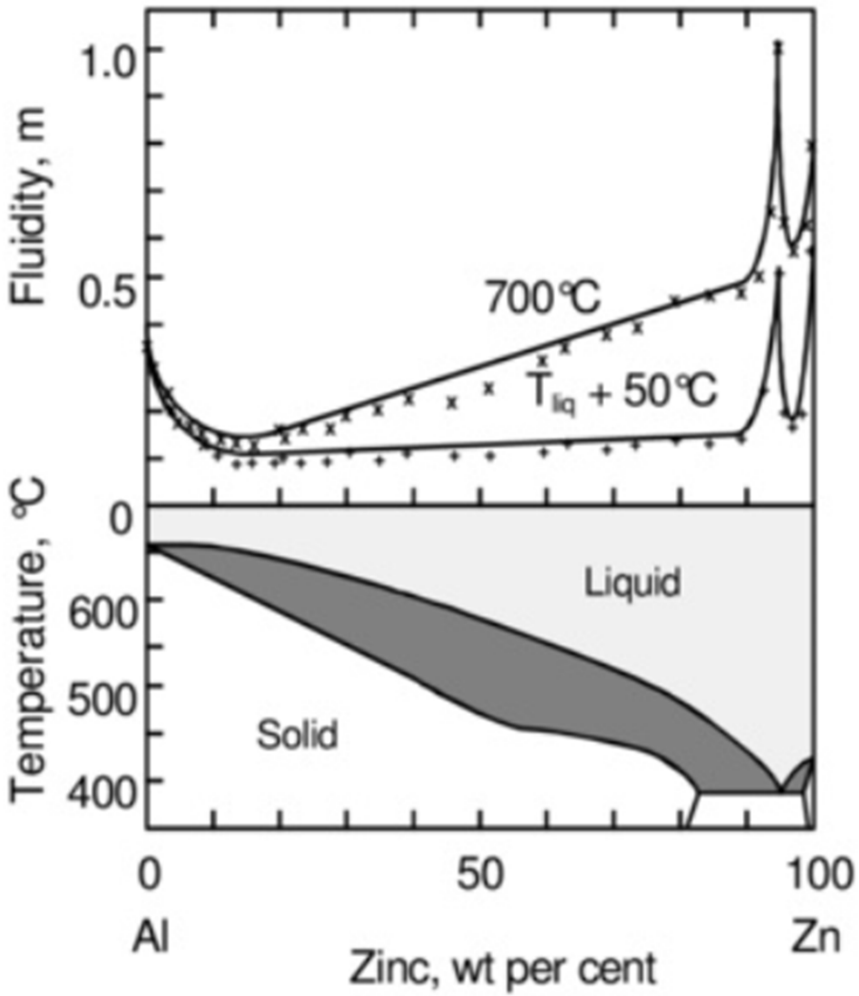




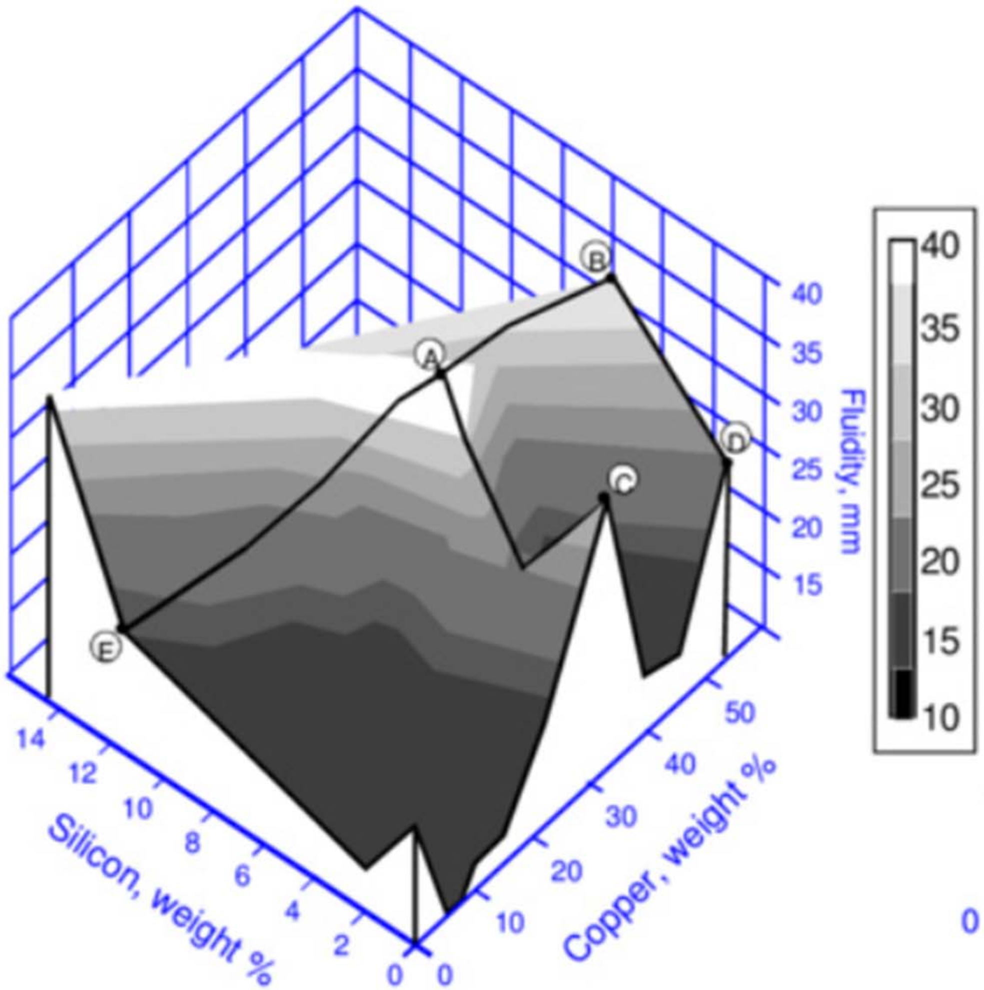
The Fluidity of Al-Si alloys



The Fluidity of Al-Zn alloys

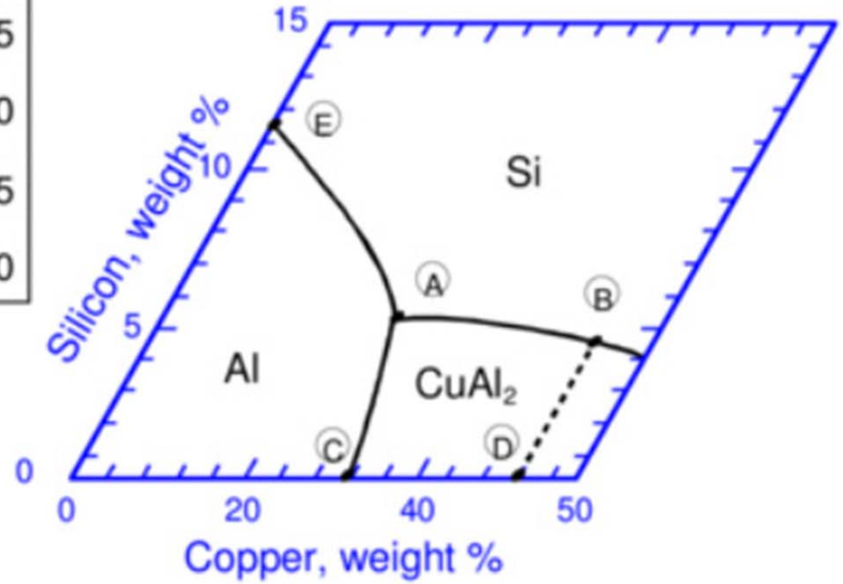


The Fluidity of Al-Cu-Si Alloys



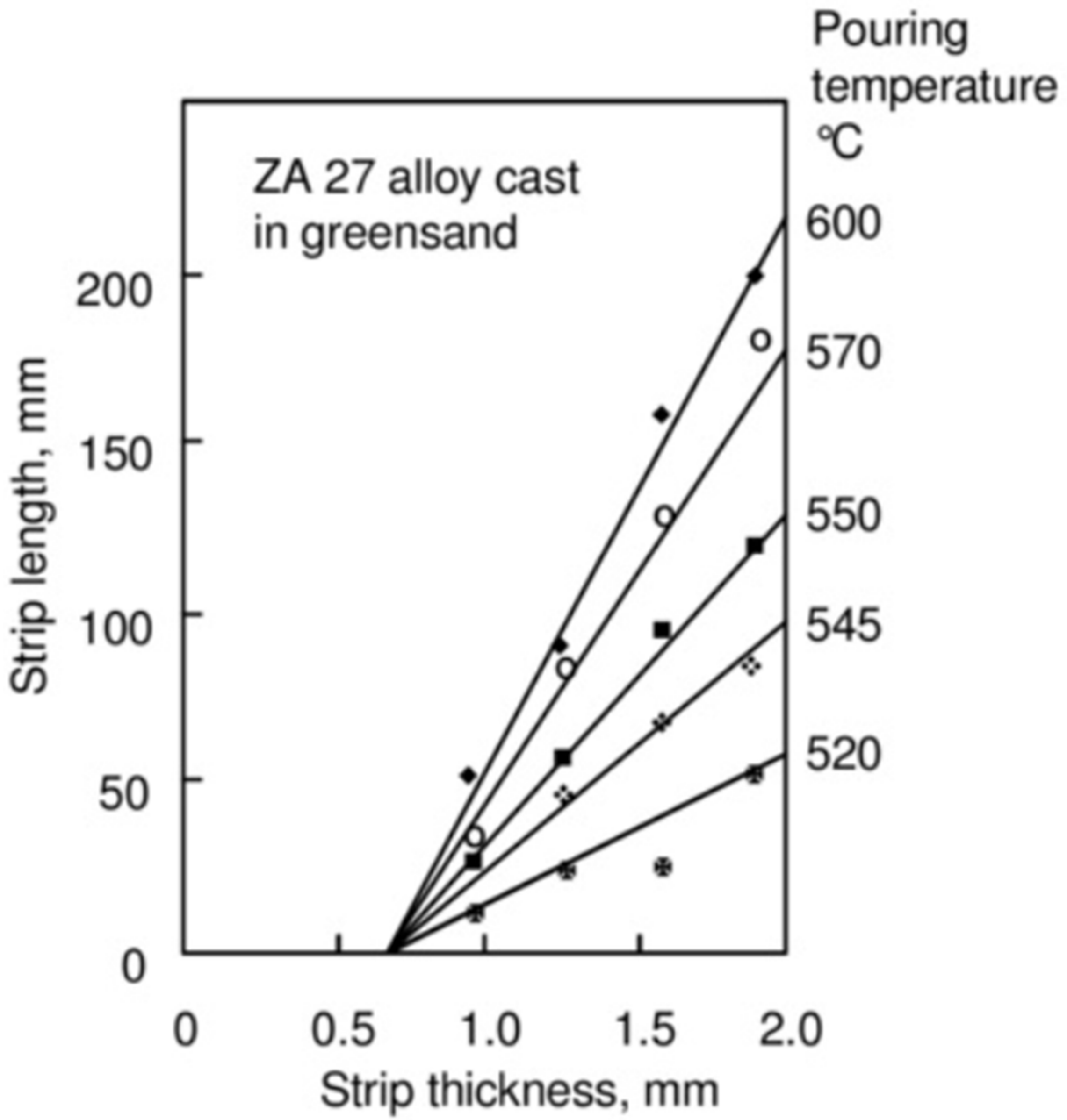
The liquidus surface of the Al-Cu-Si system

