Electromagnetics

<Chap. 8> Plane Electromagnetic waves

Section 8.5 ~ 8.8

(1st of **week 4**)

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Textbook: Field and Wave Electromagnetics, 2E, Addison-Wesley

Chap. 8 | Contents for 1st class of week 4

Sec 5. Flow of electromagnetic power and the Poynting vector

Sec 6. Normal incidence on a plane conducting boundary

Electromagnetic power

- EM waves carry ***EM power*** with them (***Energy*** is ***transported by EM waves*** through space to distant receiving points)

Transfer rate of EM energy vs. E, H

- Derivation

$$\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H})=\boldsymbol{H}\cdot(\nabla\times\boldsymbol{E})-\boldsymbol{E}\cdot(\nabla\times\boldsymbol{H})$$ (Proof of identity; HW)

$$=-\boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial t}-\boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial t}-\boldsymbol{E}\cdot\boldsymbol{J}$$ ←

$$\nabla\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t}$$

$$\nabla\times\boldsymbol{H}=\boldsymbol{J}+\frac{\partial\boldsymbol{D}}{\partial t}$$

By assuming a simple medium whose ε, μ, and σ are not functions of t,

$$\boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial t}=\boldsymbol{H}\cdot\frac{\partial(\mu\boldsymbol{H})}{\partial t}=\frac{1}{2}\frac{\partial(\mu\boldsymbol{H}\cdot\boldsymbol{H})}{\partial t}=\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right)$$ ←

$$\frac{\partial}{\partial t}(\boldsymbol{A}\cdot\boldsymbol{B})=\boldsymbol{A}\cdot\frac{\partial\boldsymbol{B}}{\partial t}+\frac{\partial\boldsymbol{A}}{\partial t}\cdot\boldsymbol{B}$$ Product rule

$$\boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial t}=\boldsymbol{E}\cdot\frac{\partial(\varepsilon\boldsymbol{E})}{\partial t}=\frac{1}{2}\frac{\partial(\varepsilon\boldsymbol{E}\cdot\boldsymbol{E})}{\partial t}=\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon E^2\right)$$

$$\boldsymbol{E}\cdot\boldsymbol{J}=\boldsymbol{E}\cdot(\sigma\boldsymbol{E})=\sigma E^2$$

$$\therefore\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H})=-\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon E^2+\frac{1}{2}\mu H^2\right)-\sigma E^2$$

Chap. 3 | Electric potential energy (review)

Potential energy of a continuous charge distribution

$$W_e = \frac{1}{2}\int_{V'} \rho V \, dv \quad \text{(J)}$$ ← ***Refer Section 3-11 for derivation***

where V is the potential at a point where the volume charge density is ρ
and V' is the volume of the region where ρ exists.

- W_e in terms of field quantities ***E*** and ***D*** without explicitly knowing ρ

$$W_e = \frac{1}{2}\int_{V'} \rho V \, dv = \frac{1}{2}\int_{V'} (\nabla \cdot \boldsymbol{D}) V \, dv$$

$$= \frac{1}{2}\int_{V'} \nabla \cdot (V\boldsymbol{D}) dv - \frac{1}{2}\int_{V'} \boldsymbol{D} \cdot \nabla V \, dv$$ ← $\nabla \cdot (V\boldsymbol{D}) = V\nabla \cdot \boldsymbol{D} + \boldsymbol{D} \cdot \nabla V$

$$= \frac{1}{2}\oint_{S'} V\boldsymbol{D} \cdot d\boldsymbol{s} + \frac{1}{2}\int_{V'} \boldsymbol{D} \cdot \boldsymbol{E} \, dv$$

Vanishes if we choose V' to be a very large sphere of a radius $R \to \infty$ since $V \propto \frac{1}{R}$, $|\boldsymbol{D}| \propto \frac{1}{R^2}$, $|d\boldsymbol{s}| \propto R^2$

- For a linear medium where $\boldsymbol{D} = \varepsilon \boldsymbol{E}$

$$\therefore W_e = \frac{1}{2}\int_{V'} \boldsymbol{D} \cdot \boldsymbol{E} \, dv$$

$$\therefore W_e = \frac{1}{2}\int_{V'} \varepsilon E^2 \, dv = \frac{1}{2}\int_{V'} \frac{D^2}{\varepsilon} dv$$

Chap. 6 | Magnetic potential energy (review)

Generalized magnetic energy of a continuous distribution of current

$$W_m = \frac{1}{2}\int_{V'} \boldsymbol{A}\cdot\boldsymbol{J}\,dv' \quad \text{(J)}$$ ← ***Refer to Section 6-12 for derivation***

where *V'* is the volume of the loop or the linear medium where ***J*** exists

- W_m in terms of field quantities ***B*** and ***H*** without explicitly knowing ***J***

$$\nabla\cdot(\boldsymbol{A}\times\boldsymbol{H}) = \boldsymbol{H}\cdot(\nabla\times\boldsymbol{A}) - \boldsymbol{A}\cdot(\nabla\times\boldsymbol{H})$$

$$\rightarrow \boldsymbol{A}\cdot(\nabla\times\boldsymbol{H}) = \boldsymbol{H}\cdot(\nabla\times\boldsymbol{A}) - \nabla\cdot(\boldsymbol{A}\times\boldsymbol{H})$$

$$\rightarrow \boldsymbol{A}\cdot\boldsymbol{J} = \boldsymbol{H}\cdot\boldsymbol{B} - \nabla\cdot(\boldsymbol{A}\times\boldsymbol{H})$$

Thus,

$$W_m = \frac{1}{2}\int_{V'} \boldsymbol{A}\cdot\boldsymbol{J}\,dv' = \frac{1}{2}\int_{V'} \boldsymbol{H}\cdot\boldsymbol{B}\,dv' - \frac{1}{2}\int_{V'} \nabla\cdot(\boldsymbol{A}\times\boldsymbol{H})\,dv'$$

$$= \frac{1}{2}\int_{V'} \boldsymbol{A}\cdot\boldsymbol{J}\,dv' = \frac{1}{2}\int_{V'} \boldsymbol{H}\cdot\boldsymbol{B}\,dv' - \frac{1}{2}\int_{S'} (\boldsymbol{A}\times\boldsymbol{H})\cdot d\boldsymbol{s}'$$

Vanishes if we choose *V'* to be a very large sphere of a radius $R \rightarrow \infty$

since $|\boldsymbol{A}| \propto \frac{1}{R}, \; |\boldsymbol{H}| \propto \frac{1}{R^2}, \; |d\boldsymbol{s}| \propto R^2$

$$\therefore W_m = \frac{1}{2}\int_{V'} \boldsymbol{H}\cdot\boldsymbol{B}\,dv' \quad \text{(J)}$$

- For a linear medium where ***H*** = ***B***/*μ*

$$W_m = \frac{1}{2}\int_{V'} \frac{B^2}{\mu}\,dv' = \frac{1}{2}\int_{V'} \mu H^2\,dv'$$

Energy transfer rate vs. E, H (cont'd)

$$\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H}) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right) - \sigma E^2$$

Energy stored in electric & magnetic fields

Ohmic power dissipated in the volume *V* due to conduction current ($\sigma\boldsymbol{E}$)

$$\int_V \nabla\cdot(\boldsymbol{E}\times\boldsymbol{H})\,dv = \oint_S (\boldsymbol{E}\times\boldsymbol{H})\cdot d\boldsymbol{s} = -\frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right)dv - \int_V \sigma E^2\,dv$$

$$\therefore \oint_S (\boldsymbol{E}\times\boldsymbol{H})\cdot d\boldsymbol{s} \longrightarrow$$

Power ***leaving*** the volume *V* enclosed by *S*

=

Decreasing electric and magnetic power

+ Dissipating ohmic power

$$\boldsymbol{P} = \boldsymbol{E}\times\boldsymbol{H} \quad (\mathrm{W/m^2})$$

Poynting's vector: "***Power density***" vector associated with an electromagnetic field
(* ***Perpendicular*** to electric and magnetic fields)

- Poynting's theorem

: $\oint_S \boldsymbol{P}\cdot d\boldsymbol{s}$ = Power ***leaving*** the enclosed volume → ***Poynting's theorem***
(Not limited to plane waves)

Electromagnetic power

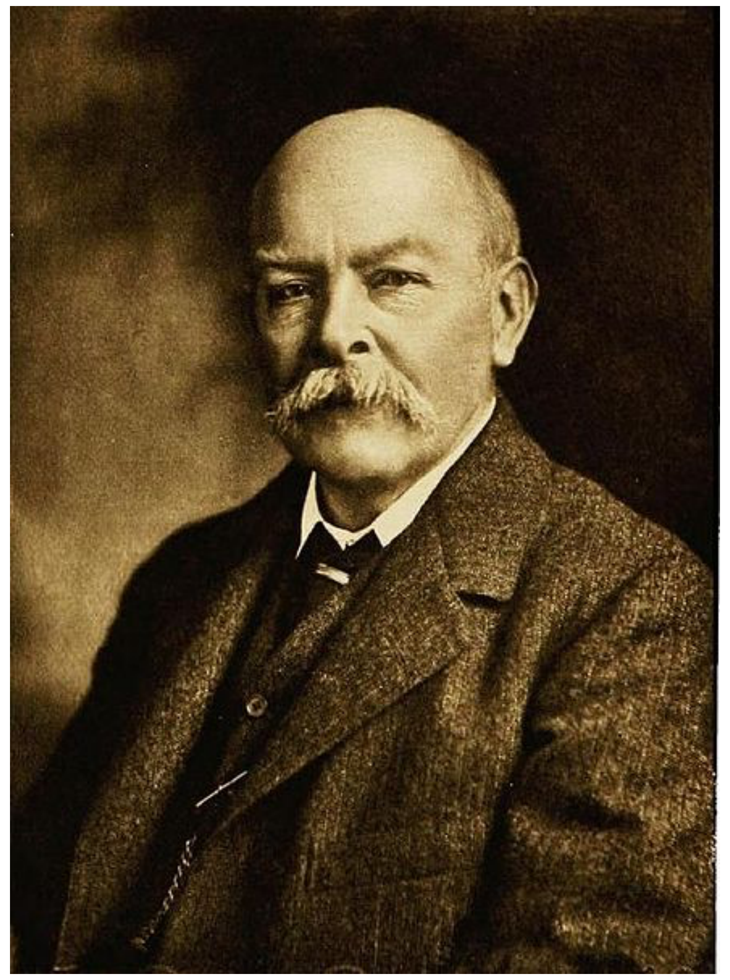
Poynting's theorem

$$-\oint_S \boldsymbol{P}\cdot d\boldsymbol{s} = \frac{\partial}{\partial t}\int_V (w_e + w_m)\,dv + \int_V p_\sigma\,dv$$: Power ***flowing into*** a closed surface = ***Increasing*** stored electric + magnetic + ohmic power

where $w_e = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}\varepsilon \boldsymbol{E}\cdot\boldsymbol{E}^*$ = ***Electric energy*** density

$w_m = \frac{1}{2}\varepsilon H^2 = \frac{1}{2}\varepsilon \boldsymbol{H}\cdot\boldsymbol{H}^*$ = ***Magnetic energy*** density

$p_\sigma = \sigma E^2 = J^2/\sigma = \sigma \boldsymbol{E}\cdot\boldsymbol{E}^* = \boldsymbol{J}\cdot\boldsymbol{J}^*/\sigma$ = ***Ohmic power*** density



John Henry Poynting (1852~1914)

Special cases

- Poynting vector in lossless medium (σ=0)

$$-\oint_S \boldsymbol{P}\cdot d\boldsymbol{s} = \frac{\partial}{\partial t}\int_V (w_e + w_m)\,dv$$

- Poynting vector in static case

$$-\oint_S \boldsymbol{P}\cdot d\boldsymbol{s} = \int_V p_\sigma\,dv$$

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Chap. 8 | Electromagnetic power

Instantaneous expression for Poynting vector $(\boldsymbol{P}=\boldsymbol{E}\times\boldsymbol{H})$

- Instantaneous expressions for time-harmonic ***E*** and ***H***-fields

$$\boldsymbol{E}(z)=\boldsymbol{a}_x E_x(z)=\boldsymbol{a}_x E_0 e^{-\gamma z}=\boldsymbol{a}_x E_0 e^{-(\alpha+j\beta)z}$$

$$\boldsymbol{E}(z,t)=\mathrm{Re}\left[\boldsymbol{E}(z)e^{j\omega t}\right]=\boldsymbol{a}_x E_0 e^{-\alpha z}\,\mathrm{Re}\left[e^{j(\omega t-\beta z)}\right]$$

$$\therefore \boldsymbol{E}(z,t)=\boldsymbol{a}_x E_0 e^{-\alpha z}\cos(\omega t-\beta z)$$

$$\boldsymbol{H}(z)=\boldsymbol{a}_y H_y(z)=\boldsymbol{a}_y \frac{E_x(z)}{\eta}=\boldsymbol{a}_y \frac{E_0}{|\eta|}e^{-\alpha z}e^{-j(\beta z+\theta_\eta)}$$

where $\eta=|\eta|e^{\theta_\eta}$ is the intrinsic impedance of the "lossless" medium

$$\boldsymbol{H}(z,t)=\mathrm{Re}\left[\boldsymbol{H}(z)e^{j\omega t}\right]=\boldsymbol{a}_y \frac{E_0}{|\eta|}e^{-\alpha z}\,\mathrm{Re}\left[e^{-j(\omega t-\beta z-\theta_\eta)}\right]$$

$$\therefore \boldsymbol{H}(z,t)=\boldsymbol{a}_y \frac{E_0}{|\eta|}e^{-\alpha z}\cos(\omega t-\beta z-\theta_\eta)$$

- Instantaneous expression for ***P*** or power density vector

$$\boldsymbol{P}(z,t)=\boldsymbol{E}(z,t)\times\boldsymbol{H}(z,t)=\mathrm{Re}\left[\boldsymbol{E}(z)e^{j\omega t}\right]\times\mathrm{Re}\left[\boldsymbol{H}(z)e^{j\omega t}\right]$$

Instantaneous expression for Poynting vector (Cont'd)

- Instantaneous expression for ***P*** or power density vector

$$\begin{cases} \boldsymbol{E}(z,t) = \boldsymbol{a_x} E_0 e^{-\alpha z}\cos(\omega t - \beta z) \\ \boldsymbol{H}(z,t) = \boldsymbol{a_y} \dfrac{E_0}{|\eta|} e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta) \end{cases}$$

$$\boldsymbol{P}(z,t) = \boldsymbol{E}(z,t)\times\boldsymbol{H}(z,t) = \mathrm{Re}\left[\boldsymbol{E}(z)e^{j\omega t}\right]\times\mathrm{Re}\left[\boldsymbol{H}(z)e^{j\omega t}\right]$$

$$= \boldsymbol{a_z}\frac{E_0^{\ 2}}{|\eta|}e^{-2\alpha z}\cos(\omega t - \beta z)\cos(\omega t - \beta z - \theta_\eta)$$

$$= \boldsymbol{a_z}\frac{E_0^{\ 2}}{2|\eta|}e^{-2\alpha z}\left[\cos\theta_\eta + \cos(2\omega t - 2\beta z - \theta_\eta)\right]$$

Time-average Poynting vector

$$\boldsymbol{P}_{av}(z) = \boldsymbol{a_z}\frac{1}{T}\int_0^T P(z,t)\,dt = \boldsymbol{a_z}\frac{1}{T}\int_0^T \frac{E_0^{\ 2}}{2|\eta|}e^{-2\alpha z}\left[\cos\theta_\eta + \cos(2\omega t - 2\beta z - \theta_\eta)\right]dt$$

$$= \boldsymbol{a_z}\frac{E_0^{\ 2}}{2|\eta|}e^{-2\alpha z}\cos\theta_\eta$$

$$\therefore \boldsymbol{P}_{av}(z) = \boldsymbol{a_z}\frac{E_0^{\ 2}}{2|\eta|}e^{-2\alpha z}\cos\theta_\eta \quad (\mathrm{W/m^2})$$

General formula for time-average Poynting vector

- Vector identity

  With given complex vectors ***A*** and ***B*** such that $\mathrm{Re}(\boldsymbol{A}) = \frac{1}{2}(\boldsymbol{A}+\boldsymbol{A}^*)$ and $\mathrm{Re}(\boldsymbol{B}) = \frac{1}{2}(\boldsymbol{B}+\boldsymbol{B}^*)$,

$$\mathrm{Re}(\boldsymbol{A})\times\mathrm{Re}(\boldsymbol{B}) = \frac{1}{2}(\boldsymbol{A}+\boldsymbol{A}^*)\times\frac{1}{2}(\boldsymbol{B}+\boldsymbol{B}^*)$$

$$= \frac{1}{4}\left[(\boldsymbol{A}\times\boldsymbol{B}+\boldsymbol{A}^*\times\boldsymbol{B}^*)+(\boldsymbol{A}\times\boldsymbol{B}^*+\boldsymbol{A}^*\times\boldsymbol{B})\right] = \frac{1}{2}\mathrm{Re}(\boldsymbol{A}\times\boldsymbol{B}+\boldsymbol{A}\times\boldsymbol{B}^*)$$

