

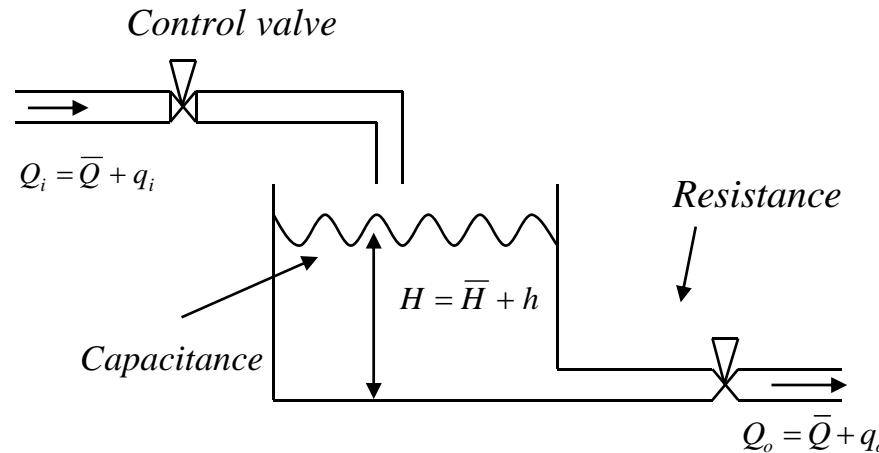
Fluid Systems and Thermal Systems



**Seoul National Univ.
School of Mechanical
and Aerospace Engineering**

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Liquid Level Systems



$$R = \frac{\text{change in level difference}}{\text{change in flow rate}} \frac{m}{m^3/s}$$

$$C = \frac{\text{change in liquid stored}}{\text{change in head}} \frac{m^3}{m}$$

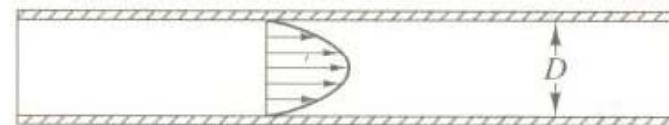
Steady state : $Q_i = \bar{Q} = Q_o = \frac{\bar{H}}{R}$

consider, $Q_o = \frac{H}{R}, \quad Q_i - Q_o = C \frac{dH}{dt}$

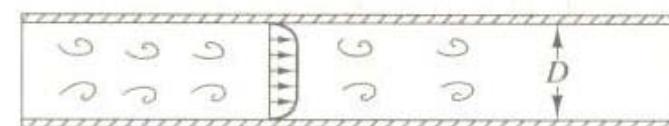


Basic Concepts

Laminar flow : When the viscous forces are dominant (slow flow, low Re) they are sufficient enough to keep all the fluid particles in line, then the flow is laminar.



Turbulent flow : It can be interpreted that when the inertial forces dominate over the viscous forces (when the fluid is flowing faster and Re is larger) then the flow is turbulent.



Basic Concepts

Reynolds number (Re) : The ratio of inertial forces to viscous forces.

It is used to determine whether a flow will be laminar or turbulent.

$$Re = \frac{\rho v D}{\mu}$$

μ : the dynamic viscosity of the fluid

ρ : density of the fluid

D : diameter

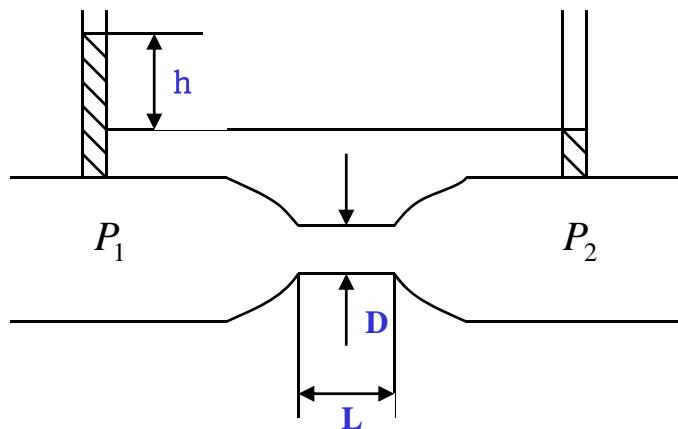
$Re < 2000$: always laminar

$Re > 4000$: always turbulent



Laminar Flow

Cylindrical pipe



$$P_1 - P_2 = \rho gh, \quad Q \frac{128vL}{g\pi D^4} = h$$

v : viscosity,

L : length of pipe

D : diameter of pipe

$$Q = \frac{h}{R} = K_l h, \quad R = \frac{128vL}{g\pi D^4} \quad [s/m^2]$$

Q : steady-state flow rate

K_l : constant

h : steady-state head



Laminar Flow

Liquid level dynamics :

$$C \frac{dH}{dt} = Q_i - Q_o$$

$$= \bar{Q} + q_i - (\bar{Q} + q_o) = q_i - q_o$$

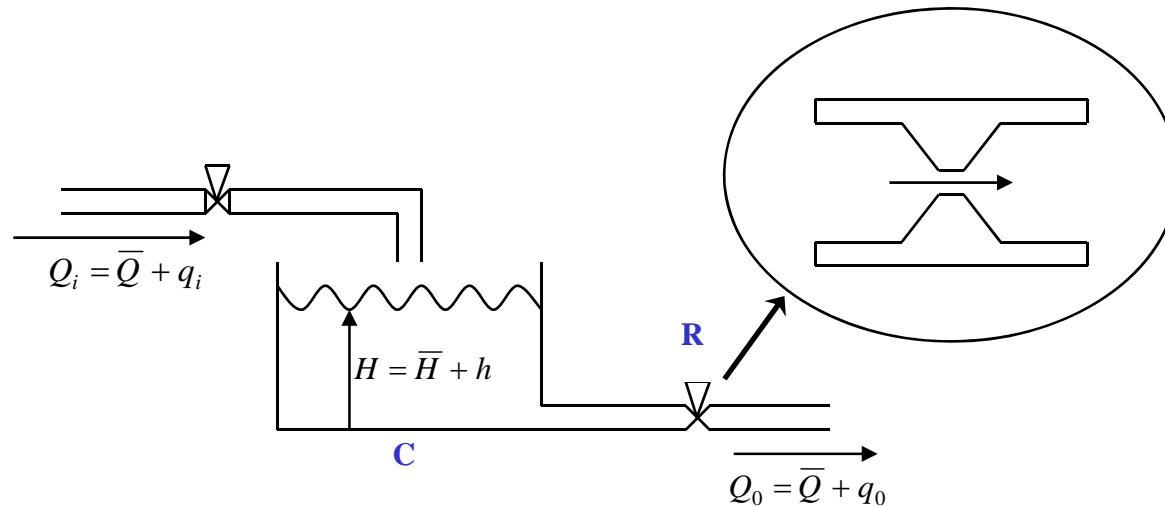
$$\frac{dH}{dt} = \frac{d}{dt}(\bar{H} + h) = \frac{dh}{dt}$$

$$Q_o = \bar{Q} + q_o = \frac{\bar{H} + h}{R}, \quad q_o = \frac{h}{R}$$

$$\rightarrow \frac{dh}{dt} = -\frac{1}{CR}h + \frac{1}{C}q_i$$



Turbulent Flow



$$Q = C_d \cdot a \cdot \sqrt{\frac{2}{\rho} (P_1 - P_2)}$$

ρ : density, a : area, C_d : discharge coefficient

$$\text{steady state: } Q_i = \bar{Q} = Q_0 = K_t \sqrt{\bar{H}}$$



Turbulent Flow

Liquid level dynamics

$$C \frac{dH}{dt} = Q_i - Q_0 = \bar{Q} + q_i - K\sqrt{H}$$

$$\frac{dH}{dt} = \frac{1}{C}Q_i - \frac{1}{C}K\sqrt{H} = f(Q_i, H)$$

$$f(Q_i, H) = f(\bar{Q}_i, \bar{H}) + \left. \frac{\partial f}{\partial Q_i} \right|_{\bar{Q}, \bar{H}} (\bar{Q}_i - \bar{Q}) + \frac{1}{2!} \left. \frac{\partial^2 f}{\partial Q_i^2} \right|_{\bar{Q}, \bar{H}} (\bar{Q}_i - \bar{Q})^2 + \dots$$

$$\dots + \left. \frac{\partial f}{\partial H} \right|_{\bar{Q}, \bar{H}} (H - \bar{H}) + \frac{1}{2!} \left. \frac{\partial^2 f}{\partial H^2} \right|_{\bar{Q}, \bar{H}} (H - \bar{H})^2 + \dots$$

$$= \frac{1}{C}\bar{Q} - \frac{1}{C}K\sqrt{\bar{H}} + \left(\frac{1}{C}q_i - \frac{K}{C \cdot 2\sqrt{\bar{H}}} h \right) + \text{high order term}$$



Turbulent Flow

$$\frac{dH}{dt} = \frac{d\bar{H}}{dt} + \frac{dh}{dt}$$

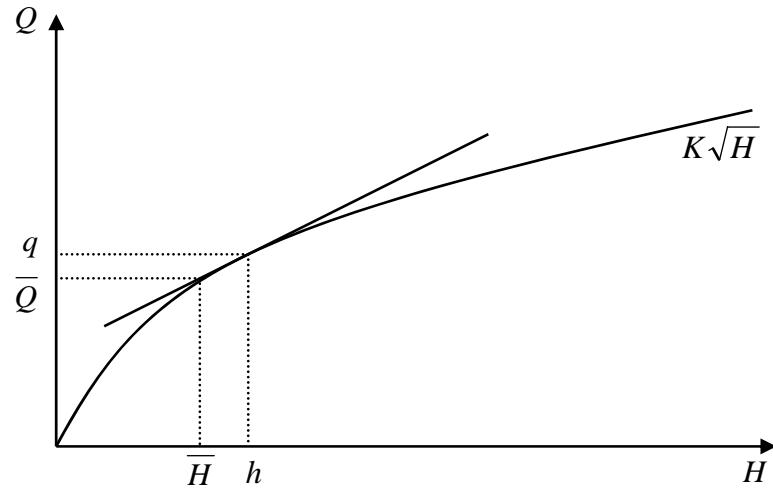
$$\Rightarrow \frac{dh}{dt} = \frac{1}{C} q_i - \frac{K}{C \cdot 2\sqrt{\bar{H}}} h$$

$$q_o = \frac{h}{R}, \quad R = \frac{2\sqrt{\bar{H}}}{K} = \frac{2\bar{H}}{\bar{Q}}, \quad (\bar{Q} = K\sqrt{\bar{H}})$$

$$\therefore \begin{cases} \frac{dh}{dt} = -\frac{1}{CR} h + \frac{1}{C} q_i \\ q_o = \frac{h}{R} \end{cases} \quad \text{for small } q_i$$



