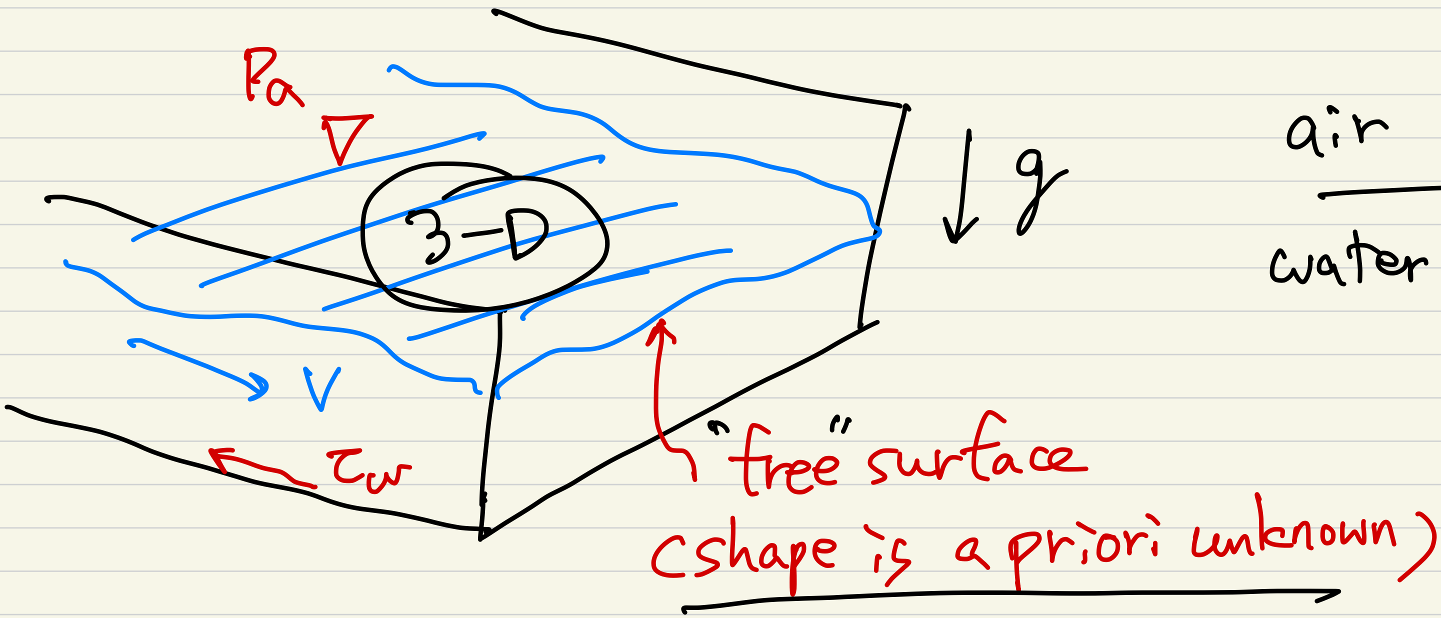
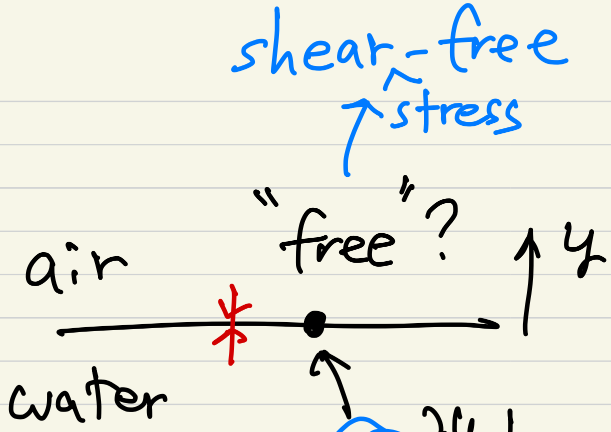
Ch. 10 Open channel flow (개수로 유동)



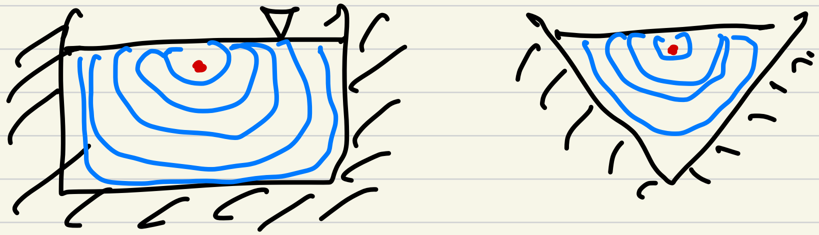


$$\tau = \mu_a \frac{\partial u}{\partial y}\Big|_a = \mu_w \frac{\partial u}{\partial y}\Big|_w \simeq 0$$