- Time-average Poynting vector

$\boldsymbol{P}(z,t) = \boldsymbol{E}(z,t)\times\boldsymbol{H}(z,t) = \mathrm{Re}\left[\boldsymbol{E}(z)e^{j\omega t}\right]\times\mathrm{Re}\left[\boldsymbol{H}(z)e^{j\omega t}\right]$. By replacing $\boldsymbol{A}\rightarrow\boldsymbol{E}(z)e^{j\omega t}$ and $\boldsymbol{B}\rightarrow\boldsymbol{H}(z)e^{j\omega t}$,

$$\mathrm{Re}\left[\boldsymbol{E}(z)e^{j\omega t}\right]\times\mathrm{Re}\left[\boldsymbol{H}(z)e^{j\omega t}\right] = \frac{1}{2}\mathrm{Re}\left[\boldsymbol{E}(z)e^{j\omega t}\times\boldsymbol{H}(z)e^{j\omega t}+\boldsymbol{E}(z)e^{j\omega t}\times\boldsymbol{H}^*(z)e^{-j\omega t}\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\boldsymbol{E}(z)\times\boldsymbol{H}(z)e^{j2\omega t}+\boldsymbol{E}(z)\times\boldsymbol{H}^*(z)\right]$$

$$\boldsymbol{P}_{av}(z) = \frac{1}{T}\int_0^T \boldsymbol{P}(z,t)\,dt = \frac{1}{2T}\int_0^T \mathrm{Re}\left[\boldsymbol{E}(z)\times\boldsymbol{H}(z)e^{j2\omega t}+\boldsymbol{E}(z)\times\boldsymbol{H}^*(z)\right]dt$$

$$= \frac{1}{2}\mathrm{Re}\left[\boldsymbol{E}(z)\times\boldsymbol{H}^*(z)\right]$$

Average power density vector of electromagnetic wave propagating "in an arbitrary direction"

$$\therefore \boldsymbol{P}_{av} = \frac{1}{2}\mathrm{Re}(\boldsymbol{E}\times\boldsymbol{H}^*) \quad (\mathrm{W/m^2})$$

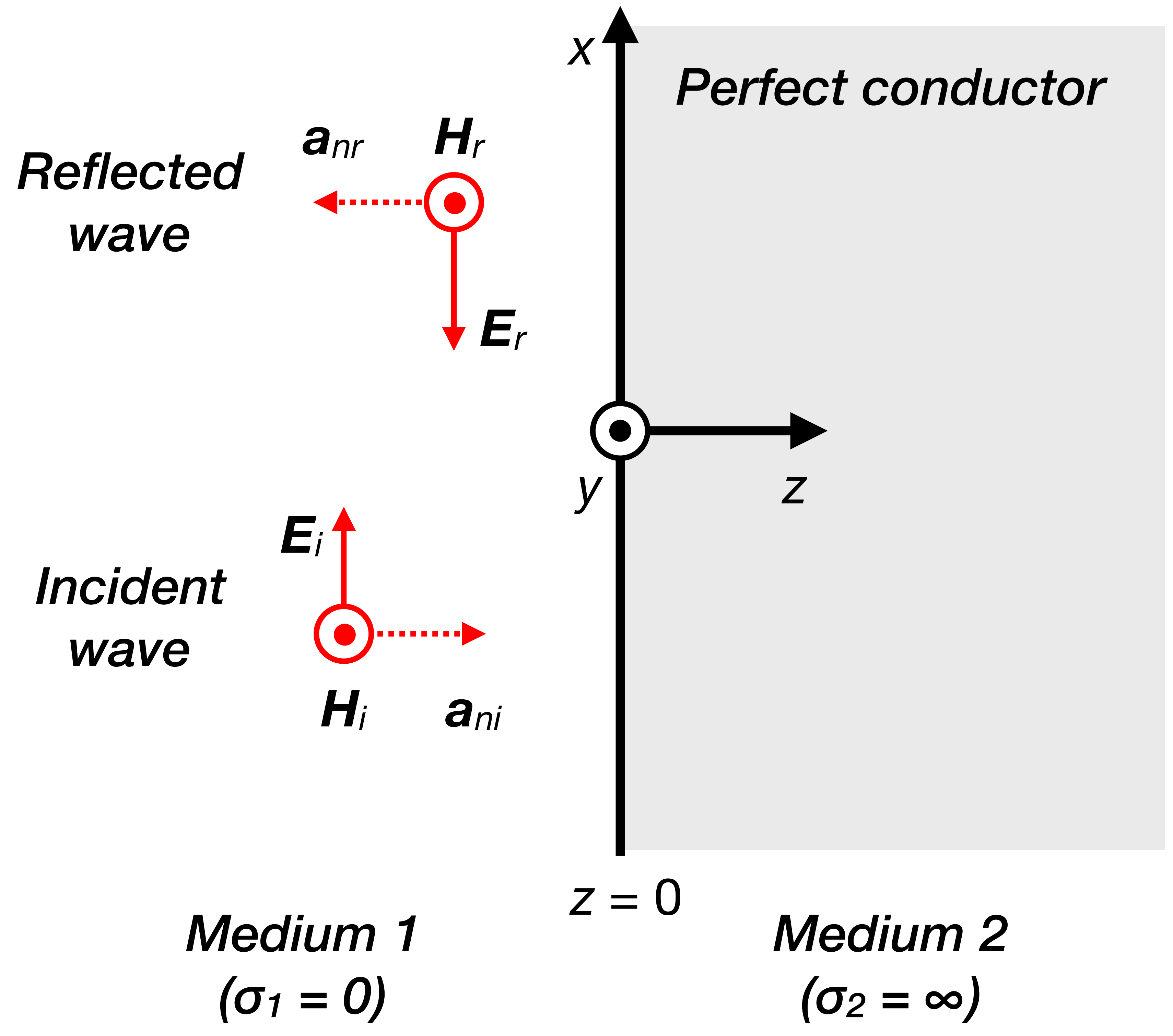
Chap. 8 | Normal incidence at a plane conducting boundary

In practical cases

- Waves propagate in ***"bounded regions"*** where several media with ***different constitutive parameters (ε, μ, and σ)*** are present

Normal incidence on a perfect conductor

- EM wave traveling in a lossless medium ***impinges on another medium with a different η*** (intrinsic impedance)
  → ***Reflection*** occurs



- ***Incident wave***

$$\begin{cases} \mathbf{E}_i(z) = \mathbf{a}_x E_{i0} e^{-j\beta_1 z}, \\ \mathbf{H}_i(z) = \mathbf{a}_y \dfrac{E_{i0}}{\eta_1} e^{-j\beta_1 z} \end{cases}$$

* Poynting vector direction: $\mathbf{a}_z$

$$\mathbf{P}_i(z) = \mathbf{E}_i(z) \times \mathbf{H}_i(z)$$

- ***Reflected wave***

$$\begin{cases} \mathbf{E}_r(z) = \mathbf{a}_x E_{r0} e^{+j\beta_1 z}, \\ \mathbf{H}_r(z) = -\mathbf{a}_y \dfrac{E_{r0}}{\eta_1} e^{+j\beta_1 z} \end{cases}$$

- ***Wave in medium 1***

$$\begin{cases} \mathbf{E}_1(z) = \mathbf{E}_i(z) + \mathbf{E}_r(z) \\ \mathbf{H}_1(z) = \mathbf{H}_i(z) + \mathbf{H}_r(z) \end{cases}$$

- ***Wave in medium 2***

$$\begin{cases} \mathbf{E}_2(z) = 0, \\ \mathbf{H}_2(z) = 0 \end{cases}$$

Normal incidence on a perfect conductor

- ***Boundary condition***: "***Continuity of tangential component*** of ***E***-field interface"

$$\mathbf{E}_1(z) = \mathbf{a}_x\left(E_{i0}e^{-j\beta_1 z} + E_{r0}e^{+j\beta_1 z}\right)$$

At $z = 0$,

$$\mathbf{E}_1(0) = \mathbf{a}_x\left(E_{i0} + E_{r0}\right) = \mathbf{E}_2(0) = 0 \quad \rightarrow \quad \therefore E_{r0} = -E_{i0}$$

$$\mathbf{E}_1(z) = \mathbf{a}_x E_{i0}\left(e^{-j\beta_1 z} - e^{+j\beta_1 z}\right) = -\mathbf{a}_x j2E_{i0}\sin(\beta_1 z) \quad \rightarrow \quad \therefore \mathbf{E}_1(z) = -\mathbf{a}_x j2E_{i0}\sin(\beta_1 z)$$

- ***Associated magnetic field***

$$\mathbf{H}_r(z) = \frac{1}{\eta_1}\mathbf{a}_{nr} \times \mathbf{E}_r(z) = \frac{1}{\eta_1}(-\mathbf{a}_z) \times \mathbf{E}_r(z)$$

$$= -\mathbf{a}_y \frac{1}{\eta_1}E_{r0}e^{+j\beta_1 z} = \mathbf{a}_y \frac{E_{i0}}{\eta_1}e^{+j\beta_1 z}$$

$$\mathbf{H}_1(z) = \mathbf{H}_i(z) + \mathbf{H}_r(z) = \mathbf{a}_y 2\frac{E_{i0}}{\eta_1}\cos(\beta_1 z) \quad \rightarrow \quad \therefore \mathbf{H}_1(z) = \mathbf{a}_y 2\frac{E_{i0}}{\eta_1}\cos(\beta_1 z)$$

No average power associated with total EM wave in medium 1

- ***Electromagnetic power*** $\mathbf{P}_{av}(z) = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}_1(z) \times \mathbf{H}_1^*(z)\right] = \mathrm{Re}\left(-\mathbf{a}_z j\frac{E_{i0}^{\ 2}}{\eta_1}\sin(2\beta_1 z)\right) = 0$

$$\therefore \mathbf{P}_{av}(z) = 0$$

Normal incidence at a plane conducting boundary

*Instantaneous expressions for **E** and **H***

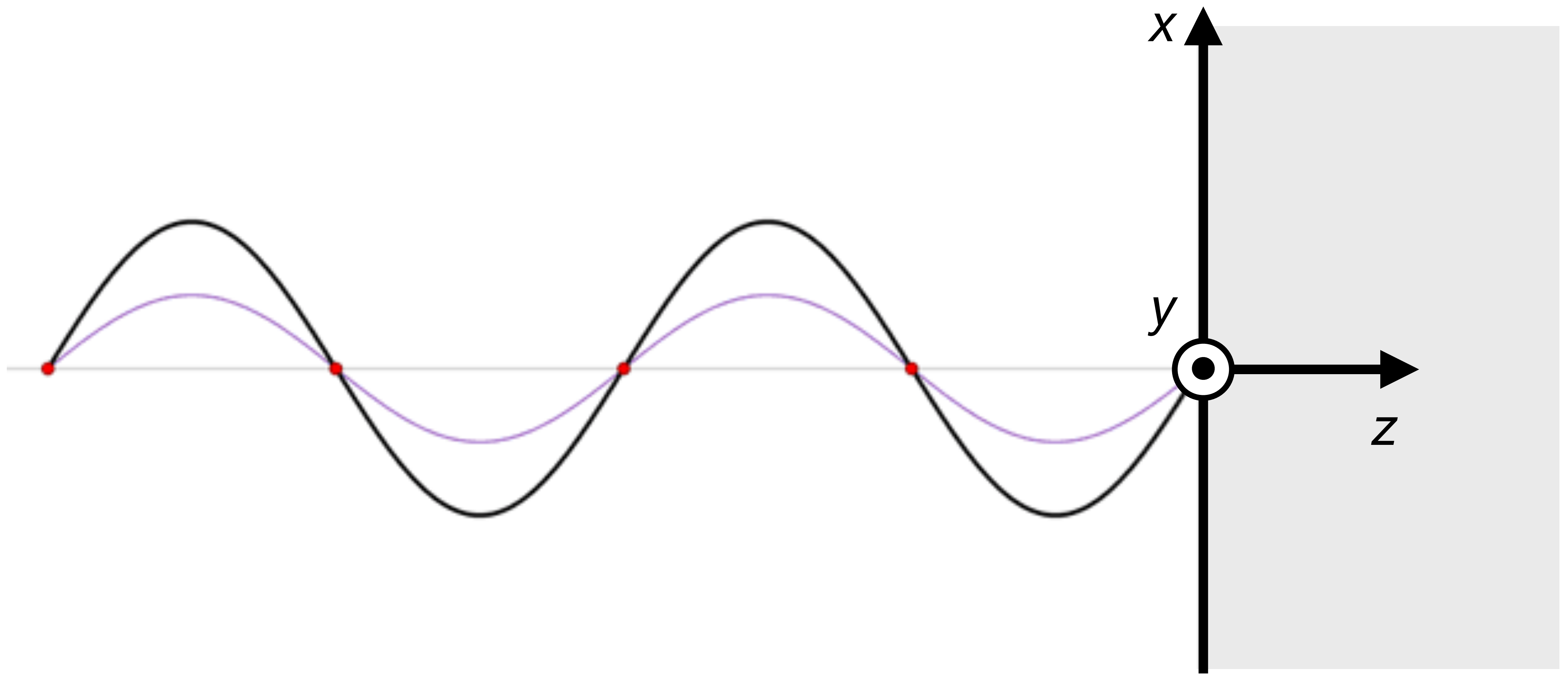
$$\boldsymbol{E}_1(z,t) = \mathrm{Re}\left[\boldsymbol{E}_1(z)e^{j\omega t}\right] = \mathrm{Re}\left[-\boldsymbol{a}_x j2E_{i0}\sin(\beta_1 z)e^{j\omega t}\right] = \boldsymbol{a}_x 2E_{i0}\sin(\beta_1 z)\sin(\omega t) = \boldsymbol{a}_x 2E_{i0}\sin(\beta_1 z)\cos\left(\omega t - \frac{\pi}{2}\right)$$

$$\boldsymbol{H}_1(z,t) = \mathrm{Re}\left[\boldsymbol{H}_1(z)e^{j\omega t}\right] = \mathrm{Re}\left[\boldsymbol{a}_y \frac{2E_{i0}}{\eta_1}\cos(\beta_1 z)e^{j\omega t}\right] = \boldsymbol{a}_y \frac{2E_{i0}}{\eta_1}\cos(\beta_1 z)\cos(\omega t)$$

$\boldsymbol{E}_1(z,t) = 0$ when $\sin(\beta_1 z) = 0$ or $\beta_1 z = -n\pi$

$\boldsymbol{H}_1(z,t) = 0$ when $\cos(\beta_1 z) = 0$ or $\beta_1 z = -(2n+1)\frac{\pi}{2}$

→ *Occurs at "Fixed Positions"*



Standing wave

- Superposition of *two wave traveling in opposite directions*
- Standing wave is ***NOT a traveling wave***
- $\boldsymbol{E}_1$ and $\boldsymbol{H}_1$ are in *time quadrature (90° phase difference)*

Electromagnetics

<Chap. 8> Plane Electromagnetic waves

Section 8.5 ~ 8.8

(2nd of **week 4**)

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Chap. 8 | Contents for 2nd class of week 4

Further review of polarization

Sec 7. Oblique incidence at a plane

Sec 6. Normal incidence on a plane conducting boundary

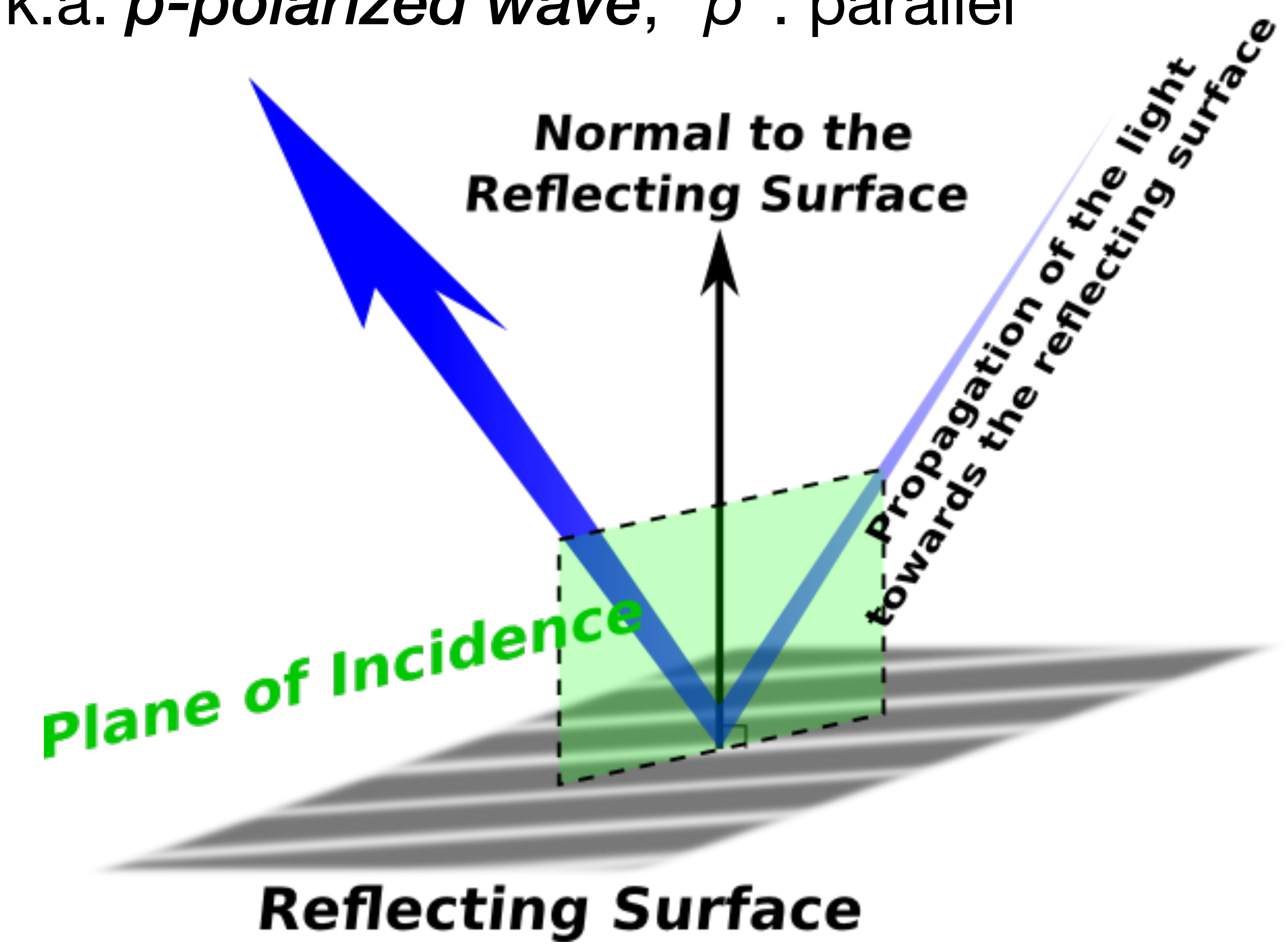
Chap. 8 | Polarization

Plane of incidence and TE & TM waves

- ***Plane of incidence***
  - Containing the ***propagation vector of the incident wave***
  - ***Normal*** to the surface

