

Electrical System



**Seoul National Univ.
School of Mechanical
and Aerospace Engineering**

Spring 2008

Development of Integrated Vehicle Control System of “Fine-X” Which Realized Free Movement.



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TOYOTA



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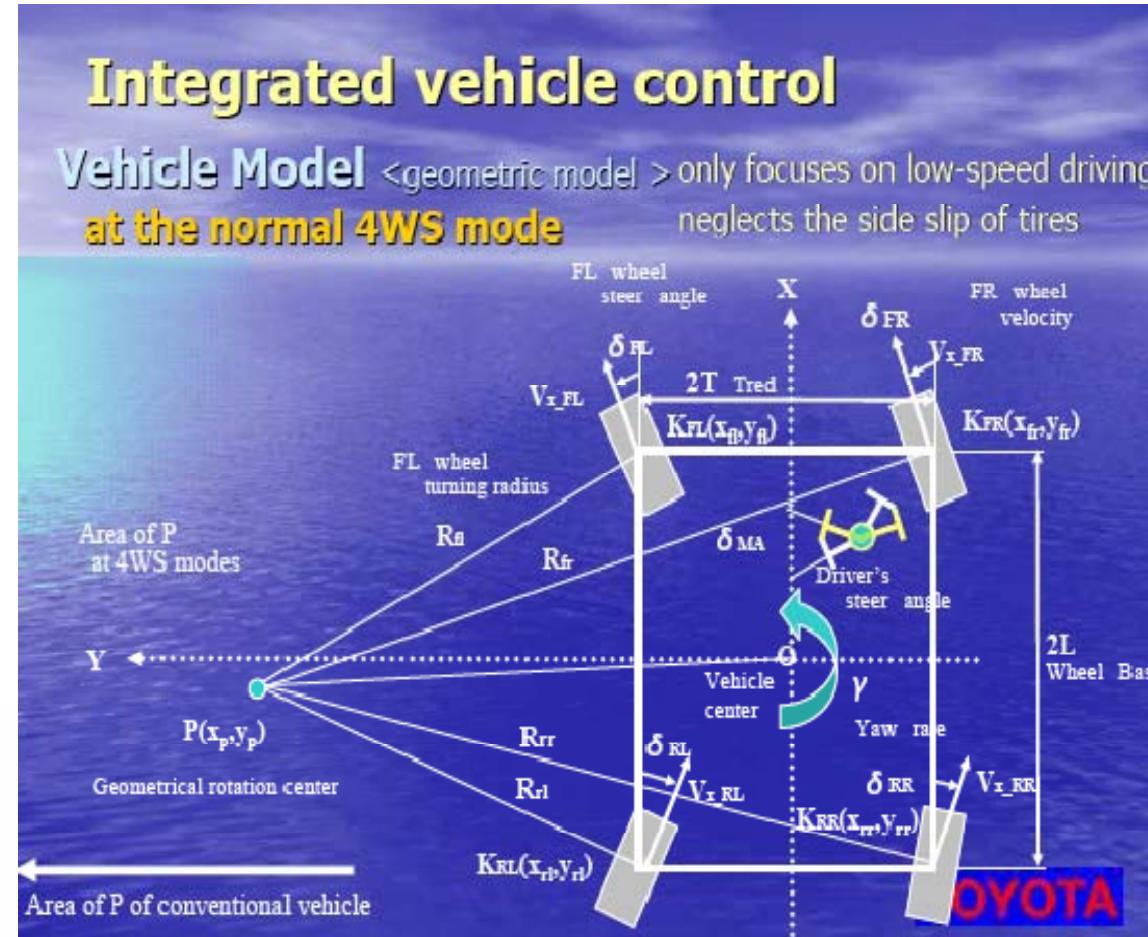
TOYOTA Freer Movement Control System



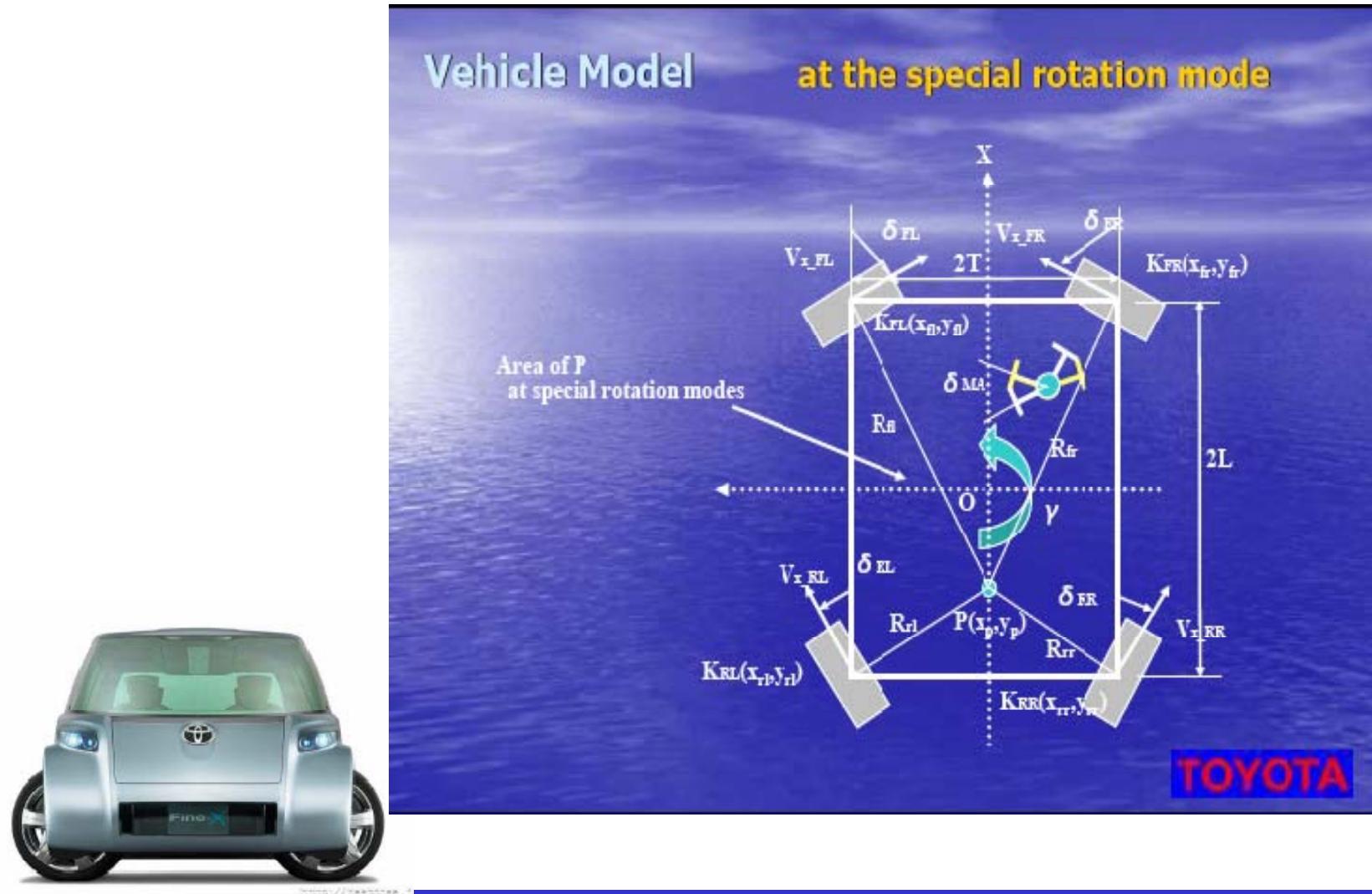
4Wheel independent drive
4wheel independent steering
4wheel independent braking
By 'wheel-in-motor'