Linearization



$$Q = \bar{Q} + q = K\sqrt{\bar{H}} + (\quad) \cdot h$$

$$y = f(x), \quad \bar{y} = f(\bar{x}), \quad x = \bar{x} + \Delta x$$

$$y = f(\bar{x}) + \left. \frac{\partial f}{\partial x} \right|_{\bar{x}} \cdot \Delta x + \frac{1}{2!} \left. \frac{\partial^2 f}{\partial x^2} \right|_{\bar{x}} \cdot \Delta x^2 + \dots$$

$$\approx \bar{y} + \left. \frac{\partial f}{\partial x} \right|_{\bar{x}} \cdot \Delta x$$

$$y - \bar{y} = \left. \frac{\partial f}{\partial x} \right|_{\bar{x}} (x - \bar{x}) = K(x - \bar{x})$$

$$y = f(x_1, x_2) \approx f(\bar{x}_1, \bar{x}_2) + \left. \frac{\partial f}{\partial x_1} \right|_{\bar{x}_1, \bar{x}_2} (x_1 - \bar{x}_1) + \left. \frac{\partial f}{\partial x_2} \right|_{\bar{x}_1, \bar{x}_2} (x_2 - \bar{x}_2)$$

$$= \bar{y} + K_1(x_1 - \bar{x}_1) + K_2(x_2 - \bar{x}_2)$$

$$\therefore y - \bar{y} = K_1(x_1 - \bar{x}_1) + K_2(x_2 - \bar{x}_2)$$



Summary of Liquid Level Systems : Laminar Flow

Laminar flow

$$\frac{dh}{dt} = -\frac{1}{CR}h + \frac{1}{C}q_i$$

$$\bar{Q} = K \cdot \bar{H} \quad R = \frac{\bar{H}}{\bar{Q}}$$

$$R = \frac{128\nu L}{g\pi D^4}$$

$$q_o = \frac{h}{R}$$



Summary of Liquid Level Systems : Turbulent Flow

Turbulent flow

$$\frac{dh}{dt} = -\frac{1}{CR} h + \frac{1}{C} q_i$$

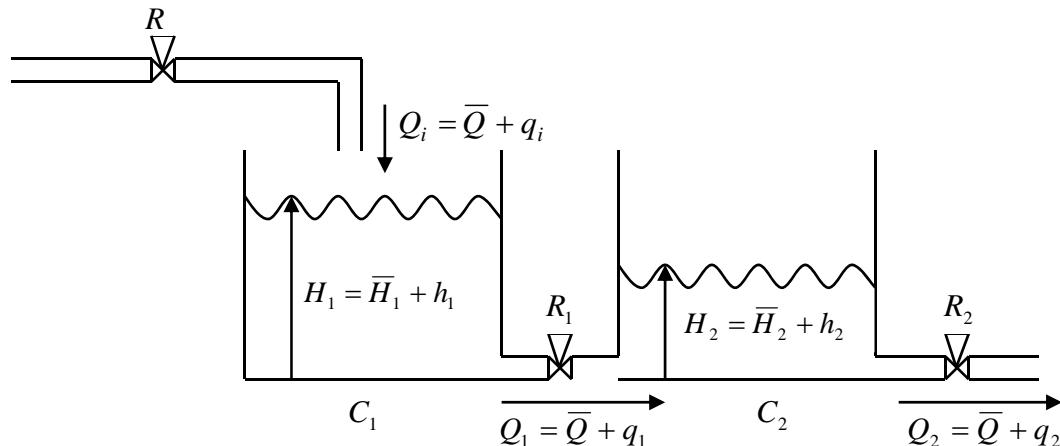
$$q_o = \frac{h}{R}$$

$$R = \frac{2\bar{H}}{\bar{Q}} = \frac{2\sqrt{\bar{H}}}{K} = \frac{2\sqrt{\bar{H}}}{C_d a \sqrt{2g}}$$

$$\left(\because \bar{Q} = K\sqrt{\bar{H}} = C_d a \sqrt{\frac{2}{\rho} \rho g \bar{H}} = C_d a \sqrt{2g} \sqrt{\bar{H}} \right)$$



Liquid Level Systems with Interaction



Steady state :

$$Q_i = \bar{Q} = Q_1 = Q_2 \\ = \frac{\bar{H}_1 - \bar{H}_2}{R_1} = \frac{\bar{H}_2}{R_2}$$

Liquid level dynamics :

$$C_1 \frac{dH_1}{dt} = \bar{Q} + q_i - (\bar{Q} + q_1) \rightarrow C_1 \frac{dh_1}{dt} = q_i - q_1$$

$$Q_1 = \bar{Q} + q_1 = \frac{1}{R_1} (\bar{H}_1 + h_1 - (\bar{H}_2 + h_2)) \rightarrow q_1 = \frac{h_1 - h_2}{R_1}$$

$$C_2 \frac{dH_2}{dt} = \bar{Q} + q_1 - (\bar{Q} + q_2) \rightarrow C_2 \frac{dh_2}{dt} = q_1 - q_2$$

$$Q_2 = \bar{Q} + q_2 = \frac{\bar{H}_2 + h_2}{R_2} \rightarrow q_2 = \frac{h_2}{R_2}$$



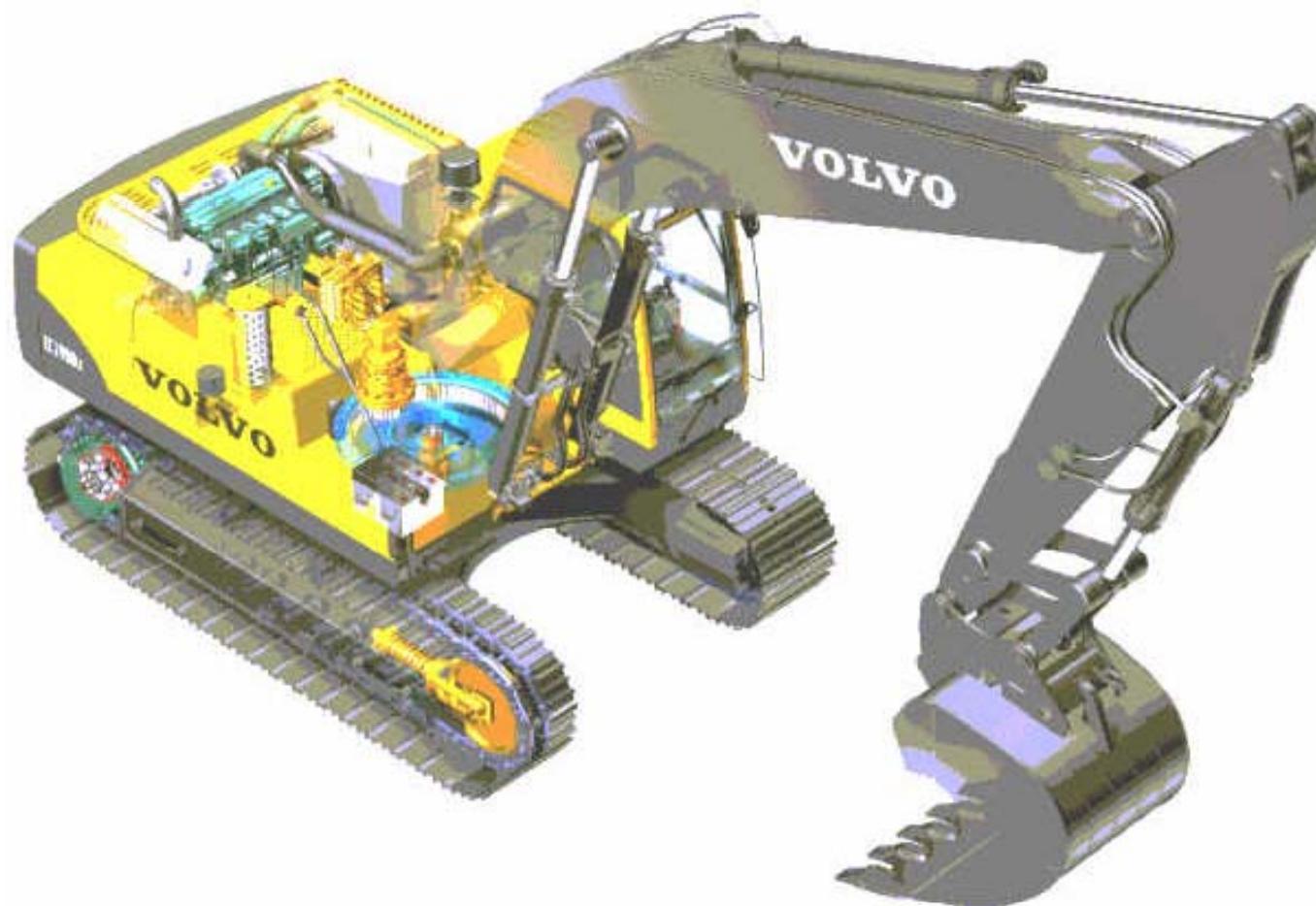
Liquid Level Systems with Interaction

$$\therefore \begin{cases} \frac{dh_1}{dt} = -\frac{1}{C_1} \left(\frac{h_1 - h_2}{R_1} \right) + \frac{1}{C_1} q_i \\ \frac{dh_2}{dt} = \frac{1}{C_2} \left(\frac{h_1 - h_2}{R_1} \right) - \frac{1}{C_2} \frac{h_2}{R_2} \end{cases}$$