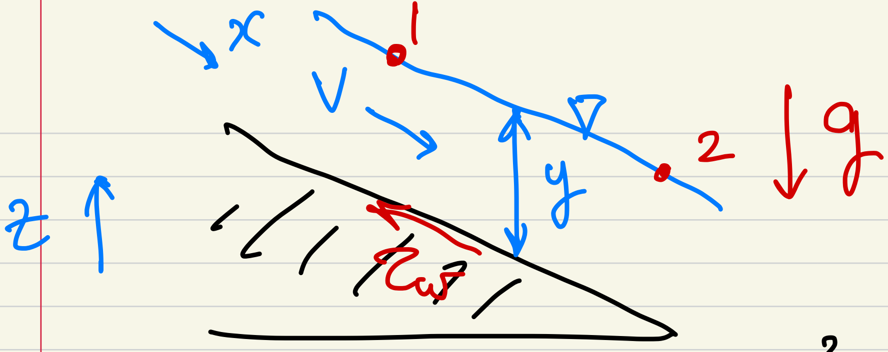
$\mu_w \gg \mu_a$

- balance between the gravity force and friction force
- depth profile changes

- 1D approximation



3D flow



$$P_1 = P_2 = P_a$$

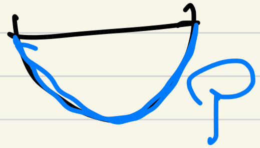
$$\frac{V_1^2}{2g} + z_1 = \frac{V_2^2}{2g} + z_2 + h_f$$

↑ friction head loss

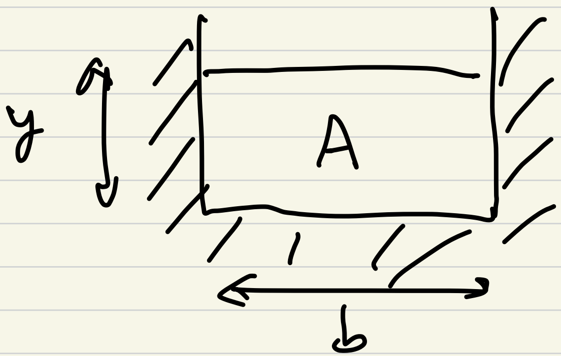
Ch.6 $h_f = f \frac{L}{d} \frac{V^2}{2g} \simeq f \frac{x_2 - x_1}{D_h} \frac{V_{avg}^2}{2g}$

↑ friction factor

$$D_h = \frac{4A}{P}$$



$Re = D_h V_{avg} / \nu \geq 10^5$ mostly turbulent



$P = b + 2y$, $A = by$) $D_h = \frac{4A}{P} = \frac{4by}{b+2y}$

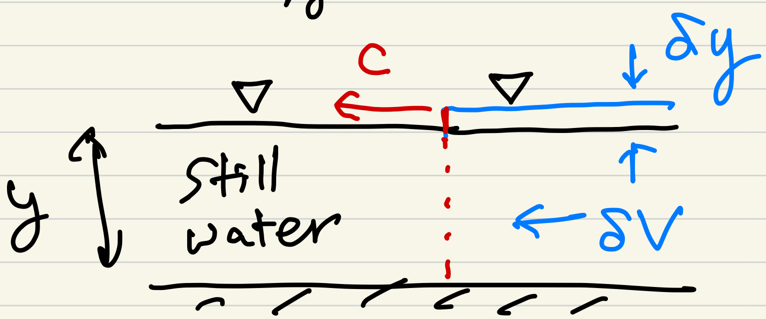
Moody chart ← accurate but seldom used

Manning's formula ← mostly used

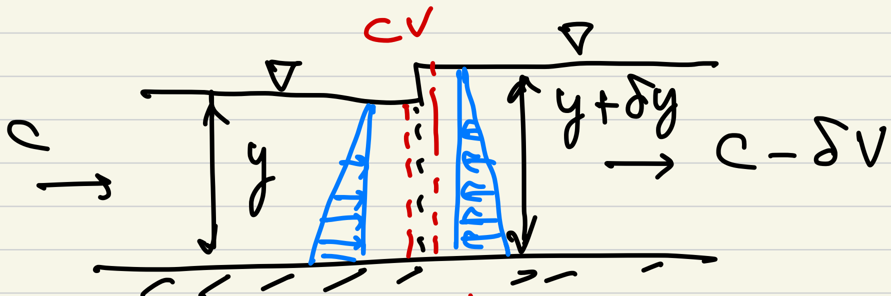
- Flow characteristics by Froude number $\left(Ma = \frac{V}{a}\right)$