- ***Transverse Electric (TE) wave***
  - ***E***-field ⊥ Plane of incidence
  - ***H***-field // Plane of incidence
  - a.k.a. ***s-polarized wave***, "*s*": senkrecht (German; vertical)

- ***Transverse Magnetic (TM) wave***
  - ***E***-field // Plane of incidence
  - ***H***-field ⊥ Plane of incidence
  - a.k.a. ***p-polarized wave***, "*p*": parallel



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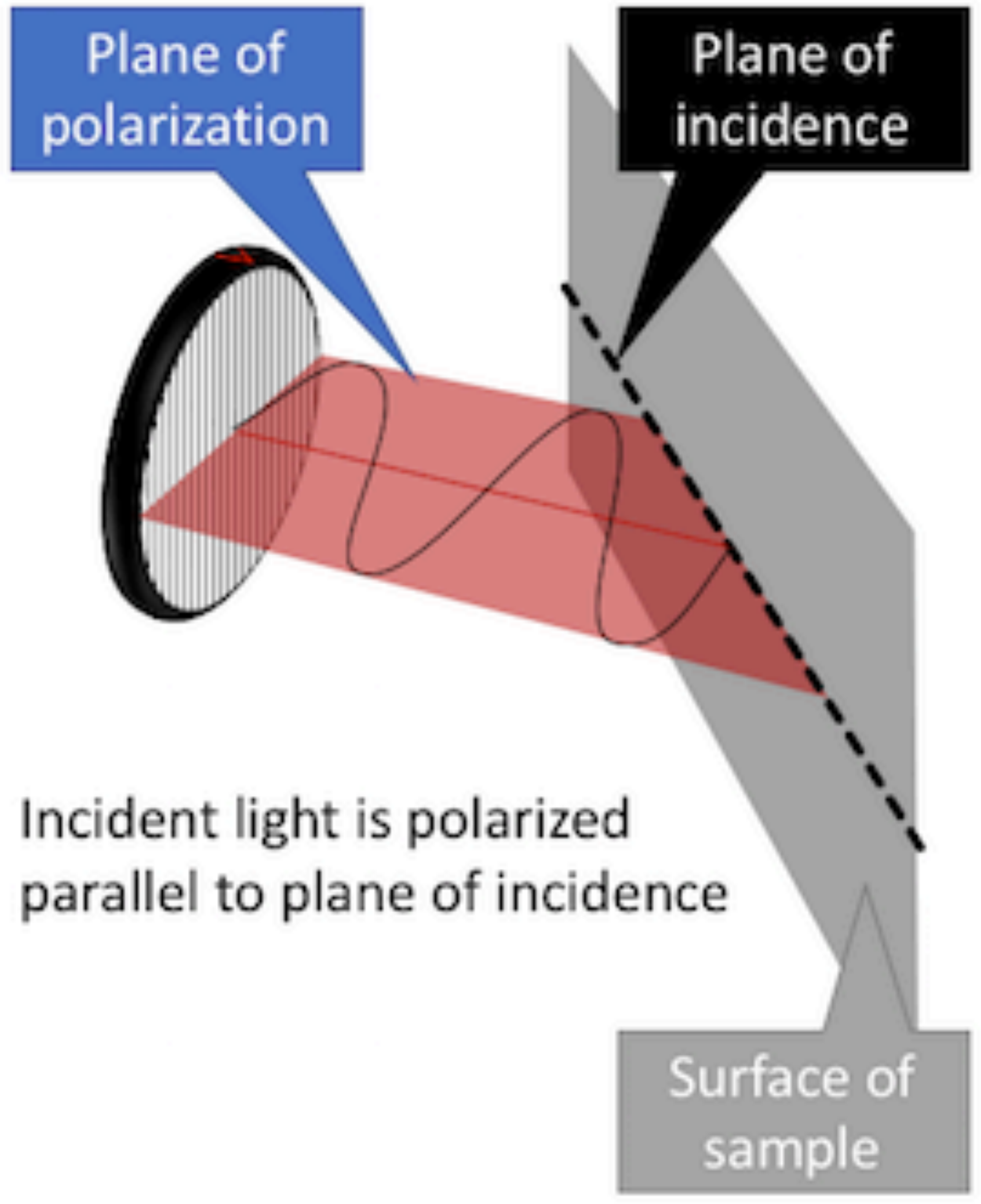
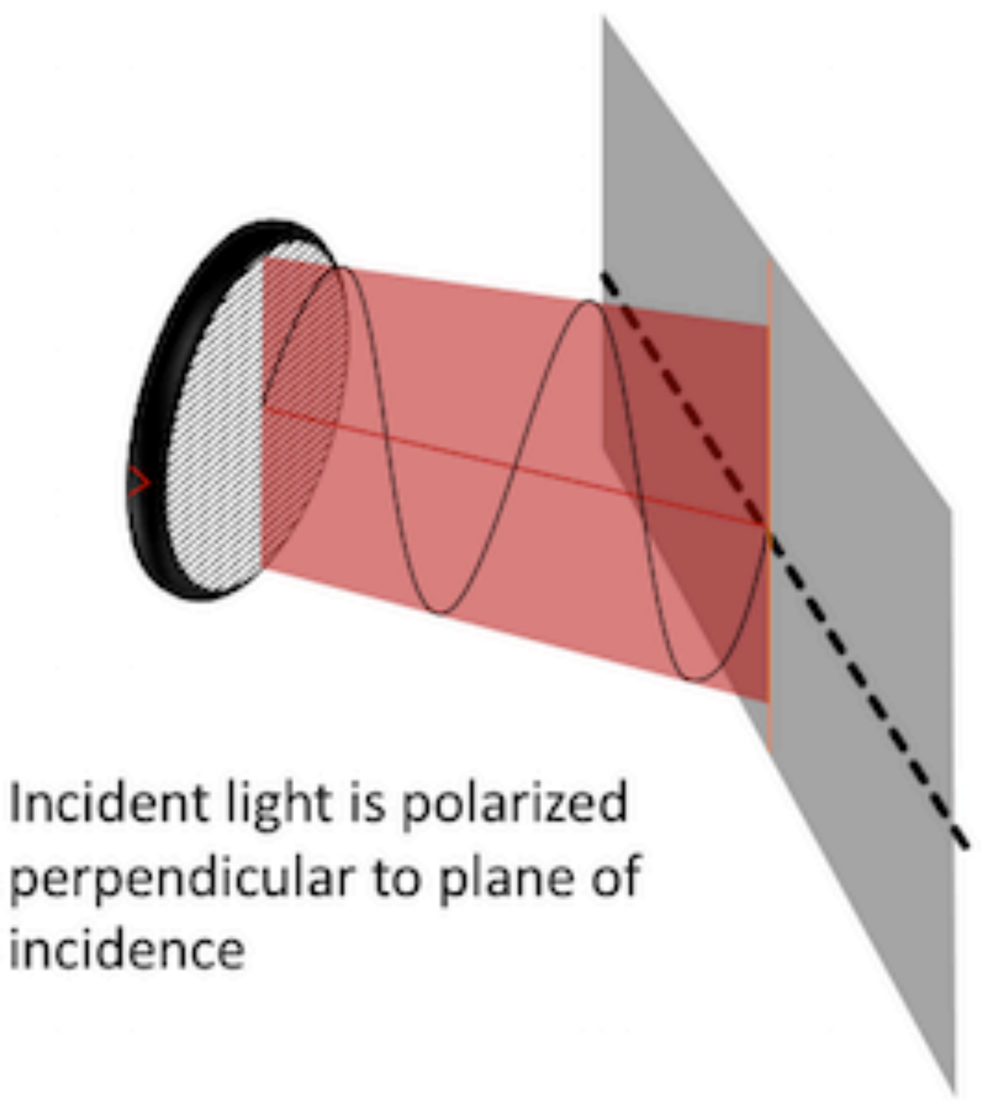
Chapter. 9	*Chapter. 10*
TEM *wave propagation (In transmission lines)*	***TE & TM*** *wave propagation (In waveguides)*

TM wave



TE wave



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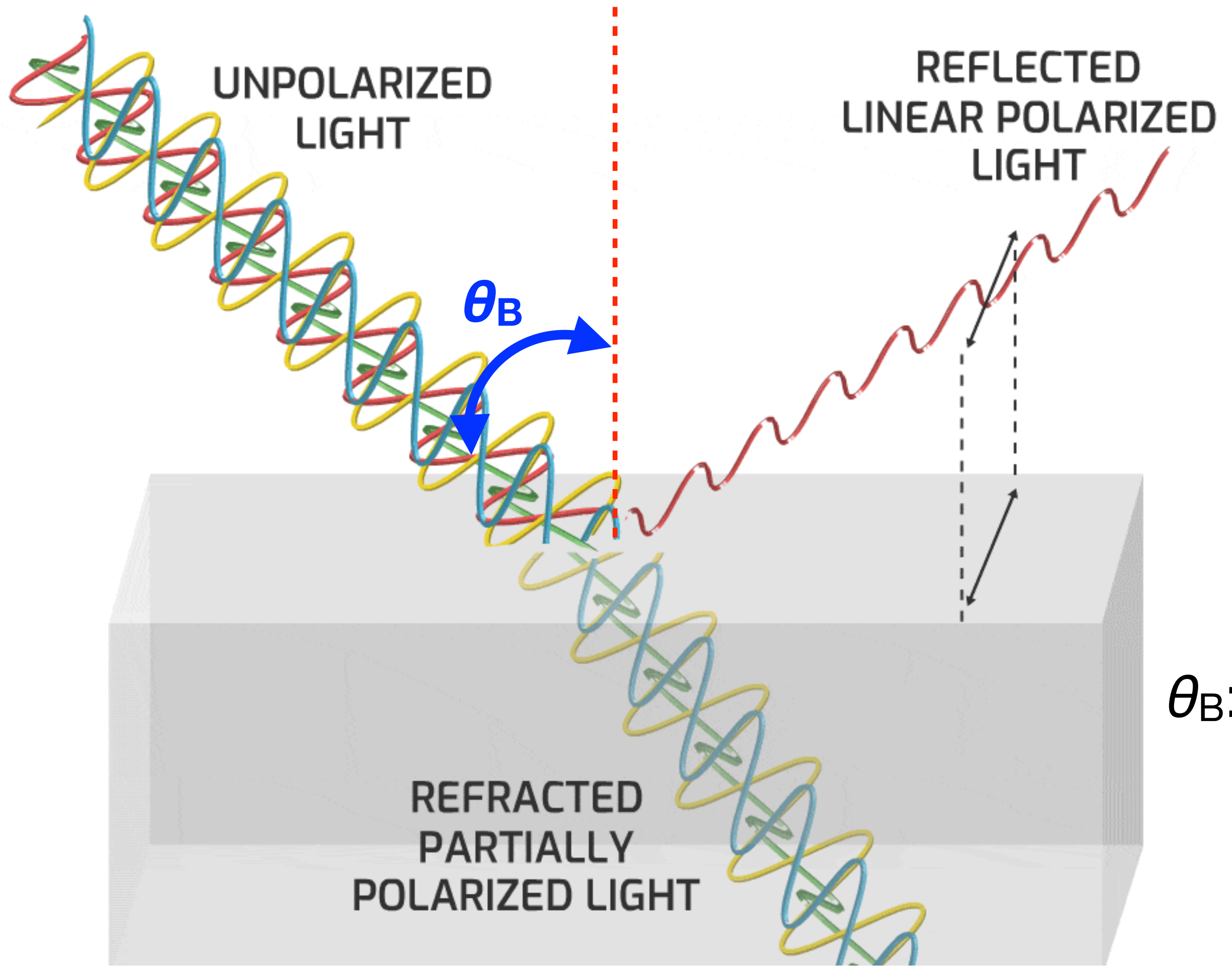
Chap. 8 | Polarization

Polarization of electromagnetic wave

- ***"Polarization" direction of EM wave = Orientation of E-field***
  - e.g.) ***TEM wave*** propagating in ***z***-direction with ***E***- and ***H***-fields oscillating in ***x***- and ***y***-directions = Polarized in "***x***-direction"

- EM waves (including light) ***characterized by their polarization***
  - Most light sources (e.g. sunlight, LED, halogen lamp and so on) = Unpolarized
  - Reflected light from surfaces = Partially polarized
  - Laser = Fully polarized

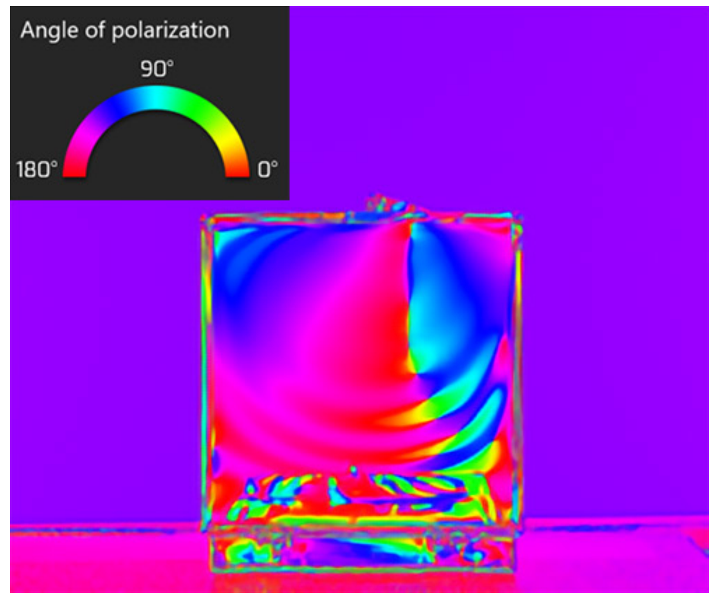
- ***Light-matter interaction*** (e.g. transmission, reflection, absorption and many) ***dependent on polarization***



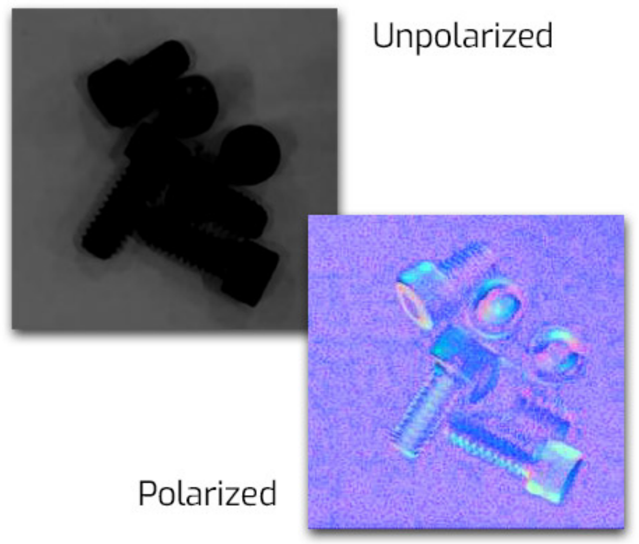
θ_B: Brewster's angle

"Oblique Incidence at a Plane Dielectric Boundary"
(Chap. 8-10, next class)

