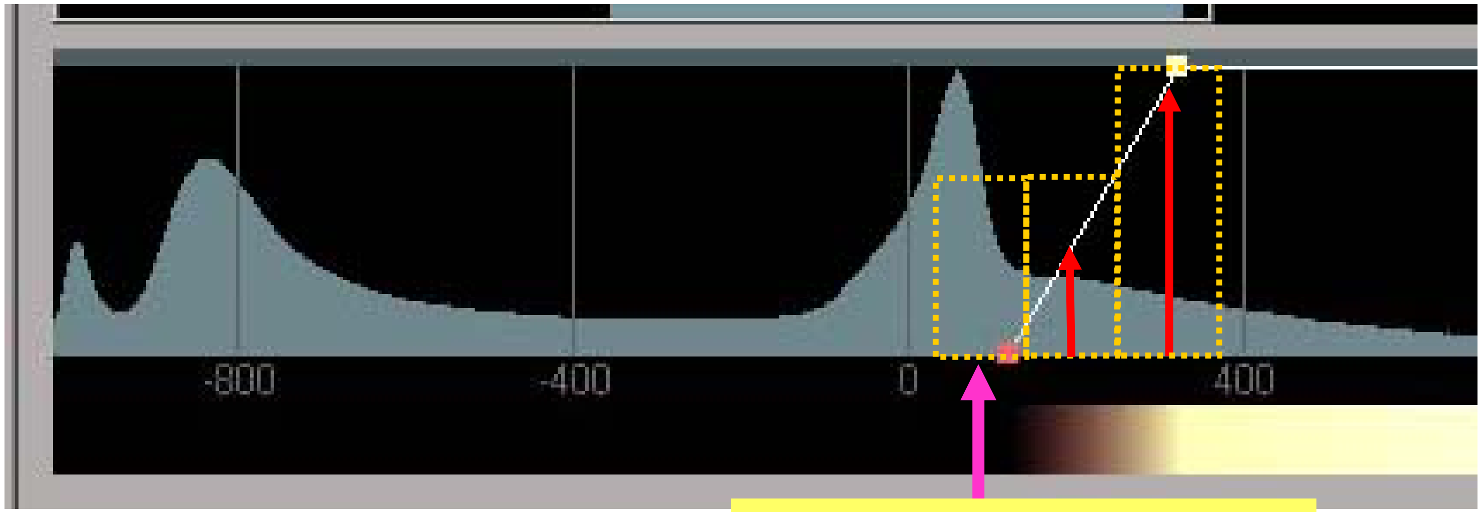
What we will cover

- Contour Tracking
- Surface Rendering
- Direct Volume Rendering
- Isosurface Rendering
- Optimizing DVR
- Pre-Integrated DVR

Image Quality Depends on

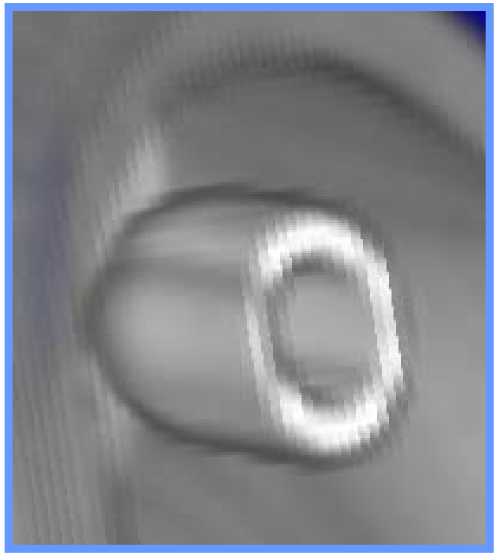
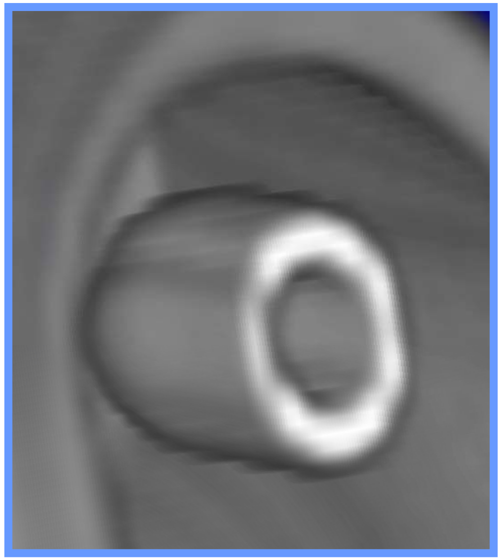
- Sampling Rate
- OTF variance
- OTF*Sampling Rate



Really nothing?