$$Fr \equiv \frac{V}{\sqrt{gy}} \qquad y: \text{water depth}$$

$\sqrt{gy}$?



b: width



cont. : $\rho c y b = \rho (c-\delta V)(y+\delta y) b \rightarrow \delta V = c \frac{\delta y}{y+\delta y}$ — ①

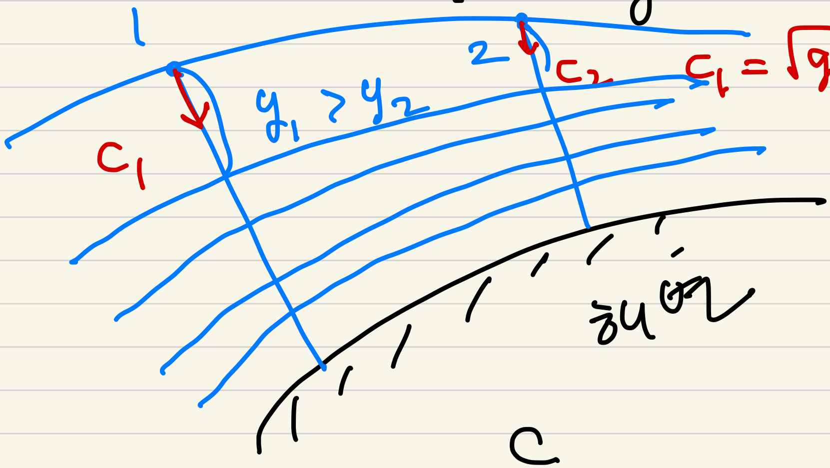
mtm : $\Sigma F = -\frac{1}{2}\rho g b (y+\delta y)^2 + \frac{1}{2}\rho g b y^2 = \rho c y b (c - \delta V - c)$

$$\rightarrow g\left(1 + \frac{1}{2}\frac{\delta y}{y}\right)\delta y = c\,\delta V \quad \text{— ②}$$

① → ② : $c^2 = gy\left(1 + \frac{\delta y}{y}\right)\left(1 + \frac{1}{2}\frac{\delta y}{y}\right)$ wave propagation speed

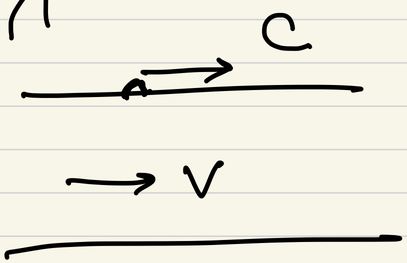
As $\delta y \uparrow$, $c \uparrow$

As $\delta y \to 0$, $c^2 = gy \rightarrow c = \sqrt{gy}$: speed of shallow water surface wave



$c_1 = \sqrt{gy_1} > c_2 = \sqrt{gy_2}$

$$F_r = \frac{V}{c}$$

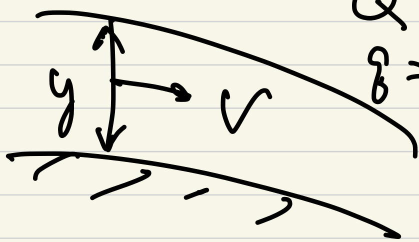


$F_r < 1$: subcritical flow

$F_r = 1$: critical "

$F_r > 1$: supercritical "