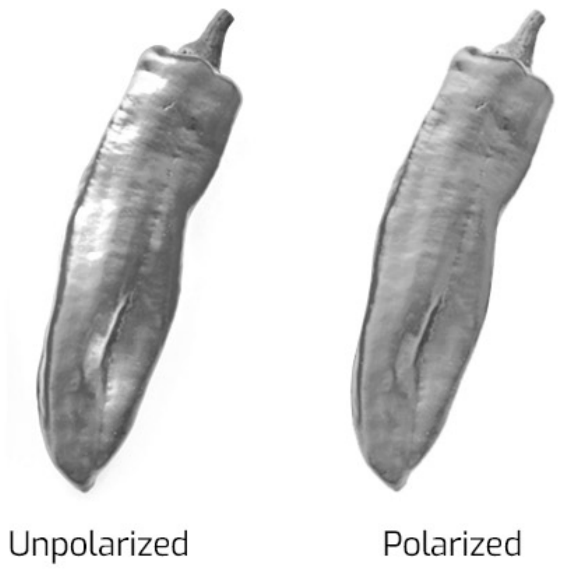
Usage of polarization detection



Non-uniformity inspection

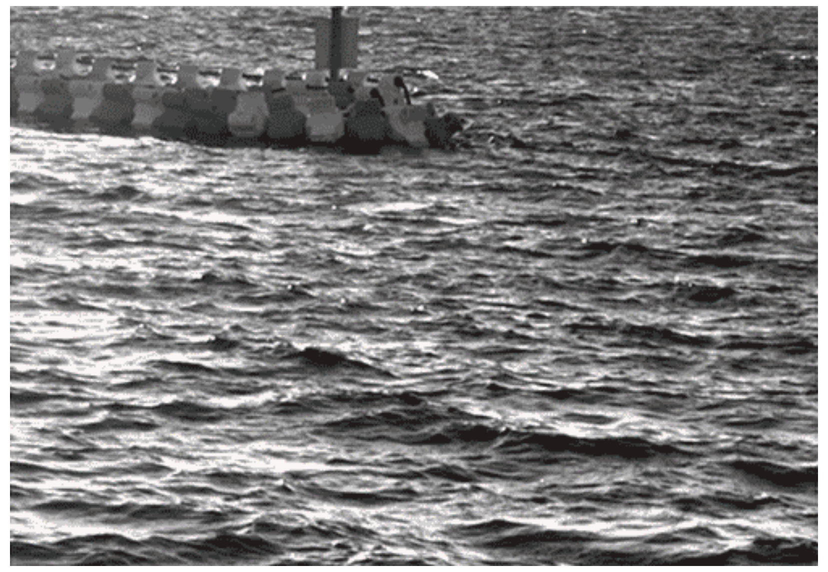


Improving contrast



Reduced reflection

Scratch inspection



Object detection

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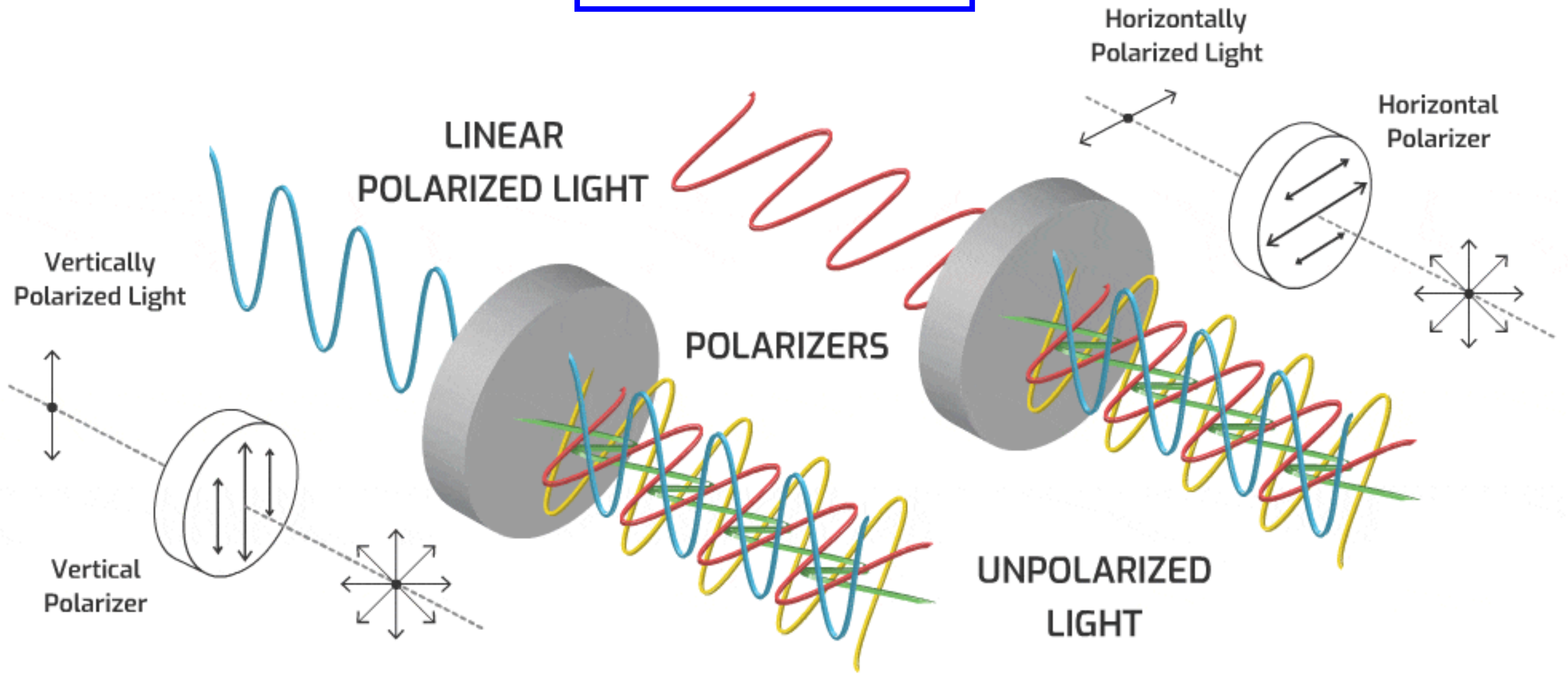
Chap. 8 | Polarization (example)

Most representative example: Polarizer

- ***Optical filter*** that allows EM wave with ***"only a specific polarization" to pass through***
- ***Blocking EM wave of other polarizations (abs or ref)***

Linear polarizer



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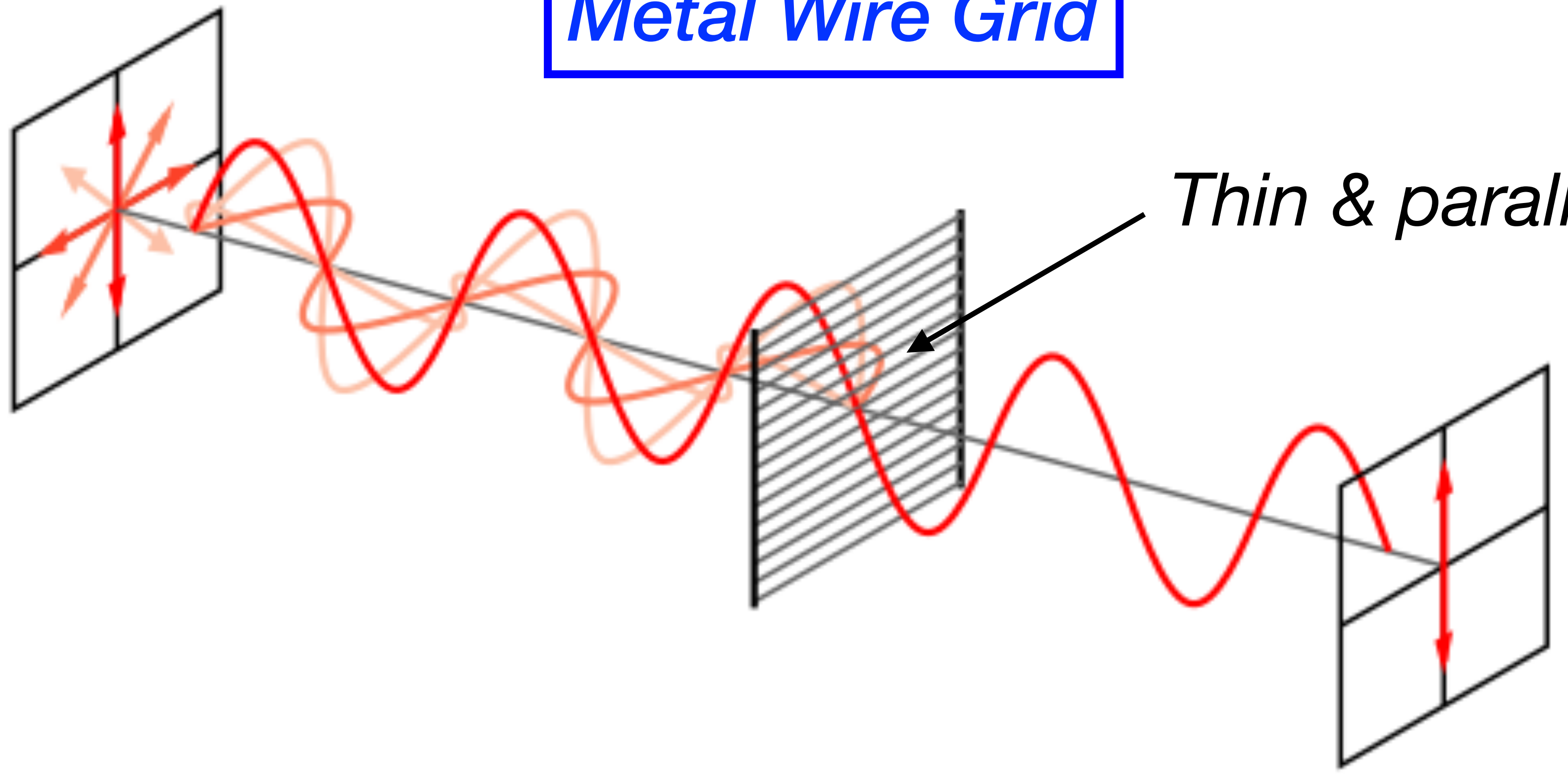
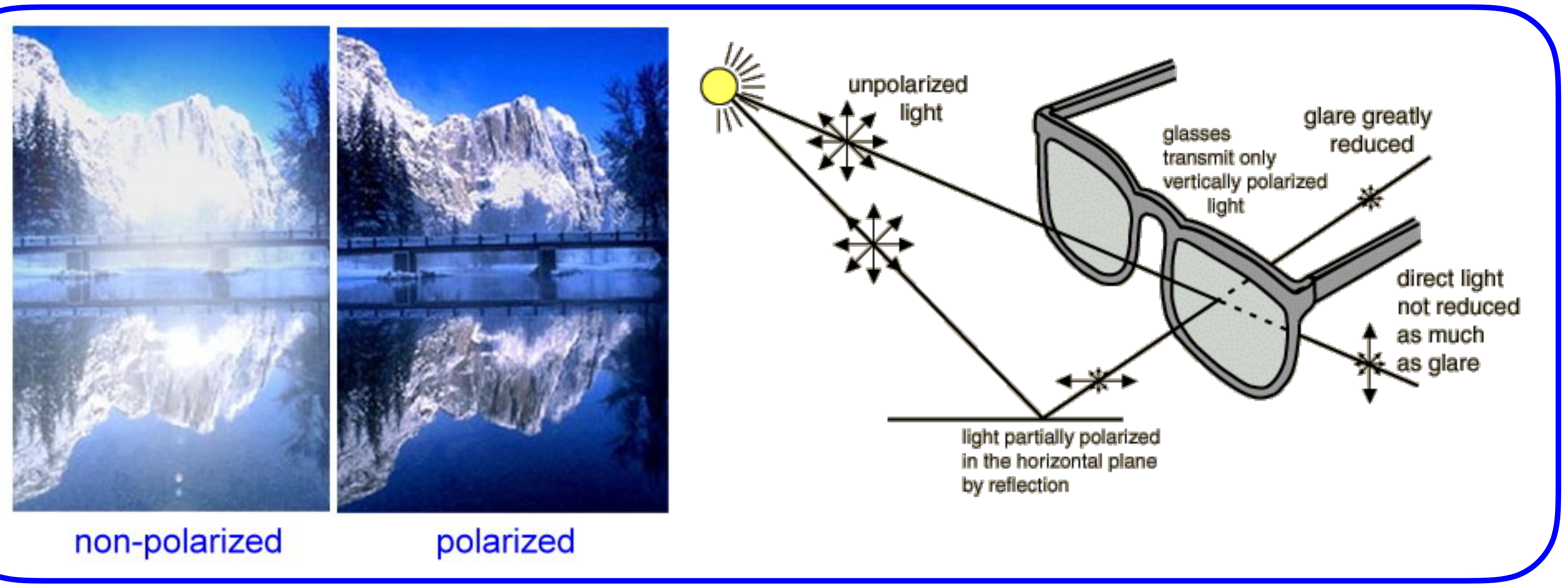
Metal Wire Grid



Thin & parallel metallic wires

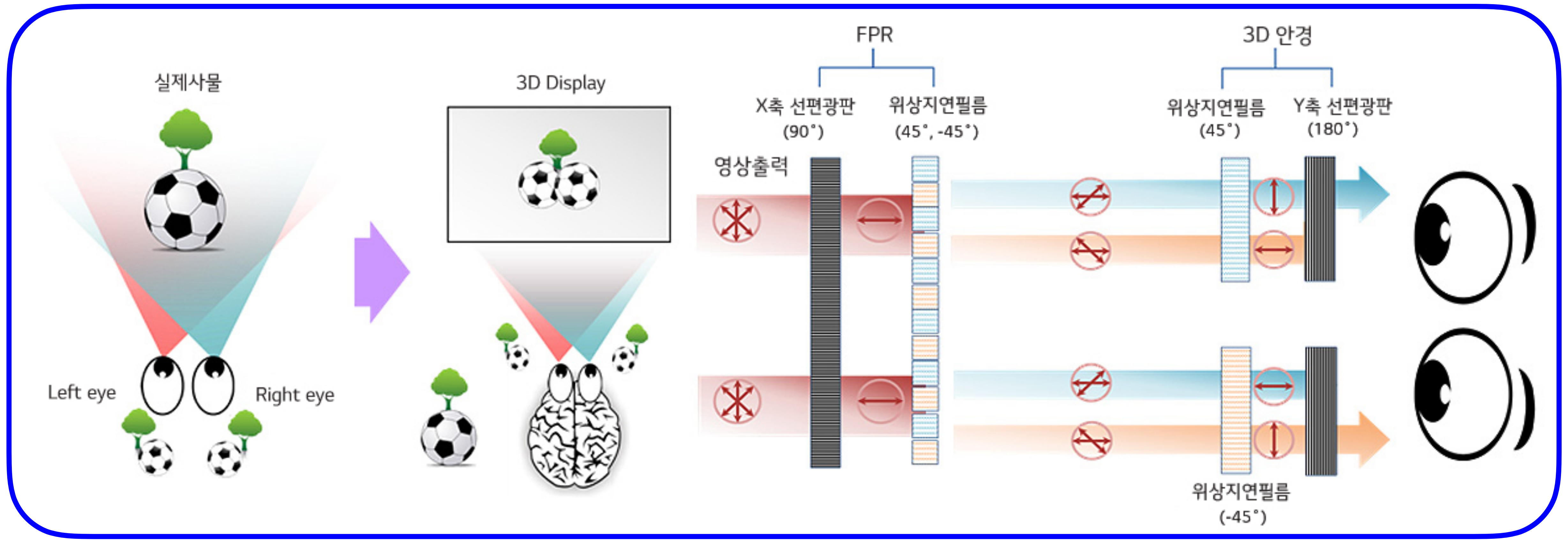
Polarized sunglasses

reduced glare



3D Display

Binocular disparity → Depth perception



https://blog.pugsgear.com/What-Exactly-Do-Affordable-Polarized-Sunglasses-Do

LG 디스플레이 블로그

Img src: flatpanelshd.com, Wikipedia

Liquid Crystal Display (LCD)