	Cu	Si	T℃	
A	27.5	5.25	524	Ternary eutectic
B	51	4.5	571	Ternary eutectic
C	33	-	548	Binary eutectic
D	51	-	591	$CuAl_2$
E	-	11.7	577	Binary eutectic

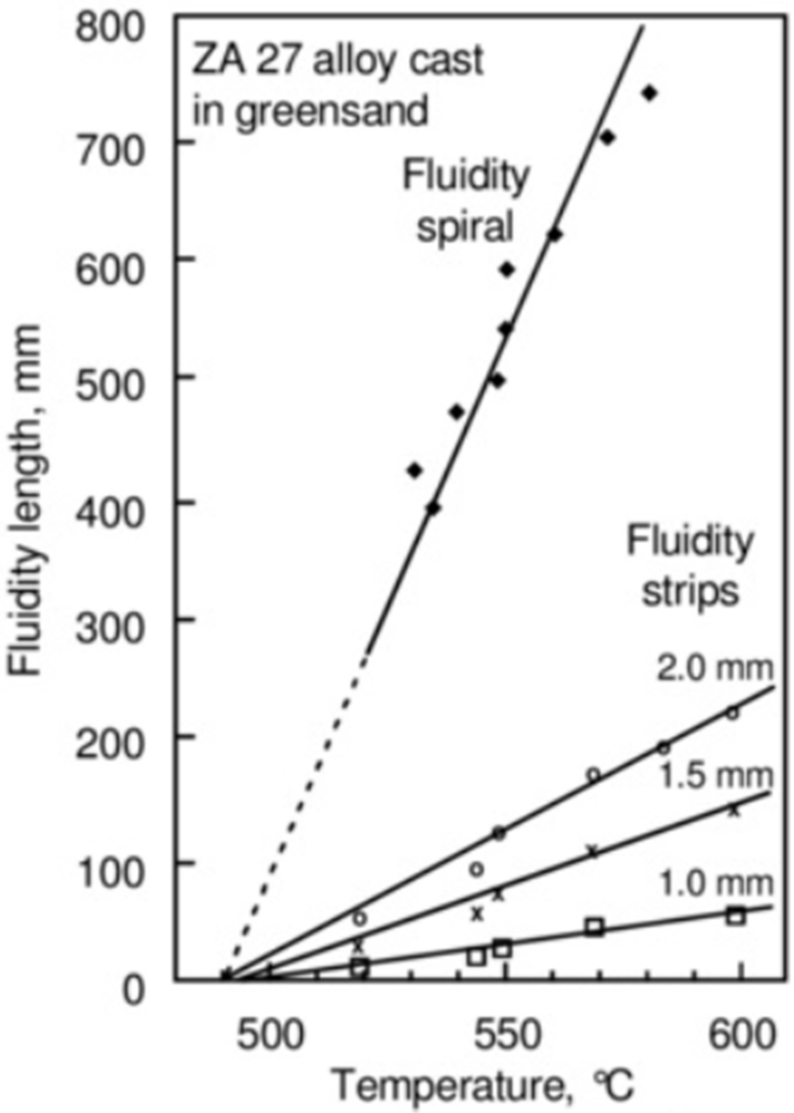


② Effect of temperature

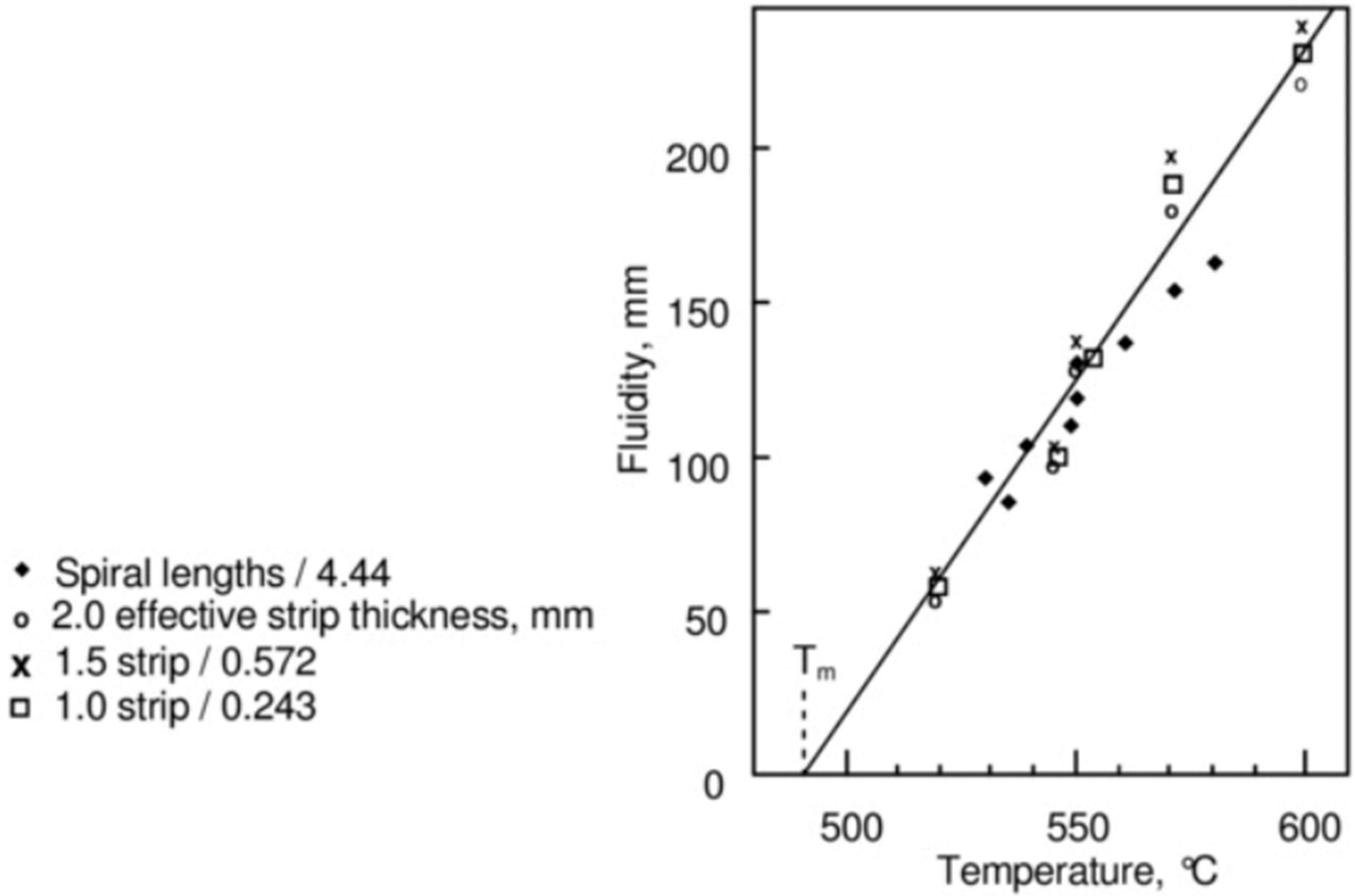
The Fluidity of ZA 27 Zinc-Aluminum Alloy



