$$\frac{T_0}{T} = 1 + \frac{k-1}{2} Ma^2$$

$$\frac{a_0}{a} = \left(\frac{T_0}{T}\right)^{\frac{1}{2}} = \left(1 + \frac{k-1}{2} Ma^2\right)^{\frac{1}{2}}$$

] adiabatic process

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}} = \left(1 + \frac{k-1}{2} Ma^2\right)^{\frac{k}{k-1}}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{k-1}} = \left(1 + \frac{k-1}{2} Ma^2\right)^{\frac{1}{k-1}}$$

] isentropic process

- (a_0, T_0, P_0, ρ_0) : stagnation values

(a^*, T^*, P^*, ρ^*) : critical values @ $Ma = 1$

sonic "

$\rho_{0_1} \neq \rho_{0_2}$

$P_{0_1} \neq P_{0_2}$

$T_{0_1} = T_{0_2}$

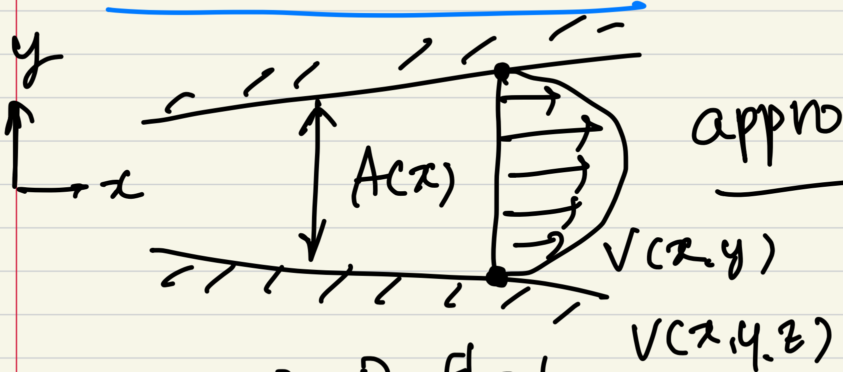
vary at irreversible adiabatic process

critical velocity $V^* = a^* = \sqrt{kRT^*} = \sqrt{\frac{2k}{k+1} R T_0}$

$Ma = 1$: $\frac{a^*}{a_0} = \left(\frac{2}{k+1}\right)^{\frac{1}{2}}$, $\frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}$, $\frac{P^*}{P_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$, $\frac{T^*}{T_0} = \frac{2}{k+1}$

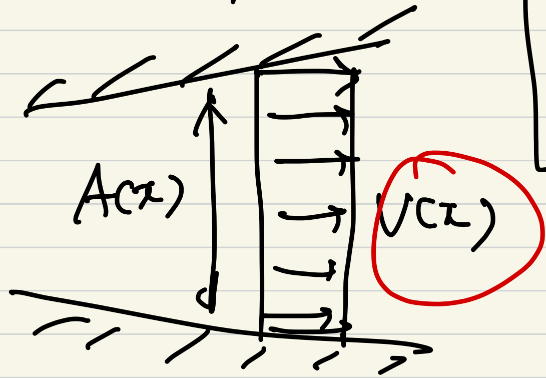
→ only fct. of k

9.4 Isentropic flow w/ area change



2-D flow
3-D "

approx. $\longrightarrow$



1-D flow

기말고사
6/11 오후 6:30-9:30
301-118
예약

cont : $\rho(x) V(x) A(x) = \dot{m} = \text{const}$

$$d\rho V A + \rho dV A + \rho V dA = 0$$

$$\rightarrow \frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$

Bernoulli eq : $\frac{dp}{\rho} + V dV = 0$

speed of sound : $\boxed{dp = a^2 d\rho}$

$$\Rightarrow \boxed{\frac{dV}{V} = \frac{dA}{A} \cdot \frac{1}{Ma^2 - 1} = -\frac{dp}{\rho V^2}}$$

isentropic process

- subsonic (Ma<1)

$\rightarrow$ $dA > 0$ $\rightarrow$ $dV < 0$, $dp > 0$, $d\rho > 0$

subsonic diffuser

$\rightarrow$ $dA < 0$ $\rightarrow$ $dV > 0$, $dp < 0$, $d\rho < 0$

subsonic nozzle

- supersonic (Ma>1)

$\rightarrow$ $dA > 0$ $\rightarrow$ $dV > 0$, $dp < 0$, $d\rho < 0$

supersonic nozzle

$\rightarrow$ $dA < 0$ $\rightarrow$ $dV < 0$, $dp > 0$, $d\rho > 0$

supersonic diffuser

when Ma=1? $\rightarrow$ $dA = 0$ @ Ma=1

subsonic $\rightarrow$ A↓ V↑ A_{min} Ma=1 $\rightarrow$ supersonic Ma>1 / $\rightarrow$ subsonic Ma<1

