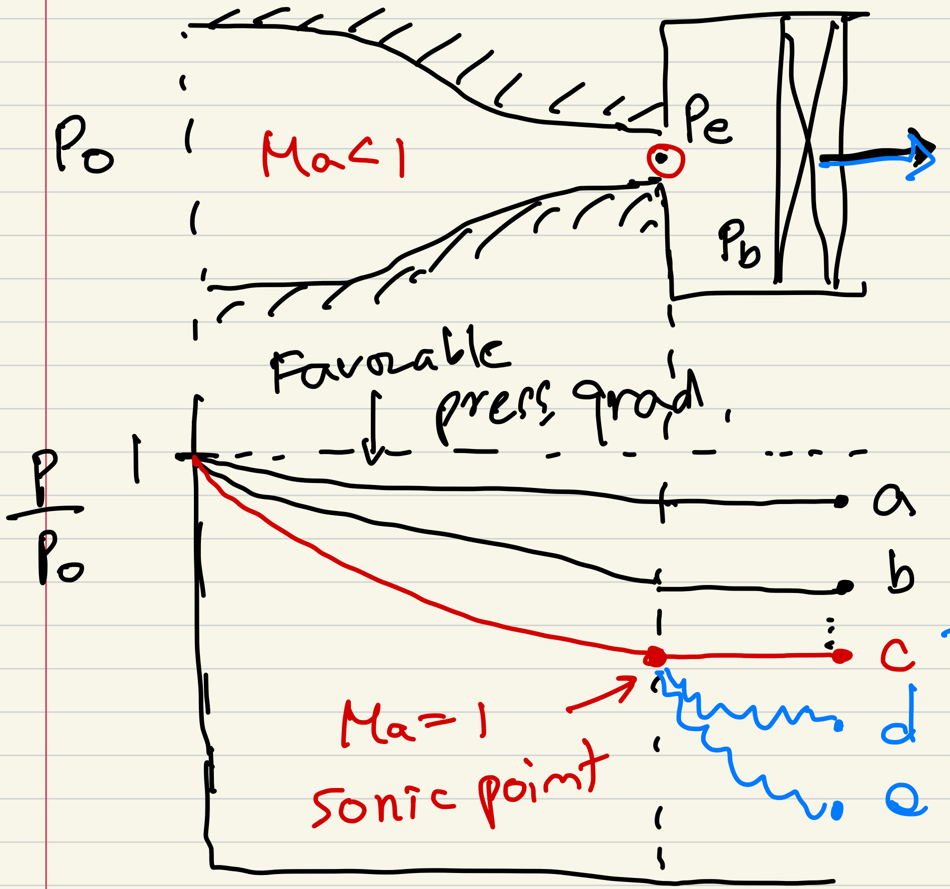
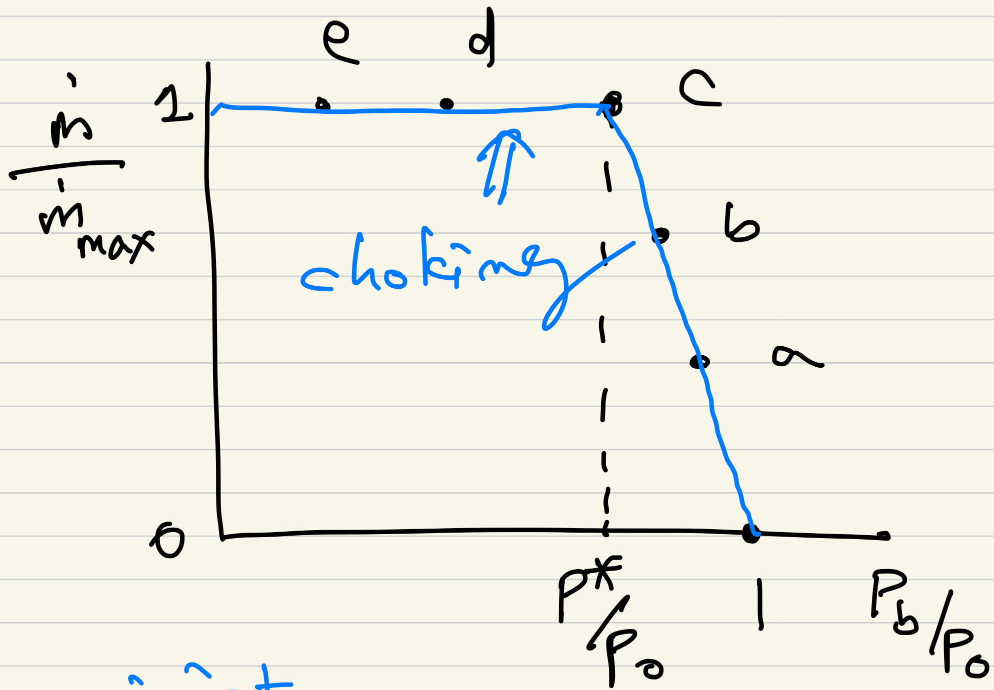
9.6 Operation of converging and diverging nozzles