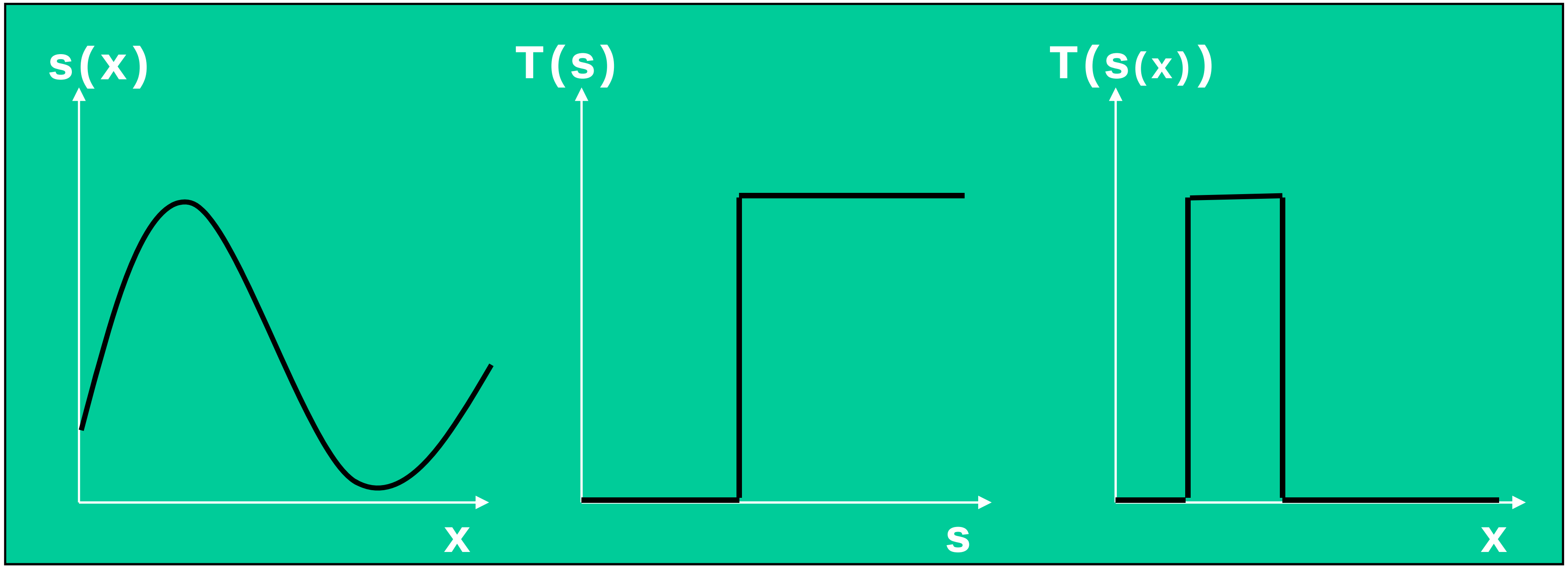
Ray-Integration

Discrete approximation of volume rendering integral will converge against correct result for ray sampling interval > 0