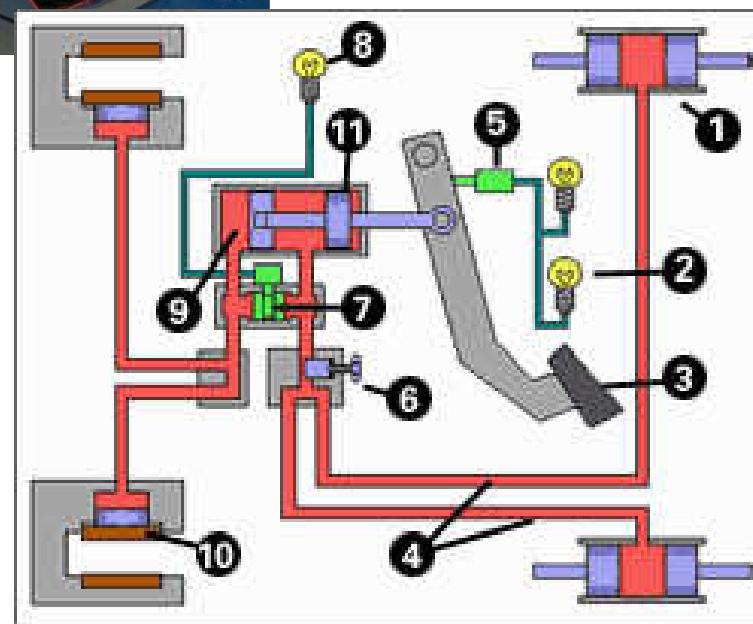
$$Transfer\ Function = \frac{Q_2(S)}{Q_i(S)} = \frac{1}{R_1 C_1 R_2 C_2 S^2 + (R_1 C_1 + R_2 C_2 + R_2 C_1) S + 1}$$



Hydraulic Systems : Excavator

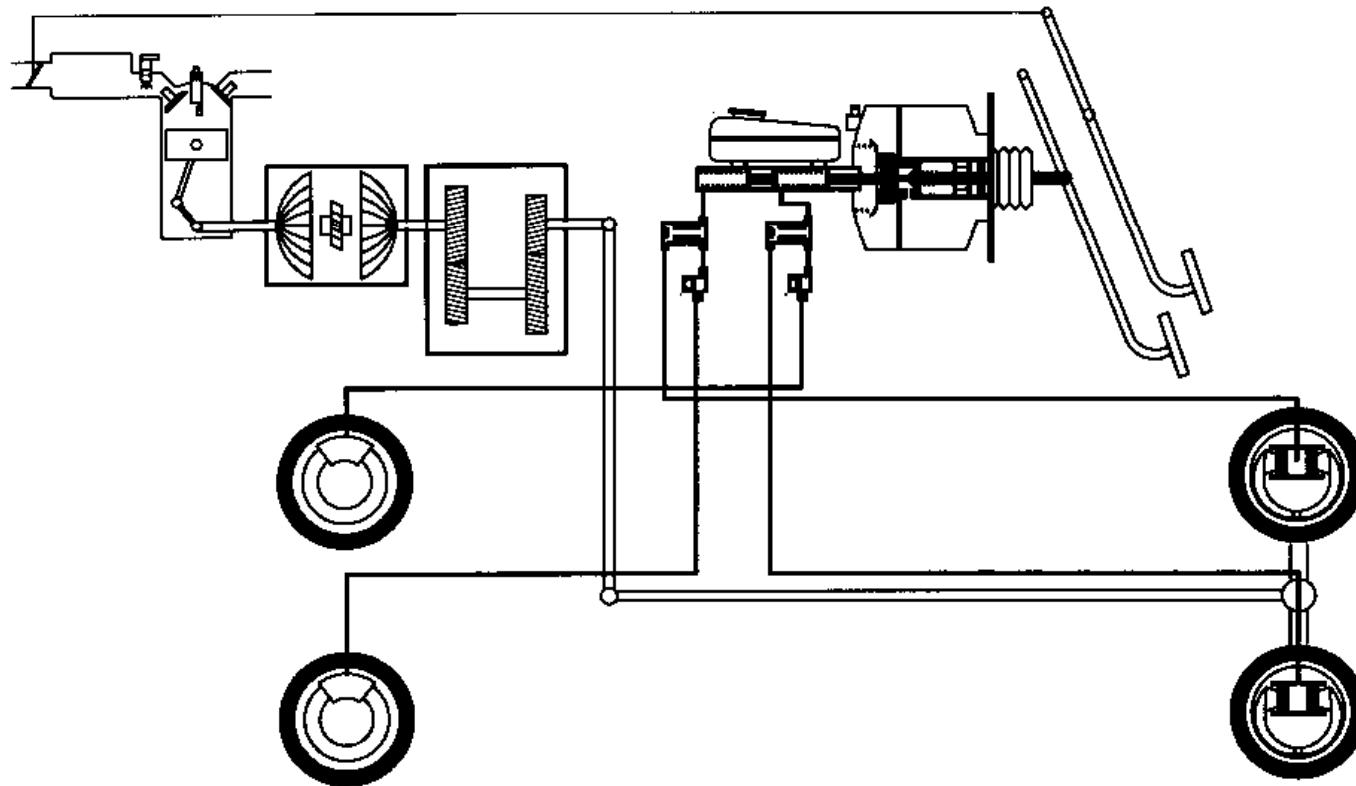


Hydraulic Systems : Brake System

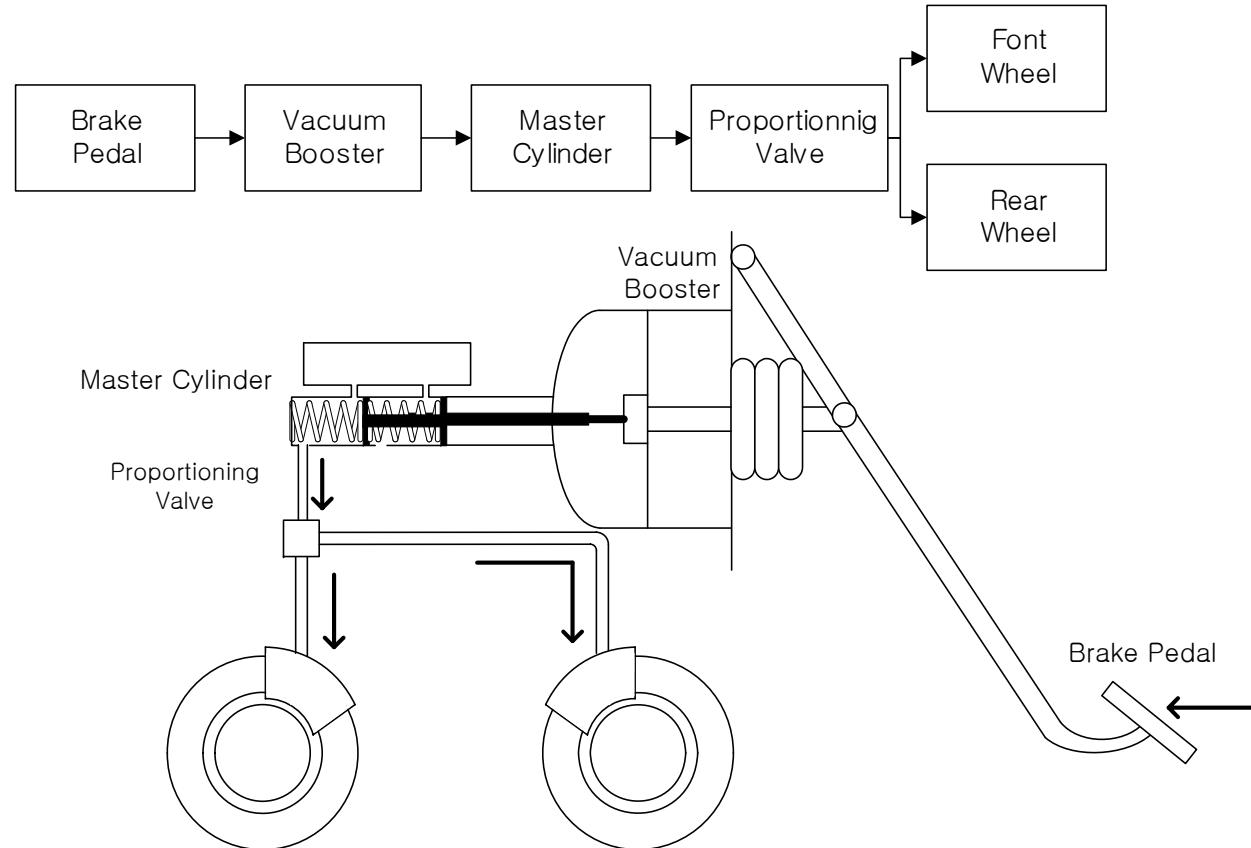


- 1) Wheel Cylinder
- 2) Brake Light
- 3) Brake Pedal
- 4) Rear Brake Lines
- 5) Stop Light Switch (Mechanical)
- 6) Front/Rear Balance Valve
- 7) Pressure Differential Valve
- 8) Brake Warning Lamp
- 9) Brake Fluid
- 10) Brake Pad
- 11) Master Cylinder