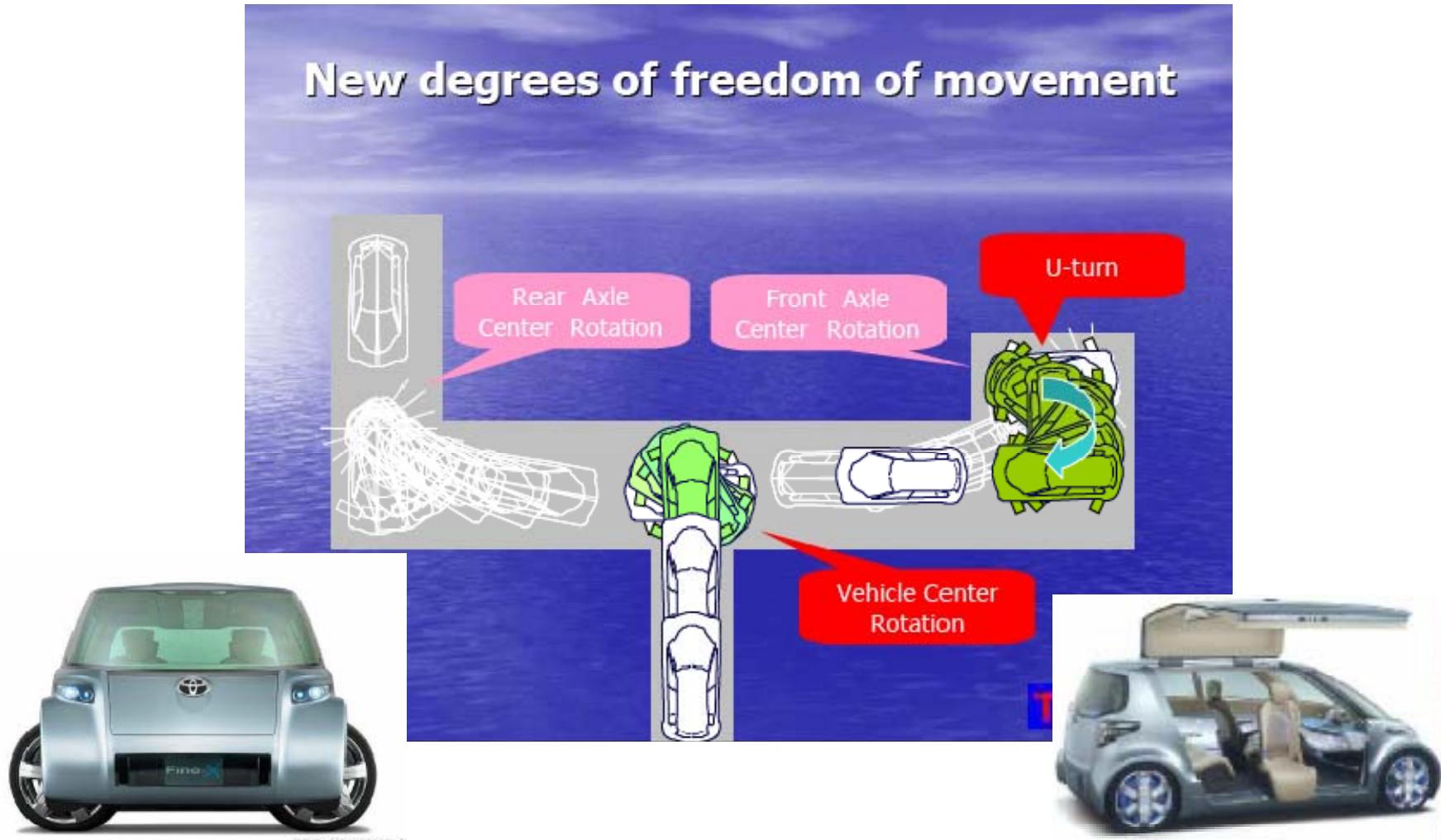
TOYOTA Freer Movement Control System



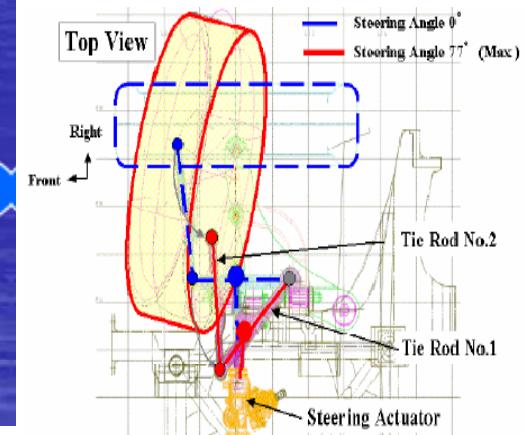
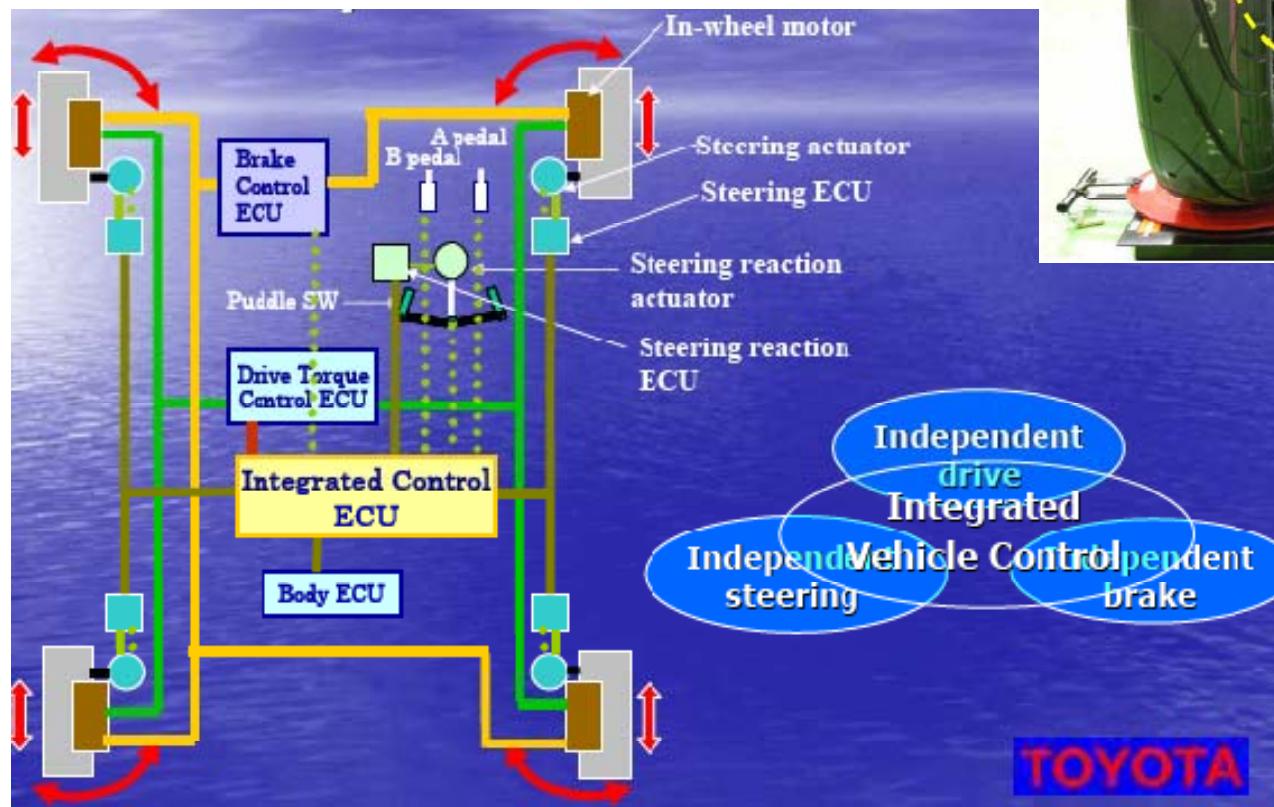
TOYOTA Freer Movement Control System



TOYOTA Freer Movement Control System for Auto-Parking



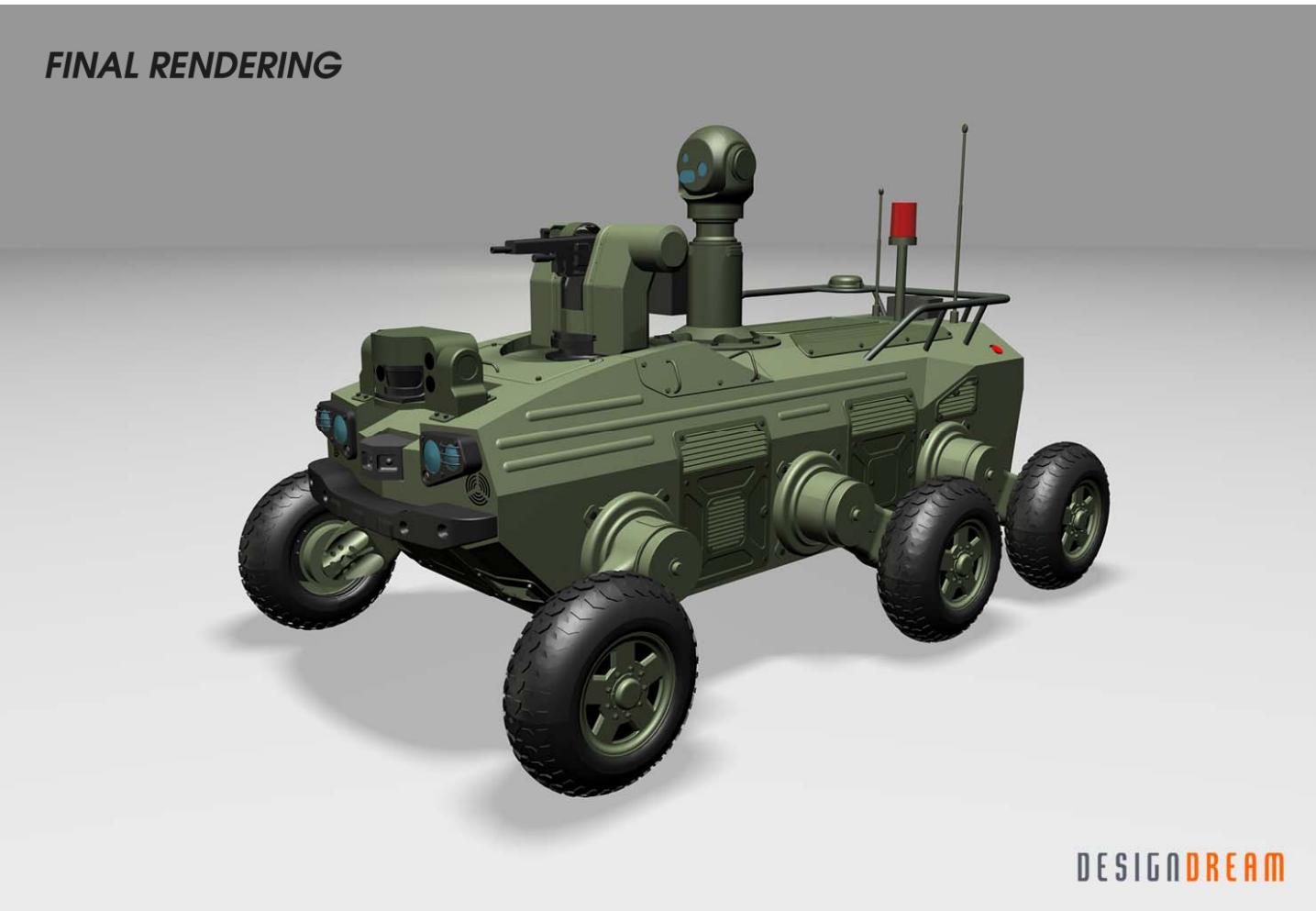
TOYOTA Freer Movement Control System for Auto-Parking