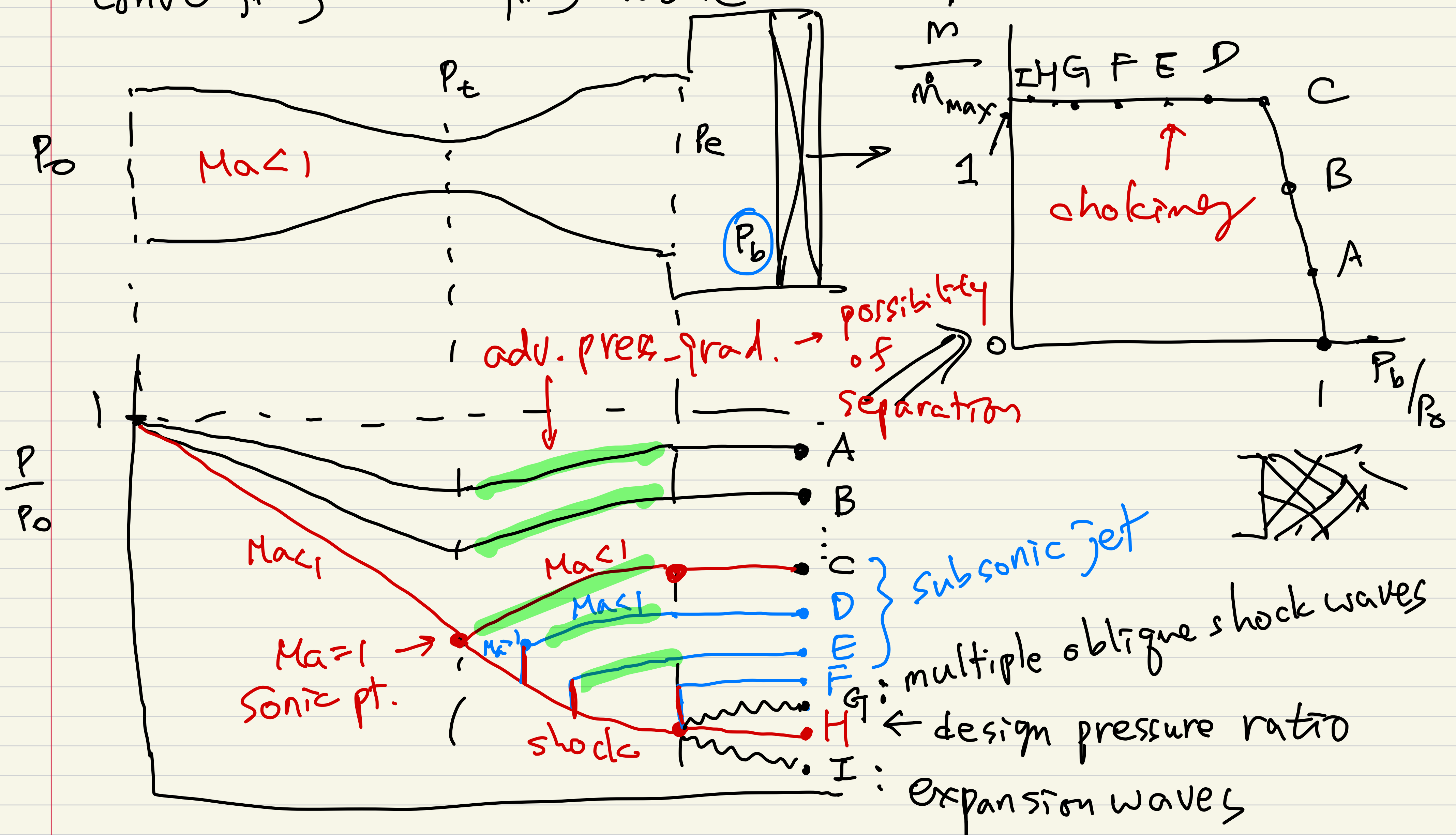
- Converging nozzle



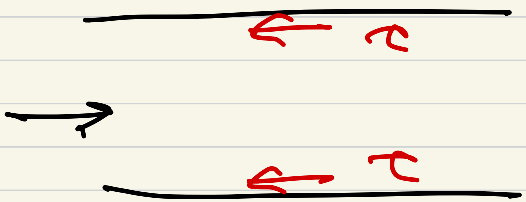
c } supersonic jet
d expansion wave



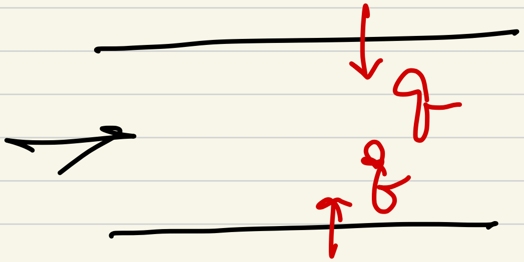
- Converging-diverging nozzle



9.7 Compressible duct flow with friction