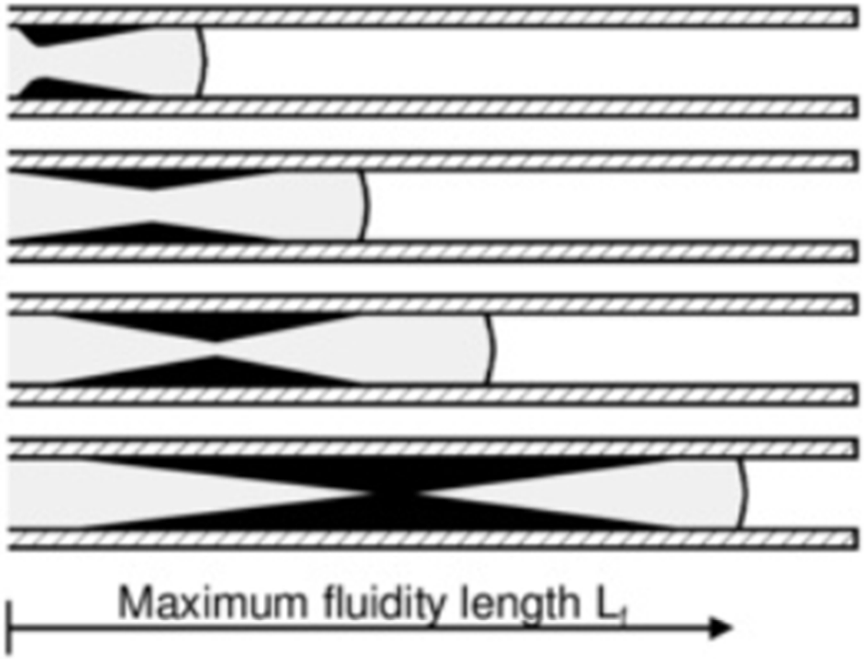
Comparison of Fluidity Measurements

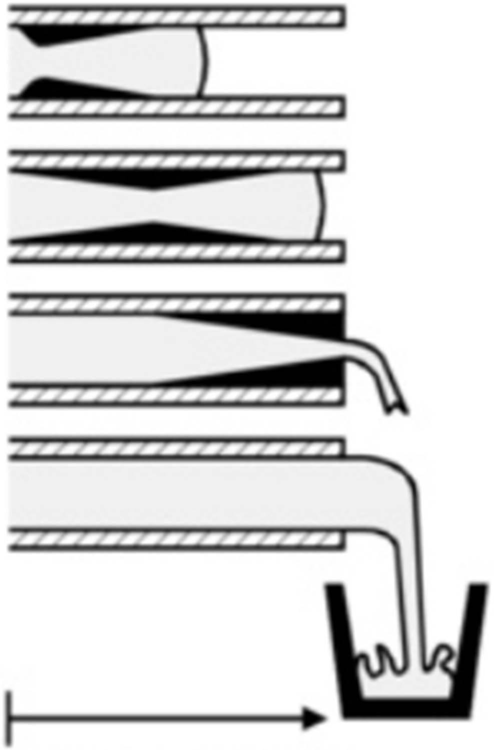


Rationalisation of Fluidity Measurement

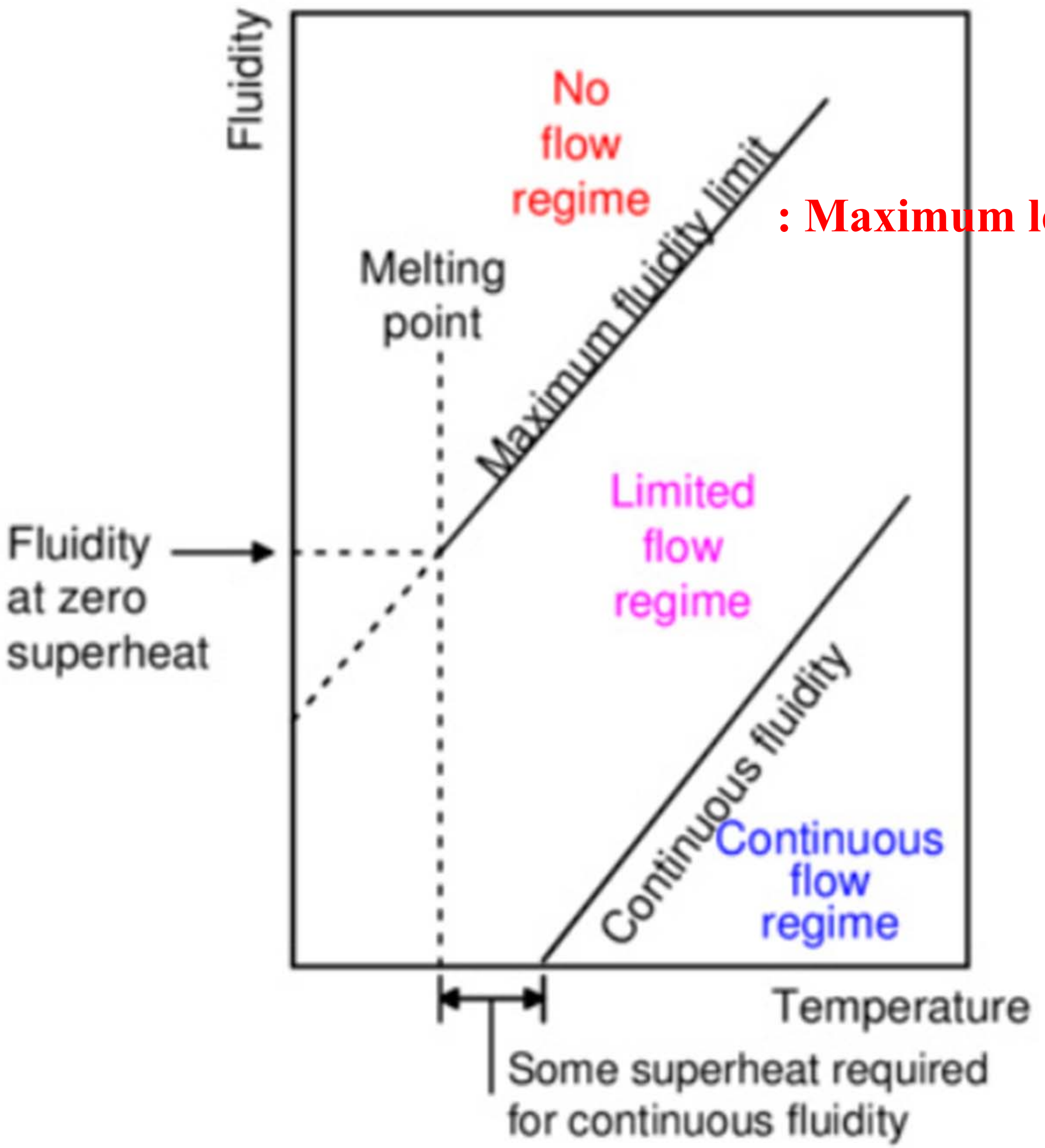


Continuous Fluidity



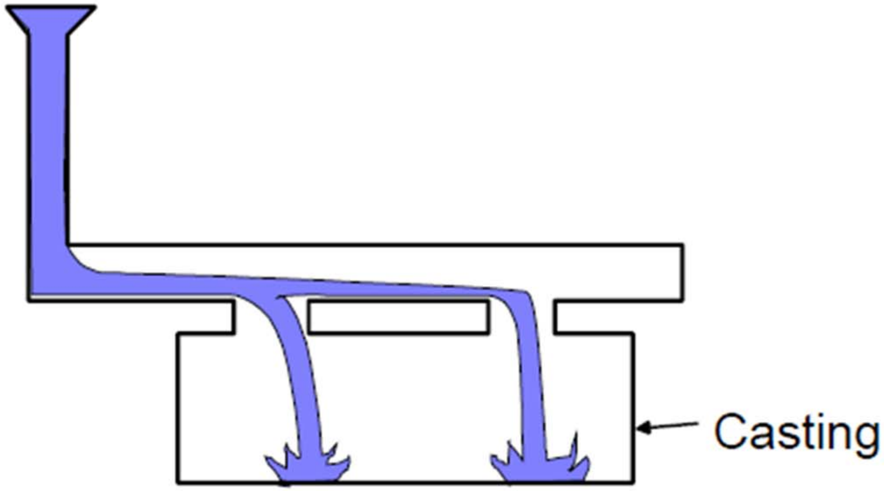


Regimes of continuous, partial and impossible flow

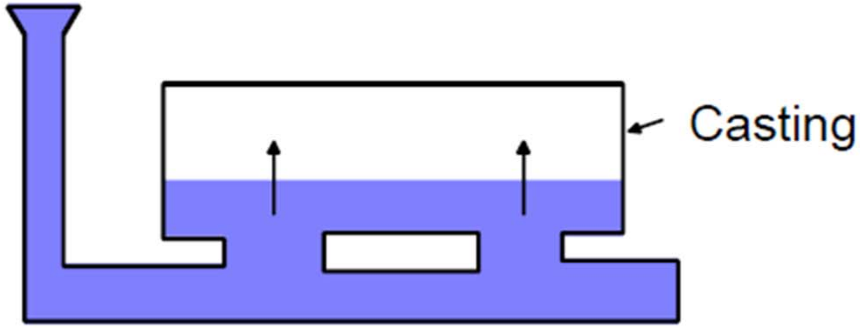


: Maximum length melt can reach

Top versus Bottom Gating

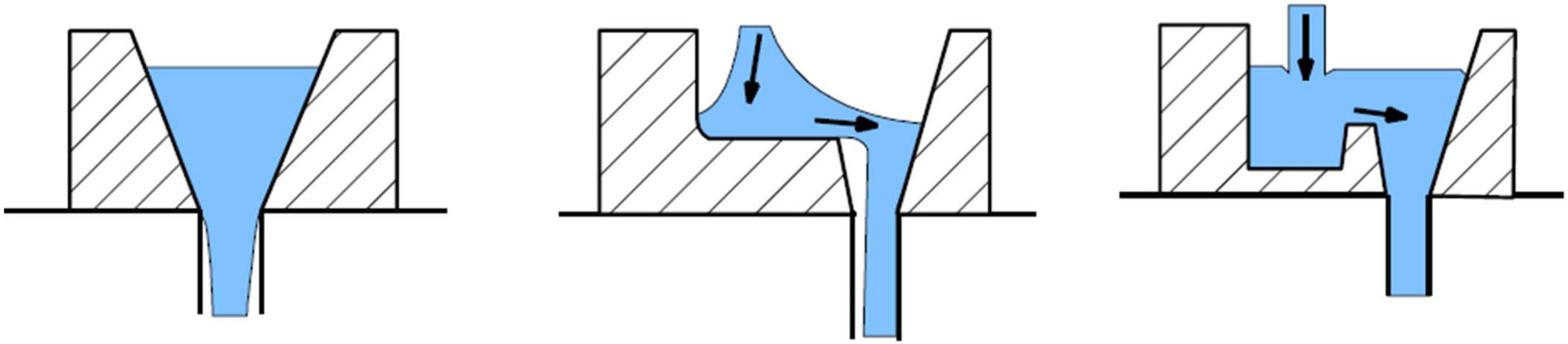


Top gating - causes turbulence



Bottom gating - prevents turbulence

Good Design 1: Pouring Basin



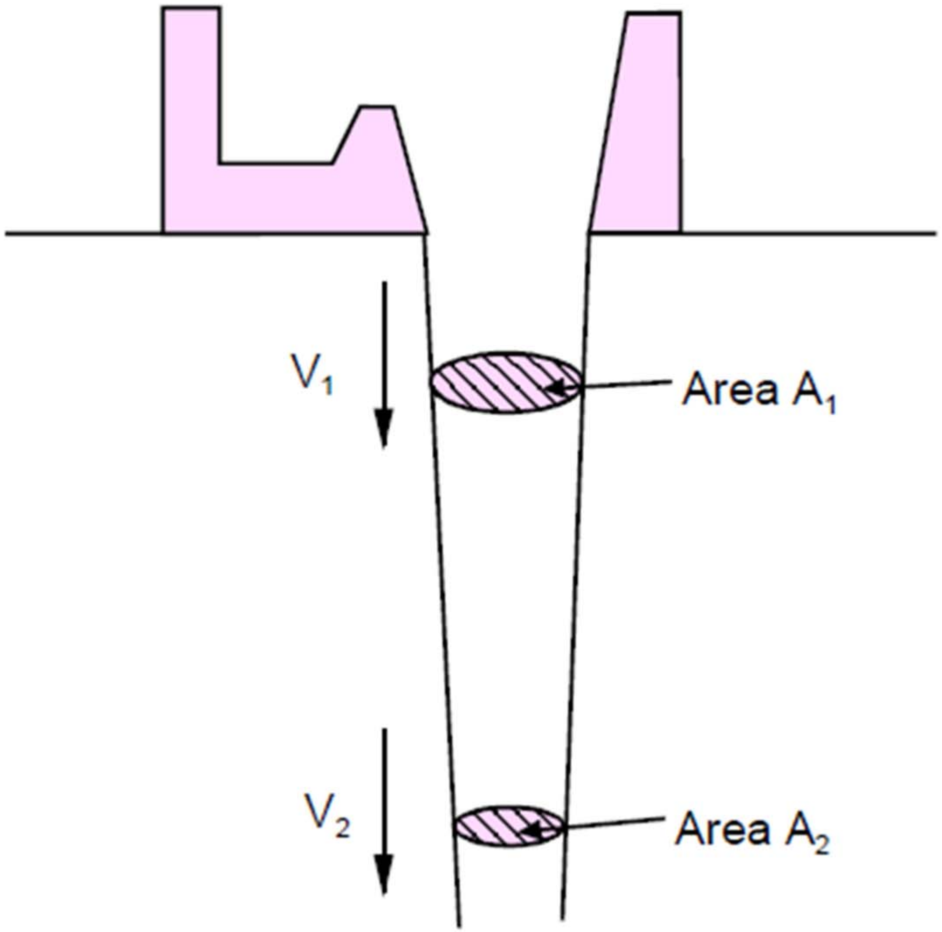
BAD
conical basin

BETTER
offset basin

BEST
offset stepped basin

Good Design 2: Tapered Sprue

탕구



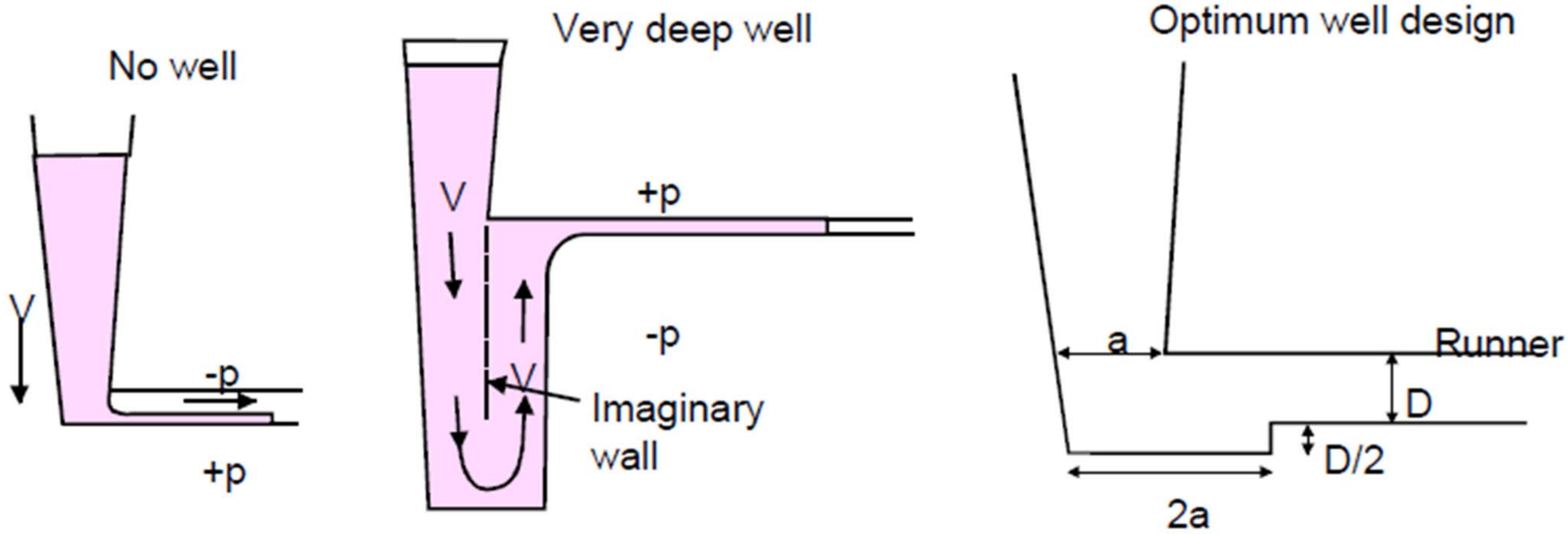
Metal accelerates from V_1 to V_2 due to gravity.

Sprue will remain full of metal if the sprue is tapered so that