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Autonomous Robot Vehicle

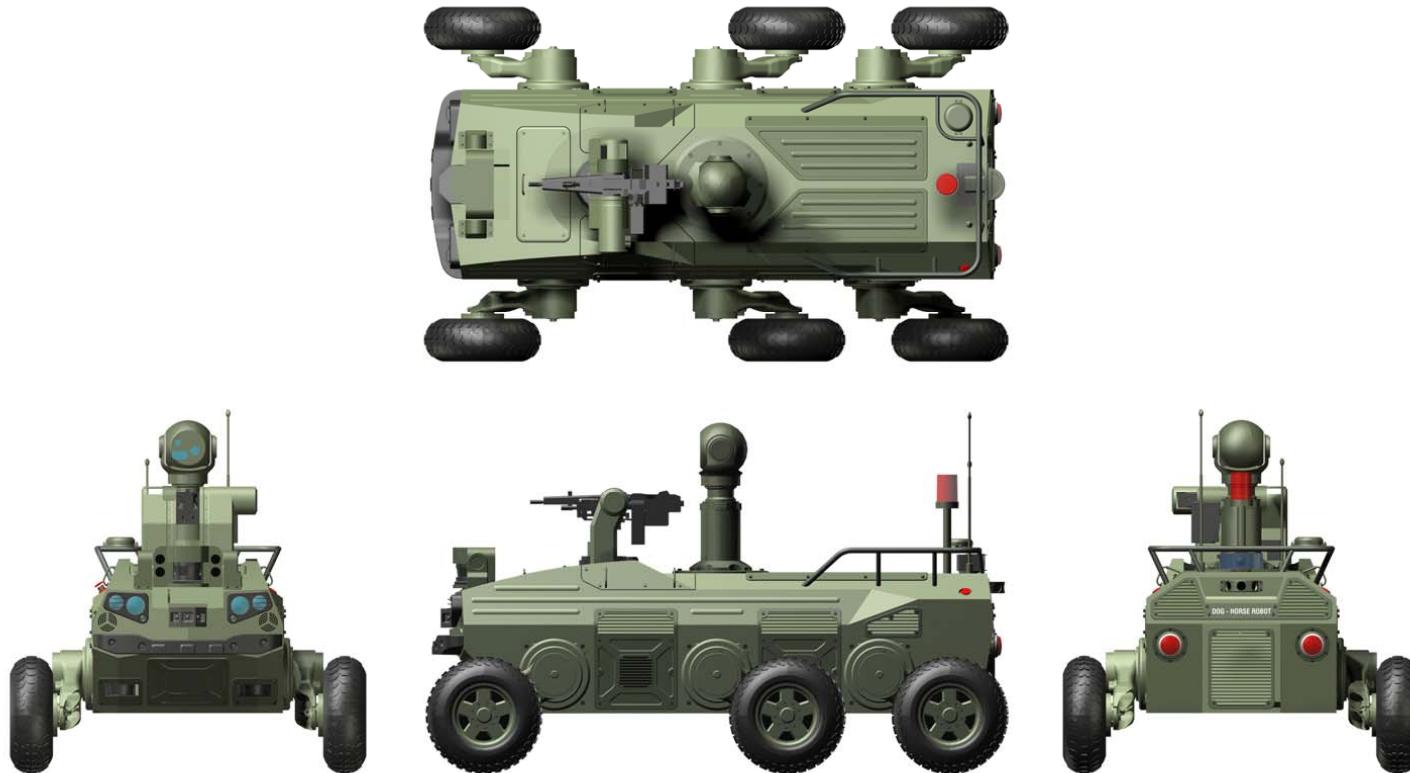


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Autonomous Robot Vehicle

FINAL RENDERING



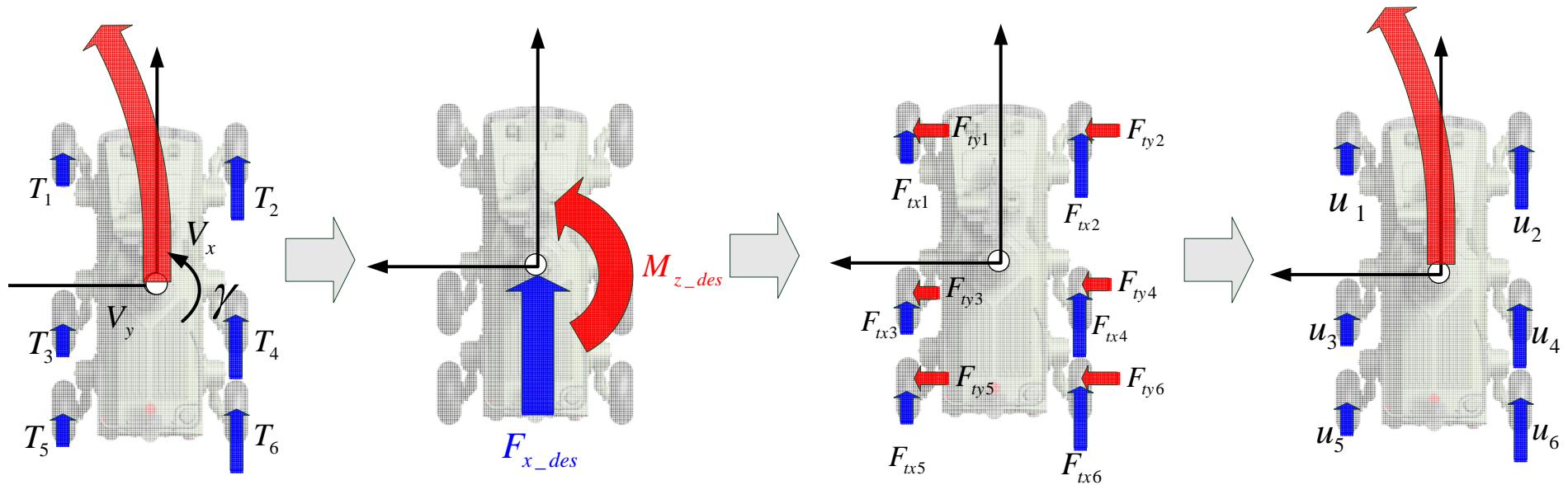
DESIGN DREAM



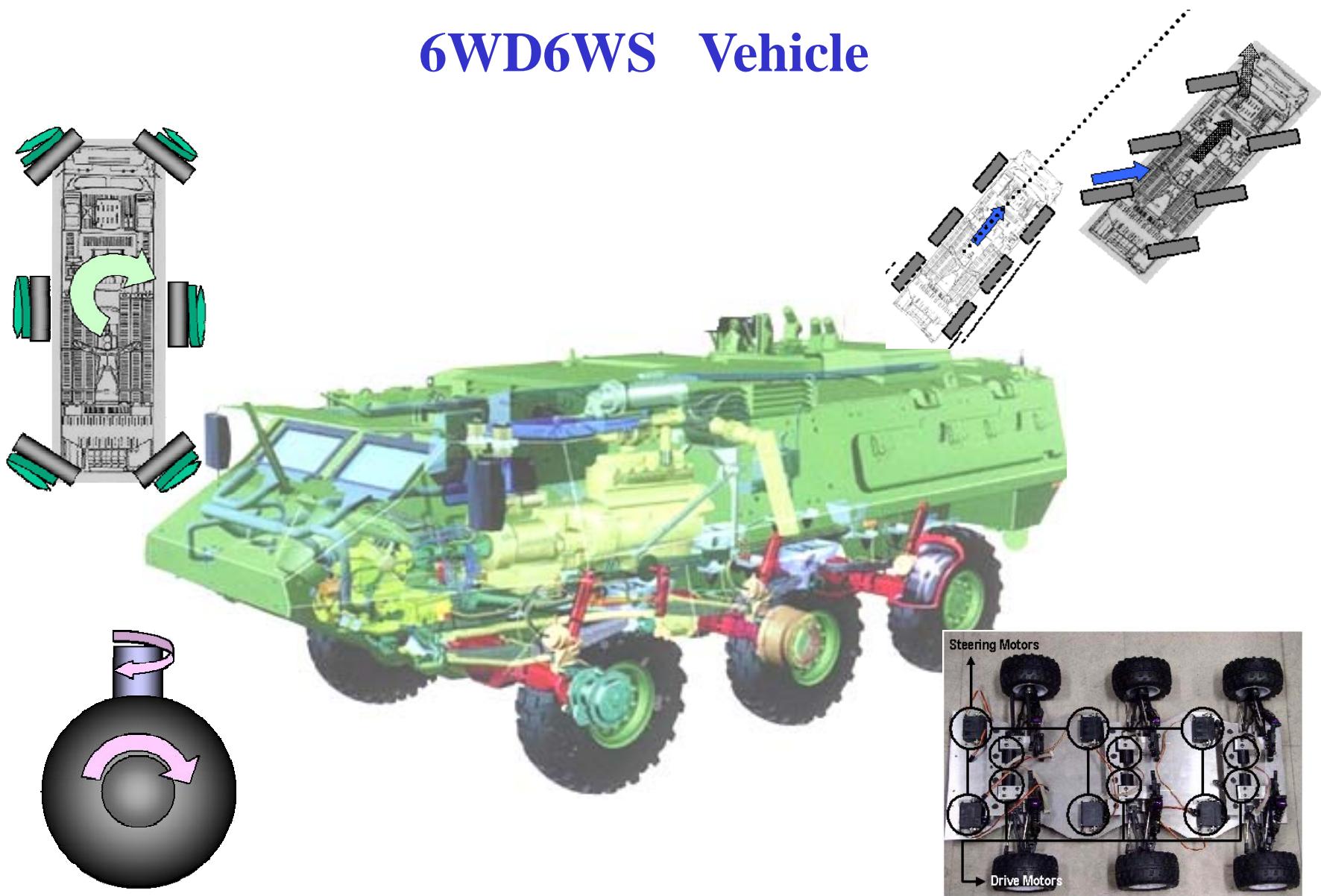
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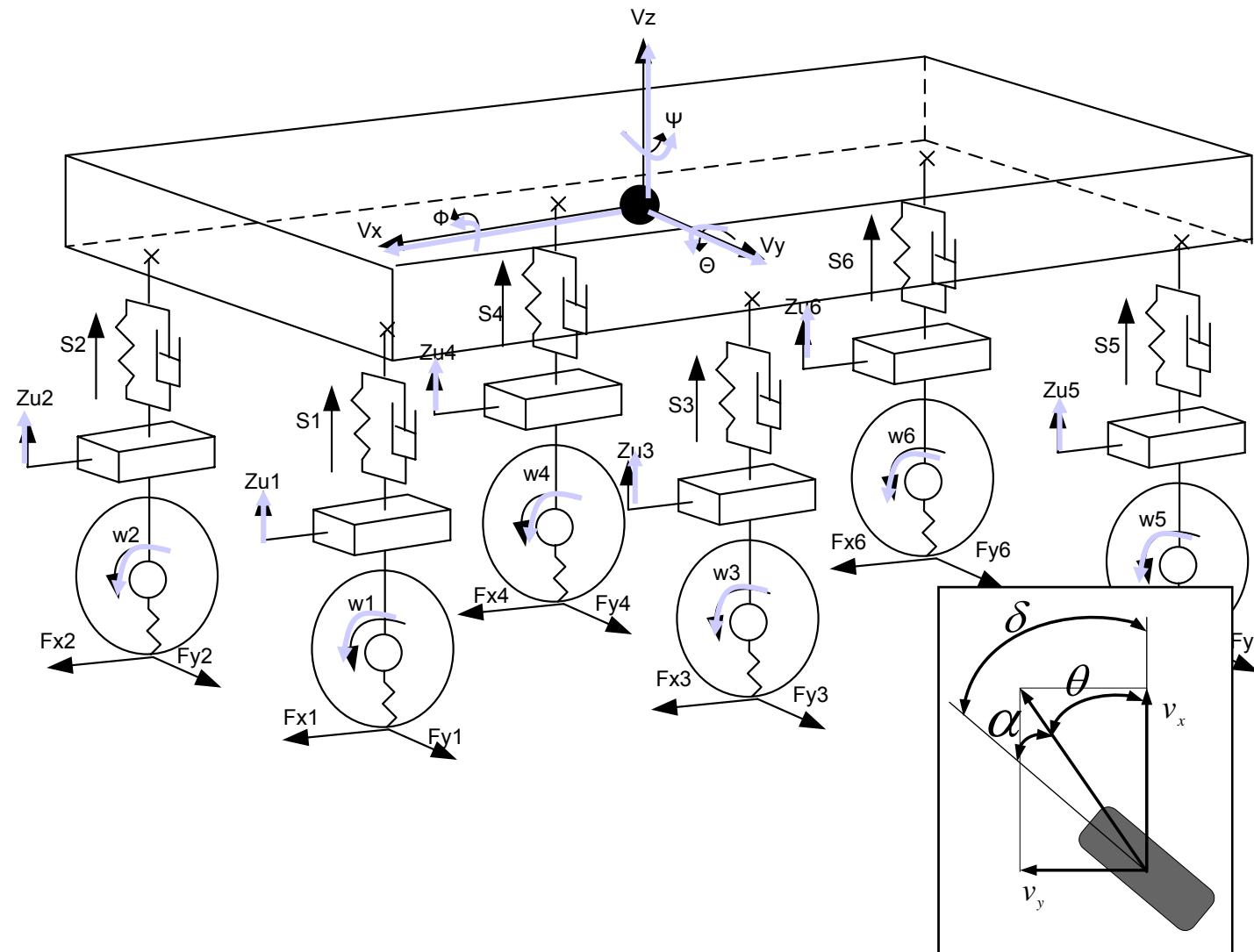
견마형 로봇 차량의 주행 제어 알고리즘



6WD6WS Vehicle



6WD6WS Vehicle



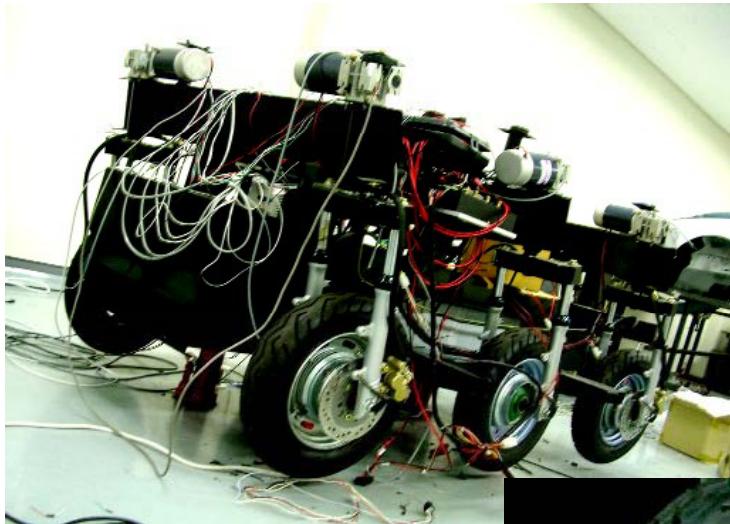
$$u = \begin{bmatrix} \Delta\delta_f \\ \delta_r \\ \delta_m \end{bmatrix}$$

$$\delta_m = X_1\beta + X_2r$$

The inset diagram shows a side view of a wheel with a steering angle δ . The vehicle's longitudinal velocity v_x is shown along the horizontal axis, and the lateral velocity v_y is shown perpendicular to it. The angle θ represents the wheel's orientation relative to the vehicle's longitudinal axis.



BLDC Wheel-in-Motor of 6WD6WS Vehicle



Sectional View of BLDC Motor



Basic Elements

- Voltage : electromotive force needed to produce a flow of current in a wire [V]
- Current : the rate of flow of charge

$$i = \frac{dq}{dt} \quad [\text{ampere}] = \frac{[\text{coulomb}]}{[\text{sec}]}$$

- Charge : (electric charge) the integral of current with respect to time [C]



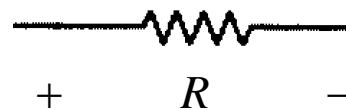
Basic Elements - Resistance

- Resistance : the change in voltage required to make a unit change in current.

$$R = \frac{\text{Change in voltage}}{\text{Change in current}} = \frac{[V]}{[A]} = [\text{Ohm}(\Omega)]$$

- Resistor

$$V_R = R \cdot i_R \quad R = \frac{V_R}{i_R}$$



Basic Elements - Capacitance

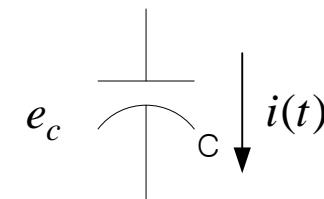
- Capacitance: the change in the quantity of electric charge required to make a unit change in voltage.

$$C = \frac{[\text{Coulomb}]}{[V]} = [\text{Farad } (F)]$$

- Capacitor: two conductor separated by non-conducting medium.

$$i = dq/dt, \quad e_c = q/C \rightarrow i = C \frac{de_c}{dt}, \quad de_c = \frac{1}{C} i dt$$

$$\therefore e_c(t) = \frac{1}{C} \int_0^t i dt + e_c(0)$$

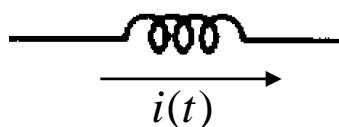


Basic Elements - Inductance

- Inductance: An electromotive force induced in a circuit, if the circuit lies in a time-varying magnetic field.