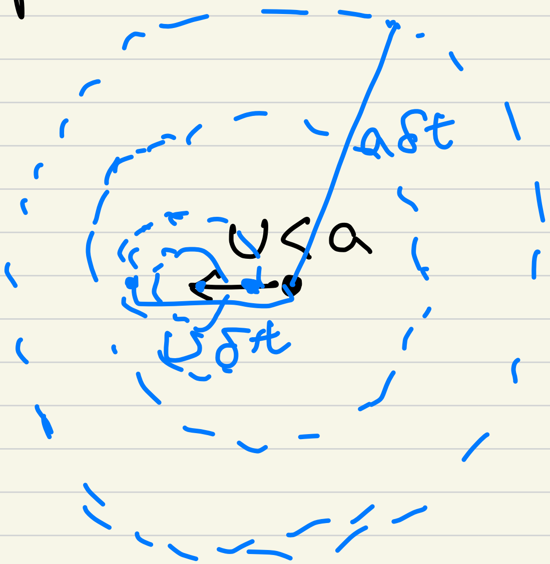


9.8 Frictionless duct flow with heat transfer

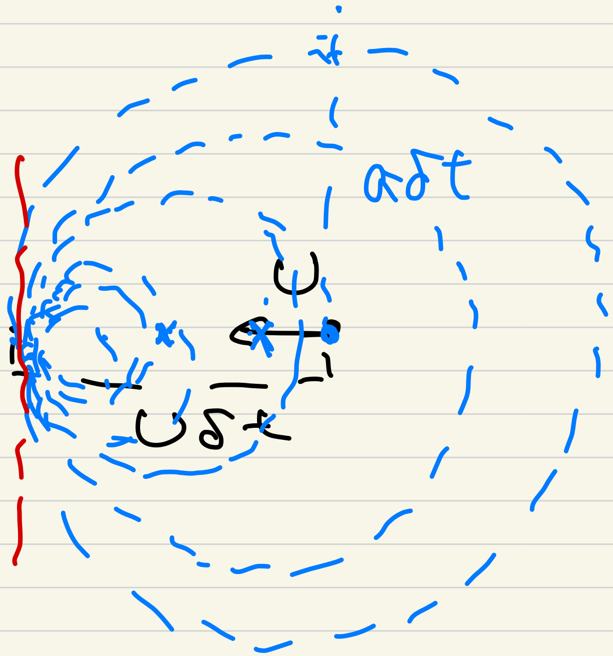


9.9 2D supersonic flow

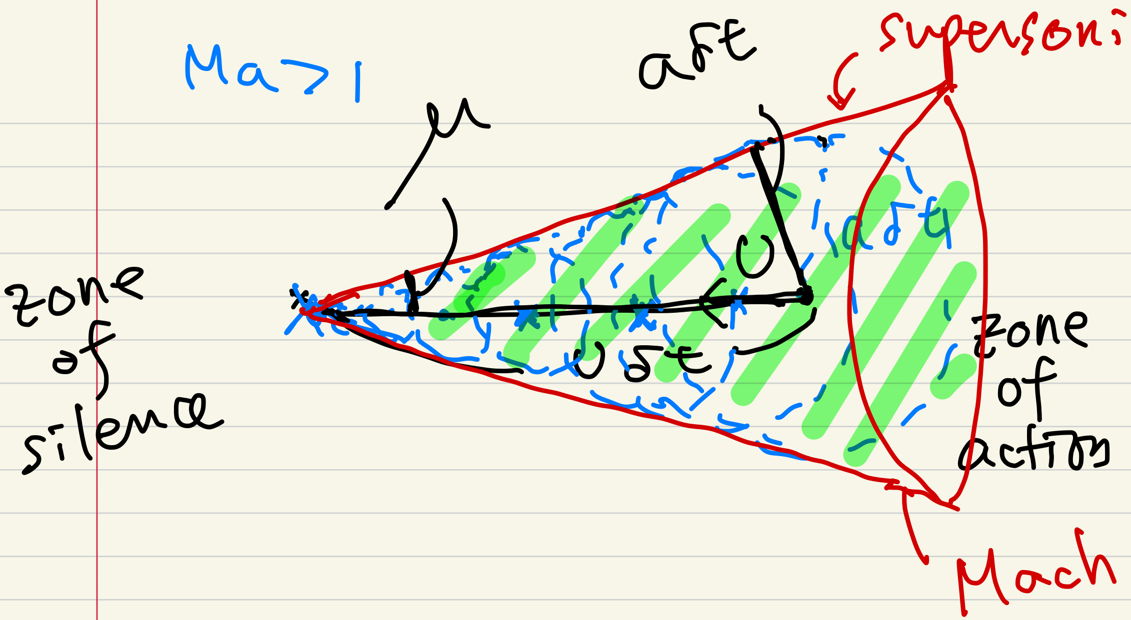
Ma<1



Ma=1

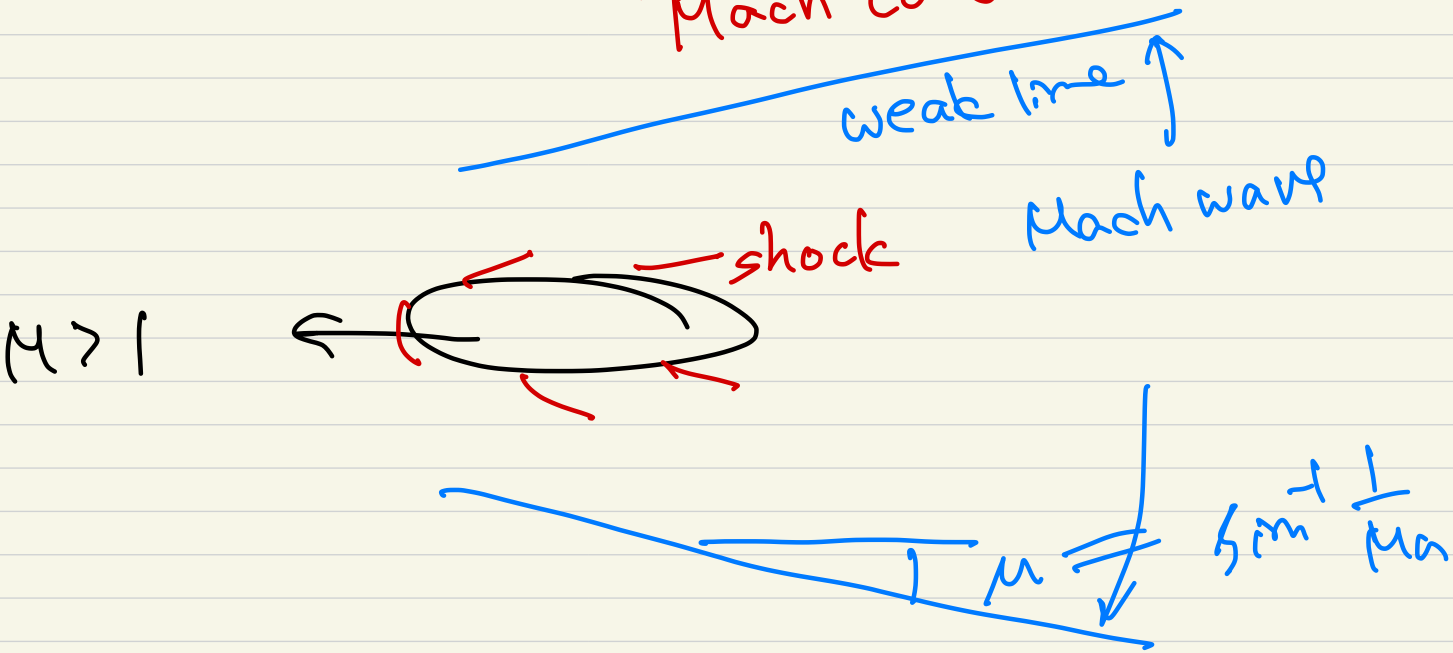


Ma > 1



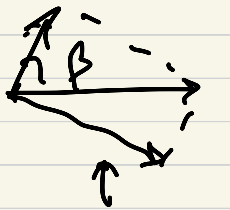