$$A_1 \cdot V_1 = A_2 \cdot V_2$$

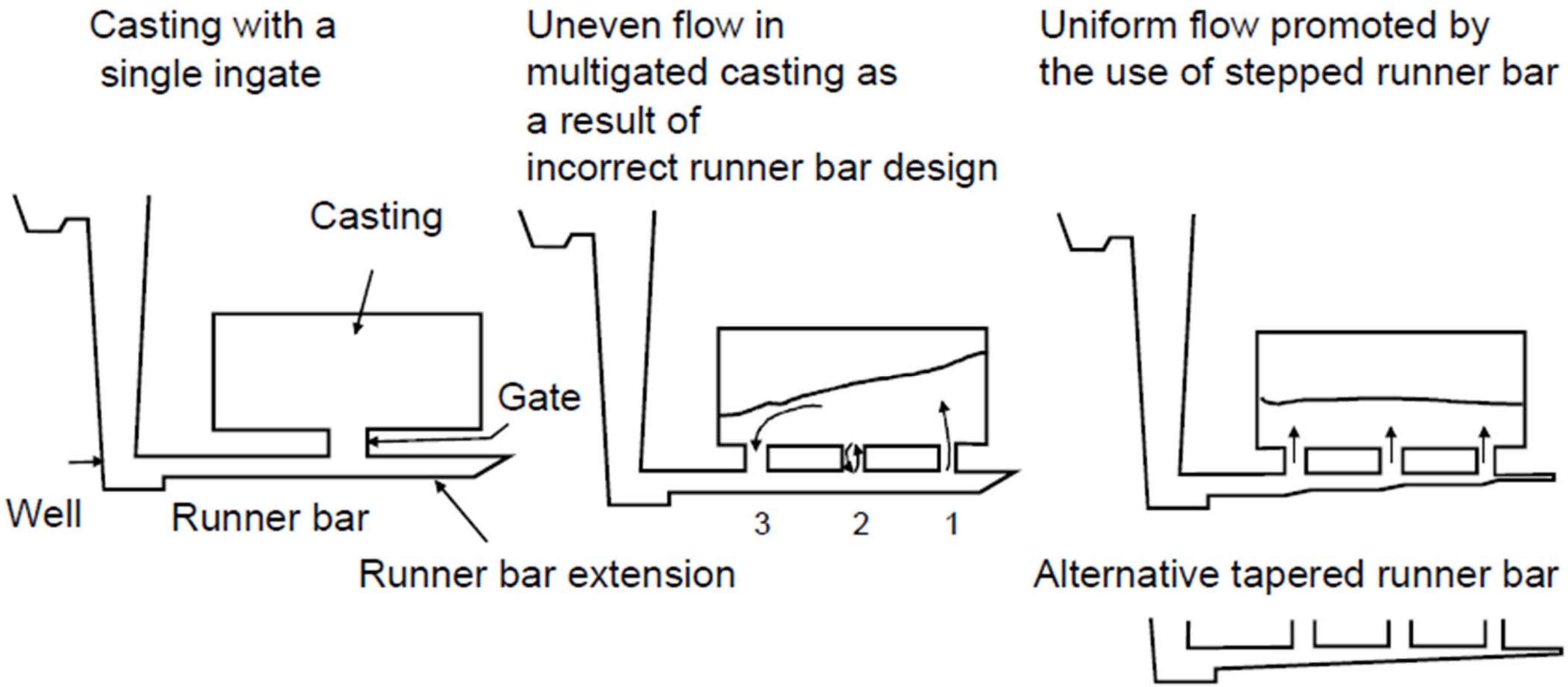
Good Design 3: Sprue Well

The sprue well helps to: (i) Declarate metal
(ii) Stop first splash
(iii) Fill runner



Good Design 4: Runner Bar and Gates

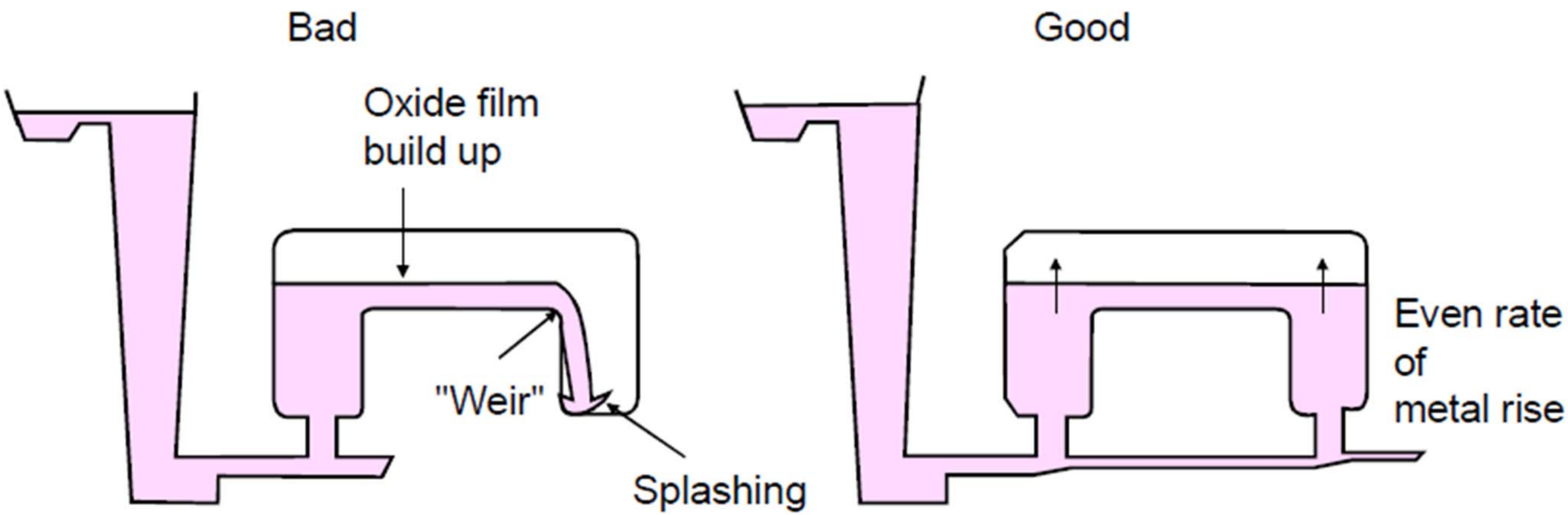
AIMS: (i) to distribute metal to lowest point(s) on a casting
(ii) to reduce metal velocity.



Good Design 4 (Continued): Runner Bar and Gates

Waterfall effects must be avoided so that:

(a) splashing is prevented
(b) the critical velocity is not exceeded
(b) the metal meniscus is never stationary



7.3. Heat Flow

Heat Balance Equation

$$K_S T_S' = K_L T_L' + v L_V$$

* Solidification rate of solid/liquid interface ∝ $\Delta T = T_e - T$ (actual temp): To maintain ΔT, latent heat generated during solidification needs to be removed.