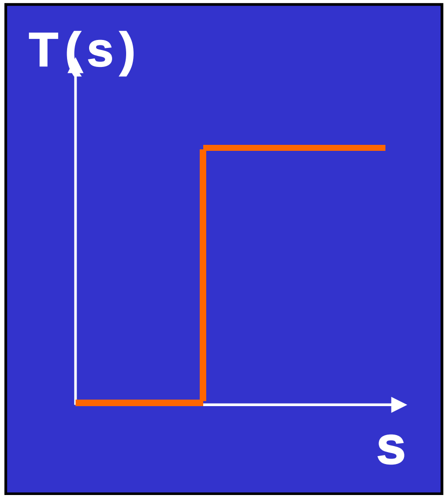
 

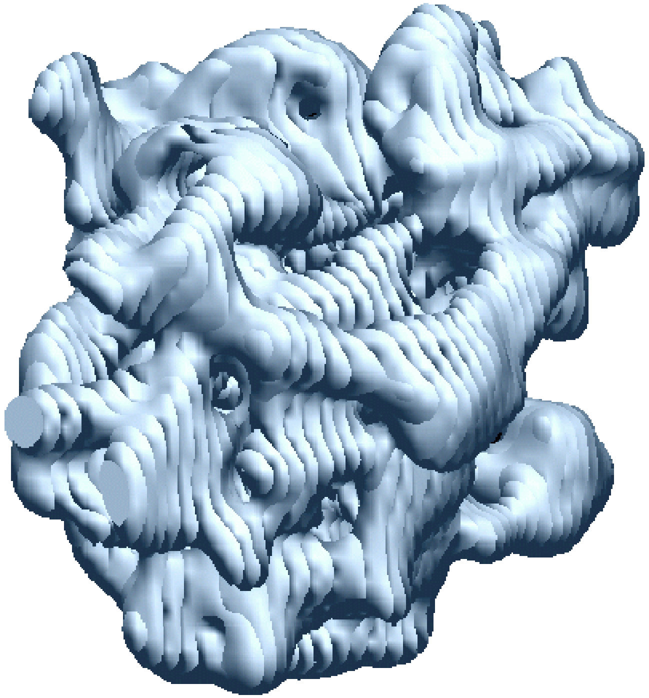
High-Frequency TFs

High frequencies in the transfer function T increase required sampling rate

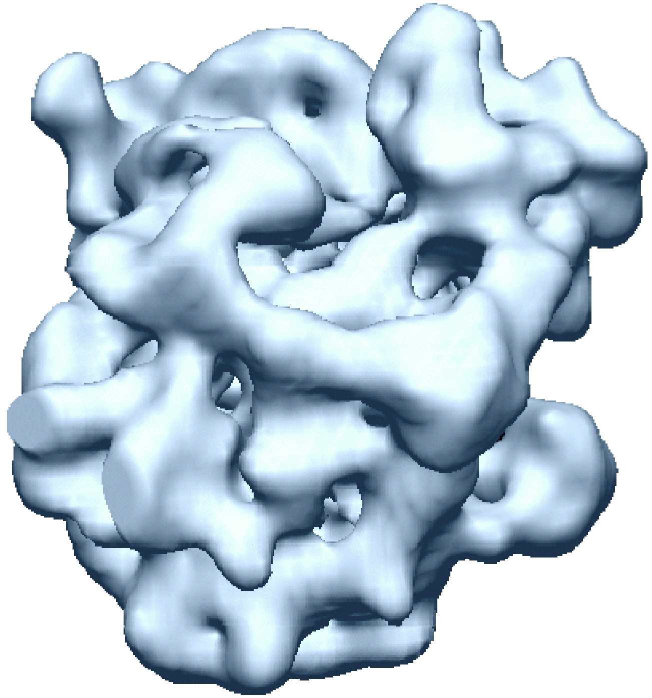


High-Frequency TFs



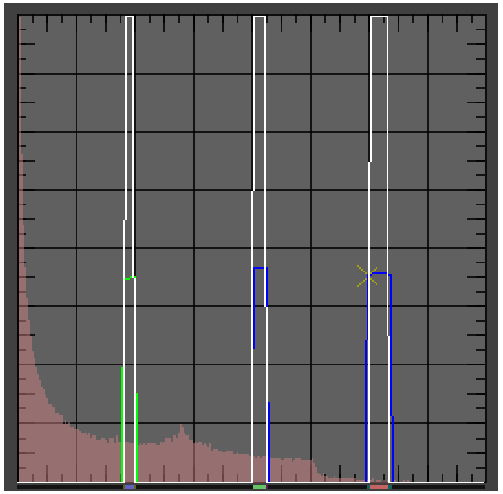


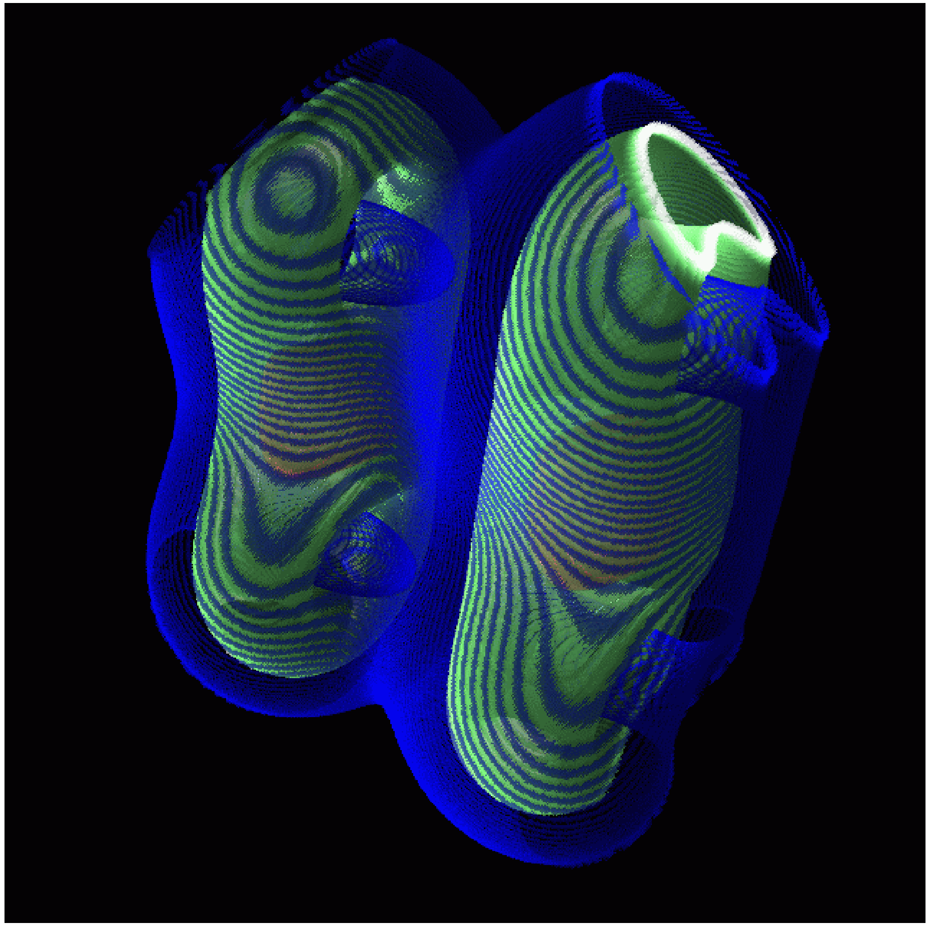
64 data slices



10 times more slices