$$\sin\mu = \frac{a\,\delta t}{\upsilon\,\delta t} = \frac{a}{\upsilon} = \frac{1}{Ma}$$

$$\mu = \sin^{-1}\frac{a}{\upsilon} = \sin^{-1}\frac{1}{Ma}$$: Mach angle



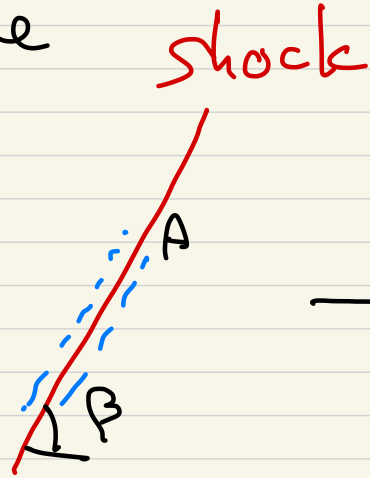
- oblique shock wave

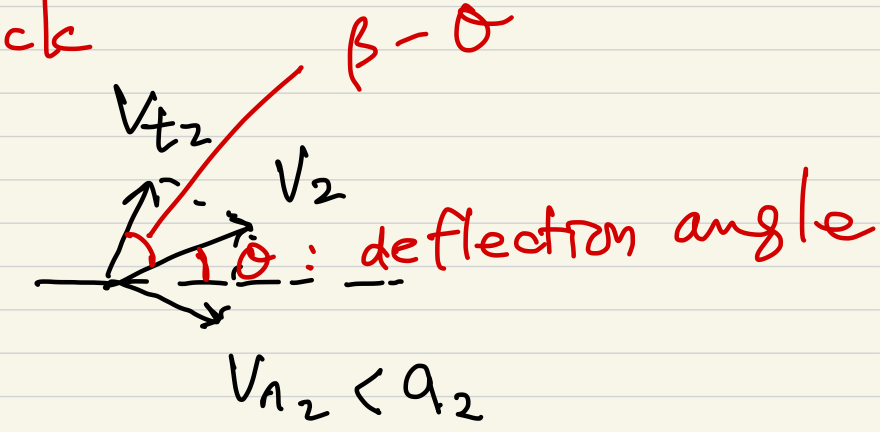
$V_{t_1} = V_1 \cos\beta$



$V_{n_1} = V_1 \sin\beta$

$V_{n_1} > a_1$





cont. : $\rho_1 V_{n_1} A = \rho_2 V_{n_2} A$

n − mtm : $P_1 - P_2 = \rho_2 V_{n_2}^2 - \rho_1 V_{n_1}^2$

t − " : $0 = \rho_1 V_{n_1} (V_{t_2} - V_{t_1}) \rightarrow V_{t_2} = V_{t_1} = V_t = \text{const}$