→ **the amount of solidification at a given time ∝ the amount of heat removed during that time, Q**

* Heat transfer in the casting process

1) **Thermal conductivity:** **Generally in pure metals and low alloys, TC decreases/ in high alloys TC increases when T increases/ TC of metal >> TC of ceramic.**

2) **Convection heat transfer:** **Convection occurs due to density difference by temperature difference in flowing of molten metal.**

3) **Radiative heat transfer:** **in high-temperature molten metal or mold surface contacting the atmosphere, radiation heat transfer in which heat energy moves in the form of electromagnetic waves should be considered.**

4) **Phase transformation and latent heat:** **Release or absorption of latent heat occurs when there is a phase transformation/ in this case, exothermic or endothermic term should be added by heat energy conservation law.**

* Possible to classify of various solidification processes according to thermal property of mold (Related to heat release associated with solidification rate)

* Heat diffusivity b = $\sqrt{K\gamma C}$

K= thermal conductivity/ γ = density/ C= specific heat

Table 7.2 (Values in feet, pounds, °F units)

Material	K	γ	C	$b = \sqrt{K\gamma C}$
Aluminum	120	170	0.26	73
Copper	224	560	0.10	112
Steel (solid)	18.4	460	0.16	37
Cast iron	20	460	0.15	37
Sand	0.90	94	0.28	3.6
Graphite, 1500°	19	140	0.29	28
1000°	67	140	0.29	52

a. For metal molds with thermal conductivity similar to solidifying metal

* n = (b of mold metal)/ (b of solidifying metal)

: Solidification of steel in cast iron mold n = 1.12 ~ close to 1.

* If the metal mold thickness is not larger than the section thickness to be cast,

Initial solidification: control by heat flow into mold

final solidification: Conduction through mold / Heat loss outside the mold

* 3 different types of metal mold

(1) Typical ingot mold: Heat release by radiation from outer surface and convection

(2) Metal mold cooled on surface relatively far from casting

→ **- Most continuous casting is made by water cooling.**
- Water cooling in the absence of mold has a similar effect.
- Arc melting also uses this type of mold.

(3) Huge metal mold compared to the solidification part
: Heat loss outside mold is not large until final solidification

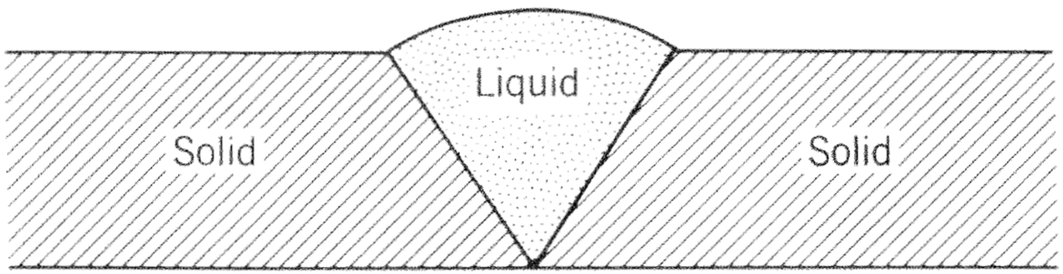


Fig. 7.2. cross section of butt weld (schematic).

B. Sand mold: Thermal conductivity is much smaller than solidifying metal

Ex) n = 0.13 → the dependence of the mold thickness is greatly reduced

(∵ heat loss from the mold surface is independent of thickness variation)

C. If the heat release is controlled by controlled heat supply control and heat sink to the liquid metal, as in the case of zone refining or single crystal growth: the process rate depends on the L-S temperature gradient and the interfacial velocity, v.

1. Solidification Rate

*** <u>The solidification rate *f*(t) is to obtain the increase in time of the solid layer in contact with the mold.</u> Although in case of <u>pure metal or eutectic</u> without solidus-liquidus interval, it is possible to calculate the heat flow by solidifying while maintaining the planar interface, heat flow calculations and interpretation of results at the <u>dendritic interface</u> are complicated due to influence by various interface conditions → Solidification rate can be calculated by measuring the Temp.-time relationship in various parts of the solidifying metal.**

Assumption: mold and metal: semi-infinite / initial liquid temperature T_m / liquid: pure metal & solid-liquid interface temperature~ constant / metal: constant temperature with mold interface

① For heat conduction in one direction in the mold (i.e., perpendicular to the planar mold wall),
Temp, θ of an element of volume at t

Thermal conductivity

$$\frac{\partial \theta}{\partial t} = \frac{K}{\gamma C} \frac{\partial^2 \theta}{\partial x^2}$$

density Heat capacity

② **ΔT** at a specific location in time t :

initial temp. at the surface, θ_0 → instantly raised to θ_i at t=0 → change to θ_m at t=t_1

$$\theta_m = \theta_0 + (\theta_i - \theta_0)\,\mathrm{erfc}\left(\frac{X}{2\sqrt{\alpha t_1}}\right)$$

where, $\alpha = \frac{K}{\gamma C}$

③ Heat removed by casting at any time t_1 :

$$\frac{\partial Q}{\partial t} = -K\left[\frac{\partial \theta}{\partial x}\right]_{x=0} = \frac{K(\theta_i - \theta_0)}{\sqrt{\pi \alpha t}} = 0.564\frac{K(\theta_i - \theta_0)}{\sqrt{\alpha t}} = 0.564b\frac{(\theta_i - \theta_0)}{\sqrt{t}}$$

By differentiation of the efrc equation

④ Total heat conducted into the mold Q up to time t : (where, b = $\sqrt{K\gamma C}$)

$$Q = b(\theta_i - \theta_0)\int_0^t \frac{0.564}{\sqrt{t}} = 1.128b(\theta_i - \theta_0)\sqrt{t}$$

⑤ Thickness of the solidified layer, D:

$$\boxed{D = q\sqrt{t}}$$

where, q = solidification constant

$$1.128 \; [b(\theta_i - \theta_0)/L\gamma'];$$

L: latent heat/ density of solidified metal: γ'

⑥ Reflection of Superheat condition: more complicated problem

Total heat to be extracted from solidifying metal = $W[L + S(\theta_c - \theta_f)]$

where W = weight of casting
θ_c = initial temperature of the liquid
θ_f = final solidification temperature

→ This amount of heat must be conducted into the mold during the time t taken for solidification.

If the area of the surface of contact of mold and metal is A, then

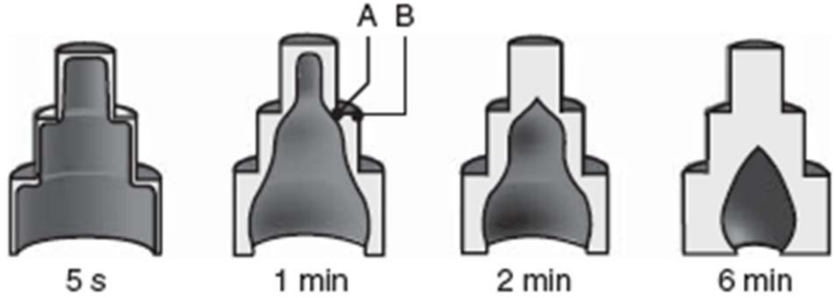
$$\sqrt{t} = \frac{W[L + S(\theta_c - \theta_f)]}{1.128A\sqrt{K\gamma C}\,(\theta_i - \theta_0)}$$

from which

$$\boxed{t = \left(\frac{V}{A}\right)^2 \times \text{constant}}$$

(Chvorinov's rule)

Solidification Time



- Total solidification time T_{TS} = time required for casting to solidify after pouring
- T_{TS} depends on size and shape of casting by relationship known as *Chvorinov's Rule*

$$T_{TS} = C_m \left(\frac{V}{A} \right)^n$$

where T_{TS} = total solidification time; V = volume of the casting; A = surface area of casting; n = exponent with typical value = 2; and C_m is *mold constant*.

Mold Constant in Chvorinov's Rule

- Mold constant C_m depends on:
  - Mold material
  - Thermal properties of casting metal
  - Pouring temperature relative to melting point
- Value of C_m for a given casting operation can be based on experimental data from previous operations carried out using same mold material, metal, and pouring temperature, even though the shape of the part may be quite different

What Chvorinov's Rule Tells Us

- Casting with a higher volume-to-surface area ratio cools and solidifies more slowly than one with a lower ratio
  - To feed molten metal to the main cavity, T_{TS} for riser must be greater than T_{TS} for main casting

- Since mold constants of riser and casting will be equal, design the riser to have a larger volume-to-area ratio so that the main casting solidifies first
  - This minimizes the effects of shrinkage

Solidification times for various shapes

Three metal pieces being cast have the same volume but different shapes: On is a sphere, one a cube, and the other a cylinder with its height equal to its diameter.
Which piece will solidify the fastest, and which one the slowest? Assume that n = 2

Solution The volume of the piece is taken as unity. Thus from Eq.

$$\text{Solidification time} \propto \frac{1}{(\text{Surface area})^2}$$

The respective surface areas are as follows:

Sphere:

$$V = \left(\frac{4}{3}\right)\pi r^3,\ r = \left(\frac{3}{4\pi}\right)^{1/3}$$

$$A = 4\pi r^2 = 4\pi\left(\frac{3}{4\pi}\right)^{2/3} = 4.84$$

Cube:

$$V = a^3,\ a = 1, \text{ and } A = 6a^2 = 6$$

Cylinder:

$$V = \pi r^2 h = 2\pi r^3,\ r = \left(\frac{1}{2\pi}\right)^{1/3}$$

$$A = 2\pi r^2 + 2\pi r h = 6\pi r^2 = 6\pi\left(\frac{1}{2\pi}\right)^{2/3} = 5.4$$

The respective solidification times are therefore

$$t_{\text{sphere}} = 0.043C,\ t_{\text{cube}} = 0.028C,\ t_{\text{cylinder}} = 0.033C$$

Hence, the cube-shaped piece will solidify the fastest, and the spherical piece will solidify the slowest.

* In the case of sand mold,

The superheat of the liquid phase is uniformly reflected throughout and delays the start of solidification until the liquid reaches the liquidus temp.