High-Frequency TFs

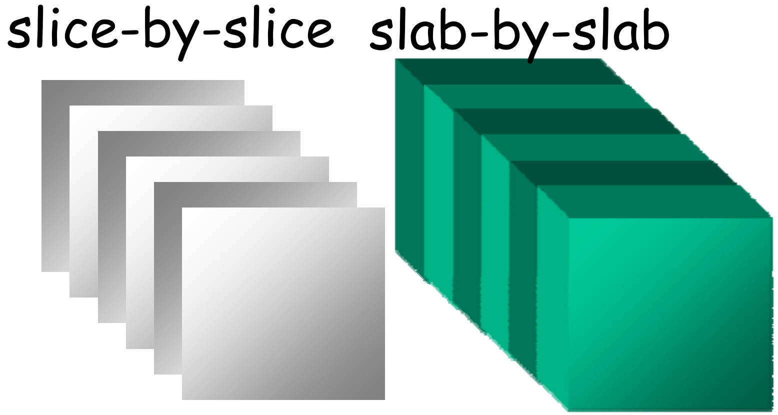


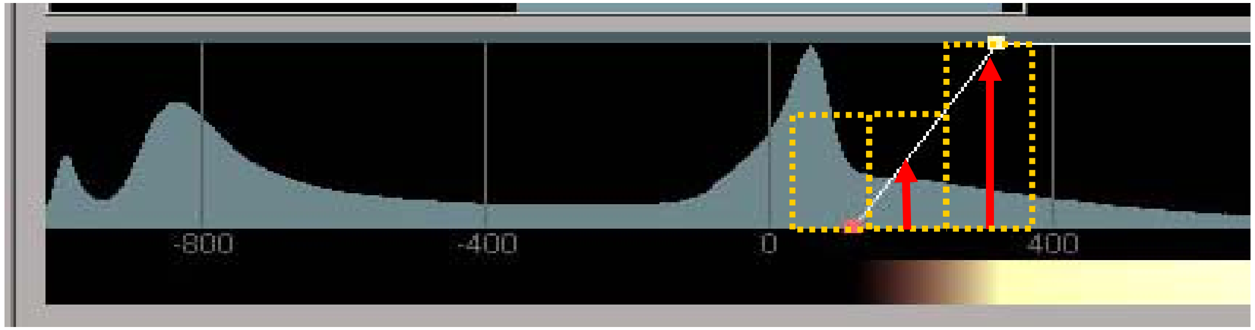


Pre-Integrated Volume Rendering

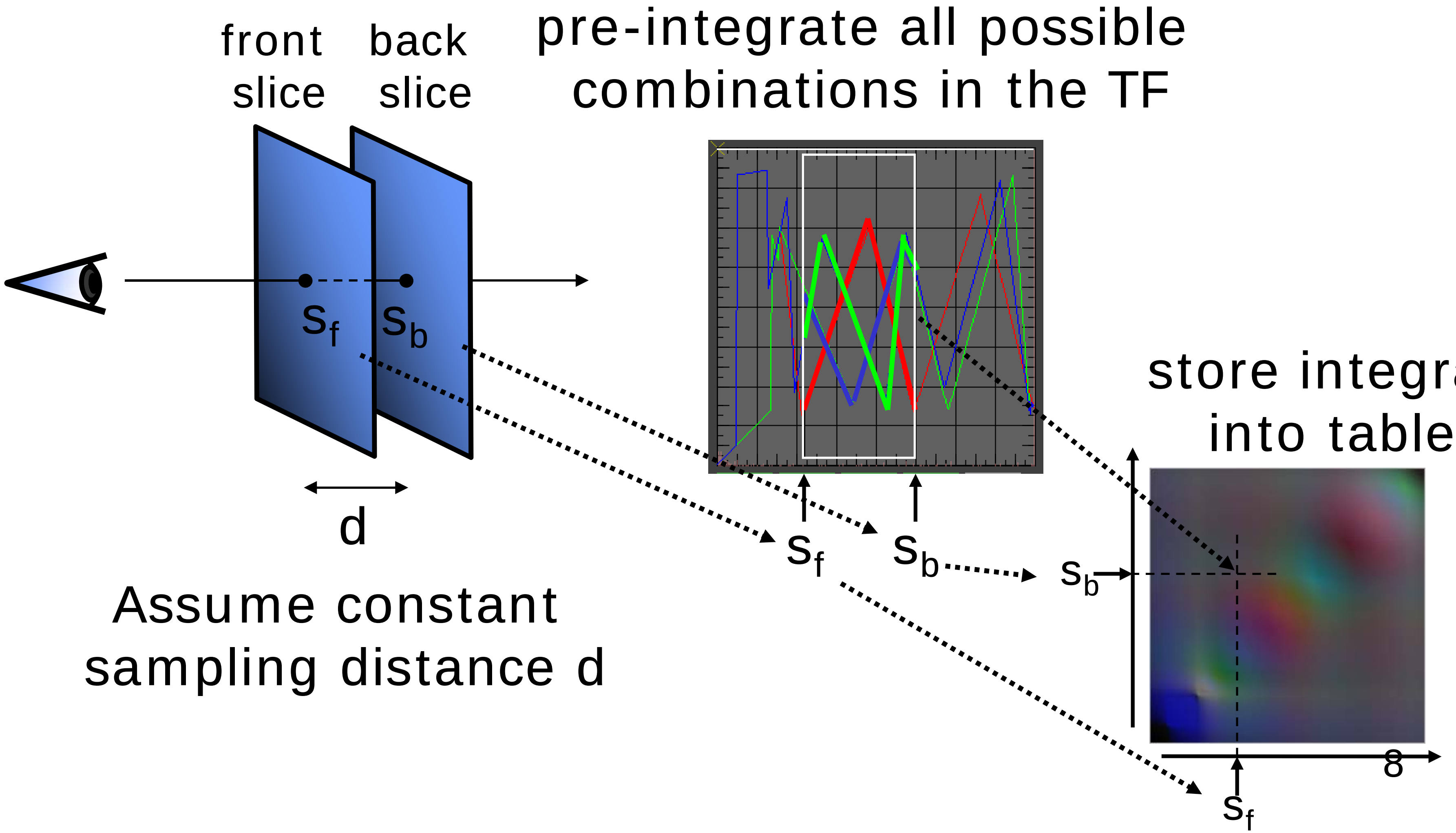
Idea: Pre-Integrated Classification

– slab-by-slab rendering

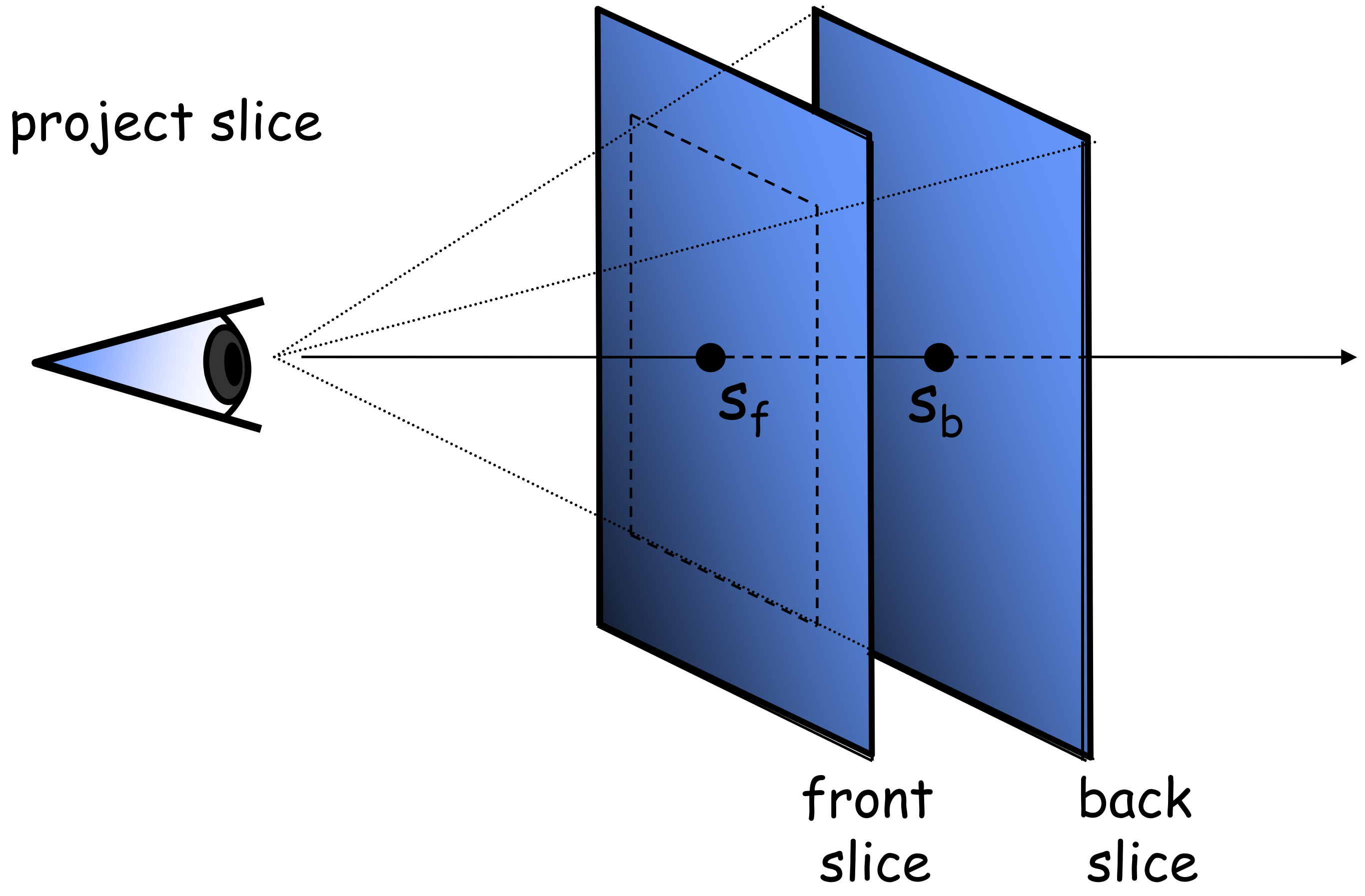




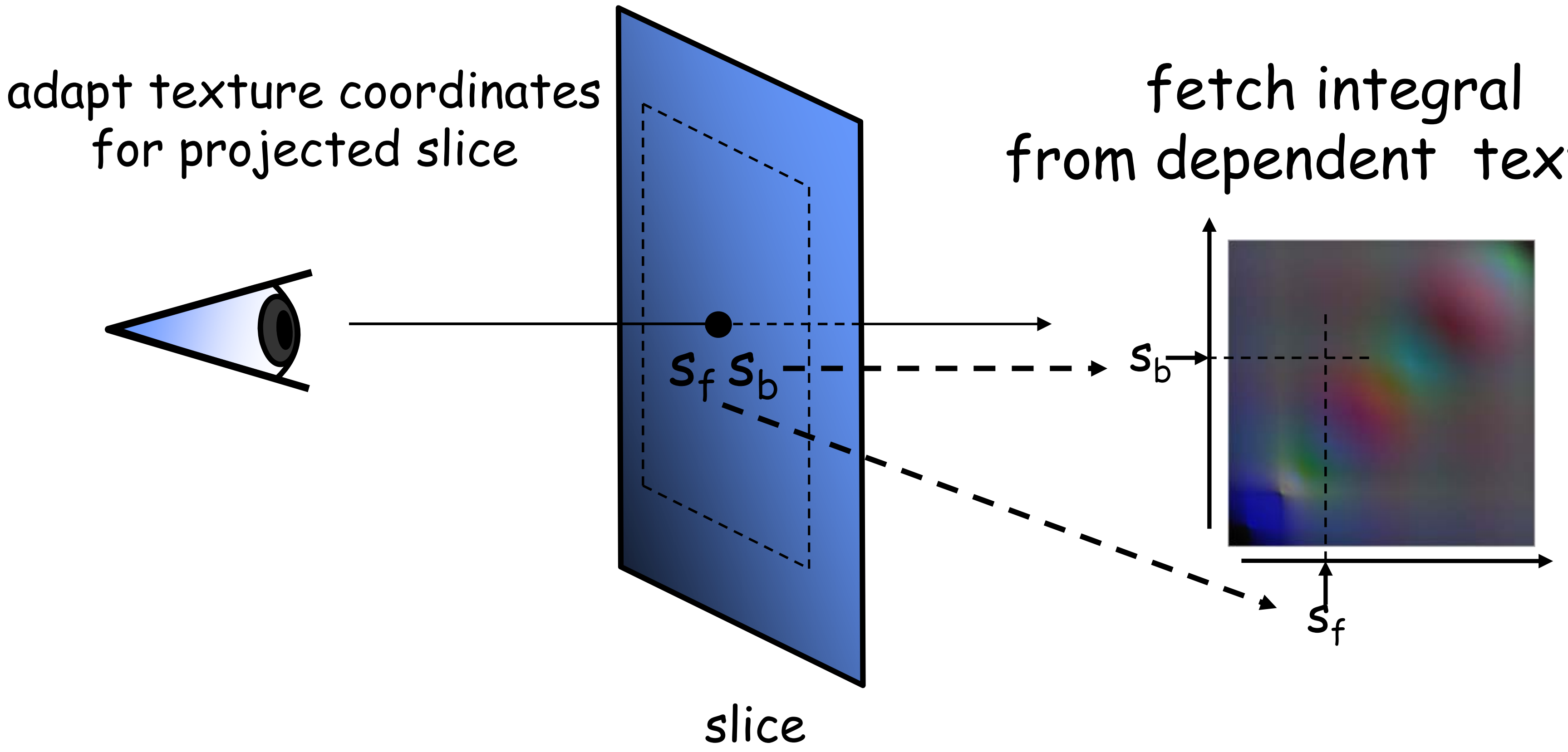
Pre-Integrated Classification