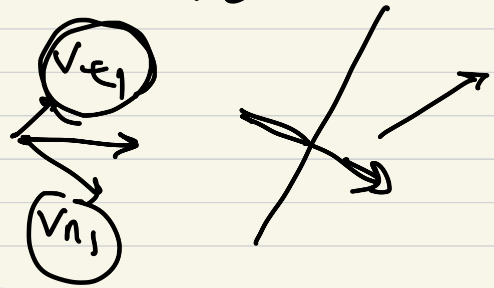
energy : $h_1 + \frac{1}{2} V_{n_1}^2 + \frac{1}{2} V_{t_1}^2 = h_2 + \frac{1}{2} V_{n_2}^2 + \frac{1}{2} V_{t_2}^2 = h_0$

Use normal shock wave formula for $V_{n_1}, V_{n_2}, P_1, P_2, \ldots$

$$Ma_{n_1} = \frac{V_{n_1}}{a_1} = Ma_1 \sin\beta$$

$$Ma_{n_2} = \frac{V_{n2}}{a_2} = Ma_2 \sin(\beta - \theta)$$

deflection angle $\theta = \tan^{-1}\frac{V_{n_1}}{V_t} - \tan^{-1}\frac{V_{n2}}{V_t}$



max θ? $\frac{\partial\theta}{\partial V_t} = 0$

$$\rightarrow \frac{V_t}{V_{n_1}} = \left(\frac{V_{n2}}{V_{n_1}}\right)^{\frac{1}{2}} = \left(\frac{1}{r}\right)^{\frac{1}{2}}$$