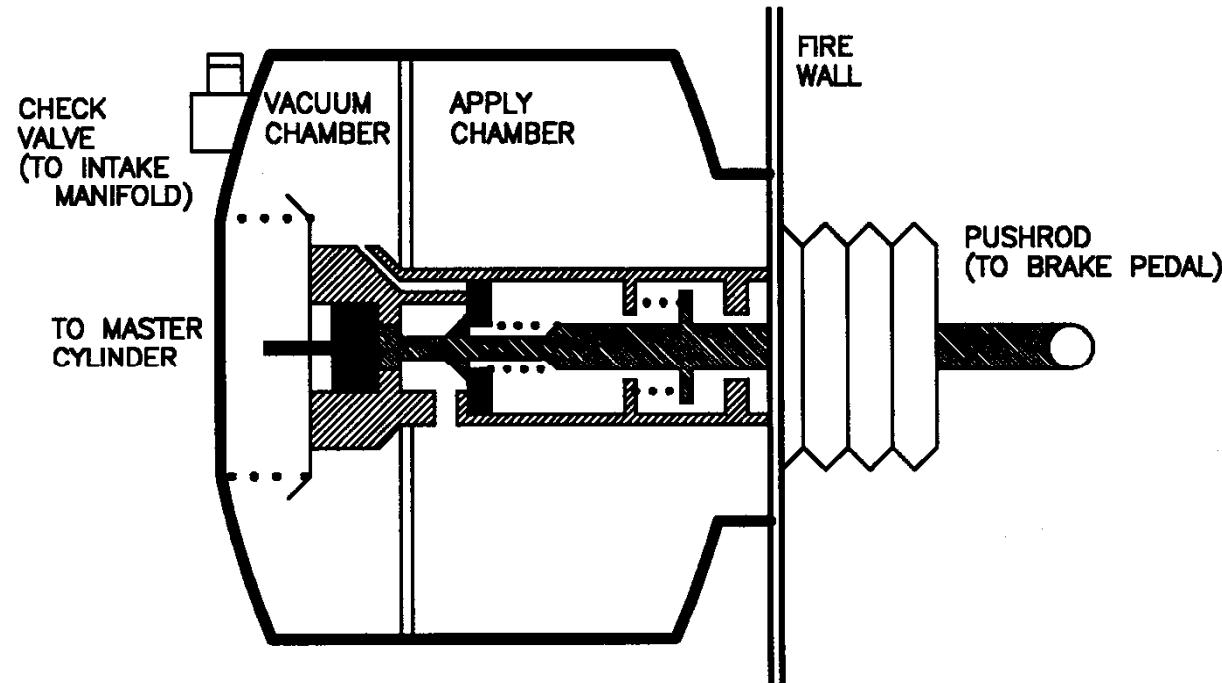
Brake Model



Fundamental structure of a hydraulic brake



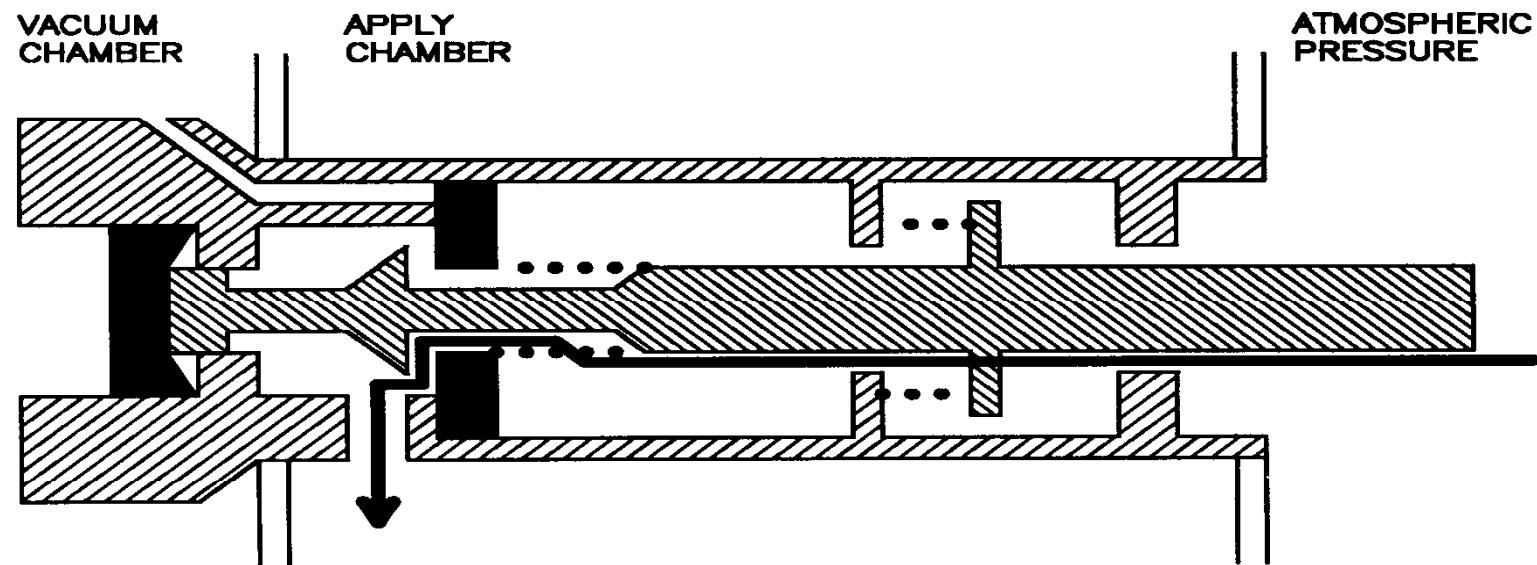
Vacuum Booster



Vacuum Booster Diagram



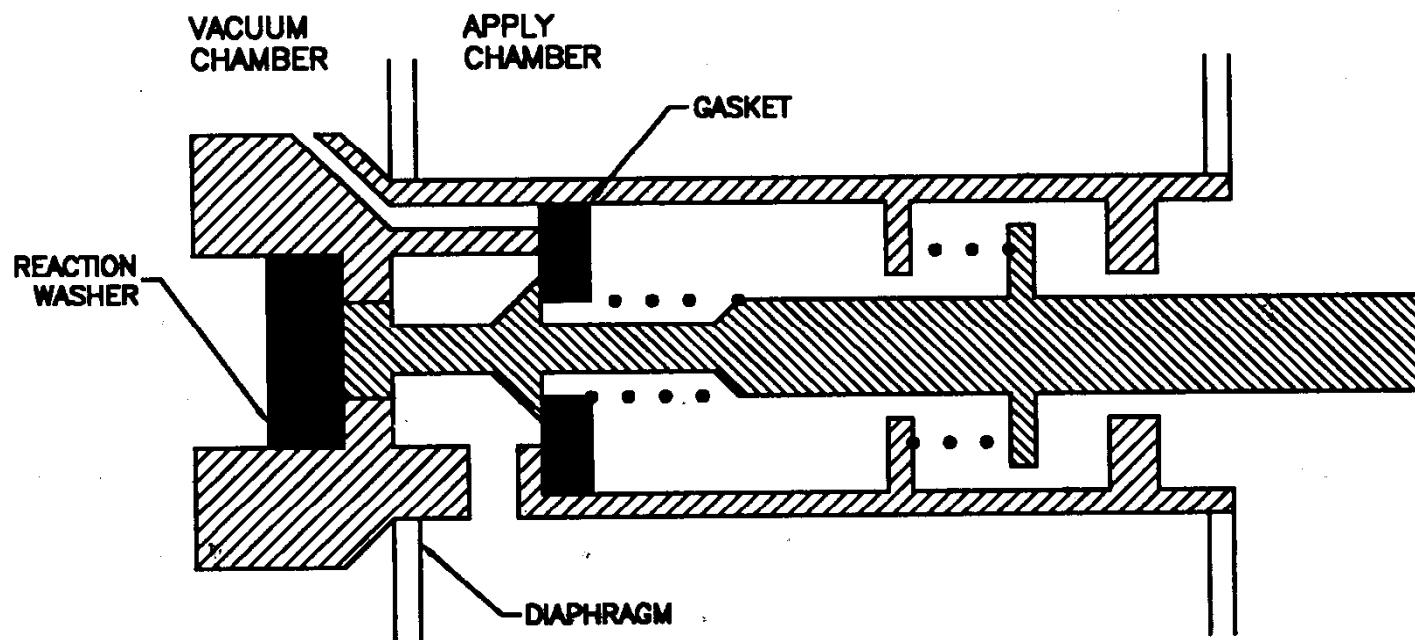
Vacuum Booster Control Valve Model



Control Valve – Apply stage



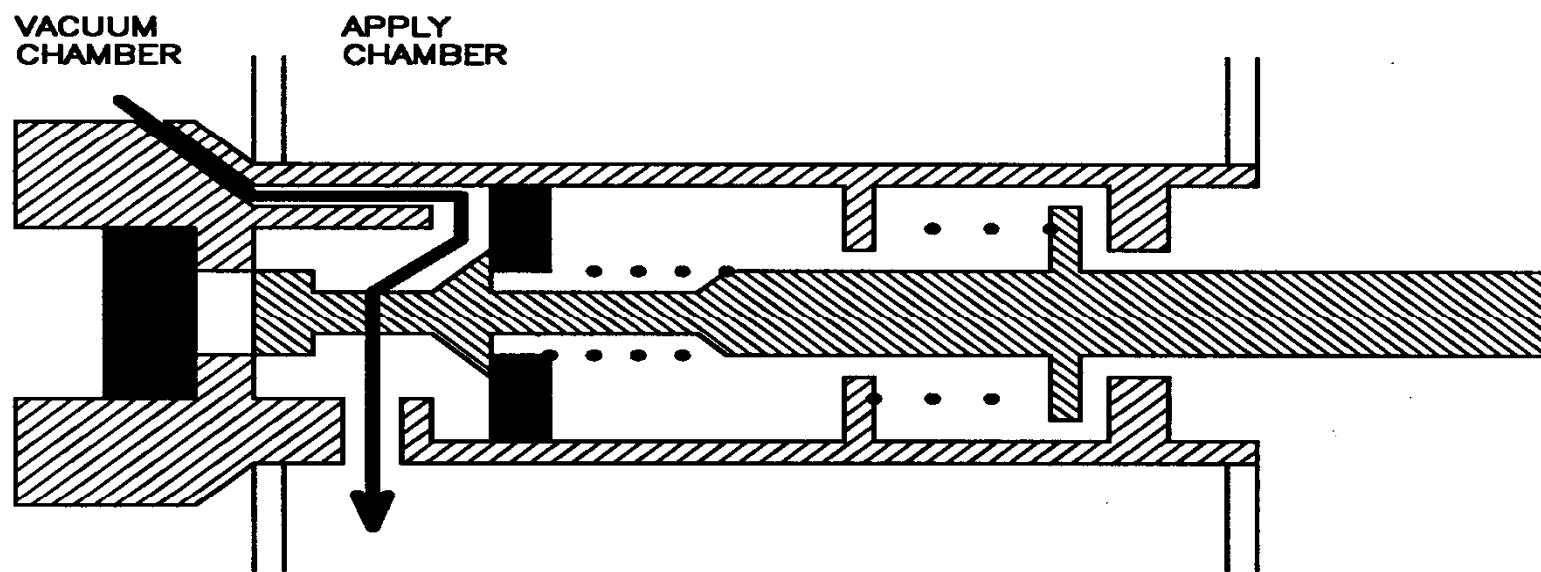
Vacuum Booster Control Valve Model



Control Valve – Hold stage



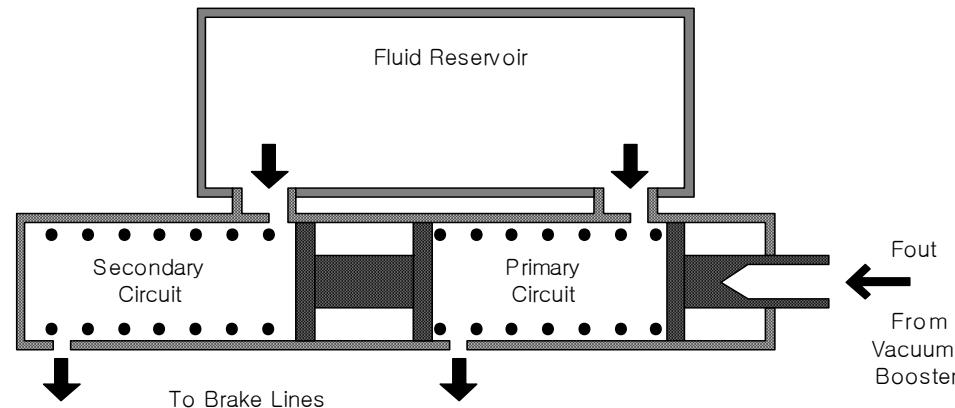
Vacuum Booster Control Valve Model



Control Valve- Release stage



Master cylinder



- Equation of motion of master cylinder piston :

$$m_{mc} \ddot{x}_{mc} = -b_{mc} \dot{x}_{mc} - F_{cs} - A_{mc} P_{mc} + F_{out} - sign(\dot{x}_{mc}) F_{loss}$$