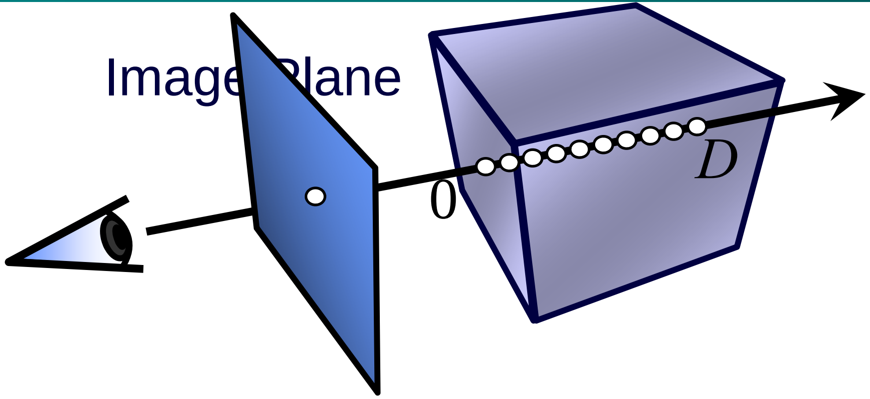
Pre-Integrated Volume Rendering

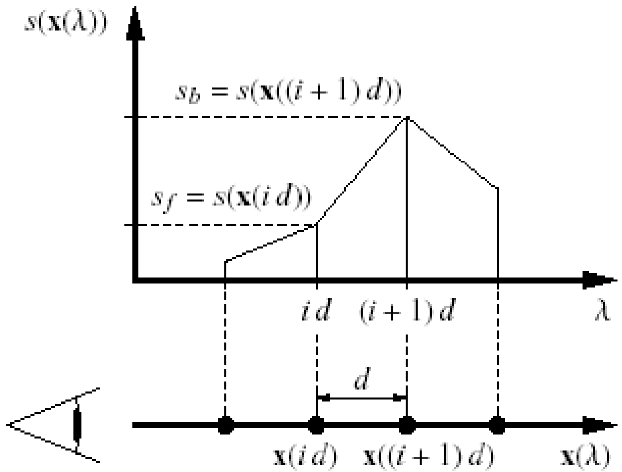


Pre-Integrated Volume Rendering



Pre-integrated Transfer Function





$$I(0,D) = \int_0^D color(x(\lambda))e^{-\tau(0,\lambda)}d\lambda$$

$$= \int_0^D \tilde{c}\left(s(x(\lambda))\right)\exp\left(-\int_0^{\lambda} \kappa(s(x(\lambda')))d\lambda'\right)d\lambda$$