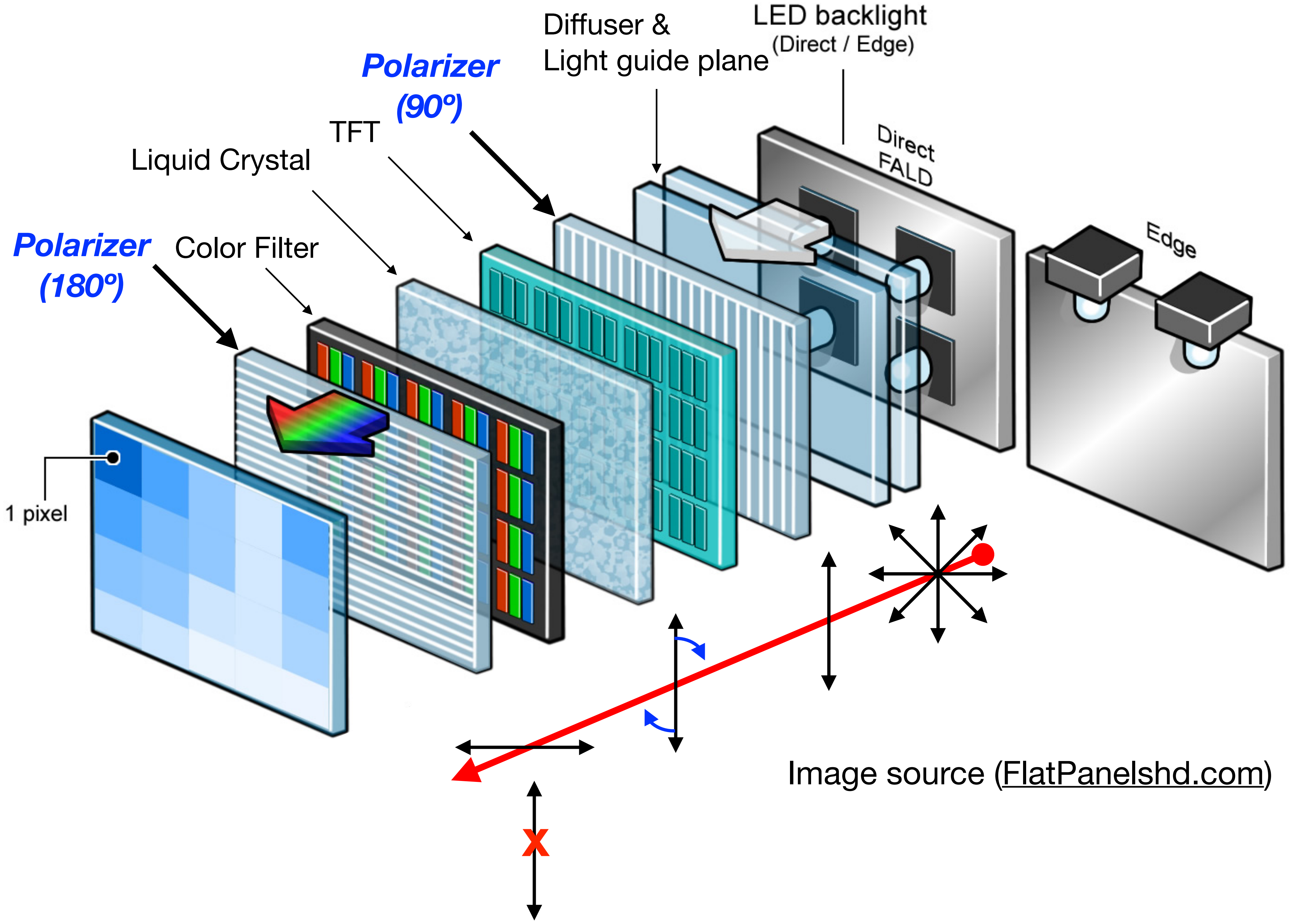
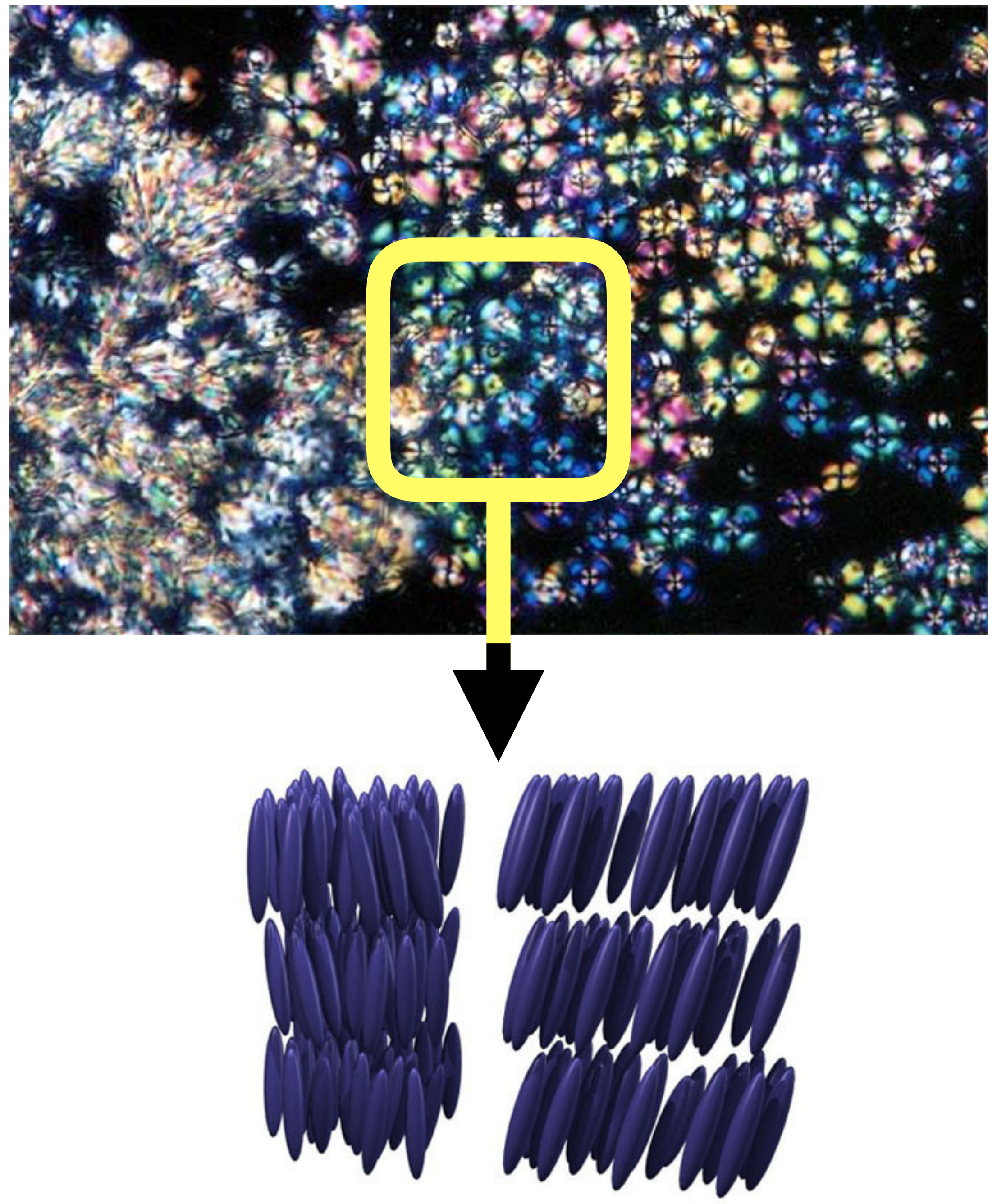
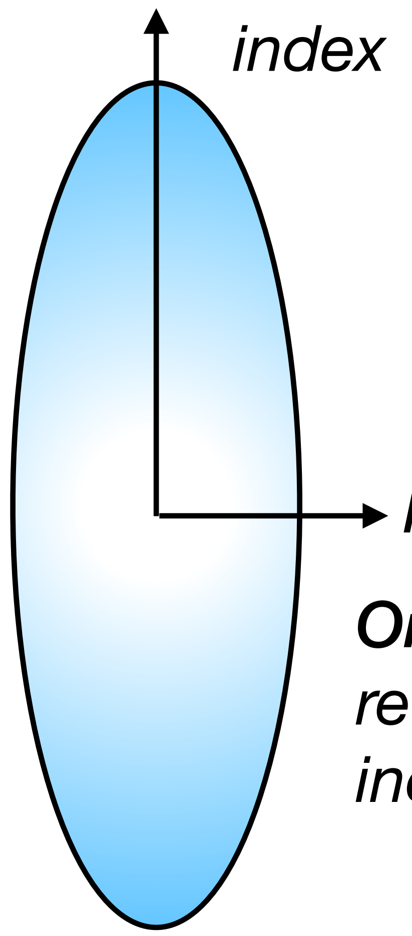


Image source (FlatPanelshd.com)



Liquid Crystals



Birefringent material
(i.e. anisotropic material)

Birefringence: Optical property of the material having ***a refractive index depending on polarization / propagation direction*** of the light

$$u_p = \frac{c}{n}, \quad n_e > n_o \quad \rightarrow \quad u_{ep} < u_{op}$$

where u_p is propagation of light in the medium
n is the refractive index of the medium
c is the speed of light

(Sec. 8-10 in next class!)

Operating Principle

- ***"Unpolarized" white light*** *generated by LED backlight unit (BLU)*
- *90º Polarizer allows only* ***"90º linearly polarized" light***
- *TFT applies E-field to liquid crystals such that they* ***rotate in desirable orientation***
- *Rotated liquid crystals* ***change the angle of polarization of light*** (θ_p)

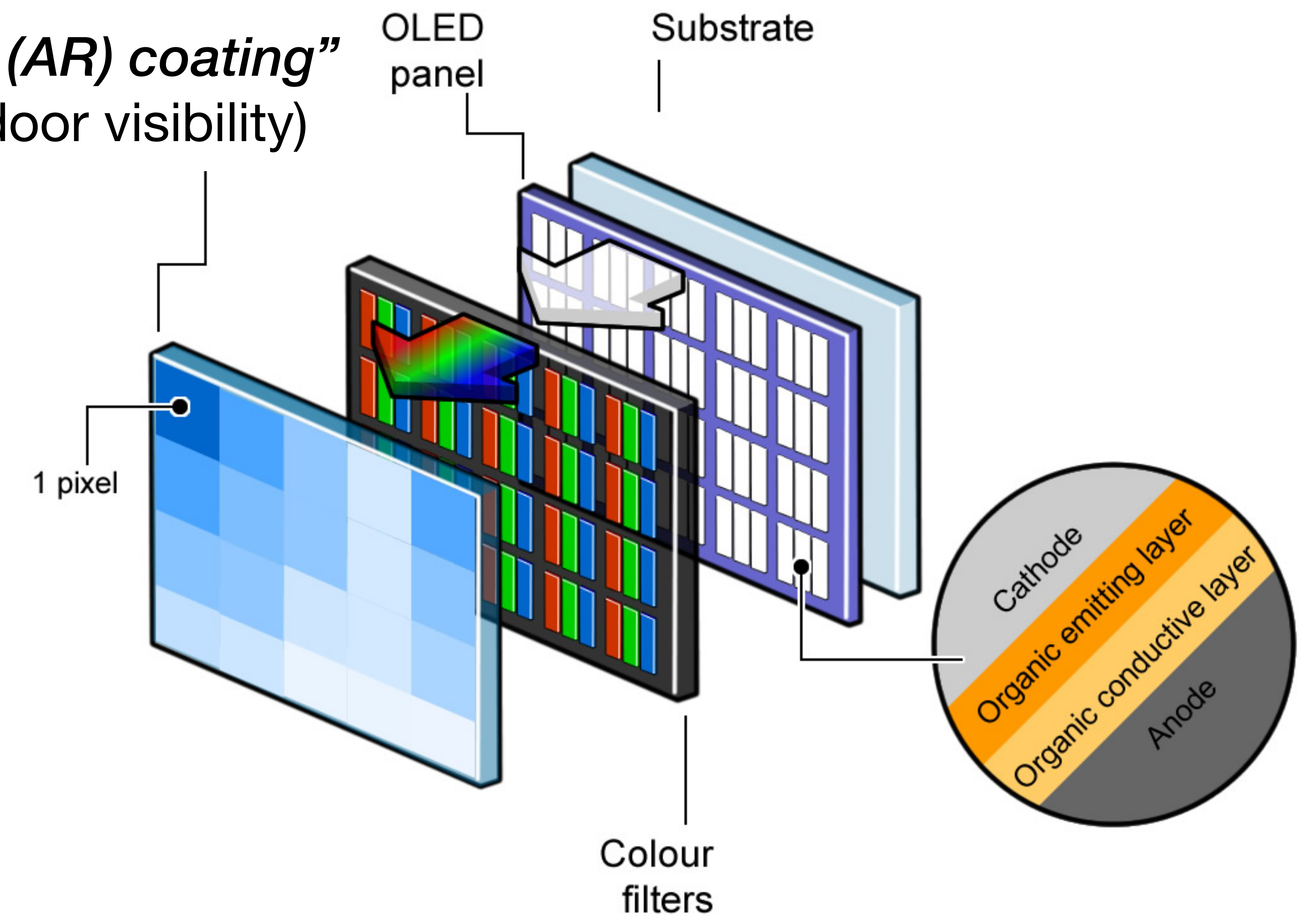
If θ_p = 90º → Completely blocked (min brightness)
If θ_p = 180º → Completely transmitted (max brightness)
If 90º < θ_p < 180º → Partially transmitted (mid brightness)

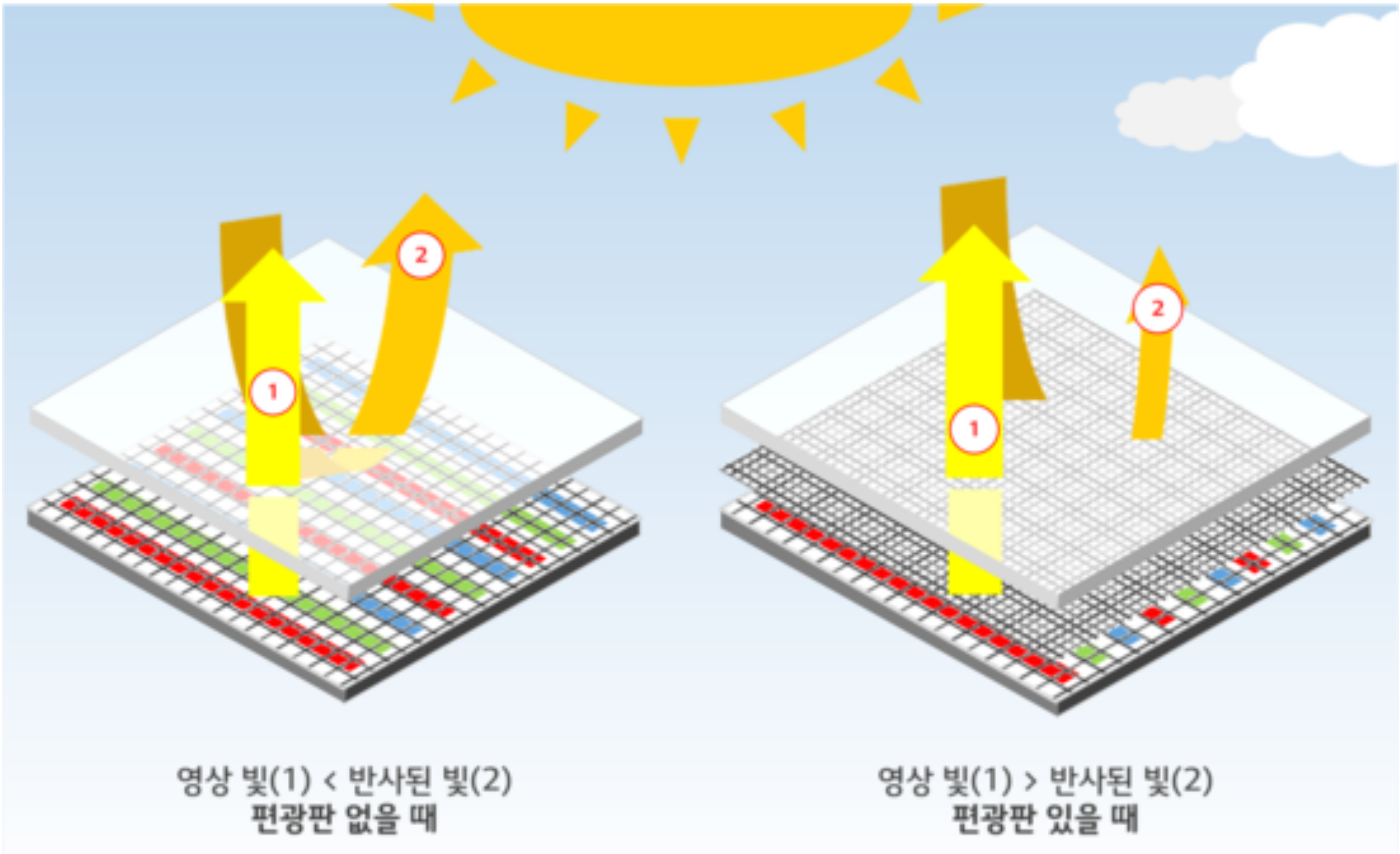
Polarization (example)