- specific energy : $E \equiv y + \frac{V^2}{2g}$

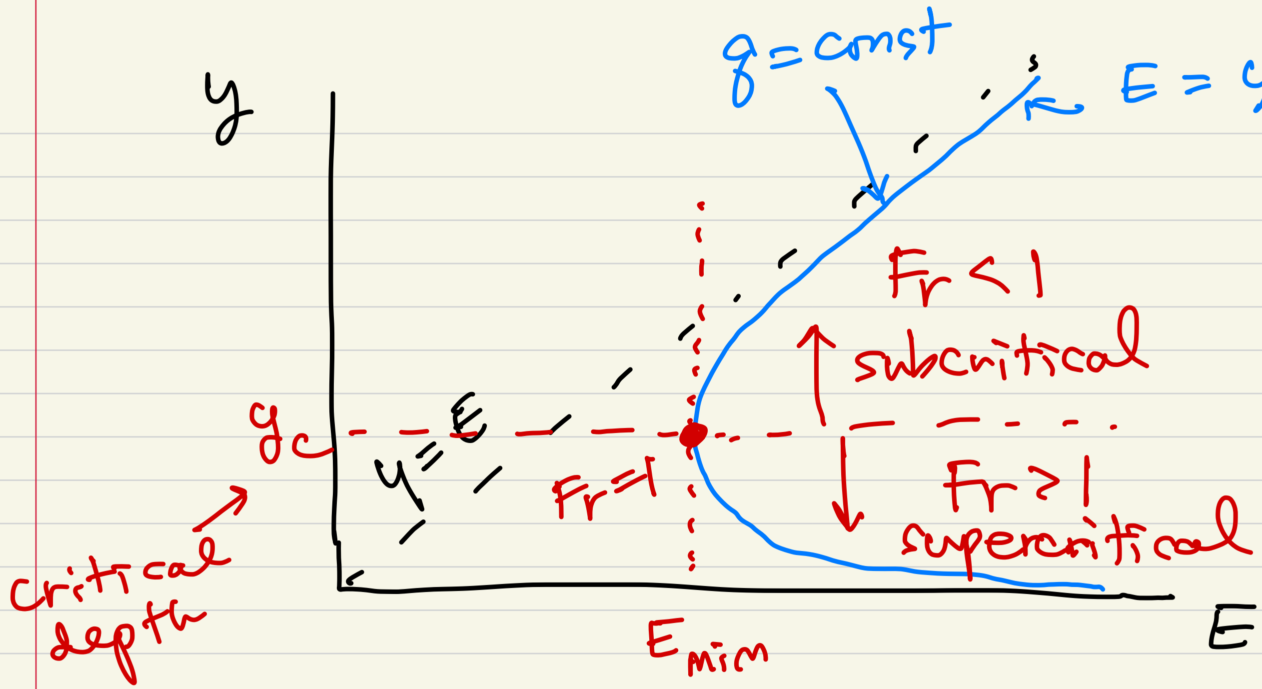


$Q = \text{const}$
$q = \text{const}$

$Q = Vyb = qb$ $\quad q = \frac{Q}{b}$: discharge per width

$\rightarrow E = y + \frac{q^2}{2gy^2}$

$$E = y + \frac{q^2}{2gy^2}$$



y_c s.t. $\frac{dE}{dy} = 0$

$$\rightarrow y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{Q^2}{b^2 g}\right)^{\frac{1}{3}}$$

$$E \rightarrow E_{min} = E(y_c) = y_c + \frac{q^2}{2gy_c^2} = \frac{3}{2} y_c$$

$$// \quad y_c + \frac{V_c^2}{2g}$$

$$V_c = \sqrt{g y_c} = c$$

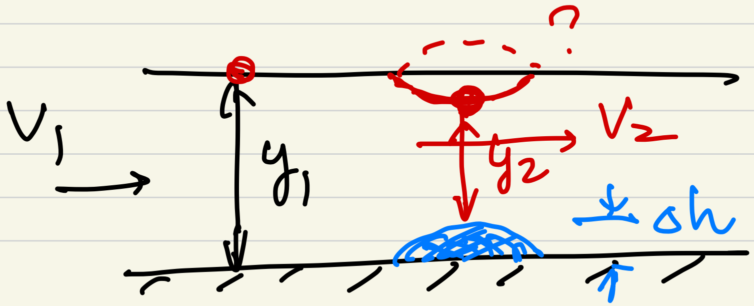
$Q = Vyb$

$\rightarrow y > y_c$: $V < V_c \rightarrow Fr < 1$

$y < y_c$: $V > V_c \rightarrow Fr > 1$

$Fr = 1$ @ $y = y_c$ ←

- Frictionless flow over a bump



cont : $V_1 y_1 = V_2 y_2$

Bernoulli eq : $\frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + (y_2 + \Delta h)$