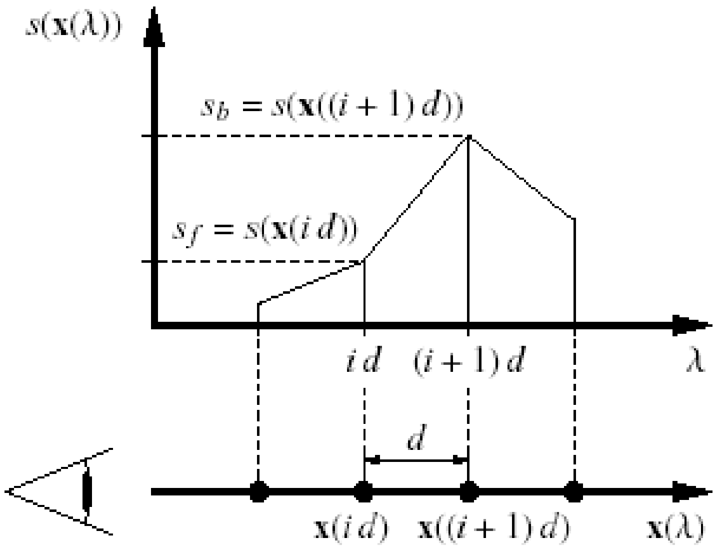
sampled density

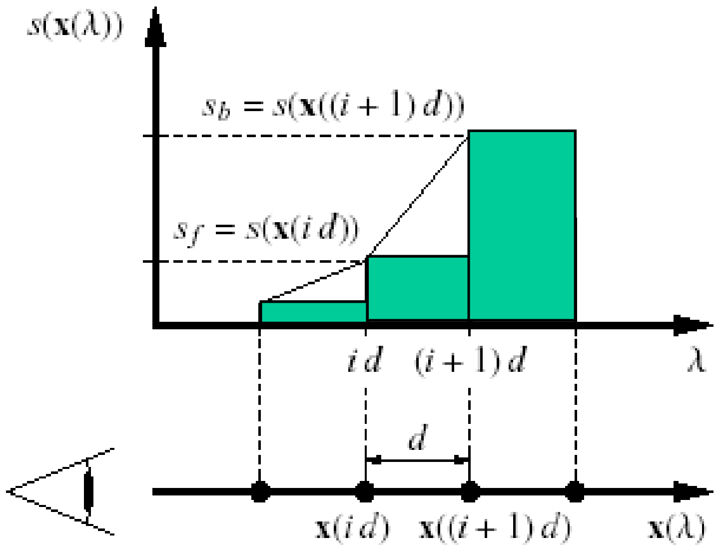
Pre-integrated Transfer Function

Approximate $\exp\left(-\int_0^{\lambda}\kappa\left(s\left(x\left(\lambda'\right)\right)\right)d\lambda'\right)$ by

$$\exp\left(-\sum_{i=1}^{\lambda/d}\kappa(s(x(i\times d)))d\right)=\prod_{i=1}^{\lambda/d}\exp\left(-\kappa\left(s(x(id))\right)d\right)=\prod_{i=1}^{\lambda/d}(1-\alpha_i)$$

where $\alpha_i \approx 1-\exp(-\kappa(s(x(i\,d)))d) \approx \kappa(s(x(i\,d)))d$





Pre-integrated Transfer Function

$$I(0,D) = \int_0^D \tilde{c}\big(s(x(\lambda))\big)\exp\left(-\int_0^\lambda \kappa\big(s(x(\lambda'))\big)d\lambda'\right)d\lambda$$

By approximating color $\tilde{\mathrm{C}} \approx \tilde{\mathrm{c}}\big(\mathrm{s}(\mathrm{x}(\mathrm{id}))\big)\mathrm{d}$

$$\rightarrow \quad I(0,n) \approx \sum_{i=0}^{n} \tilde{C}_i \prod_{j=0}^{i-1} (1-\alpha_j)$$

Back-to-front compositing

$$\tilde{C}_i' = \tilde{C}_i + (1-\alpha_i)\tilde{C}_{i+1}'$$

Pre-integrated Transfer Function

$$\alpha_i = \alpha(s_f, s_b, d) = 1 - \exp\left(-\int_{id}^{(i+1)d} \kappa(s(x(\lambda)))d\lambda\right)$$

$$\approx 1 - \exp\left(-\int_0^1 \kappa((1-\omega)s_f + \omega s_b)d\, d\omega\right)$$

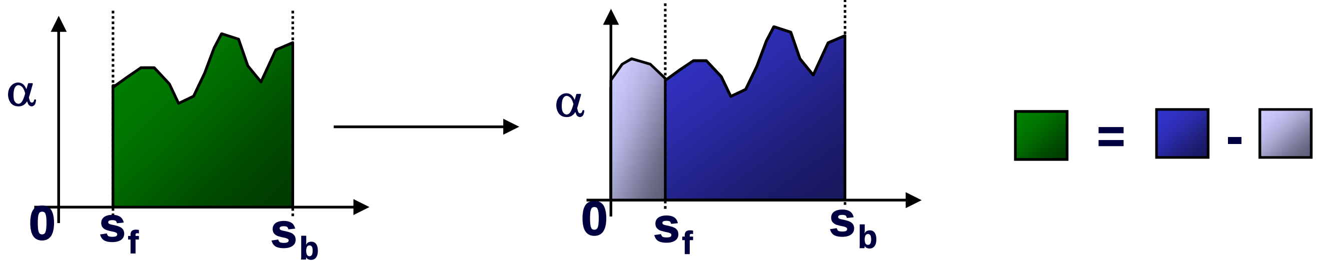
$$\widetilde{C}_i = \int_0^1 \widetilde{c}((1-\omega)s_f + \omega s_b)$$

$$\times \exp\left(-\int_0^{\omega} \kappa((1-\omega')s_f + \omega' s_b)d\, d\omega'\right)d\, d\omega$$