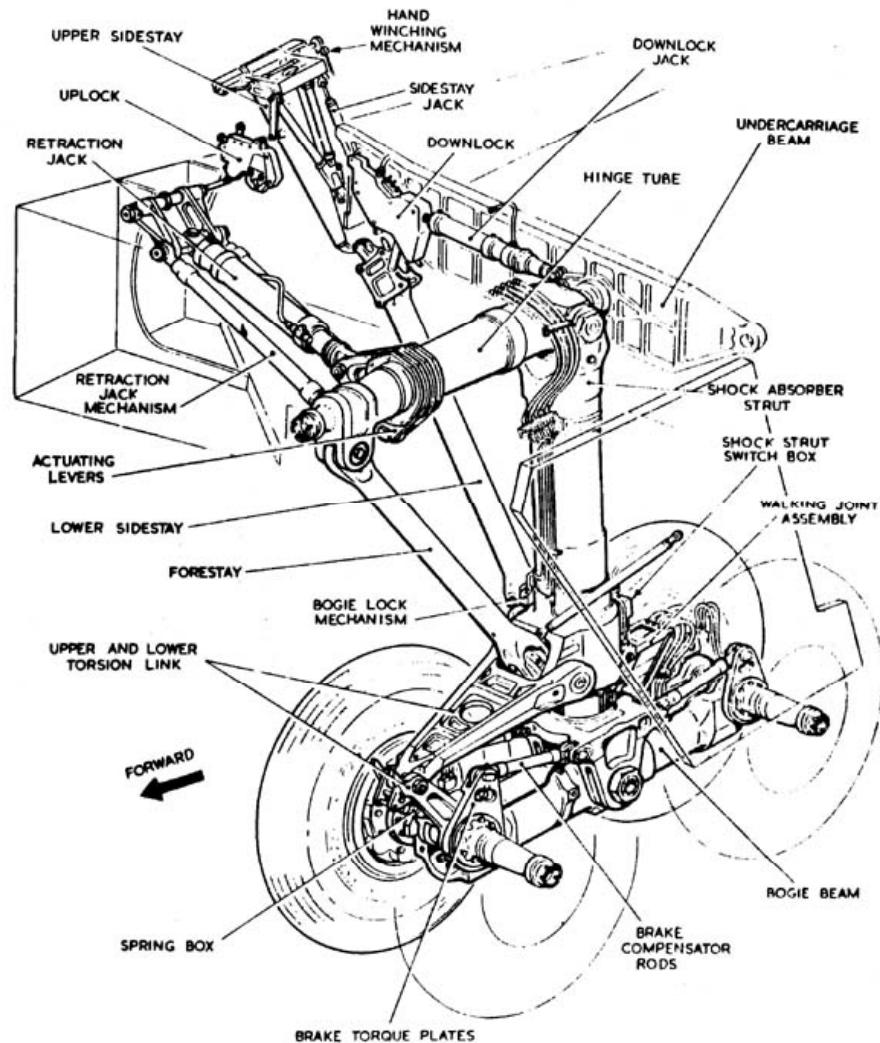
$$x_{mc} = x_{pp}$$



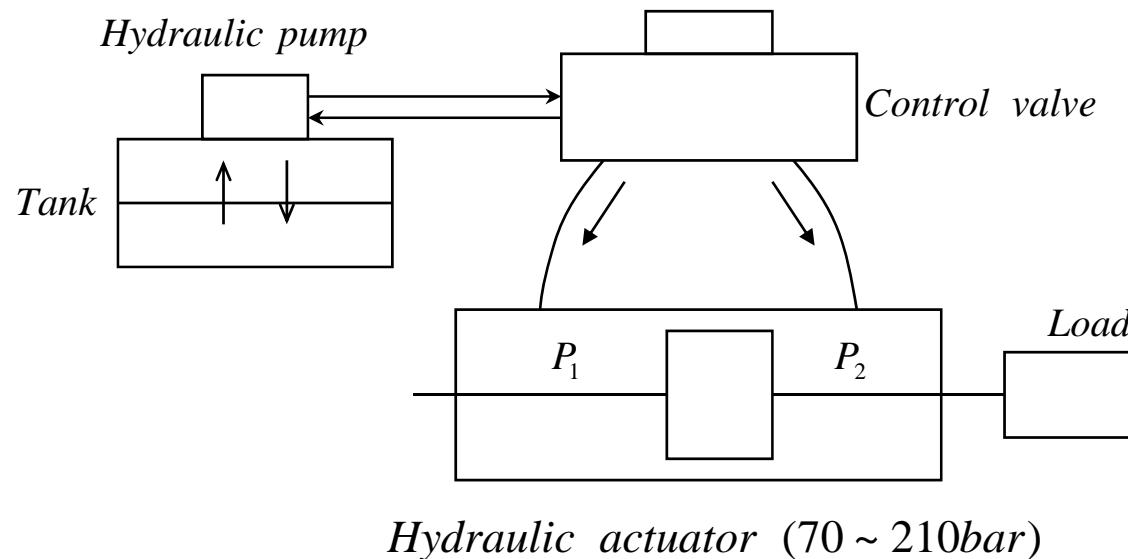
Hydraulic Systems : Landing Gear System



Landing gear system of AIRBUS A330



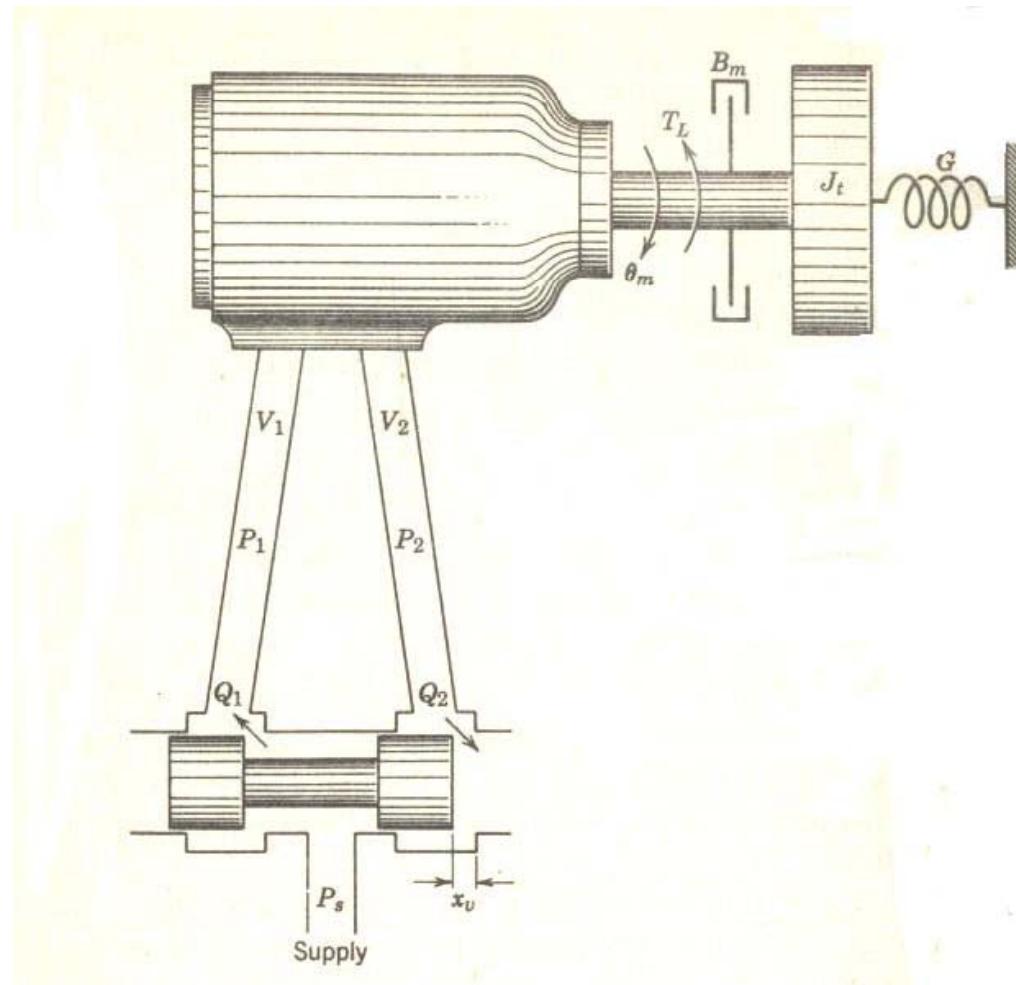
Hydraulic Systems



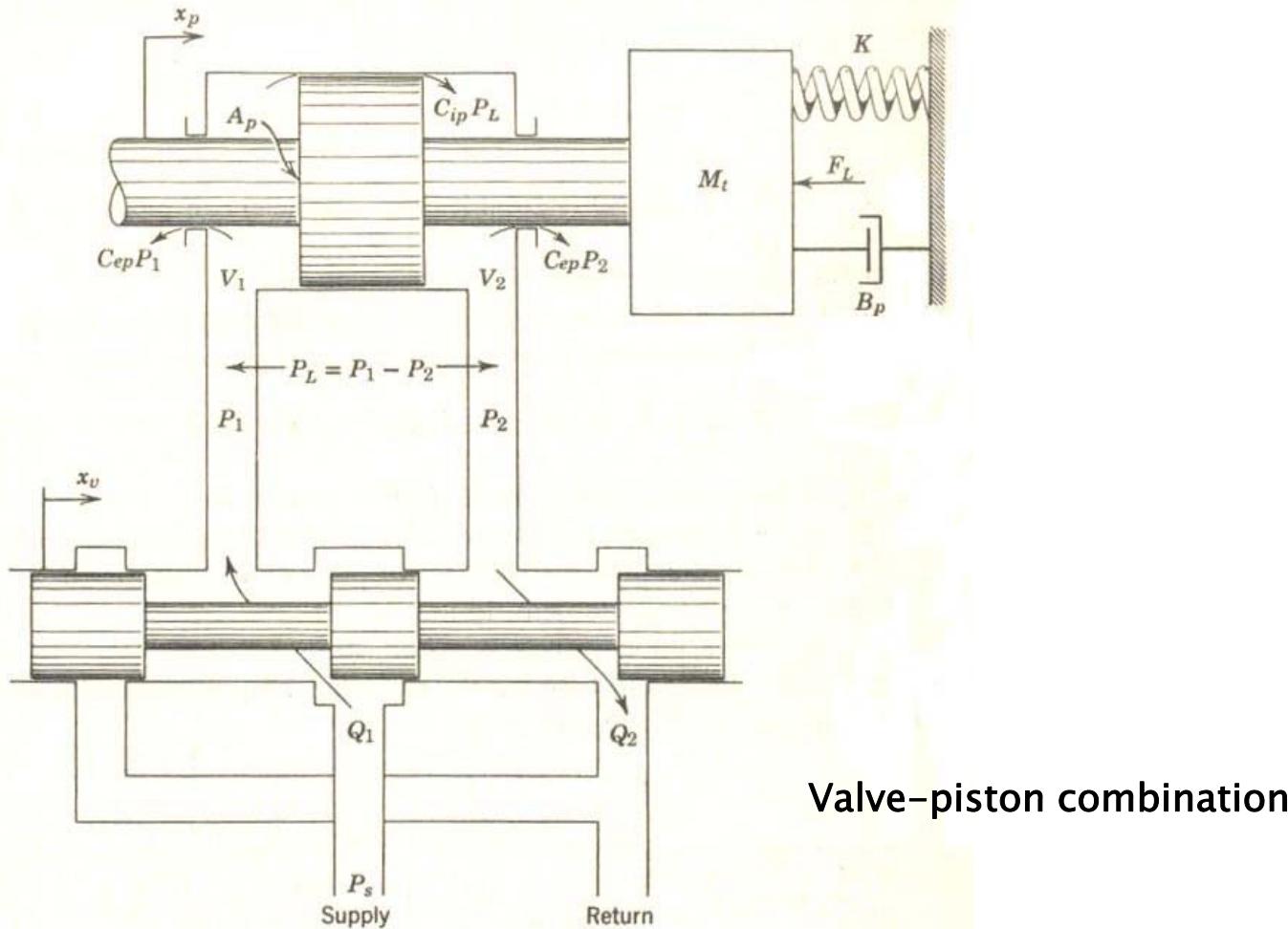