Img src: flatpanelshd.com, LGD blog, eeNews Europe, Wikipedia

Organic Light Emitting Diodes (OLED) display

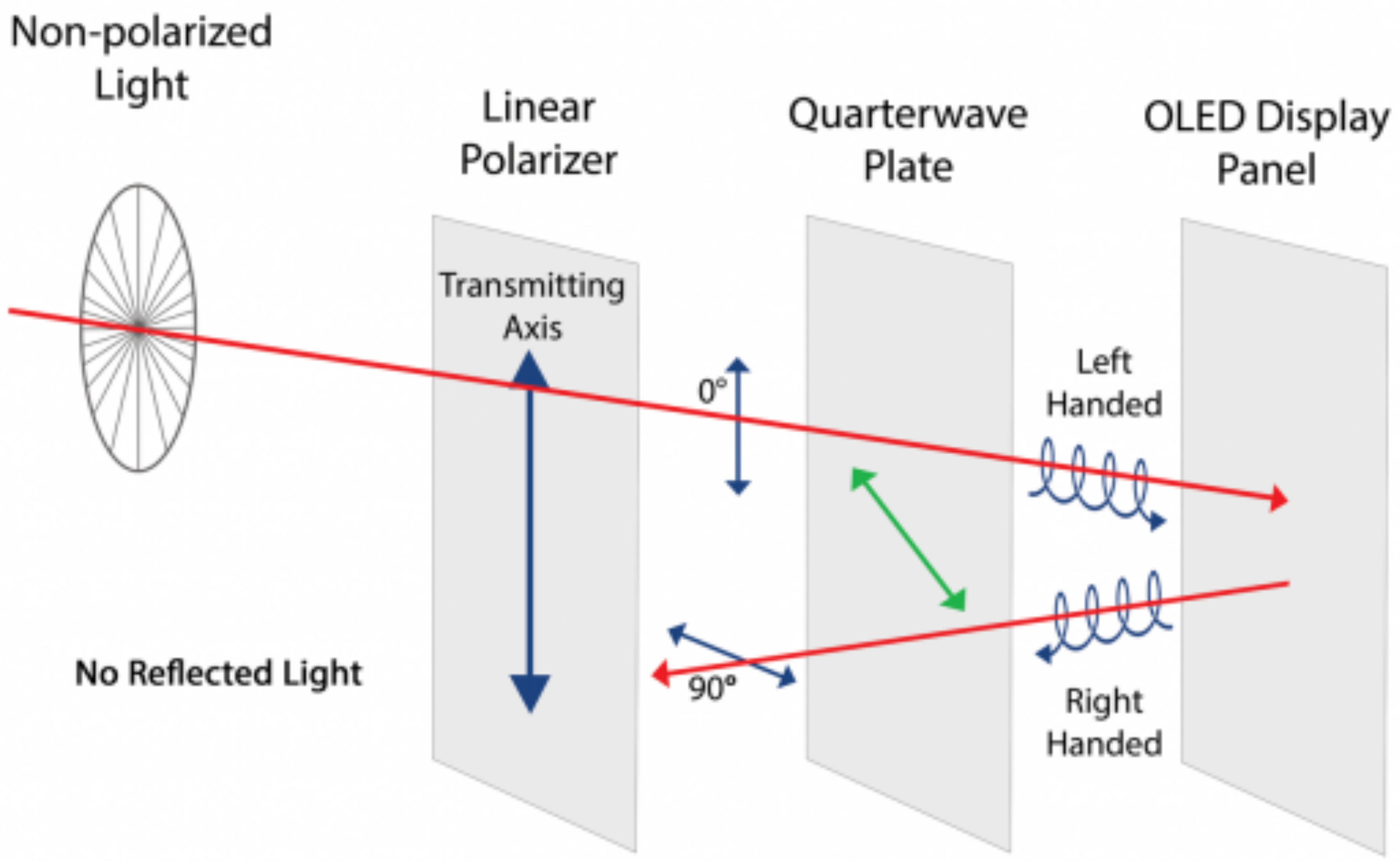
"Anti-reflecting (AR) coating"
(for better outdoor visibility)



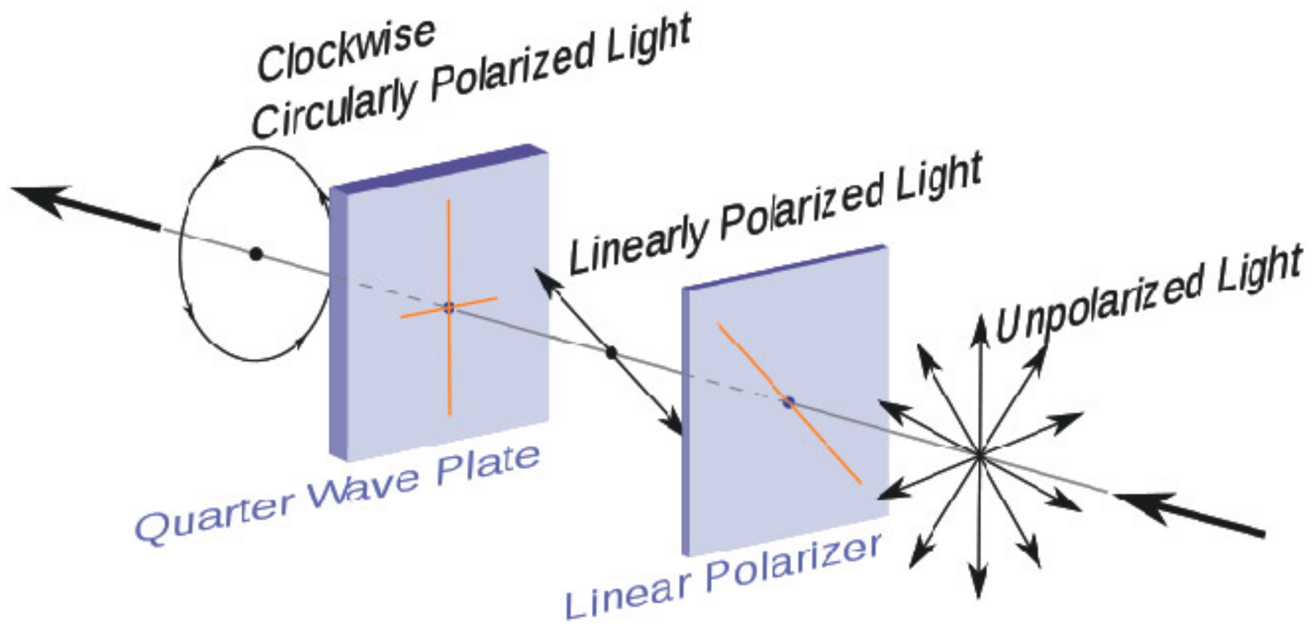


No AR coating *With AR coating*

Composition of AR coating

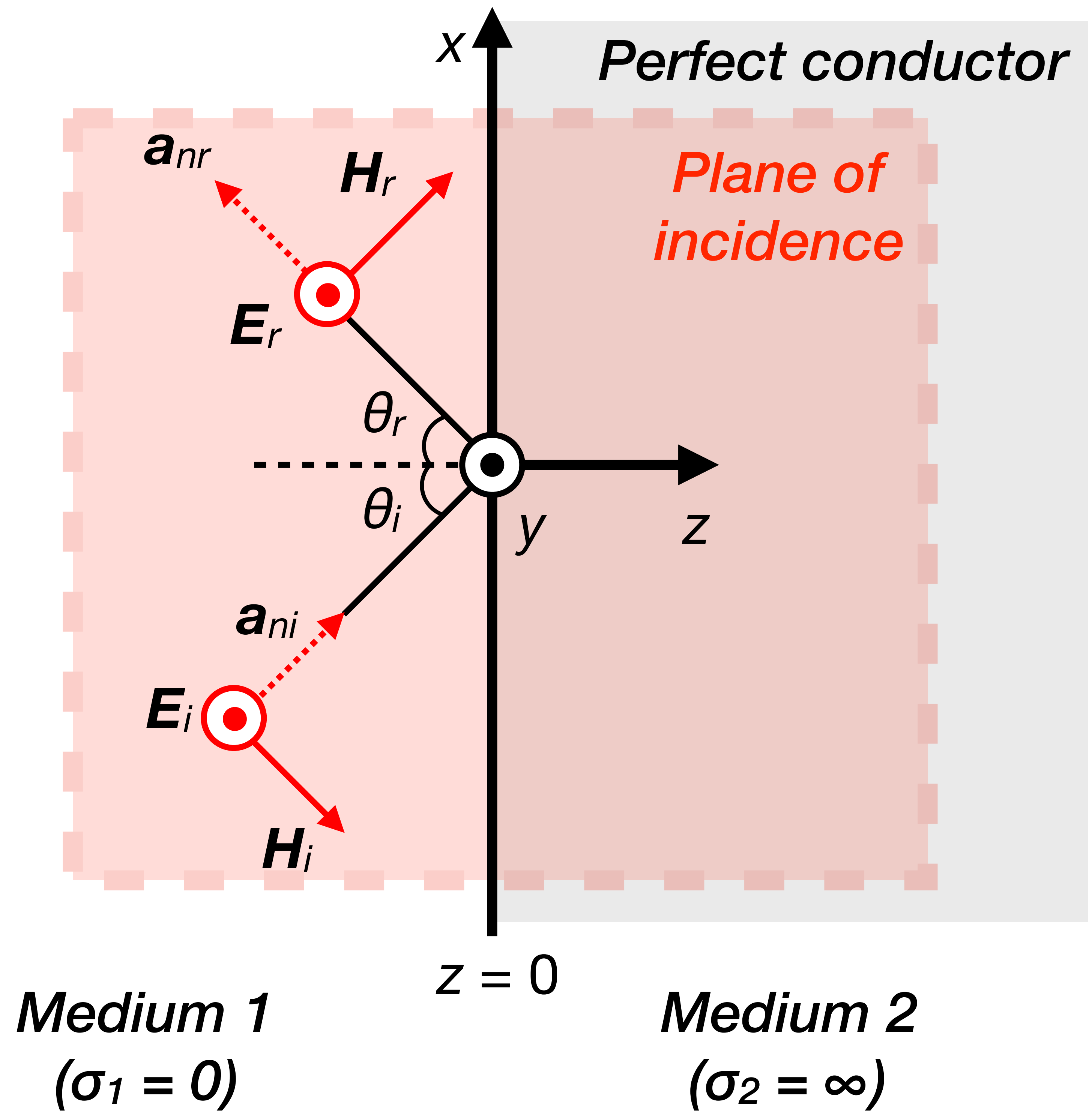


- *AR coating* = Linear polarizer + quarter-wave plate (a.k.a phase retarder by $\lambda/4$)
- ***Circularly polarized wave*** = Sum of TWO linearly polarized waves in ***both space and time quadrature***
- ***Phase change*** (180º) ***upon reflection by metal***



Oblique incidence at a plane conducting boundary

Perpendicular polarization = Transverse Electric (TE) wave



- E_i-field ⊥ Plane of incidence (*xz* plane)
- Propagation directions of incident and reflective ***E-field***

$$\boldsymbol{a}_{ni} = \boldsymbol{a}_x \sin\theta_i + \boldsymbol{a}_z \cos\theta_i$$

$$\boldsymbol{a}_{nr} = \boldsymbol{a}_x \sin\theta_r - \boldsymbol{a}_z \cos\theta_r$$

- Incident and reflective ***E-field***

$$\boldsymbol{E}_i(x,z) = \boldsymbol{a}_y E_{i0} e^{-j\beta_1 \boldsymbol{a}_{ni}\cdot\boldsymbol{R}} = \boldsymbol{a}_y E_{i0} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\boldsymbol{E}_r(x,z) = \boldsymbol{a}_y E_{r0} e^{-j\beta_1 \boldsymbol{a}_{nr}\cdot\boldsymbol{R}} = \boldsymbol{a}_y E_{r0} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

- Boundary condition (@ *z* = 0): total ***E-field*** in medium1 = total ***E-field*** in medium 2

$$\left[\boldsymbol{E}_1(x,0) = \boldsymbol{E}_i(x,0) + \boldsymbol{E}_r(x,0)\right] = \left[\boldsymbol{E}_2(x,0) = 0\right]$$

$$\rightarrow \boldsymbol{a}_y\left(E_{i0} e^{-j\beta_1 x\sin\theta_i} + E_{r0} e^{-j\beta_1 x\sin\theta_r}\right) = 0$$

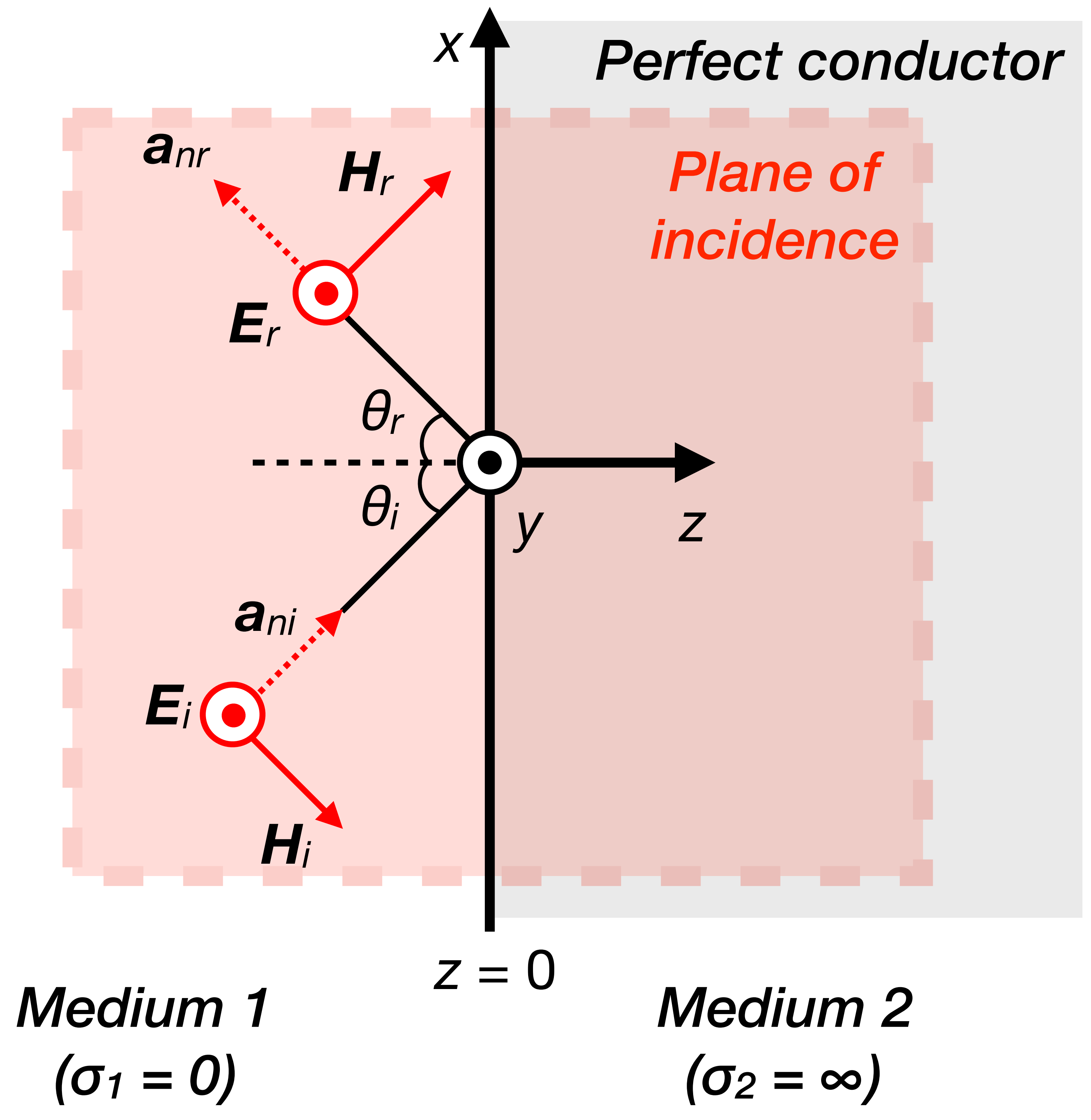
- To satisfy above condition,

$$\therefore E_{r0} = -E_{i0}, \quad \theta_r = \theta_i$$

Snell's law of reflection

Phase is shifted by 180°

Total electric and magnetic fields



- Total Electric Field $\boldsymbol{E_1}$

$$E_{r0} = -E_{i0}, \quad \theta_r = \theta_i$$

$$\boldsymbol{E_1}(x,z) = \boldsymbol{E_i}(x,z) + \boldsymbol{E_r}(x,z)$$
$$= \boldsymbol{a_y}\left(E_{i0}e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} + E_{r0}e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\right)$$
$$= -\boldsymbol{a_y}E_{i0}\left(e^{-j\beta_1 z\cos\theta_i} - e^{j\beta_1 z\cos\theta_i}\right)e^{-j\beta_1 x\sin\theta_i}$$
$$= -\boldsymbol{a_y}j2E_{i0}\sin(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i} = \boldsymbol{a_y}E_{1y}(x,z)$$