We can get $I(0,n) \approx \sum_{i=0}^{n} \widetilde{C}_i \prod_{j=0}^{i-1} (1-\alpha_j)$

with pre-computed color and opacity values

The approximated opacity of the *i-th* segment



$$\alpha_i = 1 - \exp(-\int_{id}^{(i+1)d} \kappa(s(x(\lambda)))d\lambda)$$

$$\approx 1 - \exp(-\int_0^1 \kappa((1-\omega)s_f + \omega s_b)d\, d\omega)$$

$$= 1 - \exp\left(-\frac{d}{s_b - s_f}\int_{s_f}^{s_b} \kappa(s)ds\right)$$

$$= 1 - \exp\left(-\frac{d}{s_b - s_f}\left(T(s_b) - T(s_f)\right)\right) \quad \text{where, } T(s) = \int_0^s \kappa(s)ds$$

The approximated color of the *i-th* segment

$$\tilde{C}_i = \int_0^1 \tilde{c}((1-\omega)s_f + \omega s_b)$$
$$\times \exp(-\int_0^{\omega} \kappa((1-\omega')s_f + \omega' s_b)d\ d\omega')d\ d\omega$$

Self-attenuation within segment

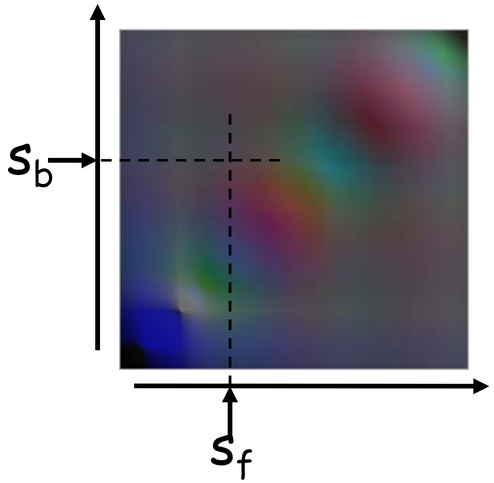
Neglect the self-attenuation

$$\tilde{C}_i \approx \int_0^1 \tilde{c}((1-\omega)s_f + \omega s_b)d\ d\omega$$
$$= \frac{d}{s_b - s_f}\int_{s_a}^{s_b} \tilde{c}(s)ds$$
$$= \frac{d}{s_b - s_f}(K(s_b) - K(s_f)) \quad \text{, where } K(s) = \int_0^s \tilde{c}(s)ds$$

12-bit Image Handling

- 2D Pre-integration Table with fixed *d*
  - 12bits per S_f, S_b
  - Construct Pre-integrated Table requires too much time and space

$$\text{Lookup table}: 2^{12} \times 2^{12} \times (R, G, B, \alpha) \times 8bit = 64MByte$$



$$\alpha_i = \alpha(s_f, s_b, d)$$
$$\approx 1 - \exp\left(-\frac{d}{s_b - s_f} \left(T(s_b) - T(s_f) \right) \right)$$

Pre-integrated classification

- 2D Pre-integration Table with a fixed *d*

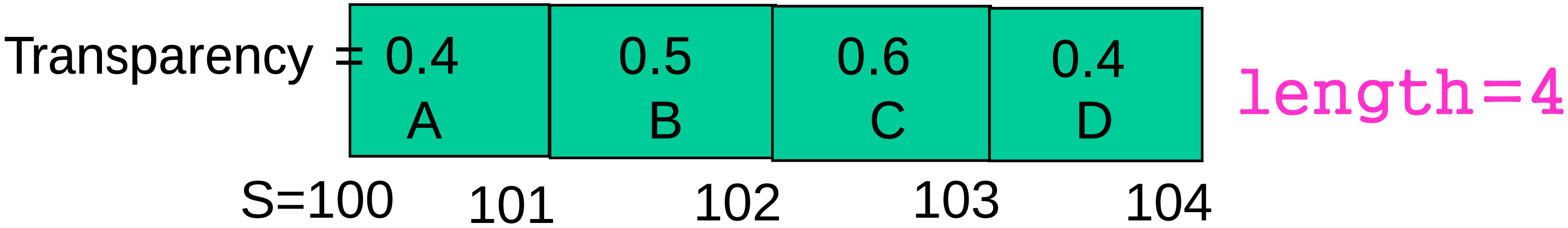
$$1-\alpha_i = 1-\alpha\left(s_f, s_b, d\right) \approx \exp\left(-\frac{d}{s_b - s_f}\left(T\left(s_b\right) - T\left(s_f\right)\right)\right)$$

$$= \exp\left(-\frac{d}{s_b - s_f}\sum_{s=s_f}^{s_b}\kappa(s)\right) = \sqrt[|s_b - s_f|]{\exp\left(-d\sum_{s=s_f}^{s_b}\kappa(s)\right)}$$

$$= \sqrt[|s_b - s_f|]{\prod_{s=s_f}^{s_b}\exp(-\kappa(s)d)}$$

Note that $\exp(-\kappa(s)d) \approx 1-\kappa(s)d = \text{transparency}(s)d$

Opacity Computation



Opacity of a material with four blocks

$$\alpha(ABCD)?$$

$$\alpha = 1 - 0.4 \times 0.5 \times 0.6 \times 0.4 \quad \text{(o)}$$

$$\alpha = 1 - \sqrt[4]{0.4 \times 0.5 \times 0.6 \times 0.4} \quad \text{(x)}$$

$$\alpha_i = \alpha\left(s_f, s_b, d\right) = \alpha(100,104,1)?$$

When d=1

$$\alpha = ?$$

100 104

$$\alpha = 1 - 0.4 \times 0.5 \times 0.6 \times 0.4 \quad \text{(x)}$$

$$\alpha = 1 - \sqrt[4]{0.4 \times 0.5 \times 0.6 \times 0.4} \quad \text{(o)}$$

1D Pre-Integration Table

- Instead of integral function, we use arithmetic average term for the fast generation of Table
- Use 1D table with extra operation but less memory

$$\text{Lookup table}: 2^{12} \times (R, G, B, \alpha) \times 8bit = 16KBytes$$

$$\alpha(s_f, s_b, d) = 1 - \sqrt[|s_b - s_f|]{\prod_{s=s_f}^{s_b} \exp(-\kappa(s)d)}$$