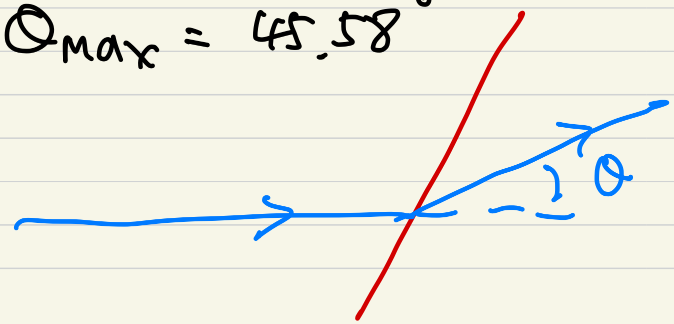
$r = V_{n_1}/V_{n_2}$

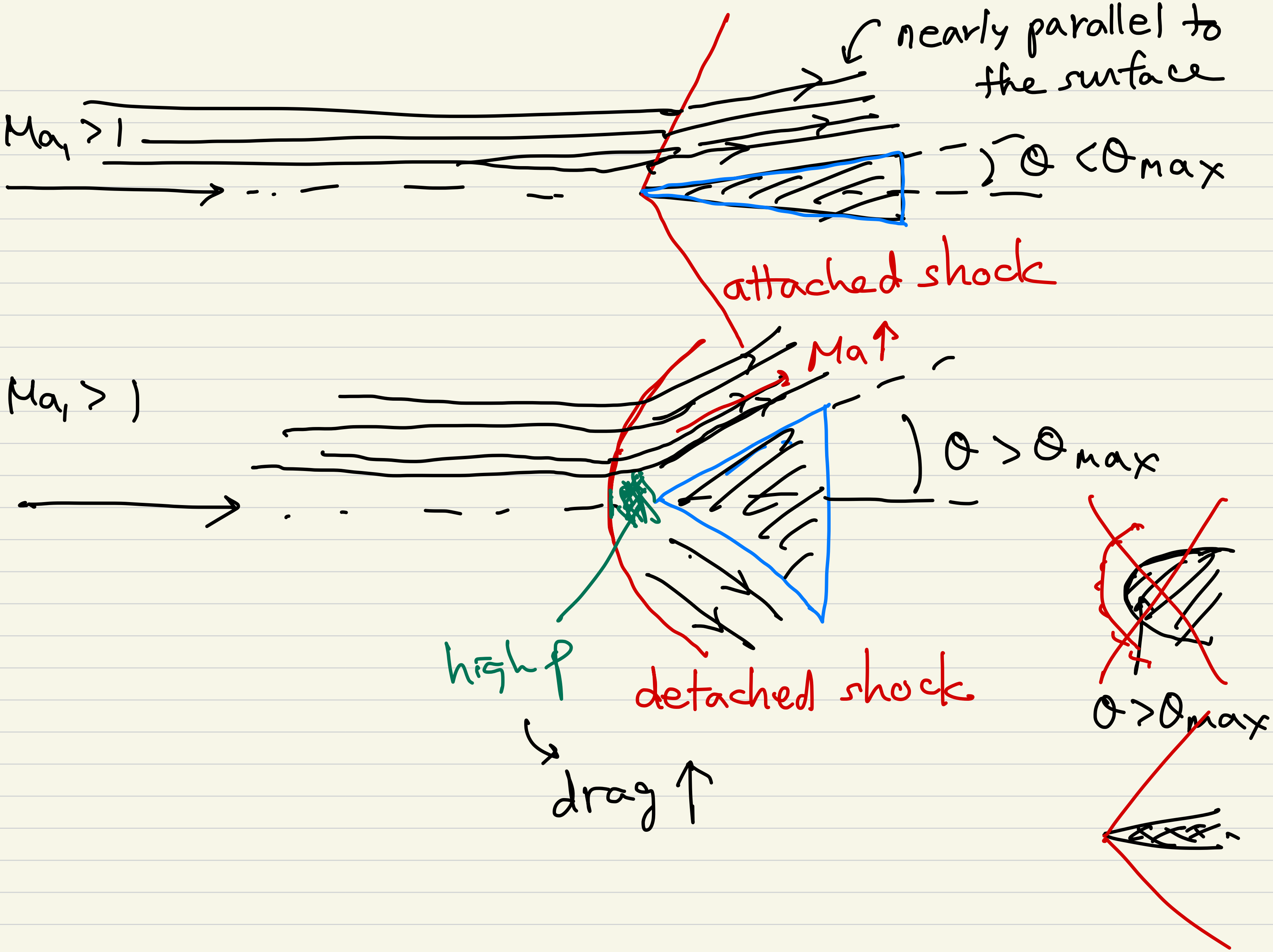
$$\rightarrow \theta_{max} = \tan^{-1}\sqrt{r} - \tan^{-1}\frac{1}{\sqrt{r}}$$

$$\left(\frac{V_t}{V_{n2}} = \frac{V_t}{V_{n_1}}\frac{V_{n_1}}{V_{n2}} = \left(\frac{V_{n2}}{V_{n_1}}\right)^{\frac{1}{2}}\left(\frac{V_{n_1}}{V_{n2}}\right) = \left(\frac{V_{n_1}}{V_{n2}}\right)^{\frac{1}{2}} = r^{\frac{1}{2}}\right)$$

If $Ma_{n_1} = 3$, $r = \frac{V_{n_1}}{V_{n2}} = 3.8571 \rightarrow \theta_{max} = 36.03°$

$Ma_{n_1} \rightarrow \infty$, $r = \frac{V_{n_1}}{V_{n2}} = 6.0 \rightarrow \theta_{max} = 45.58°$