→ "Perpendicular" to Plane of Incidence → "Transverse Electric (TE)" wave

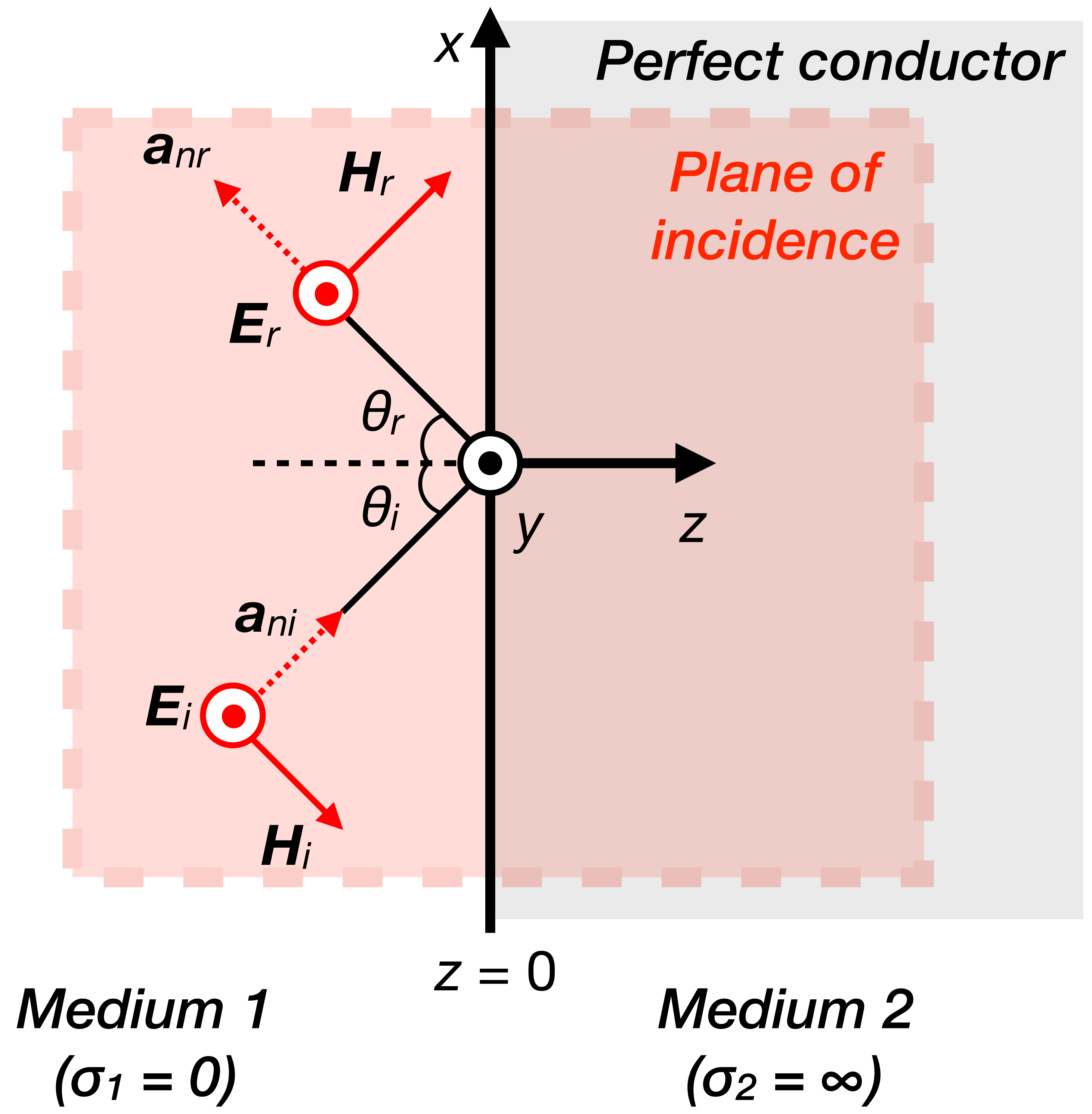
- Total Magnetic Field $\boldsymbol{H_1}$

$$\boldsymbol{a_{nr}} = \boldsymbol{a_x}\sin\theta_i - \boldsymbol{a_z}\cos\theta_i$$

$$\boldsymbol{H_1}(x,z) = \frac{1}{\eta_1}\left[\boldsymbol{a_{nr}} \times \boldsymbol{E_1}(x,z)\right] = \frac{1}{\eta_1}\left[\left(\boldsymbol{a_x}\sin\theta_i - \boldsymbol{a_z}\cos\theta_i\right)\times \boldsymbol{a_y}E_{1y}(x,z)\right]$$
$$= \left(\boldsymbol{a_x}\times\boldsymbol{a_y}\right)\frac{\sin\theta_i E_{1y}(x,z)}{\eta_1} - \left(\boldsymbol{a_z}\times\boldsymbol{a_y}\right)\frac{\cos\theta_i E_{1y}(x,z)}{\eta_1}$$
$$= -2\frac{E_{i0}}{\eta_1}\left[\boldsymbol{a_x}\cos\theta_i\cos(\beta_1 z\cos\theta_i) + \boldsymbol{a_z}j\sin\theta_i\sin(\beta_1 z\cos\theta_i)\right]e^{-j\beta_1 x\sin\theta_i}$$
$$= \frac{1}{\eta_1}\left[\boldsymbol{a_x}H_{1x}(x,z) + \boldsymbol{a_z}H_{1z}(x,z)\right]$$

Oblique incidence at a plane conducting boundary

Induced surface current on the conducting wall (Example 8-10)



- Boundary condition for *H*-field at the interface (*z*=0)

$$\mathbf{a}_{n2} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$$

$$\mathbf{J}_s(x) = (-\mathbf{a}_z) \times (\mathbf{H}_1(x,0) - \mathbf{H}_2(x,0)) = -\mathbf{a}_z \times \mathbf{H}_1(x,0)$$

- Since H_1 is given by

$$\mathbf{H}_1(x,z) = -2\frac{E_{i0}}{\eta_1}\left[\mathbf{a}_x \cos\theta_i \cos(\beta_1 z\cos\theta_i) + \mathbf{a}_z j\sin\theta_i \sin(\beta_1 z\cos\theta_i)\right]e^{-j\beta_1 x\sin\theta_i}$$

$$\mathbf{H}_1(x,0) = -\mathbf{a}_x 2\frac{E_{i0}}{\eta_1}\cos\theta_i e^{-j\beta_1 x\sin\theta_i}$$

- Thus,

$$\mathbf{J}_s(x) = -\mathbf{a}_z \times \left[-\mathbf{a}_x 2\frac{E_{i0}}{\eta_1}\cos\theta_i e^{-j\beta_1 x\sin\theta_i}\right] = \mathbf{a}_y 2\frac{E_{i0}}{\eta_1}\cos\theta_i e^{-j\beta_1 x\sin\theta_i}$$

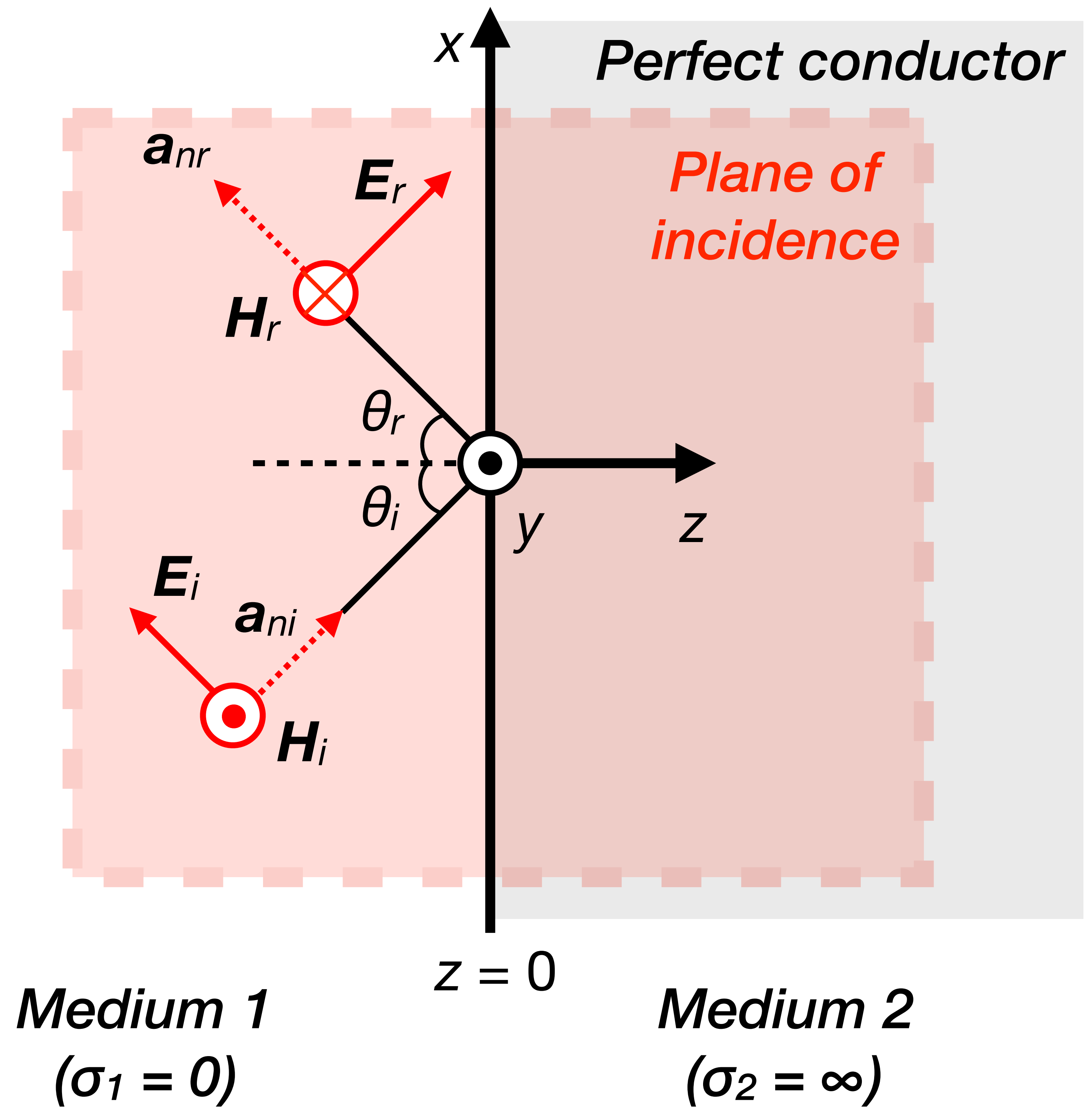
- Instantaneous expression

$$J_s(x,t) = \mathrm{Re}\left[\mathbf{J}_s(x)e^{j\omega t}\right] = \mathbf{a}_y 2\frac{E_{i0}}{\eta_1}\cos\theta_i \cos\omega\left(t - \frac{x}{c}\sin\theta_i\right)$$

- This *induced J_s* results in the *reflected wave in medium 1*
- Reflected wave ***cancels the incident wave*** in the wall
- Microscopically, incident wave absorbed by free electrons
- Oscillating electrons (J_s) re-radiate EM wave

Chap. 8 | Oblique incidence at a plane conducting boundary

Parallel polarization = Transverse Magnetic (TM) wave



- E_i-field // Plane of incidence (*xz* plane)
- Total Electric Field $\boldsymbol{E}_1$

$$\boldsymbol{E}_1(x,z)=\boldsymbol{E}_i(x,z)+\boldsymbol{E}_r(x,z)$$

$$=\boldsymbol{a}_x E_{1x}(x,z)+\boldsymbol{a}_z E_{1z}(x,z)$$ *(Refer to Sec. 8-7.2 for derivation)*

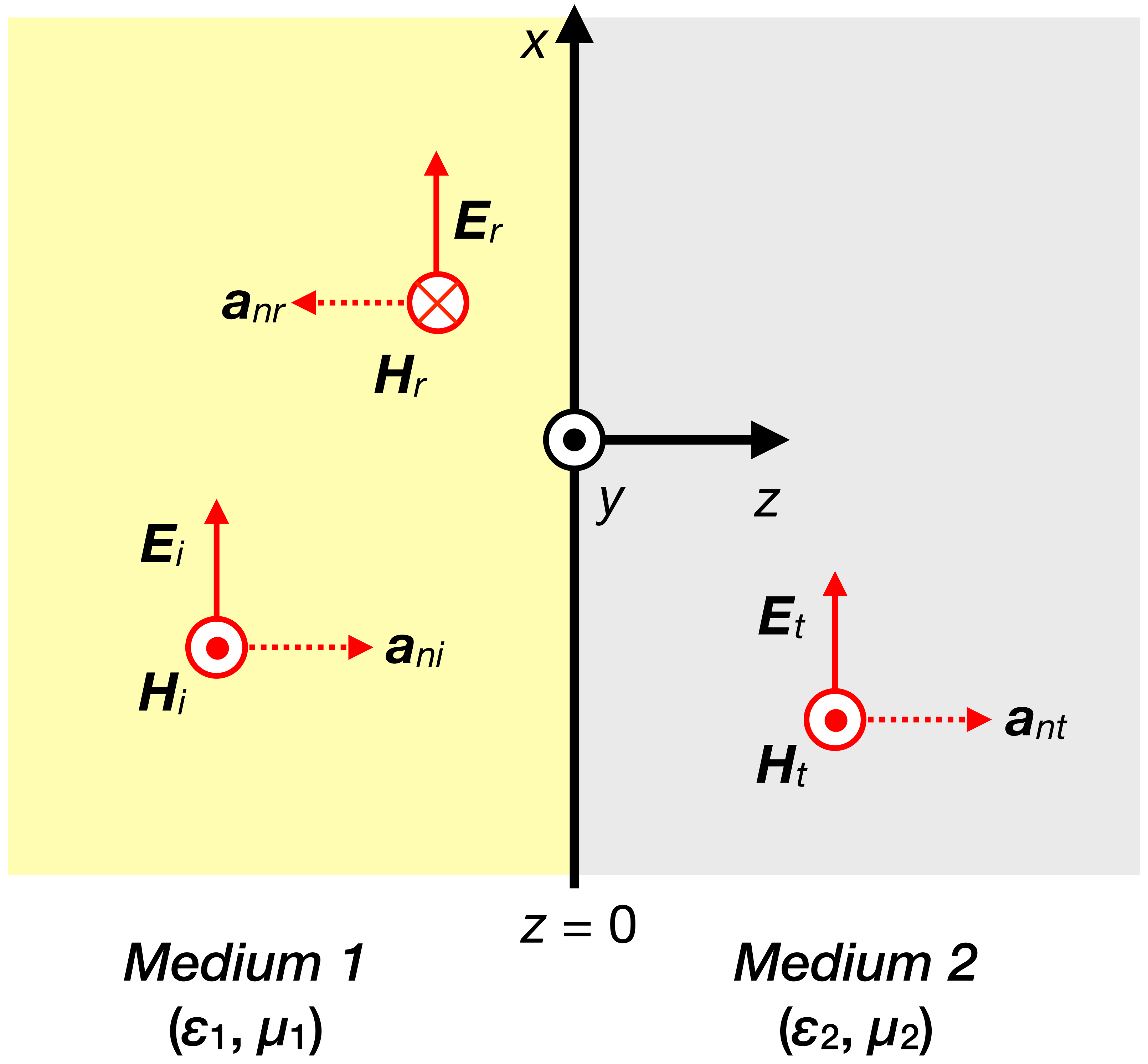
- Total Magnetic Field $\boldsymbol{H}_1$

$$\boldsymbol{H}_1(x,z)=\frac{1}{\eta_1}\left[\boldsymbol{a}_{nr}\times\boldsymbol{E}_1(x,z)\right]$$ *(Refer to Sec. 8-7.2 for derivation)*

$$=\boldsymbol{a}_y H_{1y}(x,z)$$ → *"Perpendicular" to Plane of Incidence* → *"Transverse Magnetic (TM)" wave*

Normal Incidence at a Plane Dielectric Boundary

Two different dielectric media



Both are lossless (σ_1, σ_2 = 0)

- When EM wave is incident on the interface between ***two media with different intrinsic impedance***, part of incident power is ***reflected*** and part is ***transmitted***

- Incident wave

$$\begin{cases} \mathbf{E}_i(z) = \mathbf{a}_x E_{i0} e^{-j\beta_1 z} \\ \mathbf{H}_i(z) = \mathbf{a}_z \times \frac{1}{\eta_1} \mathbf{E}_i = \mathbf{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z} \end{cases}$$

- Reflected wave

$$\begin{cases} \mathbf{E}_r(z) = \mathbf{a}_x E_{r0} e^{+j\beta_1 z} \\ \mathbf{H}_r(z) = -\mathbf{a}_z \times \frac{1}{\eta_1} \mathbf{E}_r(z) = -\mathbf{a}_y \frac{E_{r0}}{\eta_1} e^{+j\beta_1 z} \end{cases}$$

- Transmitted wave

$$\begin{cases} \mathbf{E}_t(z) = \mathbf{a}_x E_{t0} e^{-j\beta_2 z} \\ \mathbf{H}_t(z) = \mathbf{a}_z \times \frac{1}{\eta_2} \mathbf{E}_t(z) = \mathbf{a}_y \frac{E_{t0}}{\eta_2} e^{-j\beta_1 z} \end{cases}$$