$$\approx 1 - \frac{1}{|s_b - s_f|} \sum_{s=s_f}^{s_b} \exp(-\kappa(s)d)$$

T=0.4	0.6	0.3	0.2

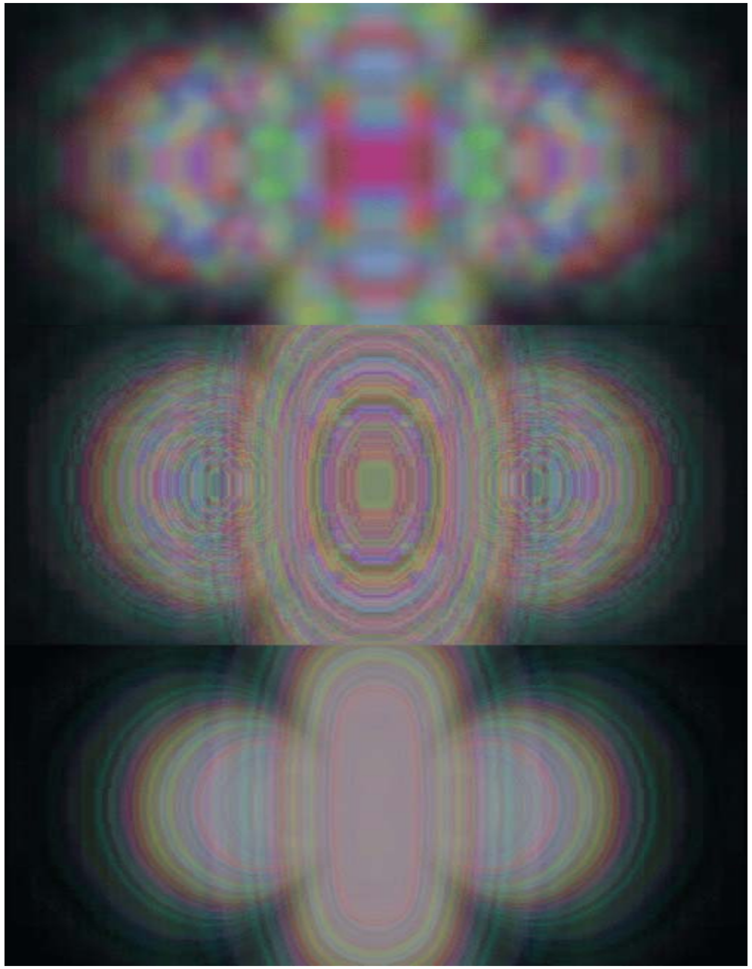
$$\alpha = 1 - (0.4 + 0.6 + 0.3 + 0.2) / 4$$

Pre-Integrated Volume Rendering

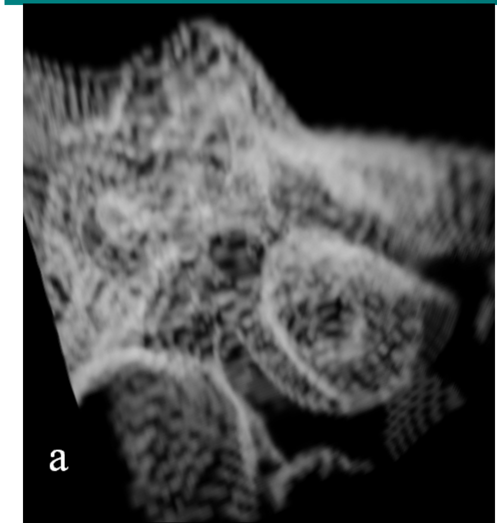
Pre-Classification

Post-Classification

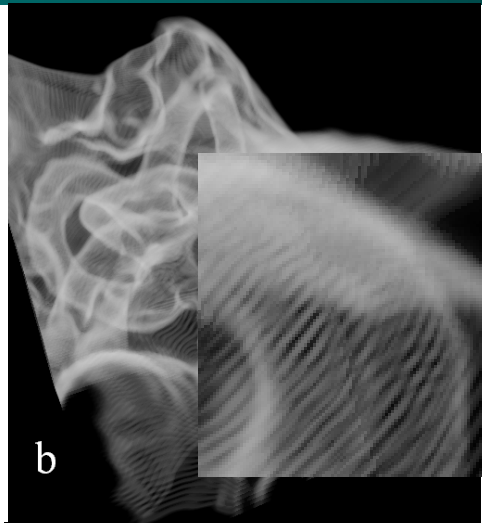
Pre-Integrated-
Classification



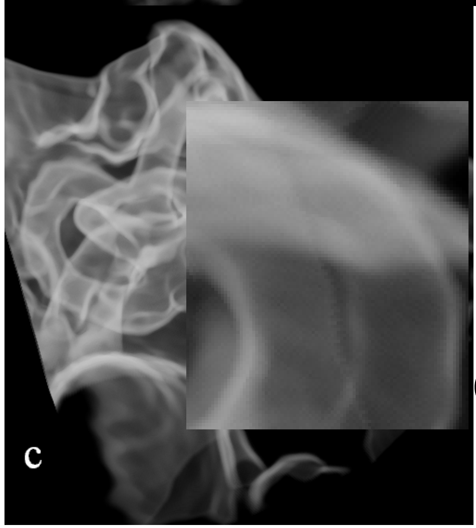
Pre-Integrated Volume Rendering



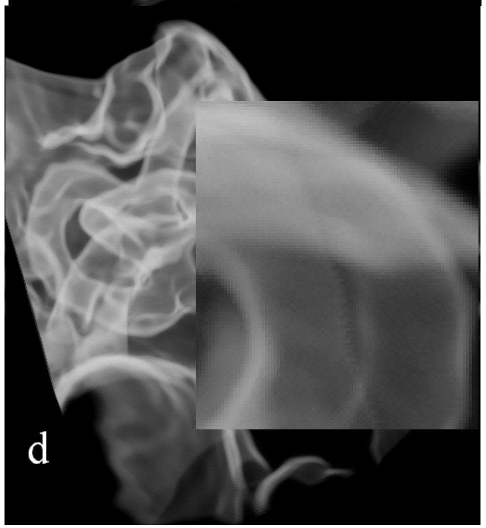
128 slices
pre-
classification



128 slices
post-
classification



284 slices
post-
classification



128 slices
pre-integrated

Pre-Integrated Volume Rendering

- Texture-based (2D/3D)
  - Pre-computed ray-segment lookup
  - Dependent texture
- Especially suited for:
  - Low resolution volume data
  - Non-linear transfer functions

Pre-Integrated Volume Rendering

- Reduce compositing time by larger re-sampling interval
- Need to re-compute the pre-integrated table whenever OTF is changed.
- Brings blurring effects
- More compositing voxels (20-30%)