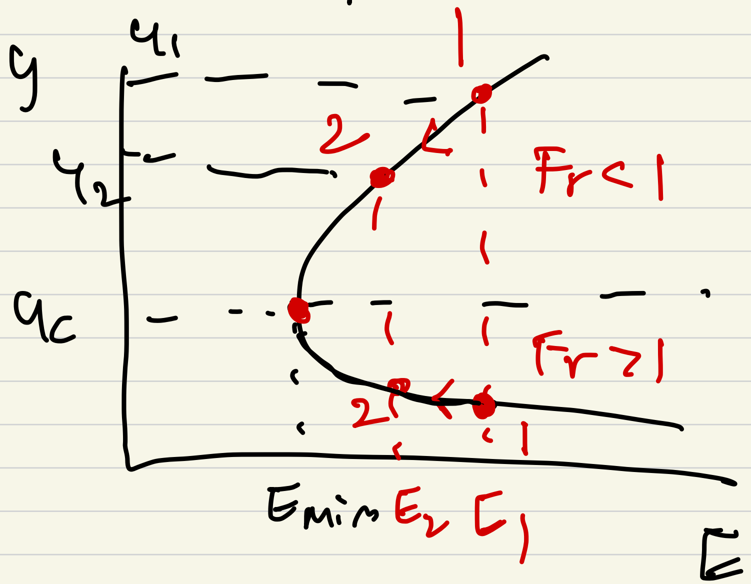
$$E_1 = E_2 + \Delta h$$

$$\begin{cases} y_2^3 - E_2 y_2^2 + \dfrac{V_1^2 y_1^2}{2g} = 0 \\ E_2 = E_1 - \Delta h \end{cases}$$

if Δh is not too large,

$Fr_1 = \dfrac{V_1}{\sqrt{g y_1}} < 1$: $(y_2 + \Delta h) - y_1 = \dfrac{1}{2g}(V_1^2 - V_2^2)$

$= \dfrac{1}{2g}(V_1 + V_2)(V_1 - V_2) < 0$



$E_2 = E_1 - \Delta h$

$\rightarrow y_2 < y_1 \rightarrow V_2 > V_1$

$\Rightarrow y_2 + \Delta h < y_1$

∴ water level down !

$Fr_1 > 1$: $E_2 < E_1 \rightarrow y_2 > y_1 \rightarrow V_2 < V_1$

$(y_2 + \Delta h) - y_1 > 0 \rightarrow y_2 + \Delta h > y_1$

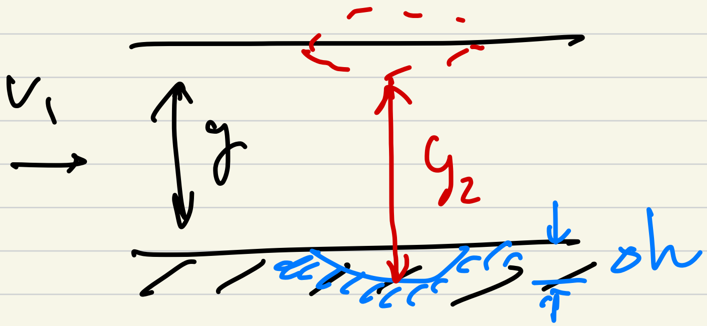
∴ water level up!

If $\Delta h = \Delta h_{max} = E_1 - E_c(min)$, flow at the crest is critical ($Fr_2 = 1$).

If $\Delta h > \Delta h_{max}$, no physical sol.!

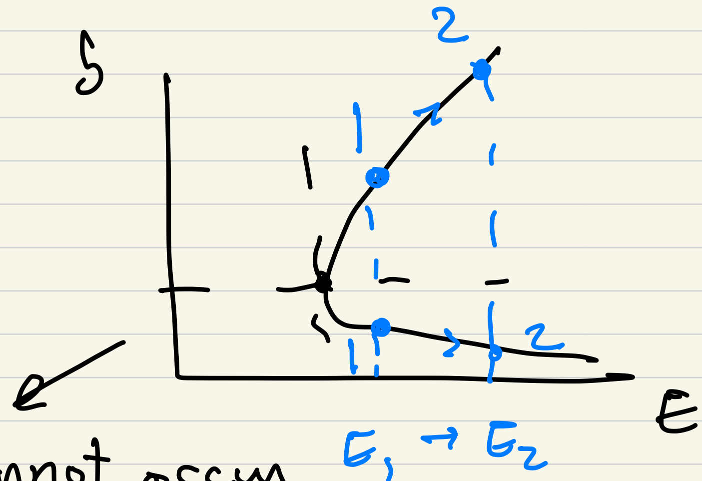
i.e. a bump too large will choke the channel and cause frictional effects, typically a hydraulic jump.