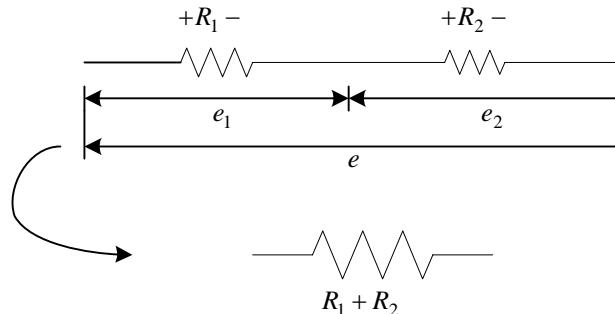
$$L = \frac{[V]}{[A/\text{sec}]} = [\text{Henry (H)}]$$

- Inductor: $e_L = L \frac{di_L}{dt}$
 $\therefore i_L(t) = \frac{1}{L} \int_0^t e_L dt + i_L(0)$



Series & Parallel Resistance

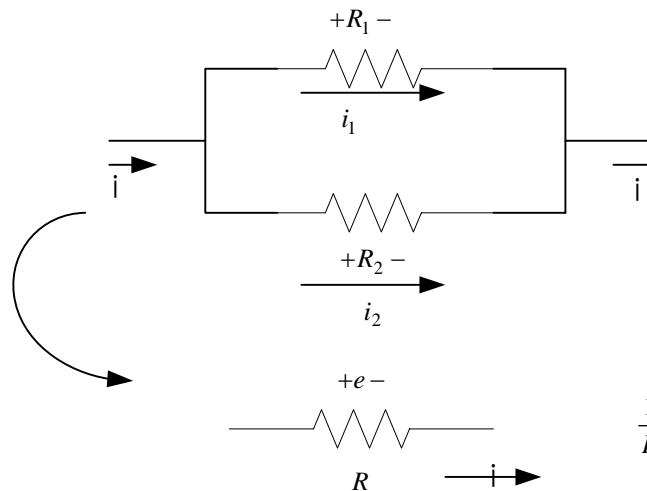
- Series Resistance



$$e_1 = iR_1, \quad e_2 = iR_2$$

$$e = e_1 + e_2 = i(R_1 + R_2)$$

- Parallel Resistance



$$-e_1 + e_2 = 0 \Rightarrow e_1 = e_2$$

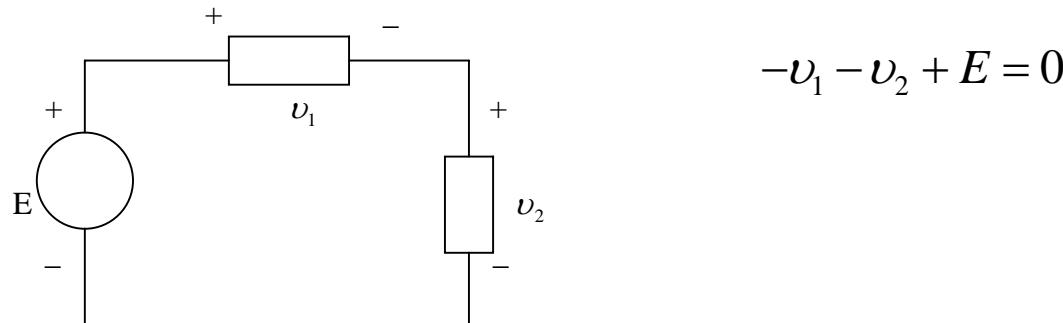
$$i = i_1 + i_2 = \frac{e_1}{R_1} + \frac{e_2}{R_2} = e\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{e}{R}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

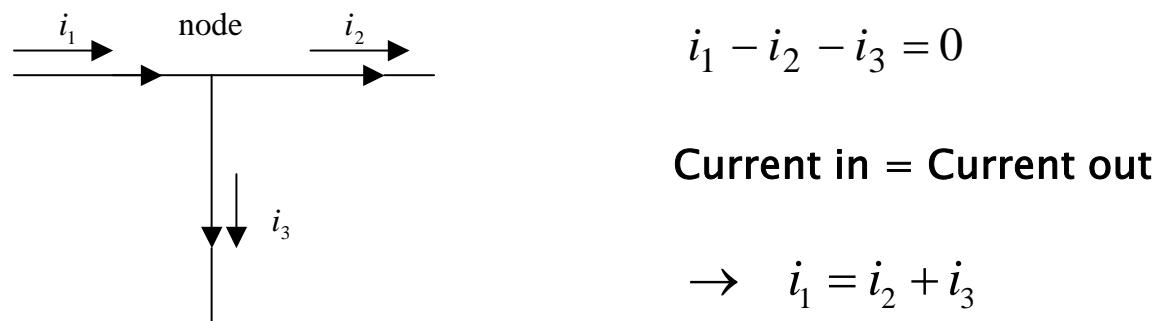


Kirchhoff's laws

1. The algebraic sum of the potential difference around a closed path equals zero.

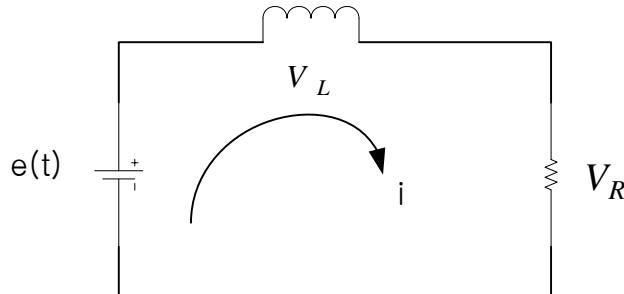


2. The algebraic sum of the currents entering (or leaving) a node is equal to zero.



Examples of Circuit Analysis

ex1) R-L Circuit

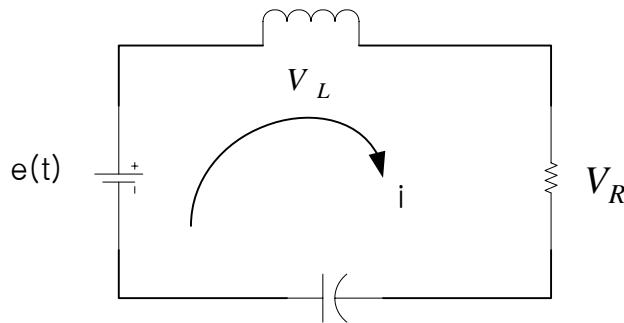


$$-V_L - V_R + e(t) = 0$$

$$V_L = L \frac{di}{dt}, \quad V_R = iR$$

$$L \frac{di}{dt} + Ri = e(t)$$

ex2) R-L-C Circuit



$$-V_L - V_R - V_C + e(t) = 0$$

$$V_L = L \frac{di}{dt}, \quad V_R = iR, \quad V_c = \frac{1}{C} \int i dt + V_C(t)$$

$$\frac{dV_c}{dt} = \frac{1}{C} i$$

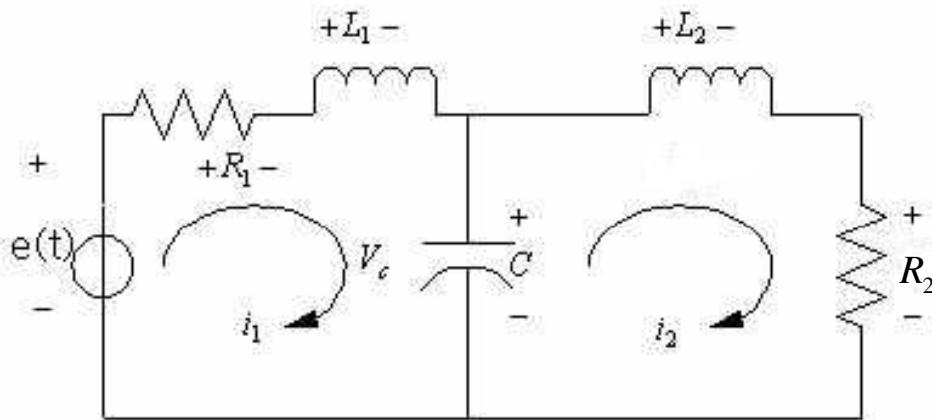
$$L \frac{di}{dt} + Ri = e(t) - V_C(t)$$



Examples of Circuit Analysis

ex3) Multiple electric circuit

By Kirchhoff's voltage law



$$-R_1 i_1 - L_1 \frac{di_1}{dt} - v_c + e(t) = 0$$

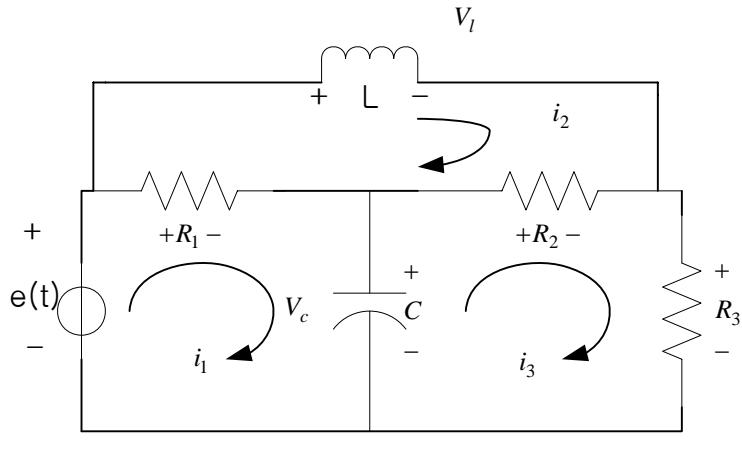
$$-L_2 \frac{di_2}{dt} - R_2 i_2 + v_c = 0$$

$$\begin{cases} \frac{dv_c}{dt} = \frac{1}{C}(i_1 - i_2) \\ \frac{di_1}{dt} = -\frac{1}{L_1}(v_c + R_1 i_1 - e(t)) \\ \frac{di_2}{dt} = \frac{1}{L_2}(-R_2 i_2 + v_c) \end{cases}$$



Examples of Circuit Analysis

ex4) Multiple electric circuit



$$v_L = L \frac{di}{dt}, \quad v_c = \frac{1}{c} \int i dt, \quad \frac{dv_c}{dt} = \frac{1}{c} i$$

$$-R_1(i_1 - i_2) - V_C + e(t) = 0 \quad 1)$$

$$-L \frac{di_2}{dt} - R_2(i_2 - i_3) - R_1(i_2 - i_1) = 0 \quad 2)$$

$$-R_2(i_3 - i_2) - R_3 i_3 + V_C = 0 \quad 3)$$

$$\frac{dv_c}{dt} = \frac{1}{c} (i_1 - i_3) \quad 4)$$

$$3) \rightarrow i_3 = \frac{1}{R_2 + R_3} (v_c + R_2 i_2),$$

$$1) \rightarrow i_1 = \frac{1}{R_1} (e(t) - v_c) + i_2$$

$$\begin{aligned} \frac{di_2}{dt} &= \frac{1}{L} \left[-(R_1 + R_2)i_2 + R_1 \cdot \frac{1}{R_1} (e(t) + v_c) + R_1 i_2 + R_2 \cdot \frac{1}{R_2 + R_3} (v_c + R_2 i_2) \right] \\ &= \frac{1}{L} \left(e(t) + v_c + \frac{R_2}{R_2 + R_3} v_c \right) = \frac{1}{L} \frac{2R_2 + R_3}{R_2 + R_3} v_c - \frac{R_2 R_3}{R_2 + R_3} i_2 + \frac{1}{L} e(t) \end{aligned}$$



Examples of Circuit Analysis

$$\begin{aligned}\frac{dv_c}{dt} &= \frac{1}{C} \left[\frac{1}{R_1} e(t) + \frac{1}{R_1} v_c + i_2 - \frac{1}{R_2 + R_3} v_c + \frac{R_2}{R_2 + R_3} i_2 \right] \\ &= \frac{1}{C} \frac{2R_2 + R_3}{R_2 + R_3} i_2 + \frac{1}{C} \left(\frac{1}{R_1} - \frac{1}{R_2 + R_3} \right) v_c + \frac{1}{CR_1} e(t)\end{aligned}$$

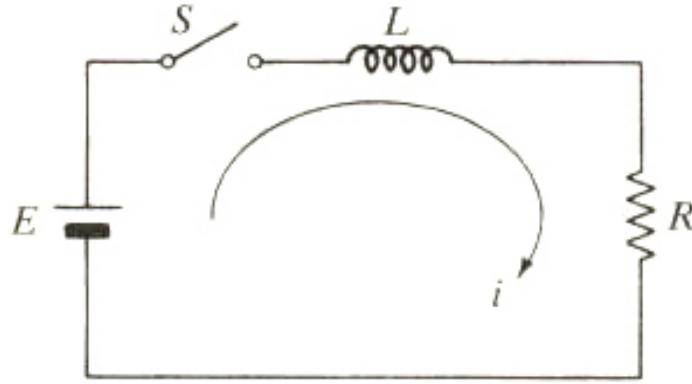
State space equation :

$$\frac{d}{dt} \begin{bmatrix} i_2 \\ v_c \end{bmatrix} = \begin{bmatrix} -\frac{R_2 R_3}{R_2 + R_3} & \frac{1}{L} \frac{2R_2 + R_3}{R_2 + R_3} \\ \frac{1}{C} \frac{2R_2 + R_3}{R_2 + R_3} & \frac{1}{C} \left(\frac{1}{R_1} - \frac{1}{R_2 + R_3} \right) \end{bmatrix} \begin{bmatrix} i_2 \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ \frac{1}{CR_1} \end{bmatrix} e(t)$$

$$y = v_0 = R_3 i_3 = \frac{R_3}{R_2 + R_3} R_2 i_2 + \frac{R_3}{R_2 + R_3} v_c = \begin{bmatrix} \frac{R_2 R_3}{R_2 + R_3} & \frac{R_3}{R_2 + R_3} \end{bmatrix} \begin{bmatrix} i_2 \\ v_c \end{bmatrix}$$