$$F = A_p \cdot (P_1 - P_2)$$



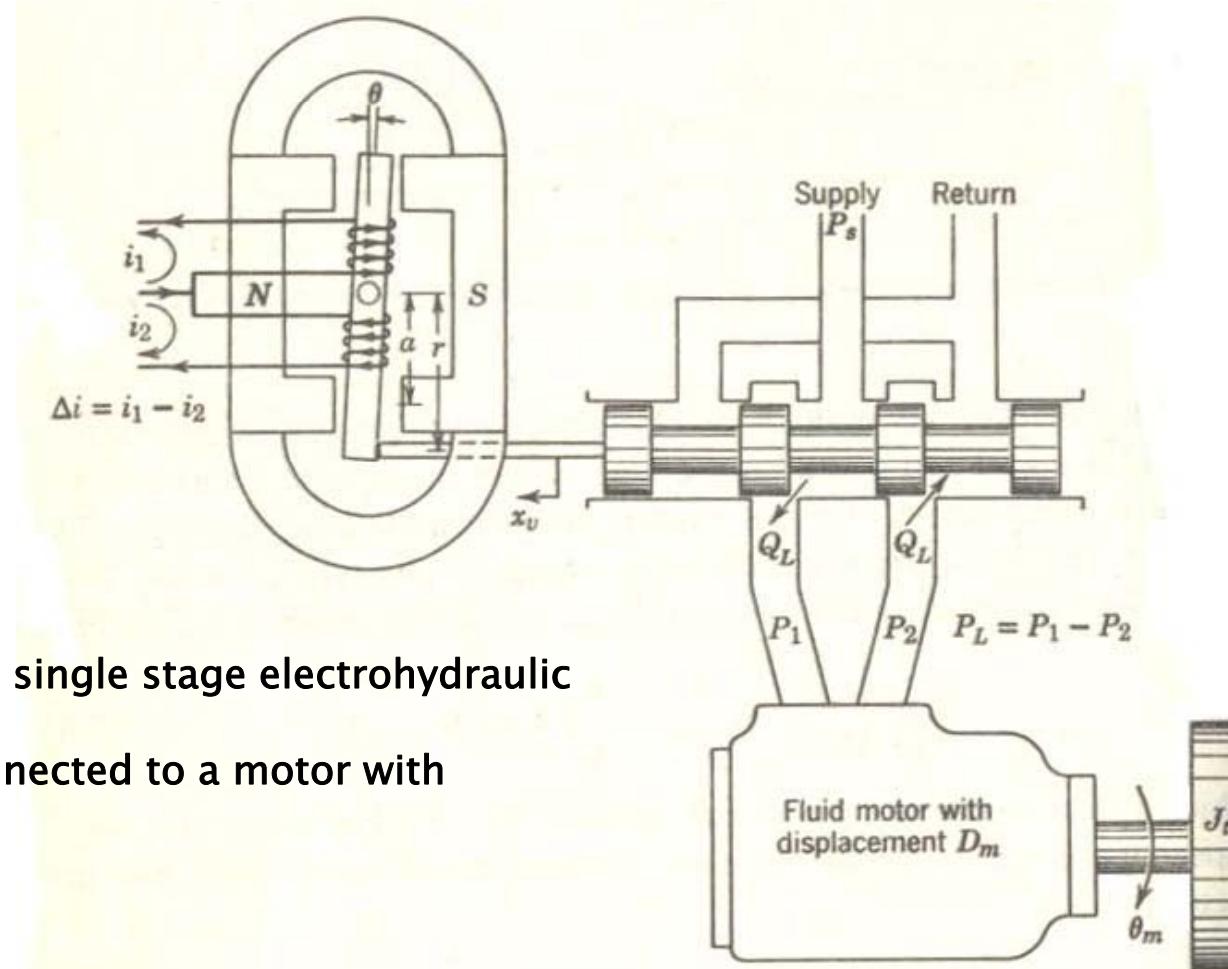
Hydraulic Systems : Valve-motor Combination



Hydraulic Systems : Valve-piston Combination



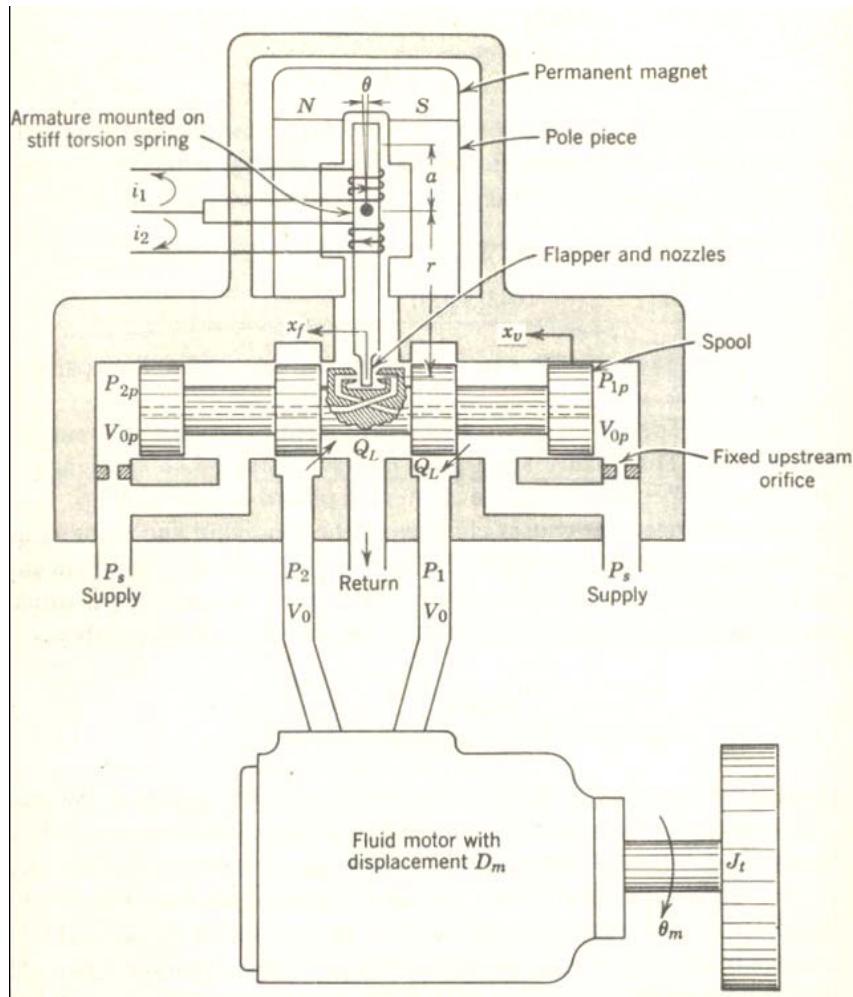
Hydraulic Systems : Single Stage Electrohydraulic Servovalve