- Boundary conditions at $z = 0$
  → Tangential ***E*** and ***H*** should be continuous

$$\mathbf{E}_i(0) + \mathbf{E}_r(0) = \mathbf{E}_t(0)$$

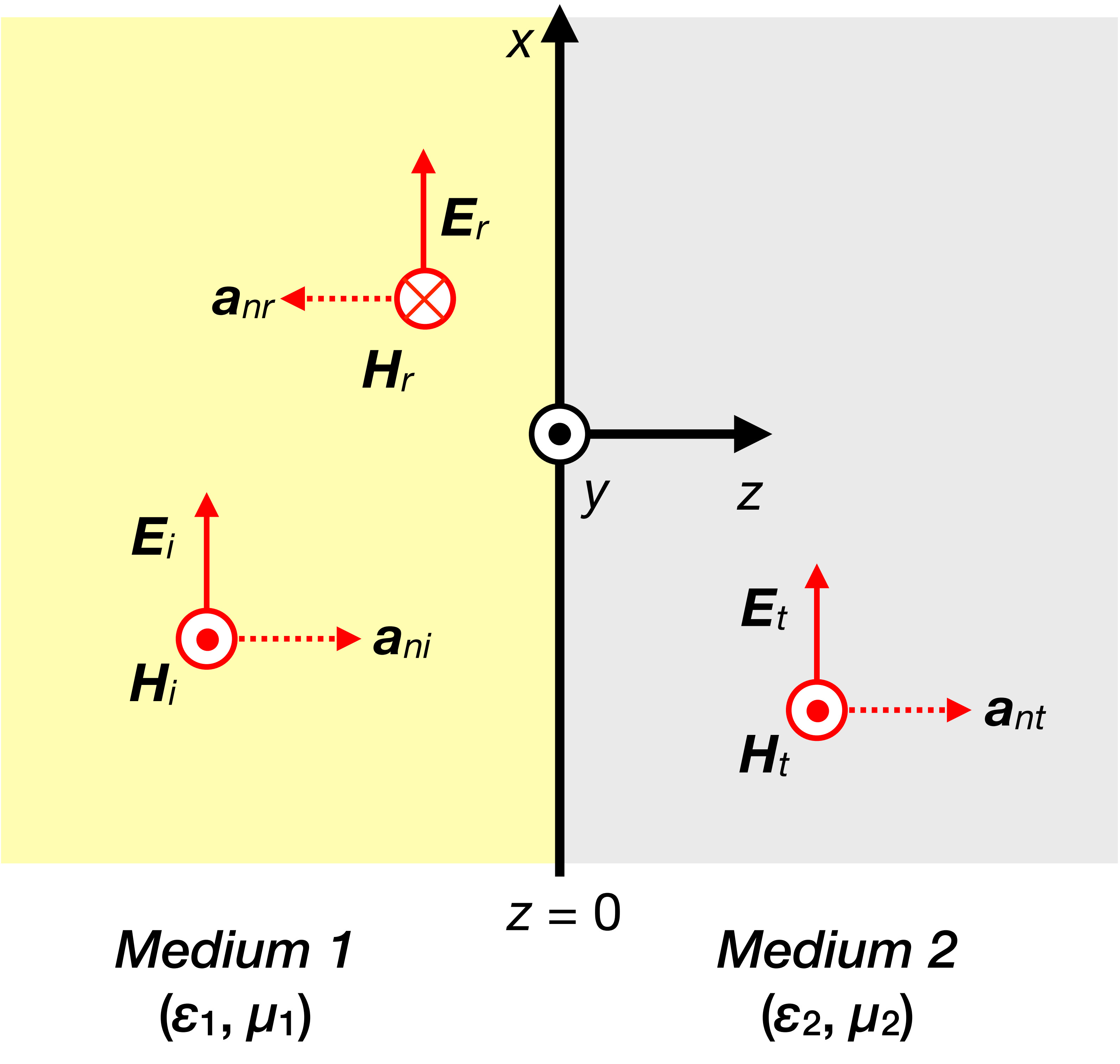
$$\rightarrow E_{i0} + E_{r0} = E_{t0}$$

$$\mathbf{H}_i(0) + \mathbf{H}_r(0) = \mathbf{H}_t(0)$$

$$\rightarrow \frac{1}{\eta_2}(E_{i0} - E_{r0}) = \frac{E_{t0}}{\eta_2}$$

Normal Incidence at a Plane Dielectric Boundary

Reflection and transmission coefficients



- Boundary conditions at $z = 0$
  → Tangential ***E*** and ***H*** should be continuous
  → 2 equations, 3 variables (E_{i0}, E_{r0}, E_{t0})

$$\boldsymbol{E}_i(0)+\boldsymbol{E}_r(0)=\boldsymbol{E}_t(0) \qquad \boldsymbol{H}_i(0)+\boldsymbol{H}_r(0)=\boldsymbol{H}_t(0)$$

$$\rightarrow E_{i0}+E_{r0}=E_{t0} \qquad \rightarrow \frac{1}{\eta_2}\left(E_{i0}-E_{r0}\right)=\frac{E_{t0}}{\eta_2}$$

- E_{r0} and E_{t0} in terms of E_{i0}

$$E_{r0}=\frac{\eta_2-\eta_1}{\eta_2+\eta_1}E_{i0} \rightarrow \boxed{\Gamma=\frac{E_{r0}}{E_{i0}}=\frac{\eta_2-\eta_1}{\eta_2+\eta_1}}$$ *Reflection Coefficient*

$$E_{t0}=\frac{2\eta_2}{\eta_2+\eta_1}E_{i0} \rightarrow \boxed{\tau=\frac{E_{t0}}{E_{i0}}=\frac{2\eta_2}{\eta_2+\eta_1}}$$ *Transmission Coefficient*

where η_1 and η_2 are intrinsic impedances for medium 1 and 2

- ***Complex Γ and τ*** (i.e. complex η_1 and η_2)
  → ***phase shift*** introduced upon transmission and reflection

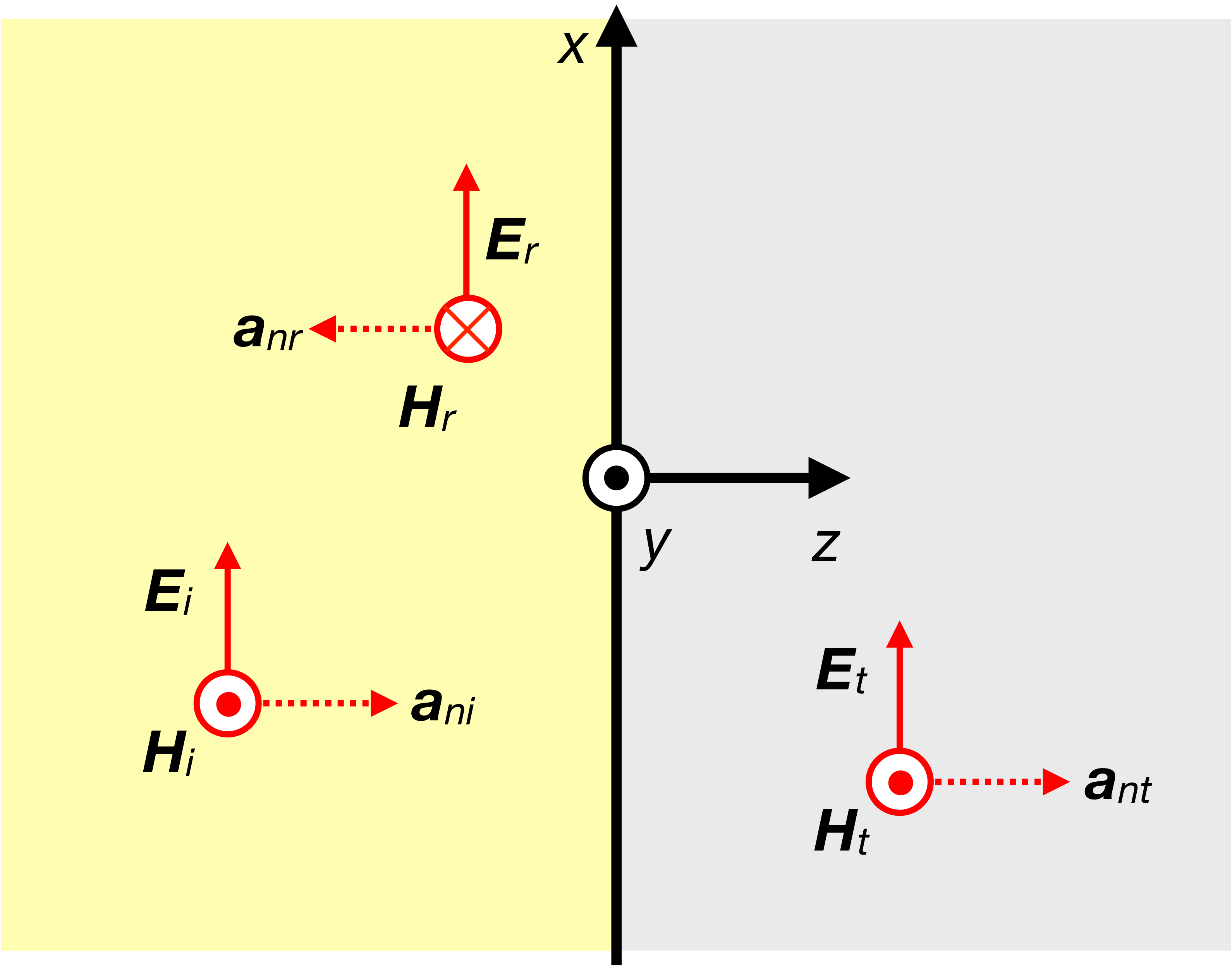
- If ***medium 2 is a perfect conductor*** (i.e. $\eta_2 = 0$)
  → $\Gamma = -1$ → $E_{r0} = -E_{i0}$
  → $\tau = 0$ → $E_{t0} = 0$

- ***Relationship between Γ and τ***

$$\boxed{\therefore 1+\Gamma=\tau}$$

Normal Incidence at a Plane Dielectric Boundary

Total E & H-fields in medium 1 (Incident + reflected)



$$\begin{cases} \boldsymbol{E}_i(z) = \boldsymbol{a}_x E_{i0} e^{-j\beta_1 z} \\ \boldsymbol{H}_i(z) = \boldsymbol{a}_y \dfrac{E_{i0}}{\eta_1} e^{-j\beta_1 z} \end{cases} \quad \begin{cases} \boldsymbol{E}_r(z) = \boldsymbol{a}_x E_{r0} e^{+j\beta_1 z} \\ \boldsymbol{H}_r(z) = -\boldsymbol{a}_y \dfrac{E_{r0}}{\eta_1} e^{+j\beta_1 z} \end{cases}$$

- Total *E*-field

  If medium 2 is not a perfect conductor → Partial transmission & reflection

$$\boldsymbol{E}_1(z) = \boldsymbol{E}_i(z) + \boldsymbol{E}_r(z) = \boldsymbol{a}_x\left(E_{i0}e^{-j\beta_1 z} + E_{r0}e^{j\beta_1 z}\right)$$

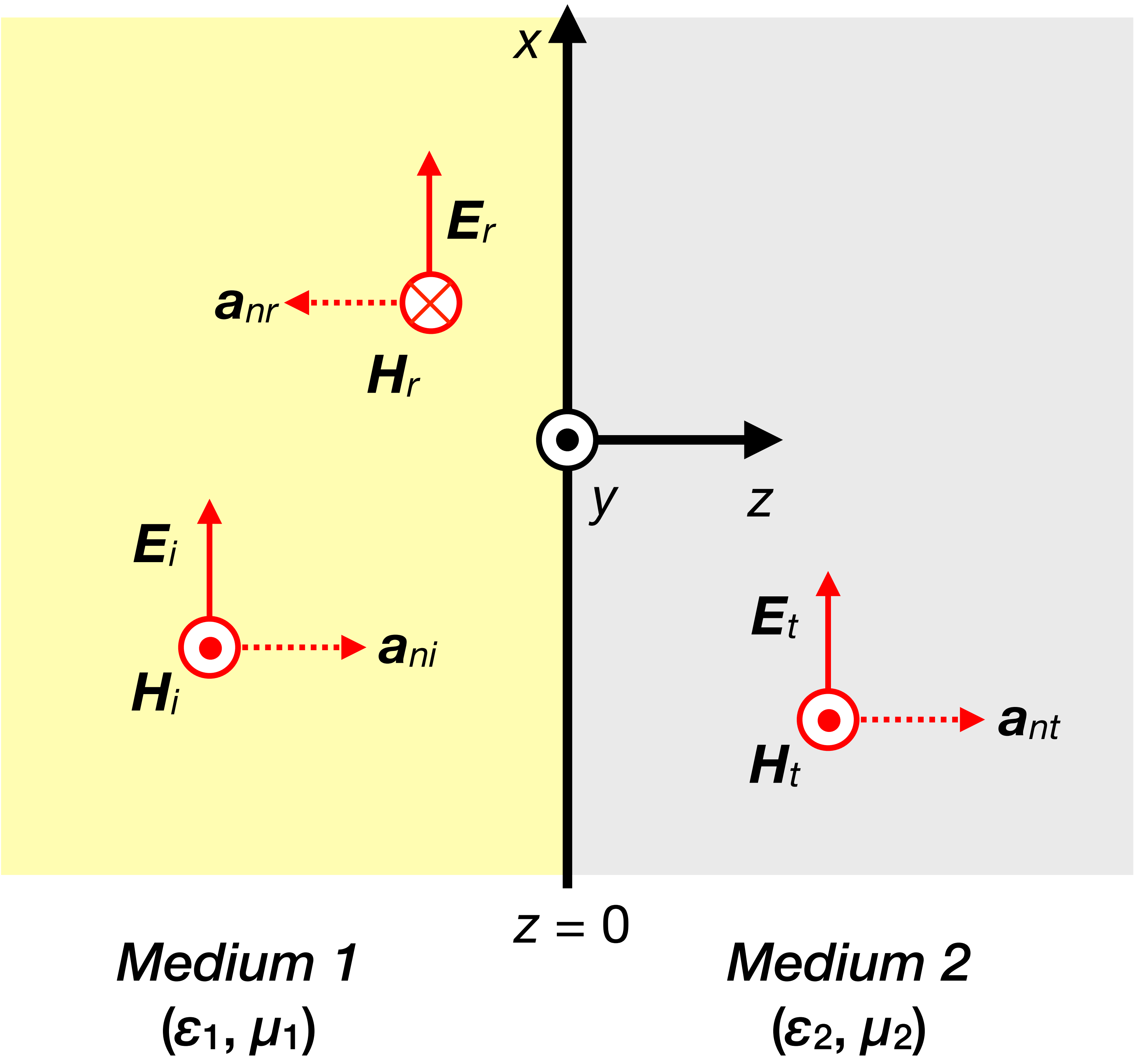
$$= \boldsymbol{a}_x E_{i0}\left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right) = \boldsymbol{a}_x E_{i0}\left[(1+\Gamma)e^{-j\beta_1 z} + \Gamma\left(e^{j\beta_1 z} - e^{-j\beta_1 z}\right)\right]$$

$$= \boldsymbol{a}_x E_{i0}\left[\underbrace{\tau e^{-j\beta_1 z}}_{\text{Traveling wave}} + \underbrace{\Gamma\left(j2\sin\beta_1 z\right)}_{\text{Standing wave}}\right] \quad \leftarrow \quad 1+\Gamma = \tau$$

- Total *H*-field

$$\boldsymbol{H}_1(z) = \boldsymbol{H}_i(z) + \boldsymbol{H}_r(z) = \boldsymbol{a}_y \frac{E_{i0}}{\eta_1}\left(e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z}\right)$$

Total E & H-fields in medium 2 (Transmitted)



$$\begin{cases} \boldsymbol{E_1}(z) = \boldsymbol{a_x} E_{i0}\left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right) \\ \boldsymbol{H_1}(z) = \boldsymbol{a_y} \dfrac{E_{i0}}{\eta_1}\left(e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z}\right) \end{cases}$$

- Total *E*-field

$$\boldsymbol{E_t}(z) = \boldsymbol{a_x} E_{t0} e^{-j\beta_1 z} = \boldsymbol{a_x} \tau E_{i0} e^{-j\beta_1 z}$$

- Total *H*-field

$$\boldsymbol{H_t}(z) = \boldsymbol{a_y} \frac{E_{t0}}{\eta_2} e^{-j\beta_1 z} = \boldsymbol{a_y} \tau \frac{E_{i0}}{\eta_2} e^{-j\beta_1 z}$$

- Time-average Poynting vector in *medium 1*

$$\boldsymbol{P_{av1}} = \frac{1}{2}\mathrm{Re}\left(\boldsymbol{E}_1 \times \boldsymbol{H}_1^*\right)$$
$$= \left(\boldsymbol{a_x} \times \boldsymbol{a_y}\right)\frac{E_{i0}^2}{\eta_1}\mathrm{Re}\left[\left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right)\left(e^{j\beta_1 z} - \Gamma e^{-j\beta_1 z}\right)\right]$$
$$= \boldsymbol{a_z}\frac{E_{i0}^2}{2\eta_1}\left(1 - \Gamma^2\right)$$

- Time-average Poynting vector in *medium 2*

$$\boldsymbol{P_{av2}} = \frac{1}{2}\mathrm{Re}\left(\boldsymbol{E}_2 \times \boldsymbol{H}_2^*\right) = \boldsymbol{a_z}\frac{E_{i0}^2}{2\eta_2}\tau^2$$

- Since *media 1 and 2* are lossless, $\boldsymbol{P_{av1}} = \boldsymbol{P_{av2}}$ $\rightarrow$

$$\therefore 1 - \Gamma^2 = \frac{\eta_1}{\eta_2}\tau^2$$