Mathematical Modeling of Electrical Systems



The switch S is closed at $t=0$

$$E - L \frac{di}{dt} - Ri = 0 \quad \text{or} \quad L \frac{di}{dt} + Ri = E$$

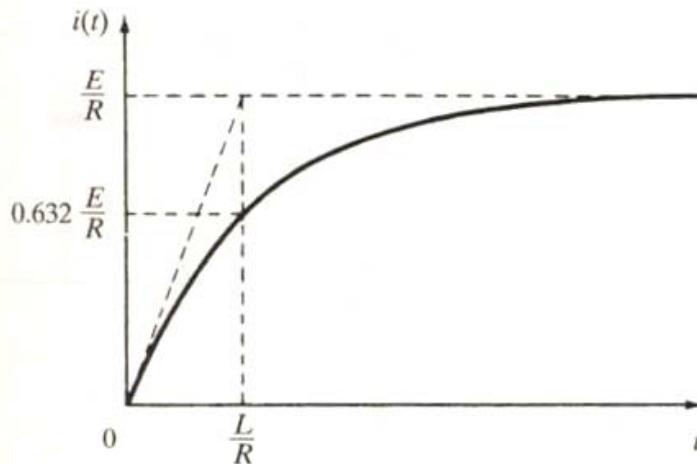
At the instant that switch S is closed,
the current $i(0) = 0$

Laplace Transformation : $L[sI(s) - i(0)] + RI(s) = \frac{E}{s}$

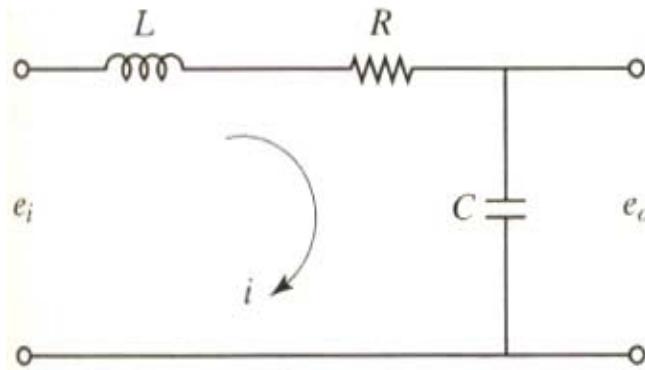
$$i(0) = 0 \quad \rightarrow \quad (Ls + R)I(s) = \frac{E}{s}$$

$$I(s) = \frac{E}{s(Ls + R)} = \frac{E}{R} \left[\frac{1}{s} - \frac{1}{s + (R/L)} \right]$$

$$\therefore i(t) = \frac{E}{R} \left[1 - e^{-(R/L)t} \right]$$



State-Space Mathematical Modeling of Electrical Systems



By Kirchhoff's voltage law

$$L \frac{di}{dt} + Ri + v_c = e_i, \quad \frac{dv_c}{dt} = \frac{1}{C} i, \quad e_o = v_c$$

Assume, initial condition is 0,

$$LsI(s) + RI(s) + V_c(s) = E_i(s), \quad sV_c(s) = \frac{1}{C} I(s)$$

$$T.F : \frac{E_o(s)}{E_i(s)} = \frac{1}{LCs^2 + RCs + 1}$$



State-Space Mathematical Modeling of Electrical Systems

Differential equation :

$$\ddot{e}_o + \frac{R}{L} \dot{e}_o + \frac{1}{LC} e_o = \frac{1}{LC} e_i$$

State variable :

$$x_1 = e_o, \quad x_2 = \dot{e}_o$$

Input and output :

$$u = e_i, \quad y = e_o = x_1$$

State-space equation :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix} u, \quad y = [1 \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

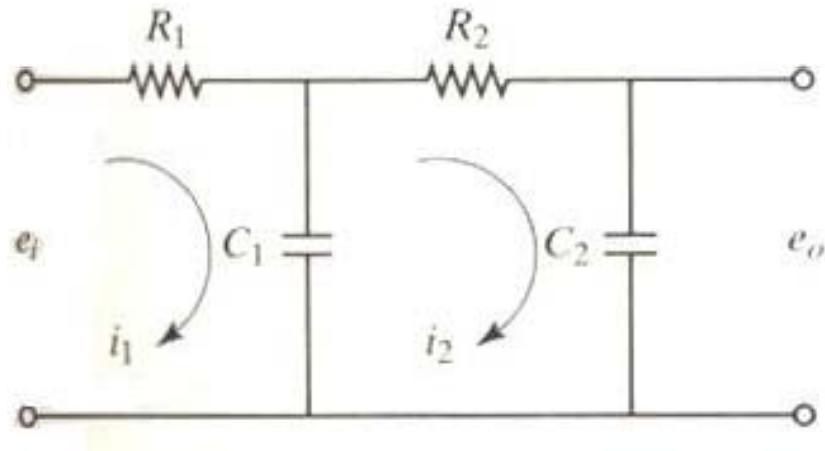


Transfer Function of Series Elements

$$\frac{1}{C_1} \int (i_1 - i_2) dt + R_1 i_1 = e_i$$

$$\frac{1}{C_1} \int (i_2 - i_1) dt + R_2 i_2 + \frac{1}{C_2} \int i_2 dt = 0$$

$$\frac{1}{C_2} \int i_2 dt = e_o$$



Laplace Transformation :

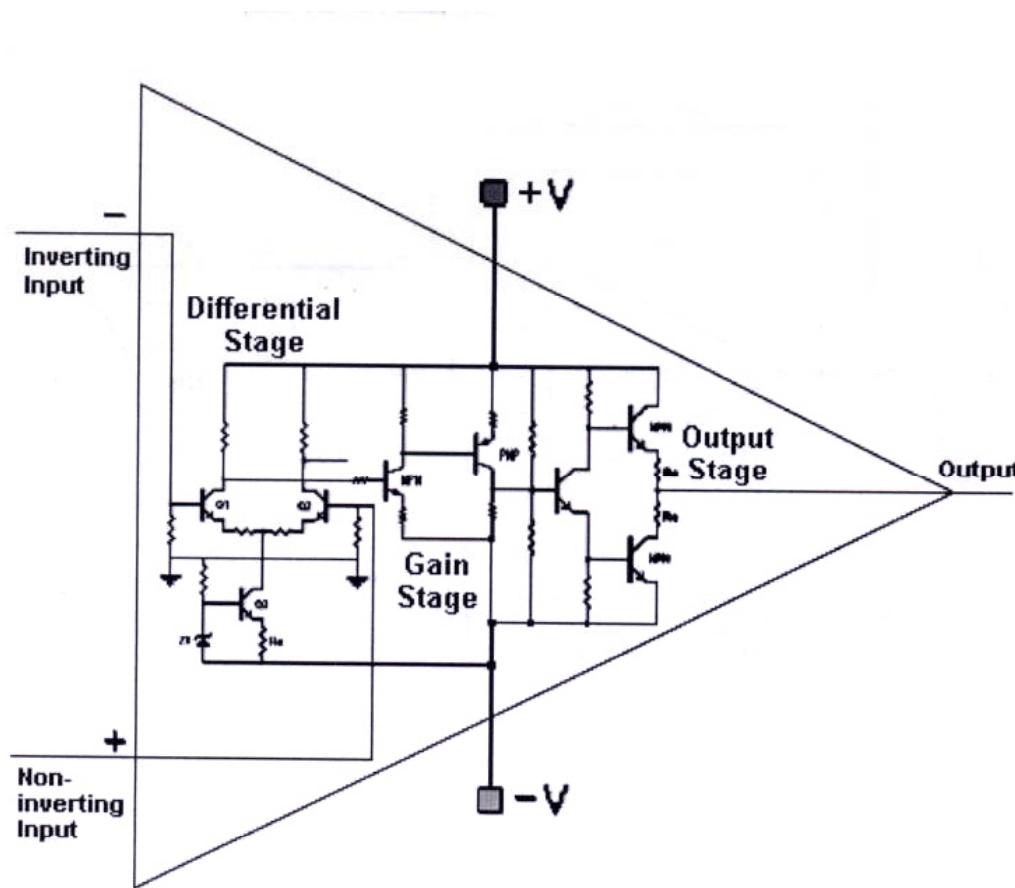
$$\frac{1}{C_1 s} [I_1(s) - I_2(s)] + R_1 I_1(s) = E_i(s), \quad \frac{1}{C_1 s} [I_2(s) - I_1(s)] + R_2 I_2(s) + \frac{1}{C_2 s} I_2(s) = 0, \quad \frac{1}{C_2 s} I_2(s) = E_o(s)$$

$$T.F : \frac{E_o(s)}{E_i(s)} = \frac{1}{(R_1 C_1 s + 1)(R_2 C_2 s + 1) + R_1 C_2 s} = \frac{1}{R_1 C_1 R_2 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2)s + 1}$$

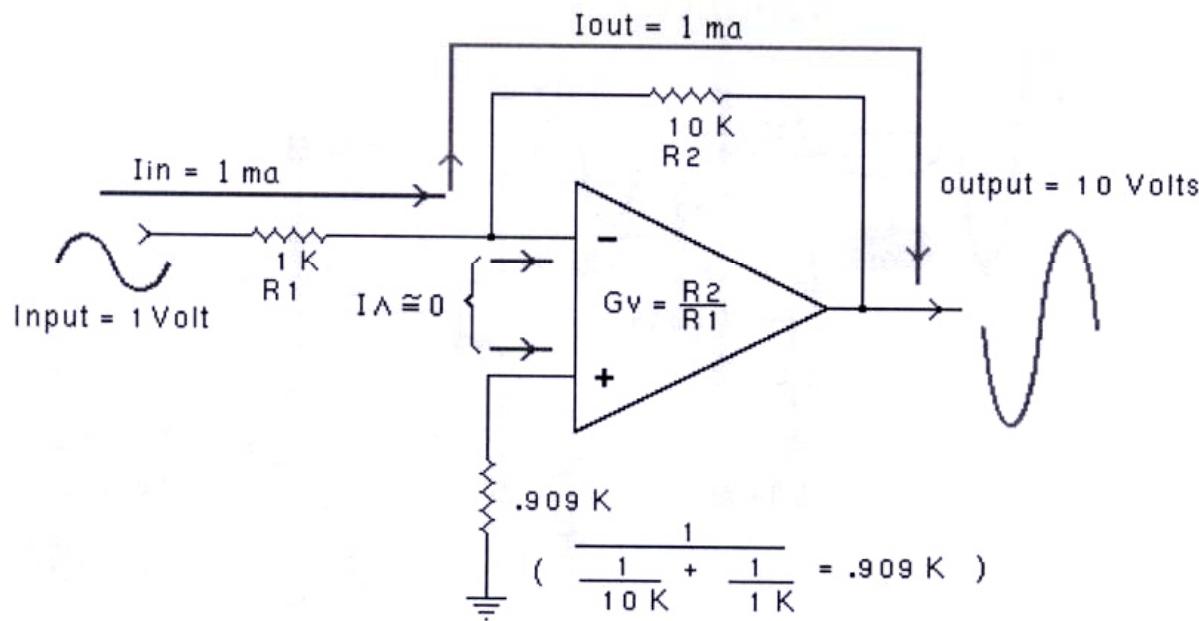
$R_1 C_2 s$ term means intersection of the two RC circuits