Schematic of a single stage electrohydraulic servovalve connected to a motor with inertia load



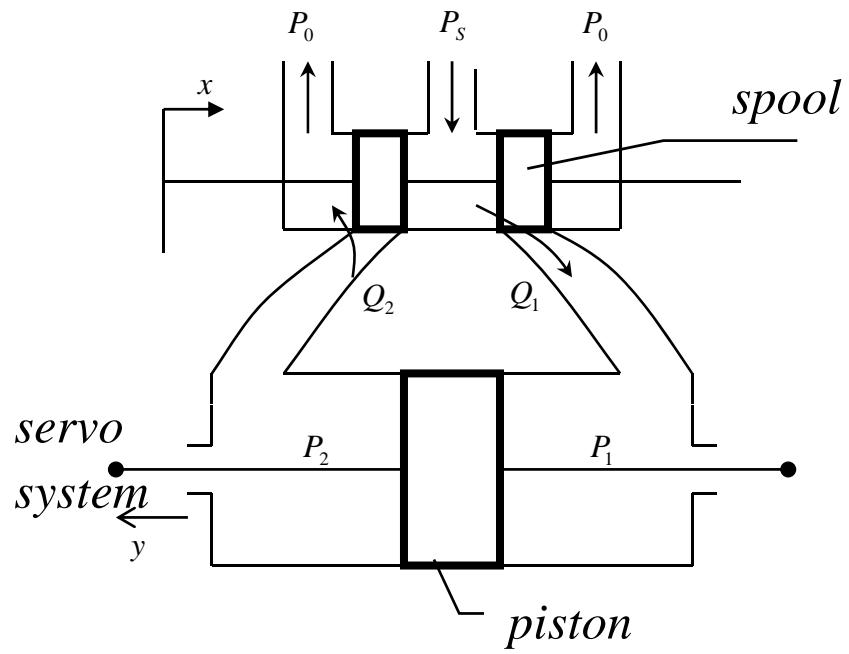
Hydraulic Systems : Two-stage Electrohydraulic Servovalve



Schematic of a two-stage electrohydraulic servovalve with force feedback controlling a motor with inertia load



Hydraulic Servo System



P_s : supply pressure

$$Q_1 = C_d \cdot a \cdot x \sqrt{\frac{2}{\rho} (P_s - P_1)} \quad [m^2 / s]$$

a : area gradient, x : displacement

ρ : density, C_d : discharge coefficient

$$Q_2 = C_d \cdot a \cdot x \sqrt{\frac{2}{\rho} (P_2 - P_0)}$$

$$= C_d \cdot a \cdot x \sqrt{\frac{2}{\rho} P_2} \quad (P_0 \approx 0)$$



Hydraulic Servo System

no leakage, no compressibility

$$Q_1 = Q_2 \rightarrow P_s - P_1 = P_2 \rightarrow P_s = P_1 + P_2$$

$$P_L = \Delta P = P_1 - P_2 \rightarrow P_s + P_L = 2P_1, \quad P_s - P_L = 2P_2$$

$$\rightarrow P_1 = \frac{P_s + P_L}{2}, \quad P_2 = \frac{P_s - P_L}{2}$$

$$Q = Q_1 = Q_2 = C_d \cdot a \cdot x \sqrt{\frac{2}{\rho} \frac{P_s - P_L}{2}} = C \cdot x \sqrt{P_s - P_L}$$

$$Q = A_p \cdot \frac{dy}{dt} = C \cdot x \sqrt{P_s - P_L}$$

$$\frac{dy}{dt} = C \cdot x \sqrt{P_s - P_L}$$



Hydraulic Servo System

$$\frac{d\bar{y}}{dt} = C \cdot \bar{x} \sqrt{P_s - \bar{P}_L}, \quad y = \bar{y} + \Delta y, \quad x = \bar{x} + \Delta x, \quad P_L = \bar{P}_L + \Delta P_L$$

$$\begin{aligned}\frac{dy}{dt} &= f(\bar{x}, \bar{P}_L) + \frac{\partial f}{\partial x} \Big|_{\bar{x} \bar{P}_L} \cdot (x - \bar{x}) + \frac{\partial f}{\partial P_L} \Big|_{\bar{x} \bar{P}_L} \cdot (P_L - \bar{P}_L) \\ &= \frac{d\bar{y}}{dt} + C \sqrt{P_s - \bar{P}_L} \cdot (x - \bar{x}) + \left(-\frac{1}{2} C \bar{x} \frac{1}{\sqrt{P_s - \bar{P}_L}} \right) \cdot (P_L - \bar{P}_L)\end{aligned}$$

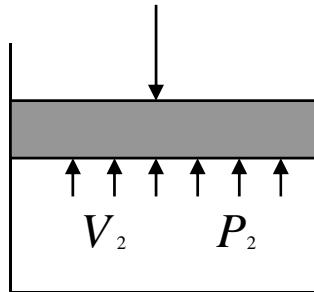
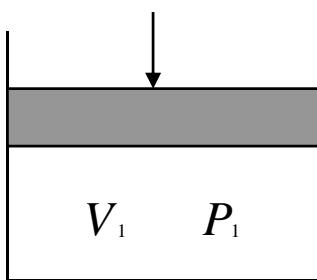
$$if \quad \bar{x} = 0, \quad \bar{P}_L = 0, \quad \frac{d\bar{y}}{dt} = 0$$

$$\frac{dy}{dt} = C \sqrt{P_s} \cdot x = K_1 \cdot x$$

$$\therefore T.F = \frac{Y(s)}{X(s)} = \frac{K_1}{S}$$



Hydraulic Servo System : Compressibility



$$\frac{\Delta V}{V_1} = \beta \Delta P$$

where, β = Compressibility

$$PV = mRT$$

$$\frac{1}{\beta} = \frac{\Delta P}{\Delta V / V}$$

$$\Delta P = dP, \quad \Delta V = V_1 - V_2 = -(V_2 - V_1) = -dV$$

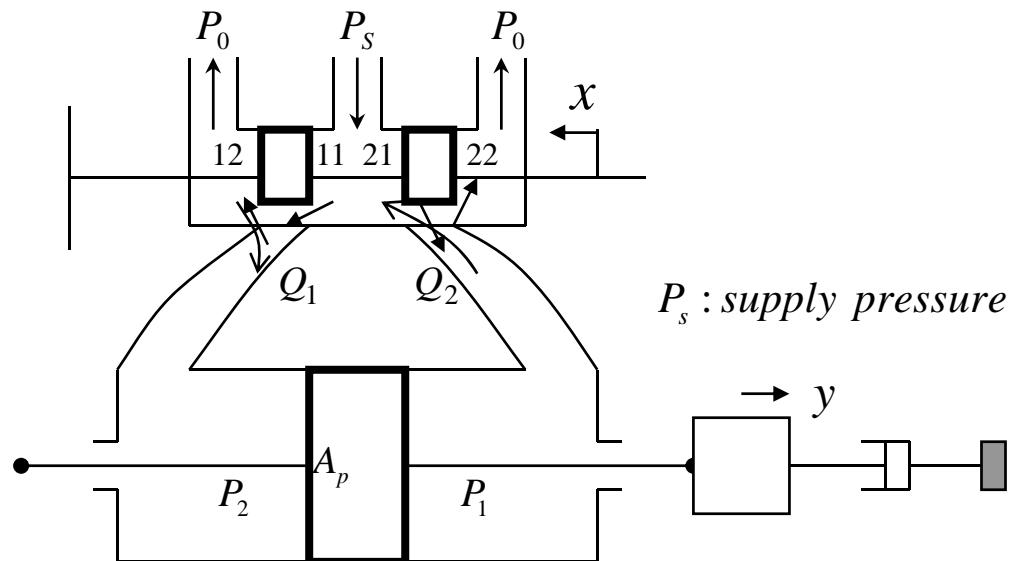
$$\frac{1}{\beta} = -V \frac{dP}{dV} = K_B \quad ; \text{ Bulk modulus}$$

$$dP = -\frac{1}{\beta} \cdot \frac{1}{V} dV = -K_B \frac{1}{V} dV$$

$$\frac{dP}{dt} = -K_B \cdot \frac{1}{V} \cdot \frac{dV}{dt}$$



Hydraulic Servo System



$$x = 0 , \quad A = A_0$$

Flow equations :

$$Q_{11} = C_d (A_0 + ax) \sqrt{\frac{2}{\rho} (P_s - P_1)}$$

$$Q_{12} = C_d (A_0 - ax) \sqrt{\frac{2}{\rho} (P_1 - 0)}$$

$$Q_1 = Q_{11} - Q_{12}$$

$$Q_{21} = C_d (A_0 - ax) \sqrt{\frac{2}{\rho} (P_s - P_2)}$$

$$Q_{22} = C_d (A_0 + ax) \sqrt{\frac{2}{\rho} (P_2 - 0)}$$

$$Q_2 = Q_{22} - Q_{21}$$



Hydraulic Servo System

Assume no leakage $Q_1 = Q_2$

$$\dot{y} = 0,$$

$$\frac{dP}{dt} = -\frac{1}{\beta} \cdot \frac{1}{V} \cdot \frac{dV}{dt}$$

$$= -\frac{1}{\beta} \frac{1}{V_1} (-Q_1)$$

$$\dot{y} \neq 0,$$

$$\frac{dp_1}{dt} = \frac{1}{\beta} \frac{1}{V_1} (Q_1 - A_p \dot{y}), \quad \frac{dp_2}{dt} = \frac{1}{\beta} \frac{1}{V_2} (-Q_2 + A_p \dot{y})$$