* Thickness of the solidified layer D in sand mold casting

$$D = q\sqrt{t} - c$$

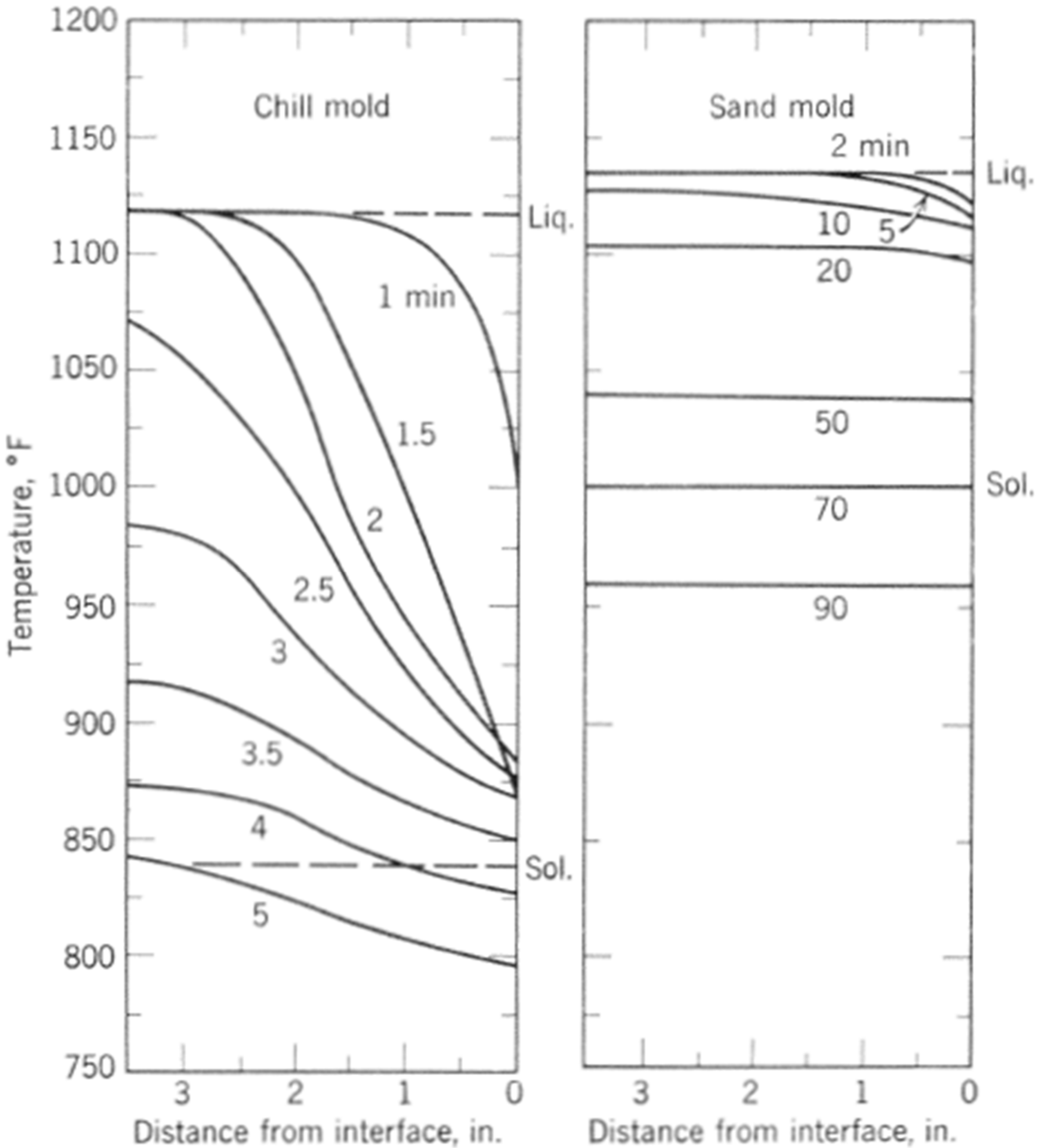


Fig. 7.3. Variation of temperature during solidification of Al 5% Mg alloy in a 7-inch square mold. (a) Metal mold, (b) sand mold.

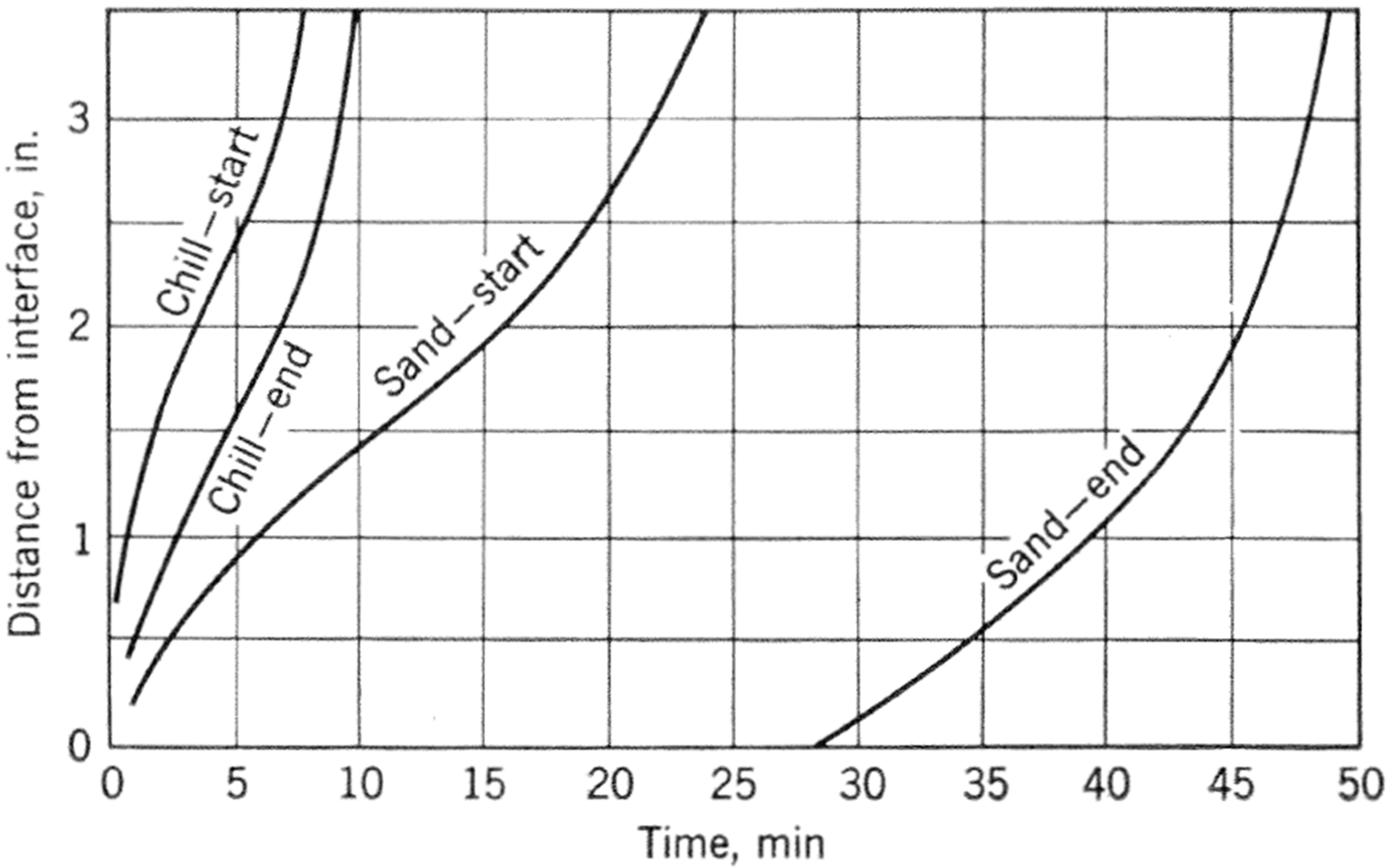
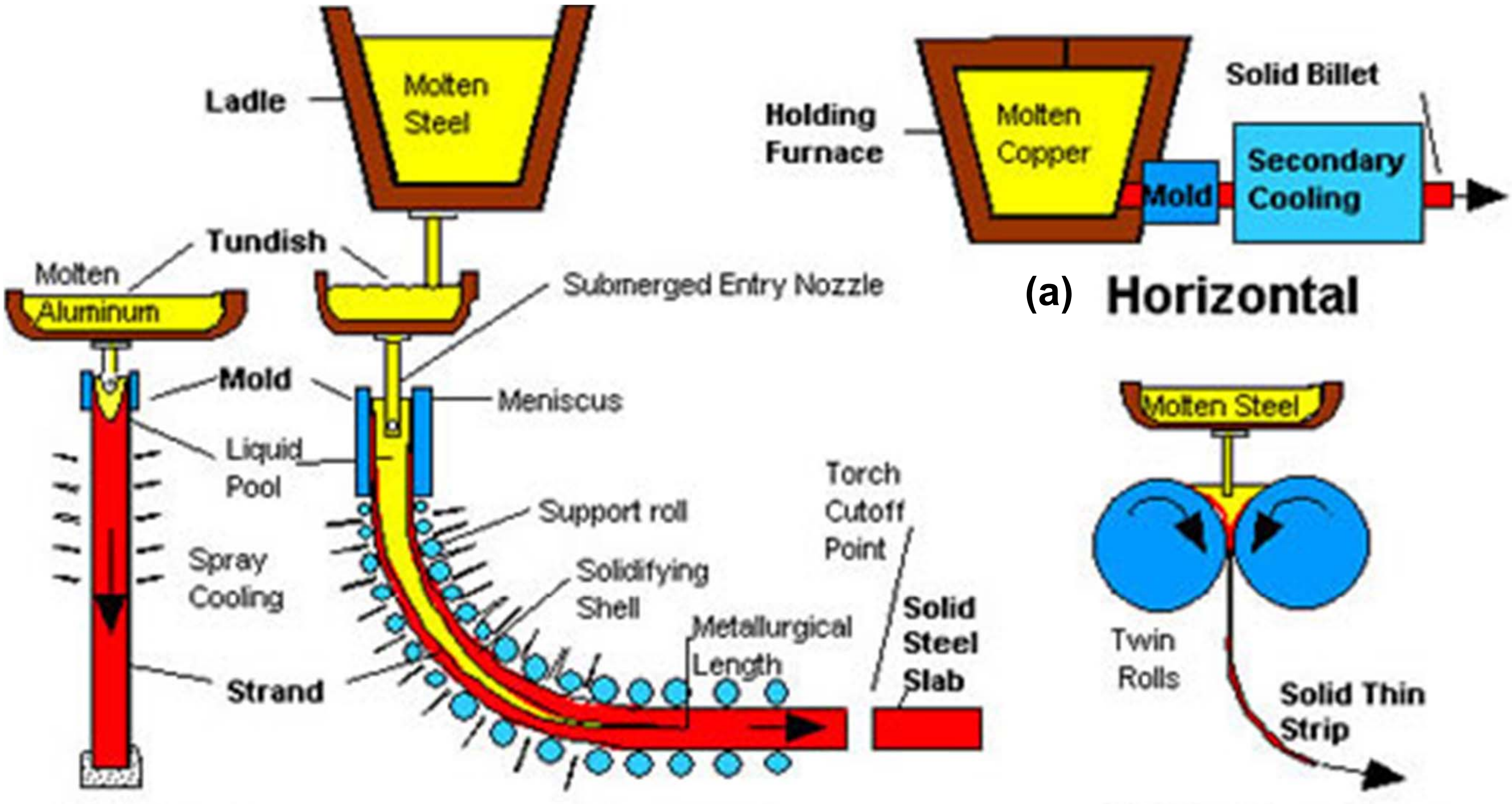


Fig. 7.4. Movement of liquidus and solidus tempeeratures during solidification of a 0.6% carbon steel.