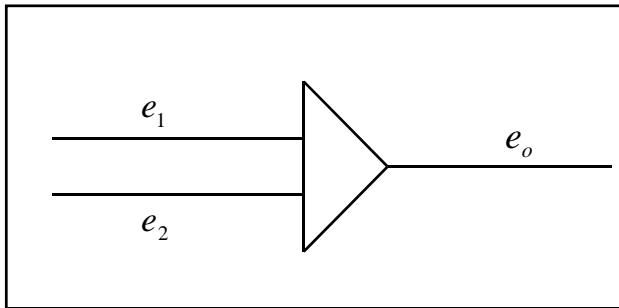
Operational Amplifiers



Operational Amplifiers



Operational Amplifiers



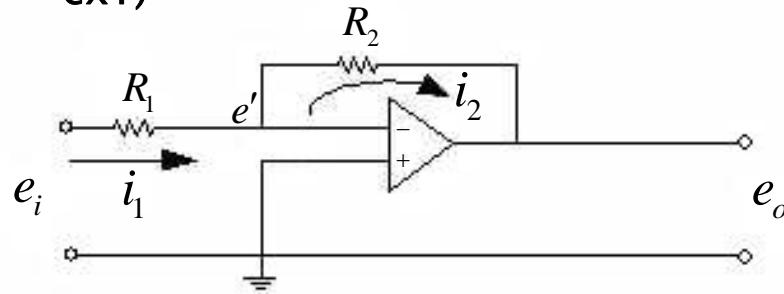
$$e_0 = K (e_2 - e_1)$$

- i) $K = 10^5 \sim 10^6$ for dc signals and ac of less than 10 Hz frequency.
- ii) $K = 1$ for ac 1 MHz ~ 50MHz
- iii) Ideal CP amps
 - $K = \infty$
 - no current flow into the input terminals
 - the output voltage is not affected by the load connected to the output terminal



Examples of Operational Amplifiers

ex1)



$$\text{i) } i_1 = \frac{e_i - e'}{R_1} \quad i_2 = \frac{e' - e_o}{R_2}$$

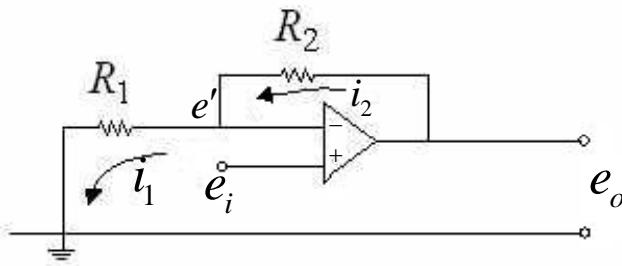
$$\text{ii) } i_{op} = 0, \quad i_1 = i_2, \quad \frac{e_i - e'}{R_1} = \frac{e' - e_o}{R_2}$$

$$\text{iii) since } e_o = K(0 - e'), \quad K \geq 1, \quad K \approx \infty \quad \Rightarrow \quad e' \approx 0$$

$$\frac{e_i}{R_1} = \frac{-e_o}{R_2} \quad \Rightarrow \quad e_o = -\frac{R_2}{R_1} e_i$$

ex2)

$$e' = e_i \quad (\because K \approx \infty)$$



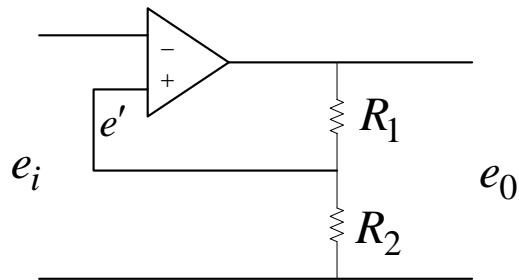
$$i_1 = i_2, \quad \frac{e_i}{R_1} = \frac{e_o - e_i}{R_2} \quad \Rightarrow \quad \left(\frac{1}{R_1} + \frac{1}{R_2} \right) e_i = \frac{1}{R_1} e_o$$

$$\Rightarrow e_o = \frac{R_1 + R_2}{R_1} e_i = \left(1 + \frac{R_2}{R_1} \right) e_i$$



Examples of Operational Amplifiers

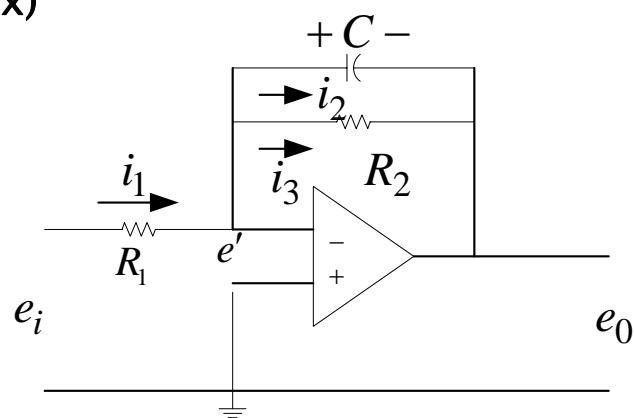
Equivalent circuit



$$e' = e_i \quad i = \frac{e_o - e_i}{R_2} = \frac{e_i}{R_1}$$

$$\Rightarrow e_o = \left(1 + \frac{R_2}{R_1}\right) e_i$$

ex)



$$e' = 0, \quad i_1 = i_2 + i_3$$

$$i_1 = \frac{e_i}{R_1}, \quad \frac{dv_c}{dt} = \frac{1}{C} i_2 = \left(-\frac{de_o}{dt} \right)$$

$$(v_c = e' - e_o = -e_o)$$

$$i_3 = \frac{e' - e_o}{R_2} = -\frac{e_o}{R_2}$$



Examples of Operational Amplifiers

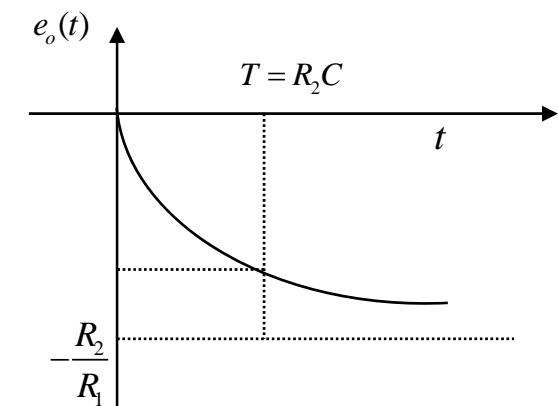
$$\Rightarrow \frac{e_i}{R_1} = C \frac{dv_c}{dt} - \frac{1}{R_2} e_o = C \frac{de_o}{dt} - \frac{1}{R_2} e_o \quad \Rightarrow \quad \frac{de_o}{dt} = -\frac{1}{CR_2} e_o - \frac{1}{CR_1} e_i$$

Laplace Transform : $E_o(s) \left(s + \frac{1}{CR_2} \right) = -\frac{1}{CR_1} E_i(s)$

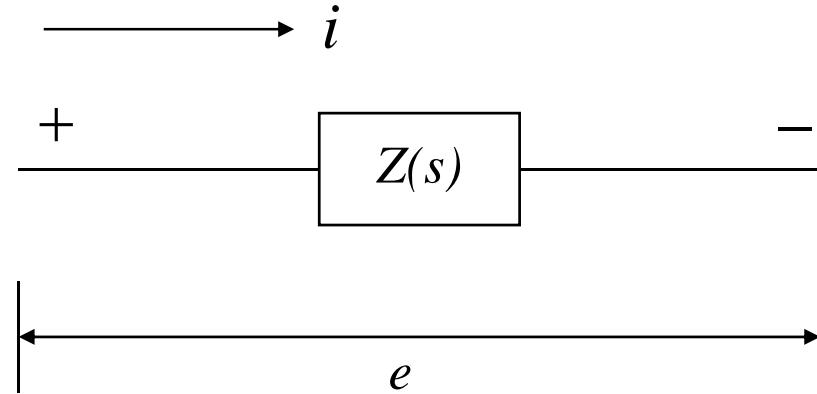
$$T.F = \frac{E_o(s)}{E_i(s)} = -\frac{1}{s + \frac{1}{CR_2}} \cdot \frac{1}{CR_1} = -\frac{R_2}{R_1} \cdot \frac{1}{R_2 C s + 1}$$

Step input response : $E_i(s) = \frac{1}{s}$, $e_i(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

$$\begin{aligned} E_o(s) &= -\frac{R_2}{R_1} \frac{1}{R_2 C s + 1} \cdot E_i(s) \\ &= -\frac{R_2}{R_1} \frac{1}{R_2 C s + 1} \cdot \frac{1}{s} = -\frac{R_2}{R_1} \left(\frac{1}{s} - \frac{1}{s + (1/R_2 C)} \right) \\ e_o(t) &= -\frac{R_2}{R_1} \left(1 - e^{(-1/R_2 C)t} \right) \end{aligned}$$



Complex Impedance



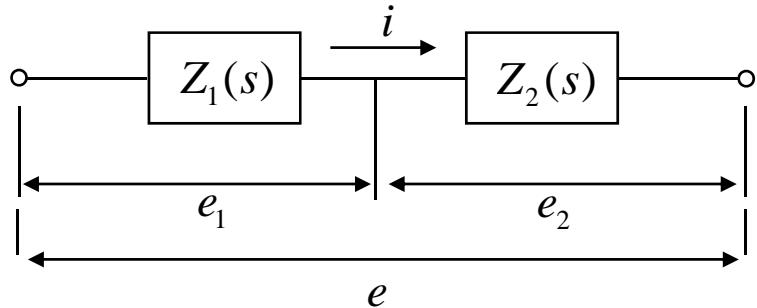
$$I(s) = \frac{E(s)}{Z(s)} \quad E(s) = Z(s)I(s)$$

$Z(s)$: complex impedance



Complex Impedance

The complex impedance $Z(s)$ of a two-terminal circuit is : the ratio of $E(s)$ to $I(s)$



$$Z(s) = \frac{E(s)}{I(s)}, \quad E(s) = Z(s)I(s)$$

$$E_1(s) = Z_1(s)I(s), \quad E_2(s) = Z_2(s)I(s)$$

$$E(s) = E_1(s) + E_2(s)$$

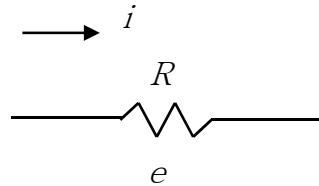
Direct derivation of transfer function,
without writing differential equations first.

$$\begin{aligned} &= Z_1(s)I(s) + Z_2(s)I(s) \\ &= (Z_1(s) + Z_2(s))I(s) \end{aligned}$$



Complex Impedance

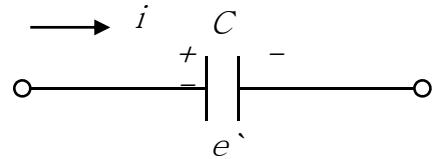
Resistance :



$$e = Ri, \quad E(s) = RI(s)$$

$$Z(s) = R$$

Capacitance :

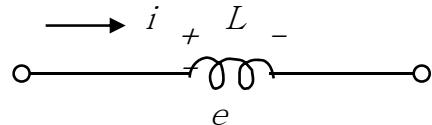


$$\frac{de}{dt} = \frac{1}{C}i$$

$$sE(s) = \frac{1}{C}I(s) \rightarrow E(s) = \frac{1}{Cs}I(s)$$

$$\therefore Z(s) = \frac{1}{Cs}$$

Inductance :



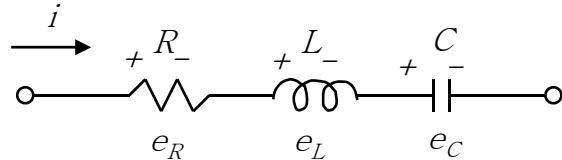
$$e = L \frac{di}{dt}, \quad E(s) = Ls I(s)$$

$$\therefore Z(s) = LS$$



Examples of Complex Impedance

ex1)

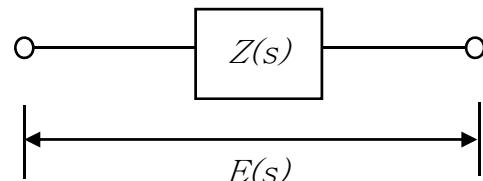


$$e_R = iR, \quad e_L = L \frac{di}{dt}, \quad \frac{de_C}{dt} = \frac{1}{C} i$$

$$e = e_R + e_L + e_C$$

$$\begin{aligned} E(s) &= E_R(s) + E_L(s) + E_C(s) \\ &= RI(s) + LsI(s) + \frac{1}{Cs} I(s) \end{aligned}$$

$$= \left(R + Ls + \frac{1}{Cs} \right) I(s)$$

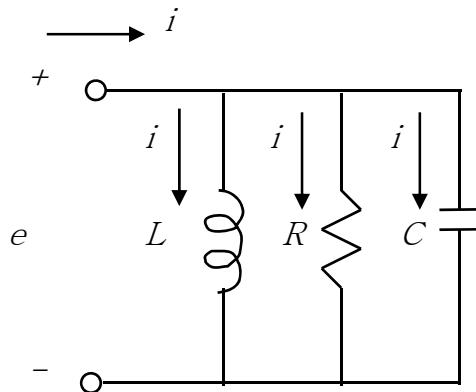


$$\therefore Z(s) = R + Ls + \frac{1}{Cs} = Z_R(s) + Z_L(s) + Z_C(s)$$



Examples of Complex Impedance

ex2)