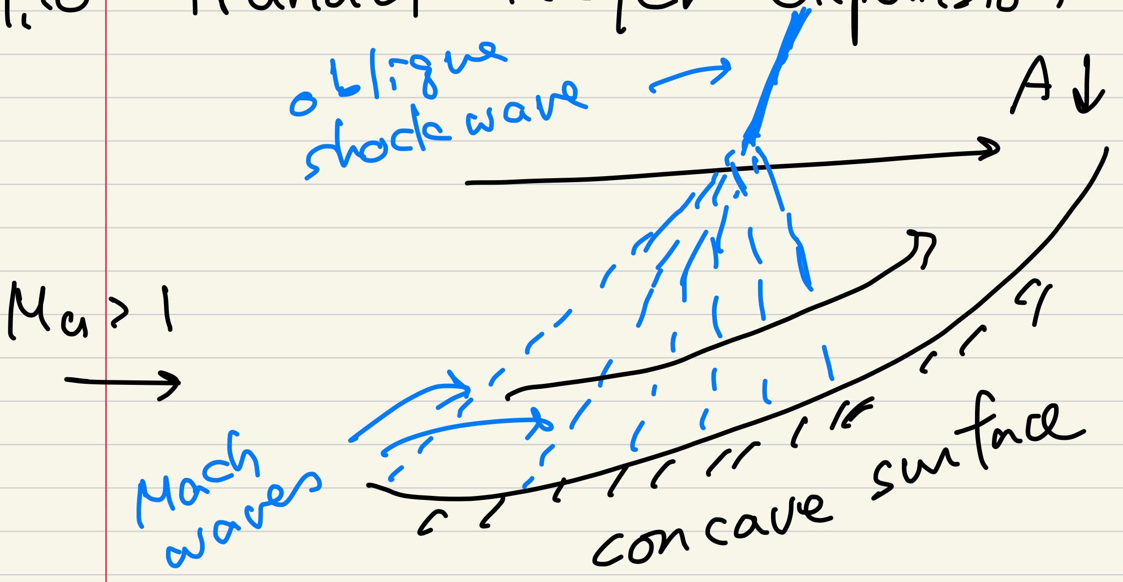


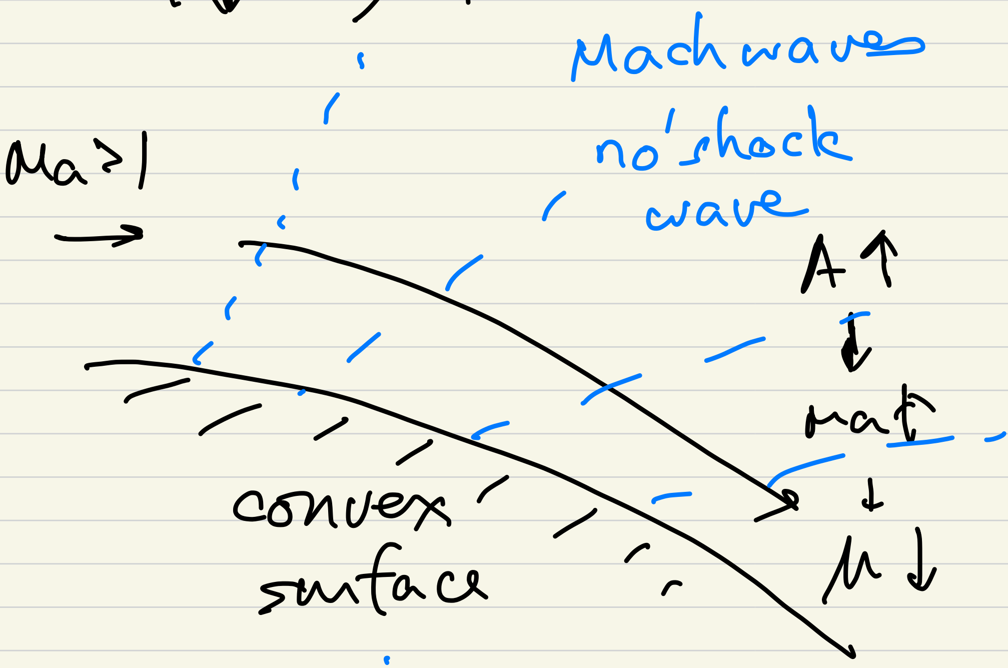


nearly parallel to the surface
$Ma_1 > 1$
$\theta < \theta_{max}$
attached shock
$Ma \uparrow$
$Ma_1 > 1$
$\theta > \theta_{max}$
high p
detached shock
drag ↑
$\theta > \theta_{max}$

9.10 Prandtl-Meyer expansion waves

$A \downarrow \rightarrow Ma \downarrow \rightarrow \mu \uparrow$





$$\mu = \sin^{-1}\frac{1}{Ma}$$