~~subsonic $\rightarrow$ V↓ Ma≠1~~

~~supersonic $\rightarrow$ V↑ Ma≠1~~

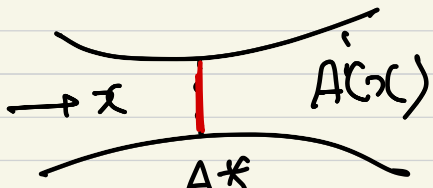
converging-diverging nozzle

- Perfect-gas relations

$$\rho V A = \rho^* V^* A^*$$

$$\rightarrow \frac{A}{A^*} = \frac{\rho^*}{\rho}\cdot\frac{V^*}{V} = \frac{\rho^*}{\rho_0}\,\frac{\rho_0}{\rho}\,\frac{\sqrt{kRT^*}}{V} = \underbrace{\frac{\rho^*}{\rho_0}}_{k}\,\underbrace{\frac{\rho_0}{\rho}}_{Ma}\,\underbrace{\frac{\sqrt{kRT}}{V}}_{Ma}\cdot\sqrt{\underbrace{\frac{T^*}{T_0}}_{k}\,\underbrace{\frac{T_0}{T}}_{Ma}}$$

$$= \frac{1}{Ma}\left[\frac{1+\frac{1}{2}(k-1)Ma^2}{\frac{1}{2}(k+1)}\right]^{\frac{1}{2}\cdot\frac{k+1}{k-1}}$$



$A(x) \rightarrow Ma(x)$

For air ($k=1.4$)

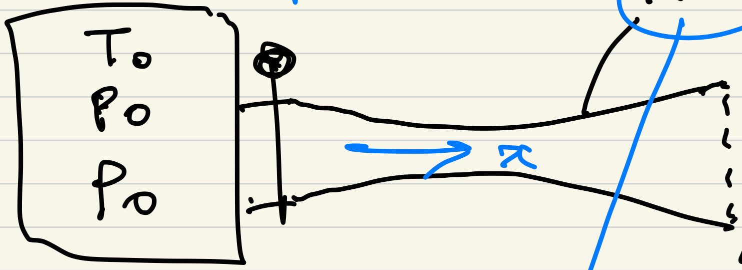
$$\frac{A}{A^*} = \frac{1}{Ma}\cdot\frac{(1+0.2Ma^2)^3}{1.728}$$

$$\frac{T_0}{T} = 1+0.2Ma^2$$

$$\frac{\rho_0}{\rho} = (1+0.2Ma^2)^{2.5}$$

$$\frac{p_0}{p} = (1+0.2Ma^2)^{3.5}$$

isentropic flow



$A(x)$

$\frac{p_0}{p_a} = \cdots$

$p_a \rightarrow Ma_a$

A_a

A^*

$T_0/T = \cdots$

$\rho_0/\rho = \cdots$

T_a, ρ_a

$T(x)$, $\rho(x)$, $p(x)$ $\leftarrow$ $Ma(x)$

* choking : for given stagnation conditions, the maximum possible mass flow passes through a duct when its throat is at the sonic condition. Then the duct is said to be choked and carry no additional mass flow.

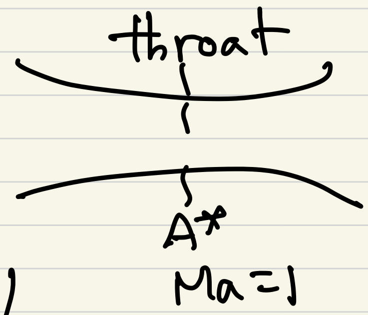
$dA=0$

$$\dot{m}_{max} = \rho^* V^* A^* = \rho_0 \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \cdot \underbrace{\sqrt{kRT^*}}_{\sqrt{kRT_0 \cdot \frac{2}{k+1}}} A^*$$

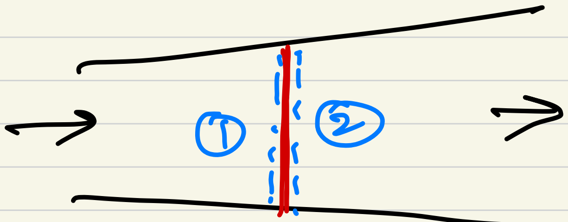
$$\rho^*/\rho_0 = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}$$

$$T^*/T_0 = 2/(k+1)$$

$$= k^{\frac{1}{2}} \left(\frac{2}{k+1}\right)^{\frac{1}{2}\cdot\frac{k+1}{k-1}} A^* \underbrace{\rho_0 (RT_0)^{\frac{1}{2}}}_{= P_0/(RT_0)^{\frac{1}{2}}}$$