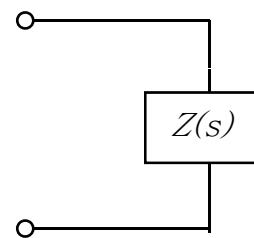


$$i = i_L + i_R + i_C \quad E(s) = Z(s)I(s)$$

$$\begin{aligned} I(s) &= I_L(s) + I_R(s) + I_C(s) \\ &= \frac{E(s)}{Z_L(s)} + \frac{E(s)}{Z_R(s)} + \frac{E(s)}{Z_C(s)} \\ &= \left(\frac{1}{Z_L(s)} + \frac{1}{Z_R(s)} + \frac{1}{Z_C(s)} \right) E(s) \end{aligned}$$

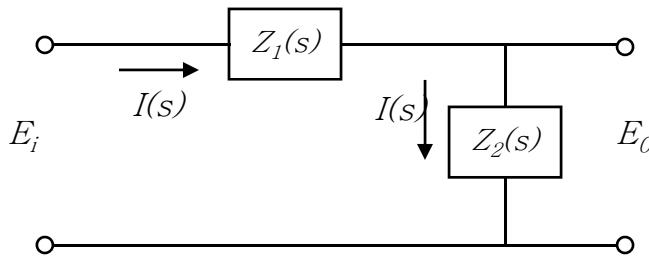
$$= \frac{1}{Z(s)} E(s)$$

$$\therefore Z(s) = \frac{1}{\frac{1}{Z_R(s)} + \frac{1}{Z_L(s)} + \frac{1}{Z_C(s)}} = \frac{1}{\frac{1}{Ls} + \frac{1}{R} + Cs}$$



Examples of Complex Impedance

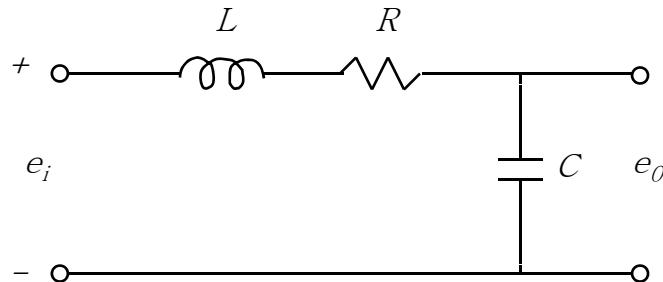
Deriving transfer functions of Electrical circuits by the use of complex impedances.



$$E_i(s) = Z_1(s)I(s) + Z_2(s)I(s), \quad E_o(s) = Z_2(s)I(s)$$

$$\frac{E_o(s)}{E_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

ex)



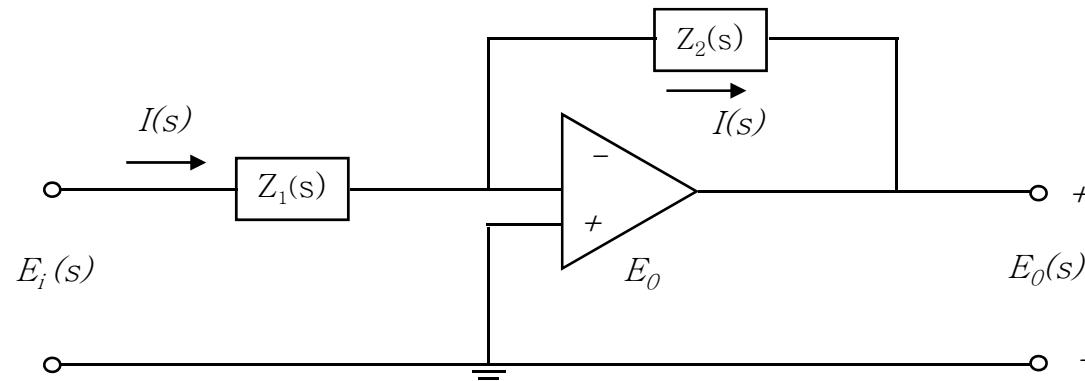
$$\frac{E_o(s)}{E_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

$$Z_1(s) = Ls + R, \quad Z_2(s) = \frac{1}{Cs}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{\frac{1}{Cs}}{Ls + R + \frac{1}{Cs}} = \frac{1}{LCs^2 + RCs + 1}$$



Examples of Complex Impedance



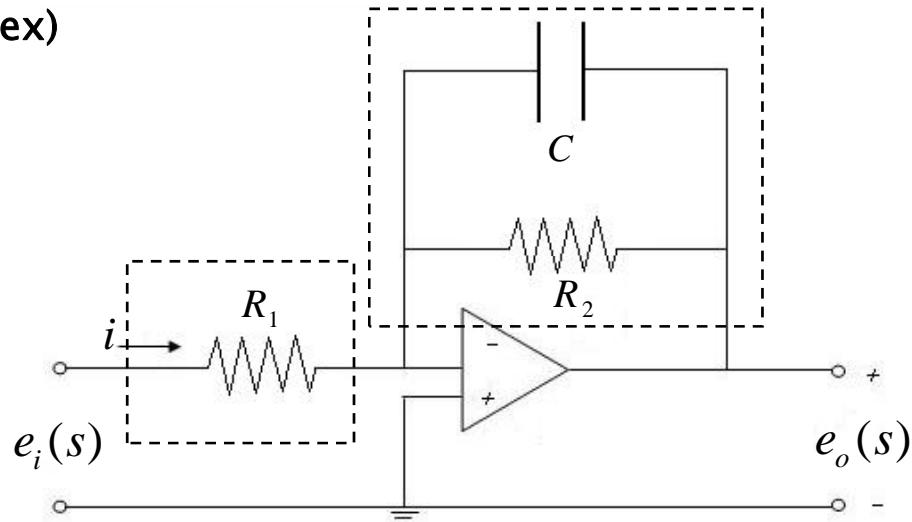
$$E_i(s) = Z_1(s)I(s), \quad E_o(s) = Z_2(s)I(s)$$

$$\frac{E_o(s)}{E_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$



Examples of Complex Impedance

ex)



$$Z_1(s) = R_1$$

$$Z_2(s) = \frac{1}{Cs + \frac{1}{R_2}} = \frac{R_2}{R_2Cs + 1}$$

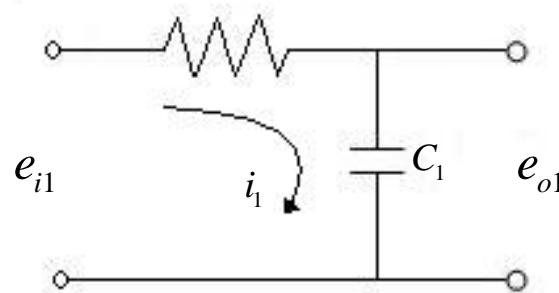
$$I(s) = Cs \cdot E(s) + \frac{1}{R_2} E(s)$$

$$\frac{E_o(s)}{E_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{R_2}{R_1} \frac{1}{R_2Cs + 1}$$

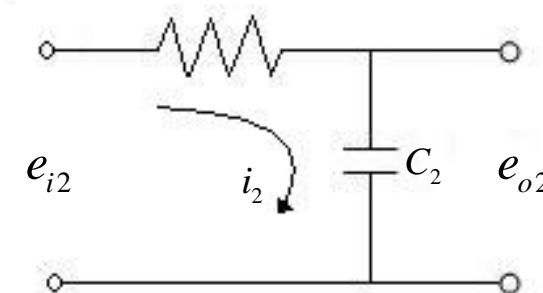


Transfer Functions of Non-loading Cascade Elements

Consider two RC circuits



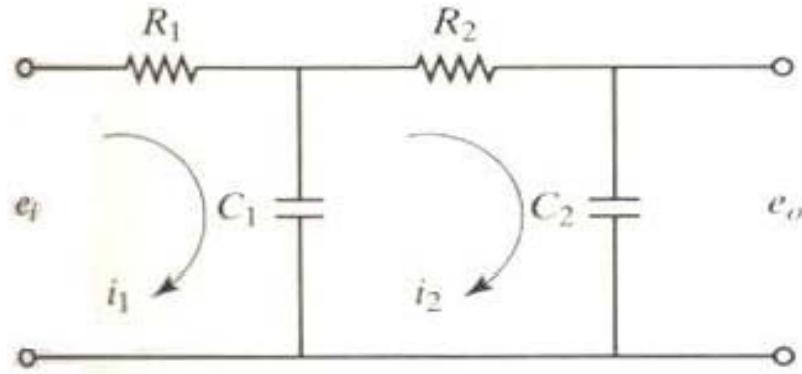
$$\frac{E_{o1}(s)}{E_{i1}(s)} = \frac{1}{R_1 C_1 s + 1} = G_1(s)$$



$$\frac{E_{o2}(s)}{E_{i2}(s)} = \frac{1}{R_2 C_2 s + 1} = G_2(s)$$



Transfer Functions of Non-loading Cascade Elements



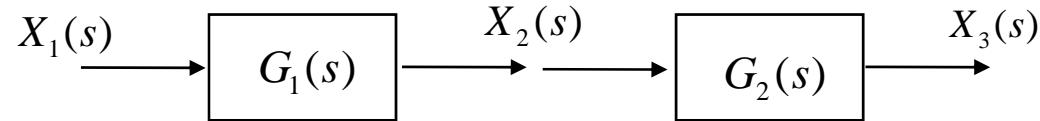
$$\frac{1}{C_1 s} [I_1(s) - I_2(s)] + R_1 I_1(s) = E_i(s), \quad \frac{1}{C_1 s} [I_2(s) - I_1(s)] + R_2 I_2(s) + \frac{1}{C_2 s} I_2(s) = 0, \quad \frac{1}{C_2 s} I_2(s) = E_o(s)$$

$$T.F : \frac{E_o(s)}{E_i(s)} = \frac{1}{(R_1 C_1 s + 1)(R_2 C_2 s + 1) + R_1 C_2 s} \neq \frac{1}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}$$

$$\therefore \frac{E_o(s)}{E_i(s)} \neq \frac{E_{o1}(s)}{E_{i1}(s)} \cdot \frac{E_{o2}(s)}{E_{i2}(s)} \quad \longrightarrow \text{Loading effect}$$



Transfer Functions of Non-loading Cascade Elements



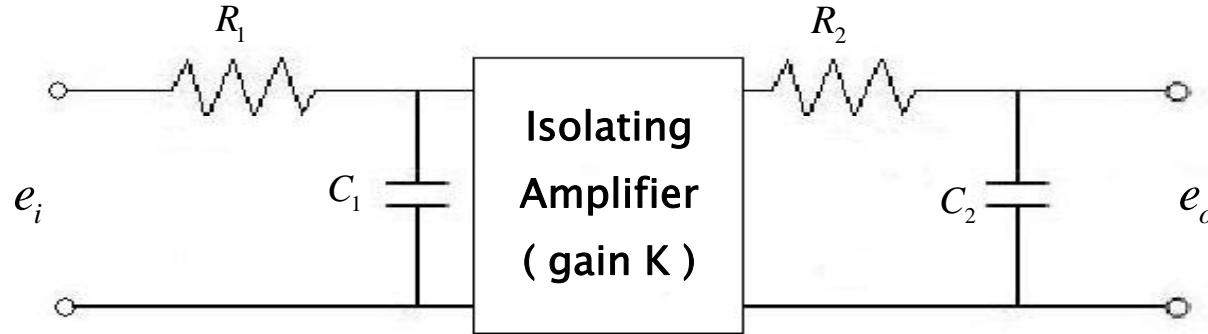
$$G(s) = \frac{X_3(s)}{X_1(s)} = \frac{X_2(s)}{X_1(s)} \cdot \frac{X_3(s)}{X_2(s)} = G_1(s)G_2(s)$$

If the "input Impedance" of the second element is infinite, the output of the first element is not affected by connecting it to the second element.