- Frictionless flow over a hollow



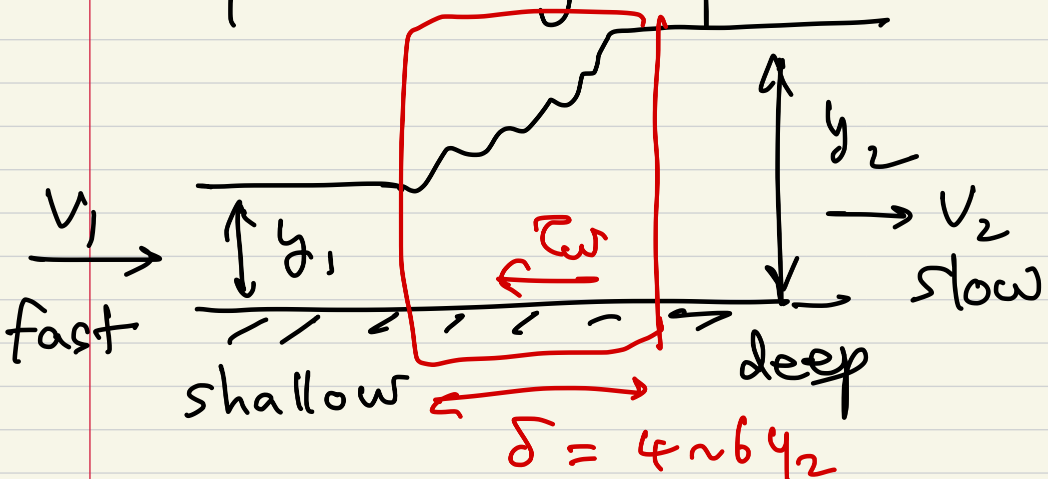
$$E_1 = E_2 - \Delta h$$

$$\rightarrow E_2 = E_1 + \Delta h$$



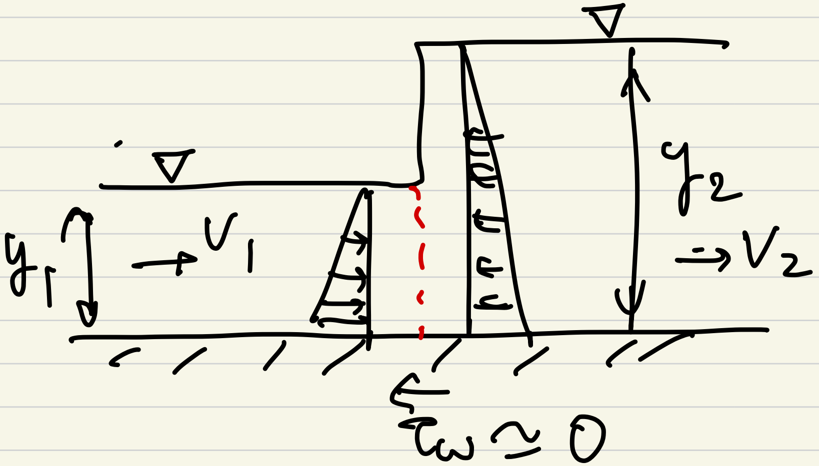
critical flow cannot occur.

- hydraulic jump



extremely turbulent

effective energy dissipator



Cont : $\rho V_1 y_1 b = \rho V_2 y_2 b$

mtm : $-\frac{1}{2}\rho g b (y_2^2 - y_1^2)$

$= \rho V_1 y_1 b (V_2 - V_1)$

$\rightarrow V_1^2 = g y_1 \eta \cdot \frac{1}{2}(1+\eta)$, $\eta = y_2/y_1$

$V_2 = V_1 y_1 / y_2$, $Fr_1 = V_1/\sqrt{g y_1}$

$$\rightarrow \boxed{\eta = \frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8 Fr_1^2}\right)} \rightarrow V_2 \rightarrow Fr_2 \sim f(Fr_1)$$

Also, $h_f = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right) = \cdots = \frac{(y_2 - y_1)^3}{4 y_1 y_2} > 0$

$\rightarrow y_2 > y_1 \rightarrow \zeta = \frac{y_2}{y_1} > 1 \rightarrow$ $Fr_1 > 1$ supercritical flow

$Fr_2 < 1 \leftarrow V_2 < V_1$

subcritical flow