9.5 Normal-shock wave



fixed normal shock

$$A_{①} = A_{②} = A$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \Rightarrow \rho_1 V_1 = \rho_2 V_2 = G = \text{const} \quad \text{—} \boxed{ⓐ}$$

mtm: $(p_1 - p_2) A = \rho_2 V_2^2 A - \rho_1 V_1^2 A$

$$\rightarrow p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2 \quad \text{—} \boxed{ⓑ}$$

$\Downarrow$

irreversible adiabatic process!

(Bernoulli eq: $p + \frac{1}{2}\rho V^2 = \text{const}$)

$\therefore$ Bernoulli eq is not valid.

energy: $h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2 = h_0 = \text{const.}$ — $\boxed{ⓒ}$

$$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2} = R, \quad h = c_p T, \quad k = c_p / c_v = \text{const.}$$

5 unknowns, 5 eqs.

→ solve them → two sols. due to V^2 term.

→ choose one s.t. $s_2 > s_1$.

ⓐ–ⓒ : $h_2 - h_1 = \frac{1}{2}(P_2 - P_1)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$: Rankine-Hugoniot relation

Since $h = c_p T = \frac{kR}{k-1} T = \frac{k}{k-1}\frac{P}{\rho}$

$$\frac{\rho_2}{\rho_1} = \frac{1+\beta P_2/P_1}{\beta + P_2/P_1} \quad \text{ⓓ}, \quad \beta = \frac{k+1}{k-1}$$

cf. $\frac{\rho_2}{\rho_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}}$ isentropic process

$T ds = c_v dT + p d\left(\frac{1}{\rho}\right)$

$T = \frac{k}{(k-1)c_p} \cdot \frac{P}{\rho}$

$$\rightarrow \frac{s_2 - s_1}{c_v} = \ln\left[\frac{P_2}{P_1}\left(\frac{\rho_1}{\rho_2}\right)^k\right]$$

If $P_2/P_1 < 1$, $s_2 - s_1 < 0 \rightarrow s_2 < s_1$!

$\rightarrow P_2 < P_1 \rightarrow$ rarefaction shock is impossible

$\Rightarrow \boxed{P_2 > P_1}$, $s_2 > s_1$ compression shock

upstream should be super-sonic.

- Mach number relations

ⓐ–ⓒ & $h = \frac{k}{k-1}\cdot\frac{P}{\rho} \Rightarrow \boxed{\frac{P_2}{P_1} = \frac{1}{k+1}\left[2k Ma_1^2 - (k-1)\right]}$

$P_2 > P_1$ only if $\boxed{Ma_1 > 1}$

(b) & $P = \rho R T \rightarrow \rho V^2 = k P Ma^2$

$$\frac{P_2}{P_1} = \frac{1 + k Ma_1^2}{1 + k Ma_2^2}$$

$$\boxed{Ma_2^2 = \frac{(k-1) Ma_1^2 + 2}{2k Ma_1^2 - (k-1)}}$$

if $Ma_1 > 1$, $\boxed{Ma_2 < 1}$

∴ subsonic

& (d) ⇒

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{(k+1) Ma_1^2}{(k-1) Ma_1^2 + 2} = \frac{V_1}{V_2}}$$

supersonic | subsonic

if $Ma_1 > 1$, $\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} > 1$

$$\boxed{\frac{T_2}{T_1} = \left[2 + (k-1) Ma_1^2\right] \frac{2k Ma_1^2 - (k-1)}{(k+1)^2 Ma_1^2}}$$

if $Ma_1 > 1$, $T_2 > T_1$

$h_{o1} = h_{o2}$, $\boxed{T_{o1} = T_{o2}}$

$$\boxed{\frac{P_{o2}}{P_{o1}} = \frac{P_{o2}}{P_2} \frac{P_2}{P_1} \frac{P_1}{P_{o1}} = \left[\frac{(k+1) Ma_1^2}{2 + (k-1) Ma_1^2}\right]^{\frac{k}{k-1}} \left[\frac{k+1}{2k Ma_1^2 - (k-1)}\right]^{\frac{1}{k-1}}}$$