Then, $G(s) = G_1(s)G_2(s)$



Transfer Functions of Non-loading Cascade Elements



This circuit has a very high input impedance

$$T.F : \frac{E_o(s)}{E_i(s)} = \left(\frac{1}{R_1 C_1 s + 1} \right) \cdot K \cdot \left(\frac{1}{R_2 C_2 s + 1} \right) = \frac{K}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}$$

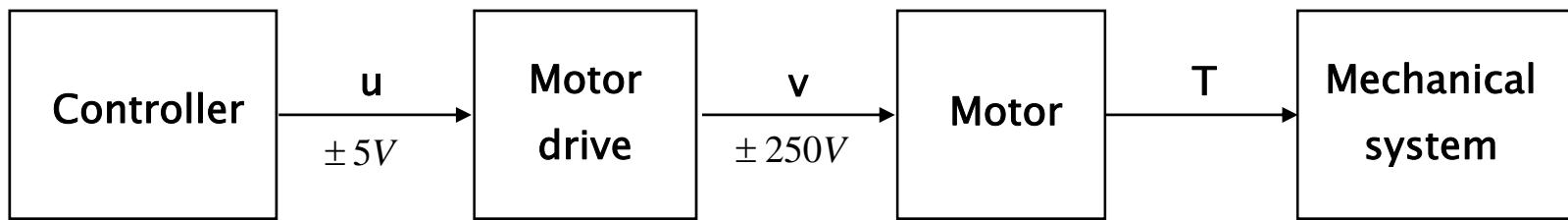
When we derive the T.F. for an isolated circuit, we implicitly assume that the output is "unloaded ". In other word, the load impedance is assumed to be Infinite.

→ No power is being withdrawn at the output.

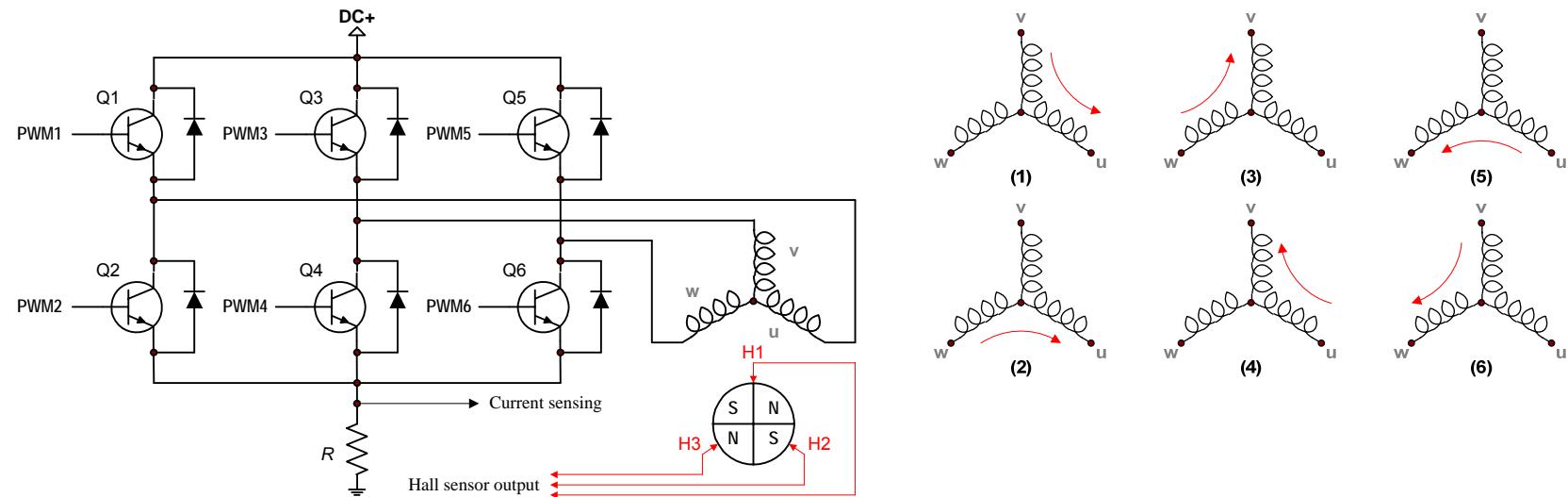


Constitution of DC Servomotor System

Motor drive system :

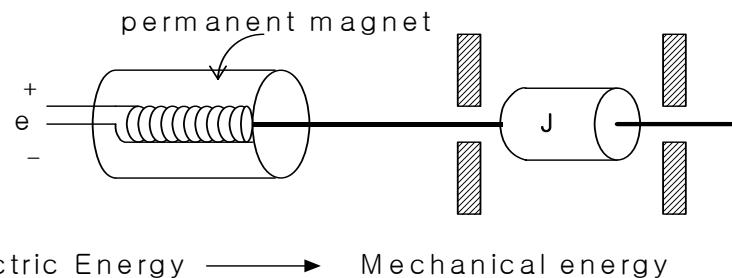


3 phase BLDC motor driver :

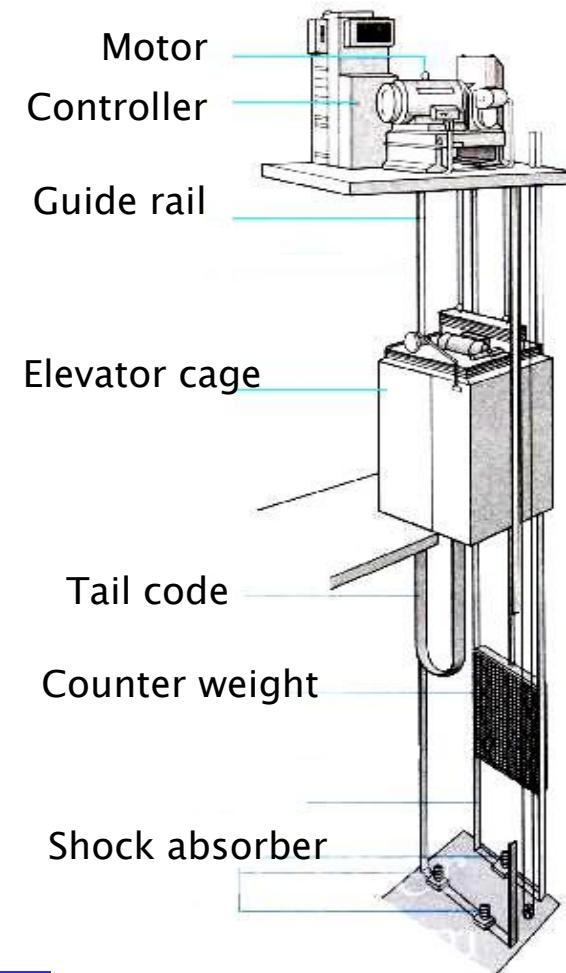


Constitution of DC Servomotor System

DC servo motor :



ex) Elevator structure :



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Spring 2008

Armature Control of DC Servomotors

Variables :

R_a : armature resistance, Ω

L_a : armature inductance, H

i_a : armature current, A

i_f : field current, A

e_a : applied armature voltage, V

e_b : back emf, V

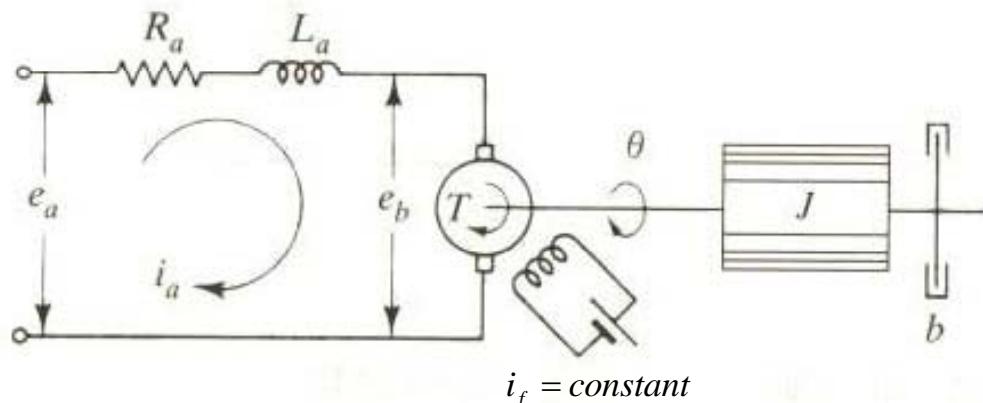
θ : angular displacement of the motor shaft, rad

T : torque developed by the motor, N- m

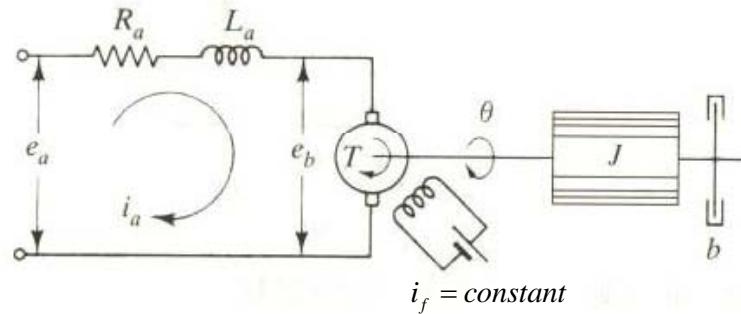
J : equivalent moment of inertia of the motor

and load referred to the motor shaft, $kg\cdot m^2$

b : equivalent viscous-friction coefficient of the motor
and load referred to the motor shaft, $N\cdot m/rad/s$



Armature Control of DC Servomotors



The torque of motor : $T = K i_a$ K : motor-torque constant

For a constant flux, the induced voltage : $e_b = K_b \frac{d\theta}{dt}$ K_b : back emf constant

Armature circuit D.E : $L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$

Inertia and friction : $J \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} = T = K i_a$



Armature Control of DC Servomotors

Laplace transforms of equations :

$$e_b = K_b \frac{d\theta}{dt}$$

$$K_b s \Theta(s) = E_b(s)$$

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$$



$$(L_a s + R_a) I_a(s) + E_b(s) = E_a(s)$$

$$J \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} = T = K i_a$$

$$(J s^2 + b s) \Theta(s) = T(s) = K I_a(s)$$

$$T.F = \frac{\Theta(s)}{E_a(s)} = \frac{K}{s(R_a J s + R_a b + K K_b)} = \frac{\frac{K}{R_a J}}{s \left(s + \frac{R_a b + K K_b}{R_a J} \right)}$$

$$= \frac{K_m}{s(T_m s + 1)}$$

$$K_m = K / (R_a b + K K_b) = \text{motor gain constant}$$

$$T_m = R_a J / (R_a b + K K_b) = \text{motor time constant}$$



Example of a DC Servomotor System

ex) servo-motor system

R_a : armature resistance, Ω

i_a : armature current, A

i_f : field current, A

e_a : applied armature voltage, V

e_b : back emf, V

θ_1 : angular displacement of the motor shaft, rad

θ_2 : angular displacement of the load shaft, rad

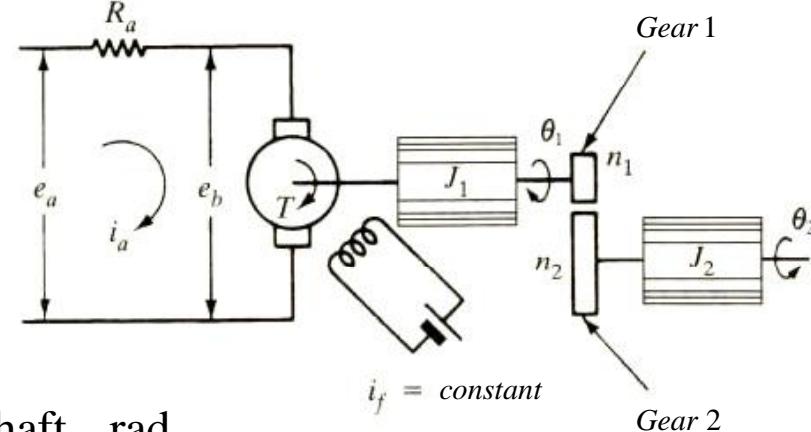
T : torque developed by the motor, N·m

J_1 : equivalent moment of inertia of the motor, $\text{kg}\cdot\text{m}^2$

J_2 : equivalent moment of inertia of the load, $\text{kg}\cdot\text{m}^2$

The torque of motor : $T = K i_a$

For a constant flux, the induced voltage : $e_b = K_b \frac{d\theta}{dt}$ K_b : back emf constant



Example of a DC Servomotor System

Armature circuit D.E : $R_a i_a + e_b = e_a$ Inertia and friction : $J_{1eq} = J_1 + \left(\frac{n_1}{n_2} \right)^2 J_2$

Laplace transforms of these equations :

$$K_b s \Theta(s) = E_b(s), \quad (L_a s + R_a) I_a(s) + E_b(s) = E_a(s), \quad (J s^2 + b s) \Theta(s) = T(s) = K I_a(s)$$

$$\begin{aligned} T.F &= \frac{\Theta(s)}{E_a(s)} = \frac{K}{s(R_a J s + R_a b + K K_b)} = \frac{\frac{K}{R_a J}}{s \left(s + \frac{R_a b + K K_b}{R_a J} \right)} \\ &= \frac{K_m}{s(T_m s + 1)} \end{aligned}$$