2. continuous casting: a number of dynamic industrial process

The molten metal is poured continuously into a water-cooled mold from which the solidified metal is continuously withdrawn in plate or rod form. (solid-liquid interface)



(a) Horizontal

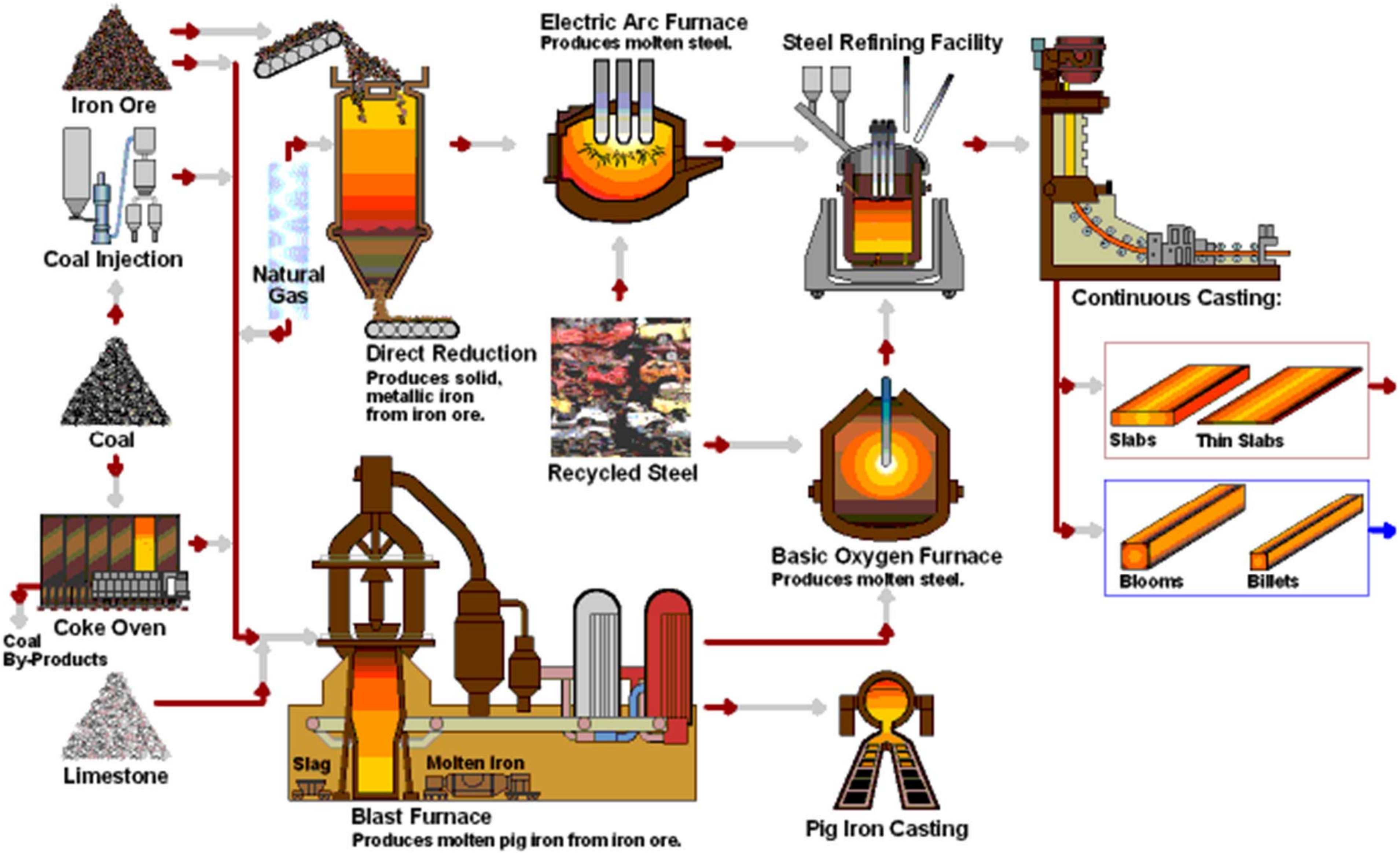
(b) Vertical

(c) Curved

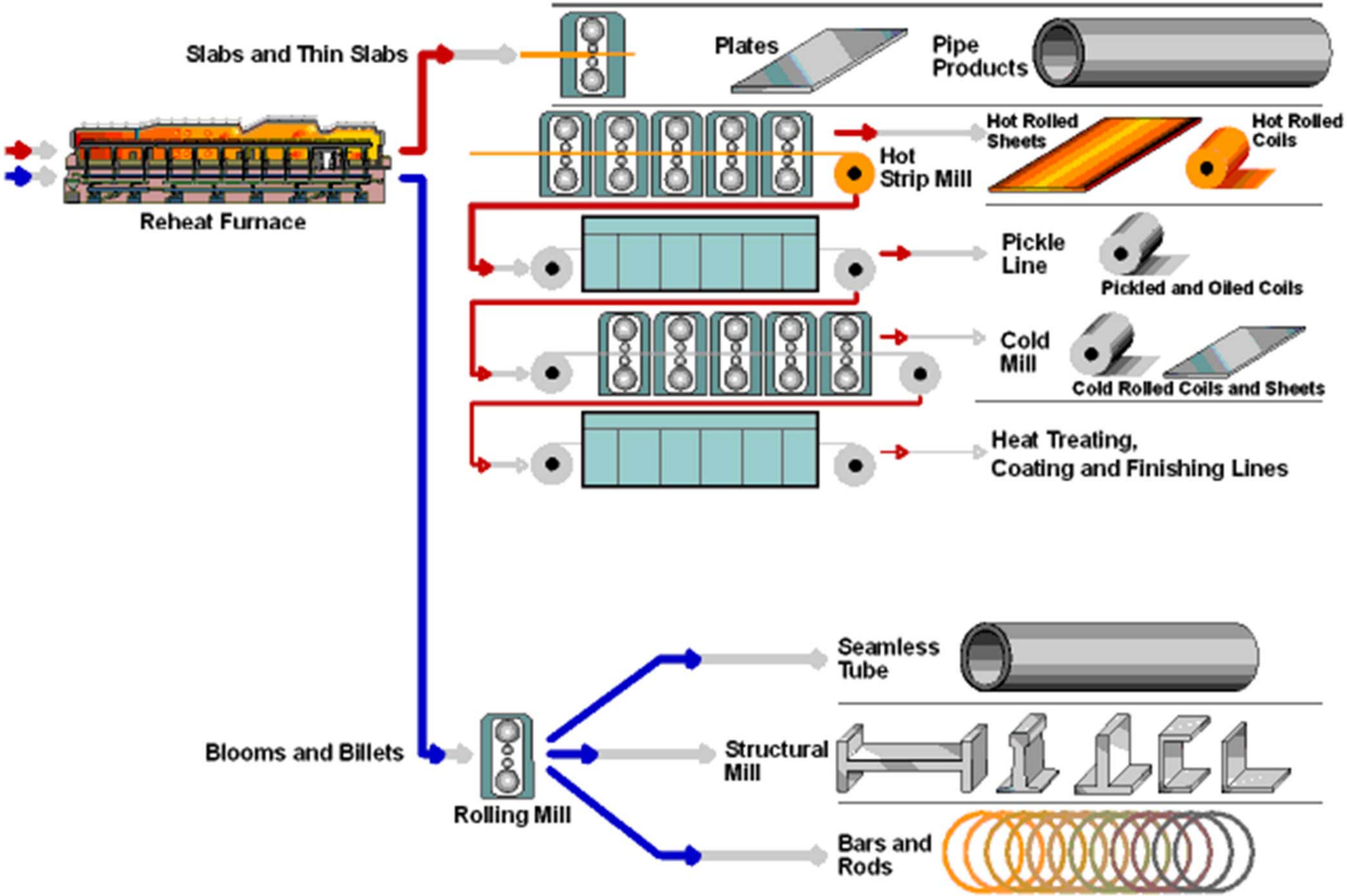
(d) Strip Casting


continuous casting

continuous casting



4.4.3 continuous casting



continuous casting: a number of dynamic industrial process

: large mass (economic advantage)/ high speed (property good)

→ **Process speed: related to latent heat removal & metal flow during solidification.**