if $Ma_1 > 1$,

$P_{o2} < P_{o1}$

$$\boxed{\frac{A_2^*}{A_1^*} = \frac{A_2^*}{A_2}\cdot\underbrace{\frac{A_2}{A_1}}_{=1}\cdot\frac{A_1}{A_1^*} = \frac{Ma_2}{Ma_1}\left[\frac{2+(k-1)Ma_1^2}{2+(k-1)Ma_2^2}\right]^{\frac{1}{2}\frac{k+1}{k-1}}}$$

if $Ma_1 > 1$,

$A_2^* > A_1^*$

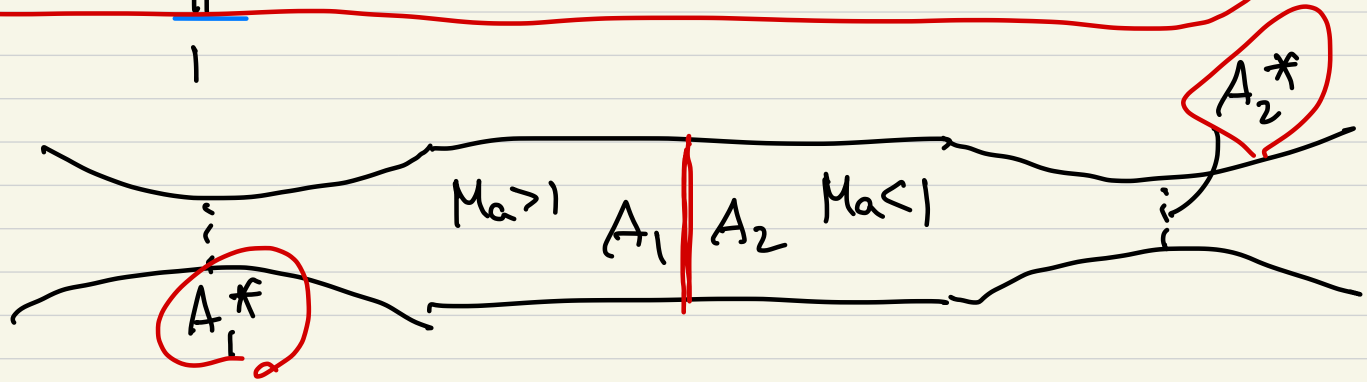
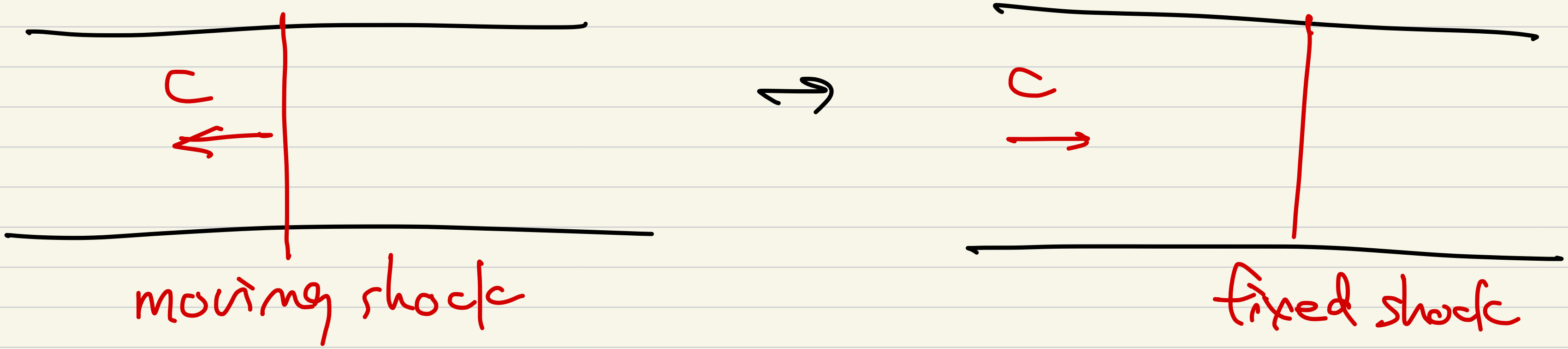


Table B.1
Isentropic process

Ma	p/p_0	ρ/ρ_0	T/T_0	A/A^*
0				
⋮				
4.0				

Table B.2
normal-shock relation
($k=1.4$)

Ma_1	Ma_2	p_2/p_1	$V_1/V_2 = \rho_2/\rho_1$	T_2/T_1	p_{02}/p_{01}	A_2^*/A_1^*
1.0						
⋮						
5.0						


c
moving shock
c
fixed shock