① Dynamic process: importance of isotherm distribution

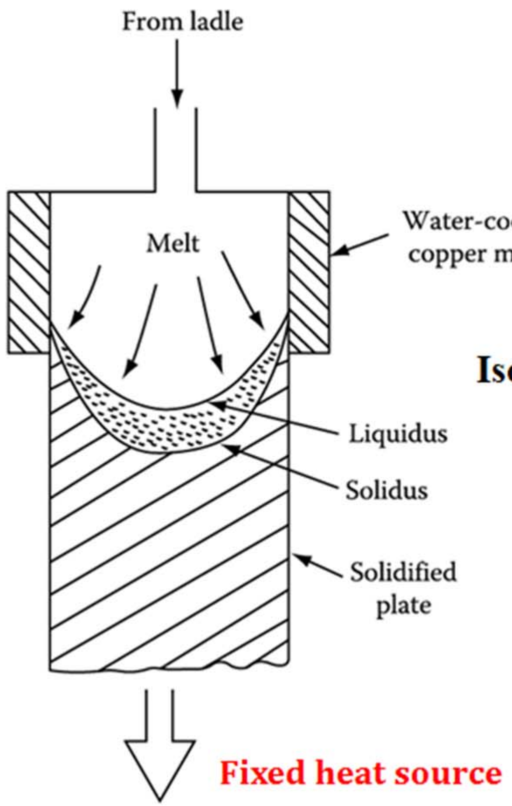


Fig. 4.44 Schematic illustration of a continuous casting process

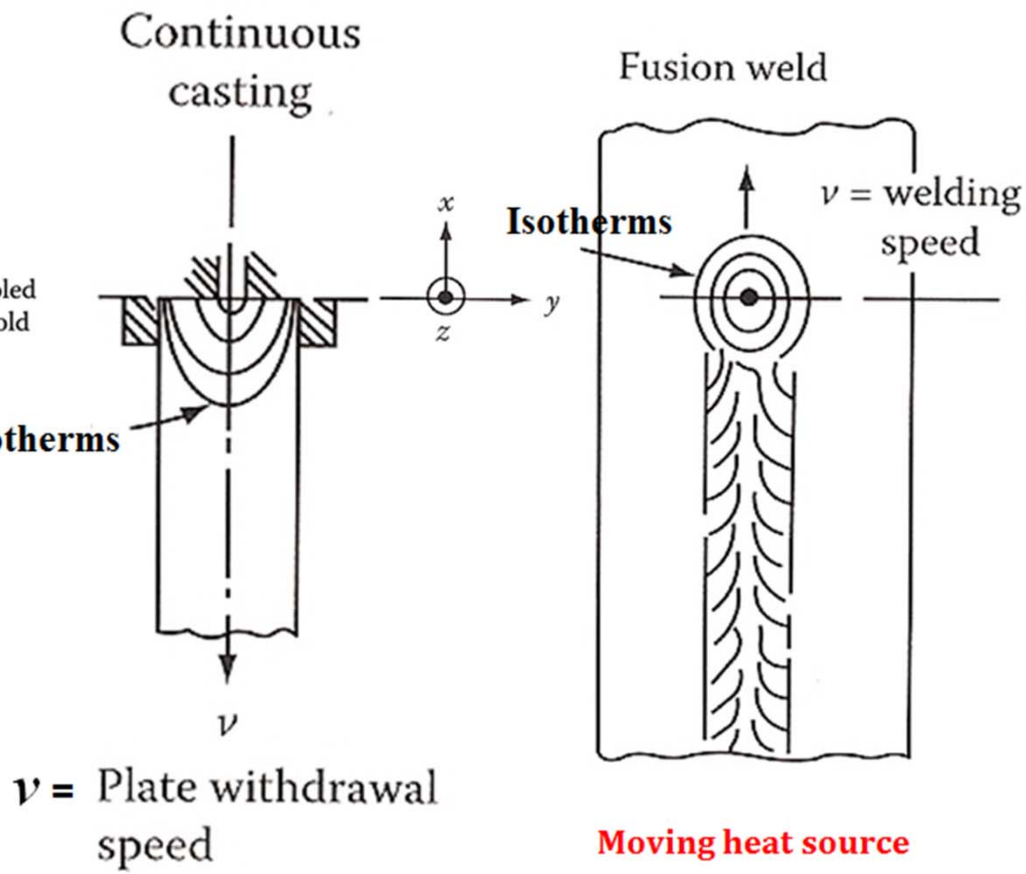
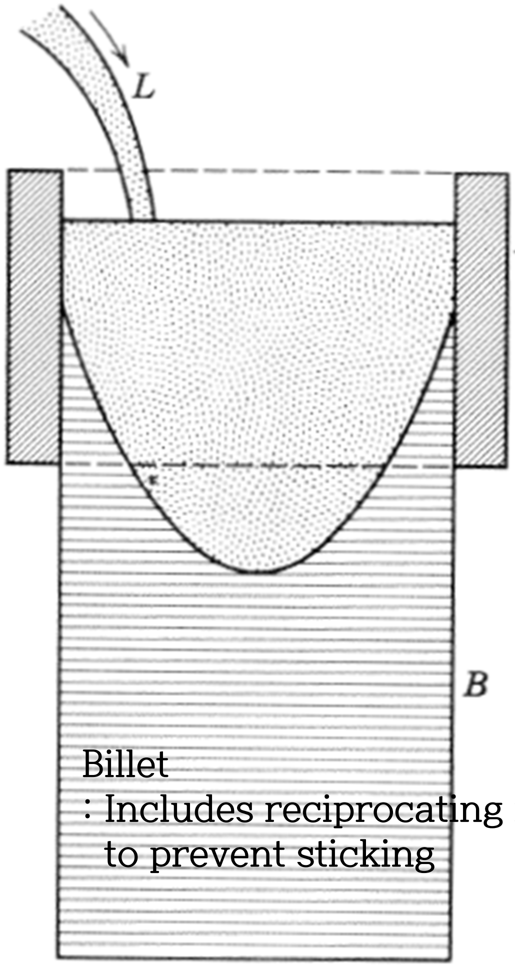


Fig. 4.45 Illustrating the essential equivalence of isotherms about the heat sources in fusion welding and continuous casting

continuous casting: a number of dynamic industrial process

② the temperature gradient is maintained in a steady state→ related to constant shape of the interface and the solidification rate (here, the solidification rate is changed by not to time but to position from the surface)



Mold
: Square or circular section

a) **cooling by water spray or water jet/**

b) **Extra cooling for the metal extracting from the mold**

Billet
: Includes reciprocating motion
to prevent sticking

Fig. 7.5. Continuous casting (schematic).

* solidification rate varies depending on the position as shown in the following figure
(∵ Since solidification occurs in a direction perpendicular to the solidification interface)

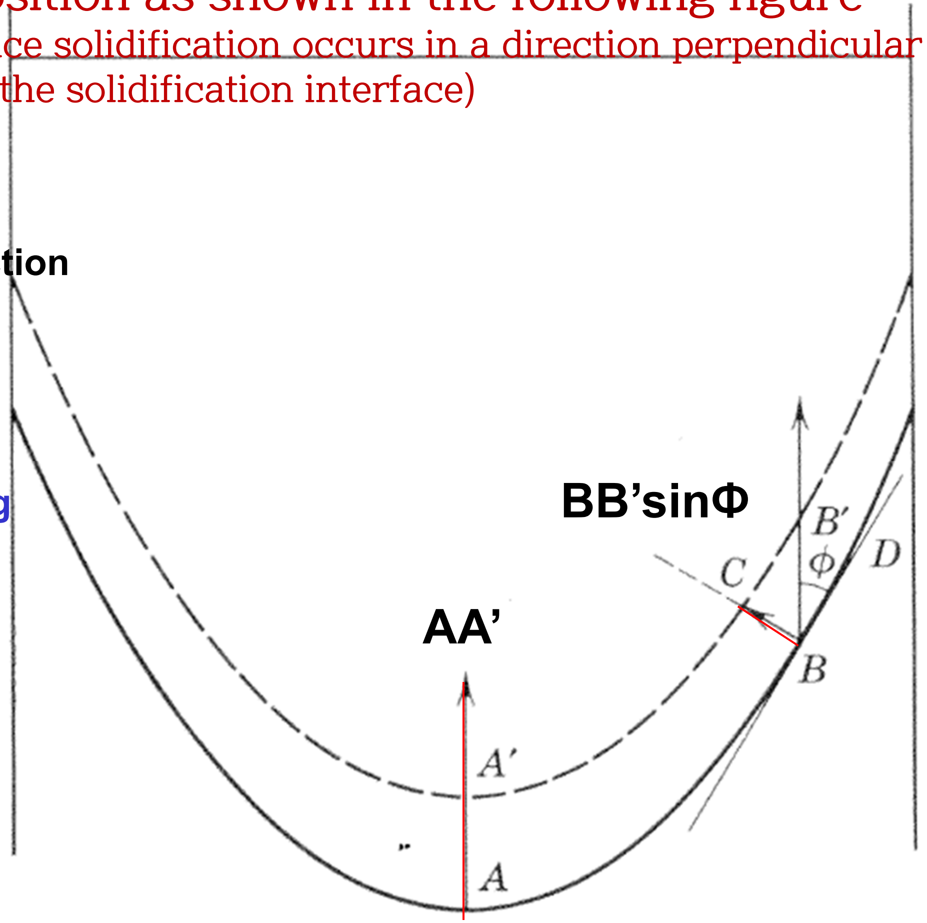


Fig. 7.6. Interface shape and rate of solidification in continuous casting.

* To obtain the interface shape of Fig. 7.6,

  in the case of max. emission of latent heat at A/
  min. emission of latent heat at D

→ For this, efficient cooling of the billet beyond the mold is necessary.

* To obtain solid-liquid interface shape of Fig. 7.7,

  At the billet center, the solidification rate becomes minimum.

→ In this case. a large tendency of segregation and porosity by shrinkage in the center line where solidification ends

→ additional cooling of the lower part of the mold is required

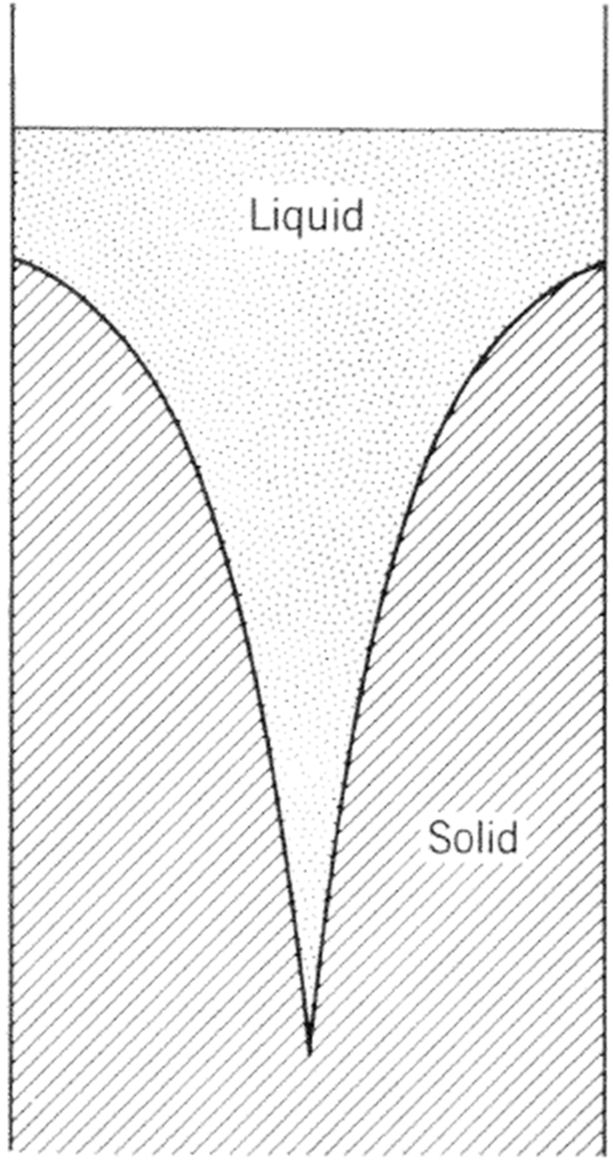


Fig. 7.7. Alternative interface shape in continuous casting