Equation of motion : $m\ddot{y} = A_p(p_1 - p_2) - b\dot{y}$

$$m\ddot{y} + b\dot{y} = A_p(p_1 - p_2)$$



Hydraulic Servo System Model

$$m\ddot{y} + b\dot{y} = A_p(p_1 - p_2)$$

$$\frac{dp_1}{dt} = \frac{1}{\beta} \frac{1}{V_1} (Q_1 - A_p \dot{y})$$

$$\frac{dp_2}{dt} = \frac{1}{\beta} \frac{1}{V_2} (-Q_2 + A_p \dot{y})$$

$$Q_1 = Q_{11} - Q_{12} = \left\{ C_d (A_0 + ax) \sqrt{\frac{2}{\rho} (P_s - P_1)} \right\} - \left\{ C_d (A_0 - ax) \sqrt{\frac{2}{\rho} (P_1 - 0)} \right\}$$

$$Q_2 = Q_{22} - Q_{21} = \left\{ C_d (A_0 + ax) \sqrt{\frac{2}{\rho} (P_2 - 0)} \right\} - \left\{ C_d (A_0 - ax) \sqrt{\frac{2}{\rho} (P_s - P_2)} \right\}$$

$$Q_1 = Q_2$$



Hydraulic Servo System : Linearization

$$Q_1 = Q_2 \Rightarrow Q_1 - Q_2 = 0$$

$$C_d (A_0 + ax) \left\{ \sqrt{\frac{2}{\rho} (P_s - P_1)} - \sqrt{\frac{2}{\rho} P_2} \right\} - C_d (A_0 - ax) \left\{ \sqrt{\frac{2}{\rho} P_1} - \sqrt{\frac{2}{\rho} (P_s - P_2)} \right\}$$

when $x = 0$,

$$C_d ax \left\{ \sqrt{\frac{2}{\rho} (P_s - P_1)} - \sqrt{\frac{2}{\rho} P_2} + \sqrt{\frac{2}{\rho} P_1} - \sqrt{\frac{2}{\rho} (P_s - P_2)} \right\} \\ + C_d A_0 \left\{ \sqrt{\frac{2}{\rho} (P_s - P_1)} - \sqrt{\frac{2}{\rho} P_2} - \sqrt{\frac{2}{\rho} P_1} + \sqrt{\frac{2}{\rho} (P_s - P_2)} \right\} = 0$$

To make an identical equation, $P_s - P_1 = P_2$, $P_1 = P_s - P_2 \Rightarrow P_s = P_1 + P_2$

$$\text{let } P_L = P_1 + P_2, \Rightarrow P_1 = \frac{P_s + P_L}{2}, \quad P_2 = \frac{P_s - P_L}{2}$$



Hydraulic Servo System : Linearization

$$Q_1 = C_d(ax + A_0) \sqrt{\frac{1}{\rho}(P_s - P_L)} - C_d(A_0 - ax) \sqrt{\frac{1}{\rho}(P_s + P_L)}$$

$$Q_L = Q_L(x, P_L)$$

Operating point : $x = 0, p_L = 0$

$$Q_1(x, p_L) = Q_1(0, 0) + \frac{\partial Q_1}{\partial x} \Big|_{x=0, p_L=0} (x - 0) + \frac{\partial Q_1}{\partial p_L} \Big|_{x=0, p_L=0} (p_L - 0) + \dots$$

$$\frac{\partial Q_1}{\partial x} \Big|_{x=0, p_L=0} = 2C_d \cdot a \sqrt{\frac{1}{\rho} p_s} = K_1$$

$$\frac{\partial Q_1}{\partial p_L} \Big|_{x=0, p_L=0} = -C_d \cdot A_0 \frac{1}{\sqrt{\rho \cdot p_s}} = -K_2$$

$$Q = K_1 x - K_2 p_L = Q_2 = Q_L$$

$$p_L = p_1 - p_2, \quad \frac{dp_L}{dt} = \frac{dp_1}{dt} - \frac{dp_2}{dt}$$



Hydraulic Servo System

$$m\ddot{y} + b\dot{y} = A_p p_L$$

$$\frac{dp_1}{dt} = \frac{1}{\beta} \frac{1}{V_1} (Q_L - A_p \dot{y}) = \frac{1}{\beta} \frac{1}{V_1} (K_1 x - K_2 p_L - A_p \dot{y})$$

$$\frac{dp_2}{dt} = \frac{1}{\beta} \frac{1}{V_2} \left\{ -(K_1 x - K_2 p_L) + A_p \dot{y} \right\}$$

$$\text{let } V_1 = V_2$$

$$\frac{dp_L}{dt} = \frac{1}{\beta} \frac{1}{V} (2K_1 x - 2K_2 p_L - 2A_p \dot{y})$$

$$\therefore \frac{Y(s)}{X(s)} = \frac{\left(\quad \right)}{\left(\quad \right)}$$

(): cubic equation form



Hydraulic Servo System

Simplification : No compressibility, No leakage

$$m\ddot{y} + b\dot{y} = p_L A_p$$

$$Q_L = K_1 x - K_2 p_L = A_p \dot{y} \quad \Rightarrow \quad p_L = \frac{1}{K_2} (K_1 x - A_p \dot{y})$$

$$\Rightarrow m\ddot{y} + \left(b + \frac{A_p^2}{K_2} \right) \dot{y} = A_p \frac{K_1}{K_2} x$$

$$\therefore \frac{Y(s)}{X(s)} = \frac{K}{s(Ts + 1)}$$

$$K = \frac{K_1 A_p}{K_2 b + A_p^2}, \quad T = \frac{m K_2}{K_2 b + A_p^2}$$



Basic Concepts of Thermal System

- Thermal Systems : Systems that involve the transfer of heat from one substance to another.
- Macroscopic viewpoint
- Lumped parameters
- Heat transfer
 - conduction (전도)/ convection (대류)/ radiation (복사)



Thermal System

- $q = K\Delta\theta$

$\Delta\theta$: temperature difference [°C]

q : heat flow rate [kcal/sec]

K : coefficient [kcal/(sec·°C)]

specific heat : α [kcal/(kg·°C)]

Heat capacitance : $C = m \cdot \alpha$ [kcal/°C]



Heat Flow Rate

Coefficient K :

$$K = \frac{kA}{\Delta x} \quad (\text{conduction})$$
$$= HA \quad (\text{convection})$$

k = thermal conductivity, [kcal / ms°C]

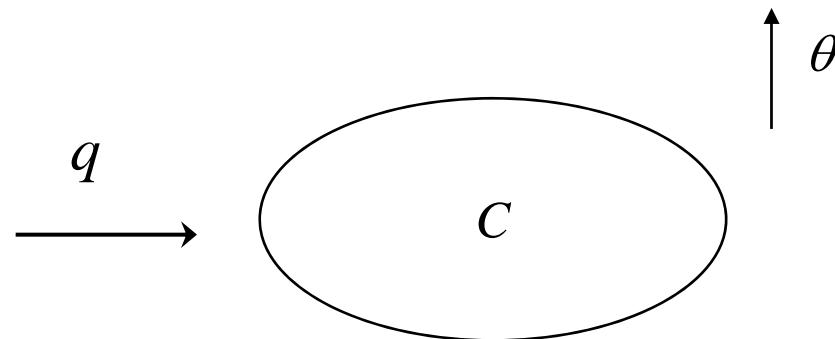
A = area normal to heat flow, [m^2]

Δx = thickness of conductor, [m]

H = convection coefficient, [kcal / $m^2 s^\circ C$]



Heat Balance Equation



$$q \cdot dt = C \cdot d\theta$$

$$\frac{d\theta}{dt} = \frac{q}{C}$$



Thermal Resistance / Capacitance

- Thermal resistance

$$R = \frac{\text{change in temperature difference } [{}^{\circ}\text{C}]}{\text{change in heat flow rate } [\text{kcal/sec}]}$$

$$q = \frac{\Delta\theta}{R}, \quad R = \frac{1}{K}$$

- Thermal capacitance

$$\begin{aligned} C &= \frac{q}{d\theta} = \frac{\text{change in heat stored } [\text{kcal}]}{\text{change in temperature } [{}^{\circ}\text{C}]} \\ &= m \cdot c \left(\text{mass } [\text{kg}] \cdot \text{specific heat } [\text{kcal}/(\text{kg} \cdot {}^{\circ}\text{C})] \right) \end{aligned}$$

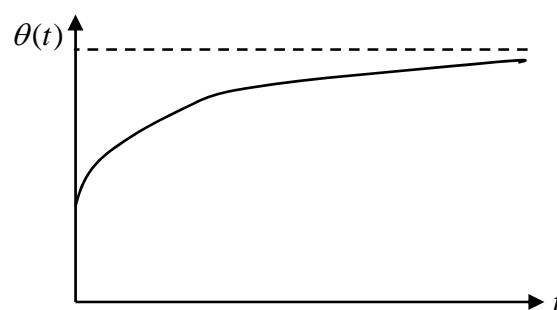
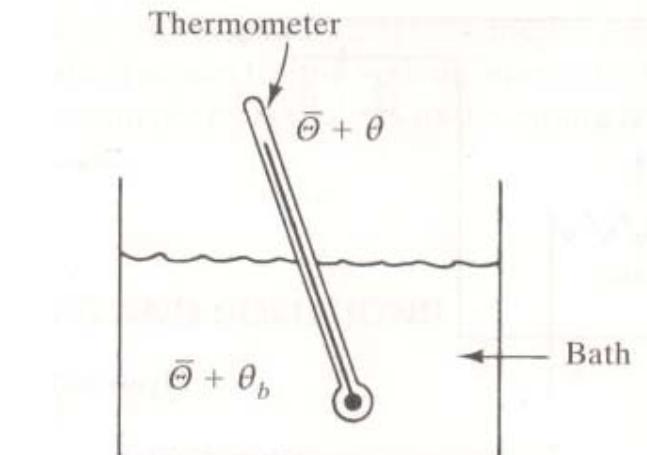
$$\frac{d\theta}{dt} = \frac{q}{C}$$



Thermal System : Thermometer System

$$q = K\Delta\theta, \quad R = \frac{1}{K}, \quad q = \frac{\Delta\theta}{R}$$

ambient temperature $\bar{\theta}$: constant
 bath temperature $\bar{\theta} + \theta_b$, θ_b : constant



$$qdt = Cd\theta$$

C : heat capacitance of the thermometer

C_b : heat capacitance of the fluid

R : thermal resistance

$$q = \frac{(\bar{\theta} + \theta_b) - (\bar{\theta} + \theta)}{R} = \frac{\theta_b - \theta}{R}$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{q}{C} = \frac{1}{RC}(\theta_b - \theta)$$

$$\therefore T.F = \frac{\theta(s)}{\theta_b(s)} = \frac{1}{RCs + 1}$$

$$\Rightarrow \theta_b(t) = \theta_b, \quad \theta(t) = \theta_b \left(1 - e^{-\frac{1}{RC}t} \right)$$



Thermal System : Thermometer System

When q_i applied,

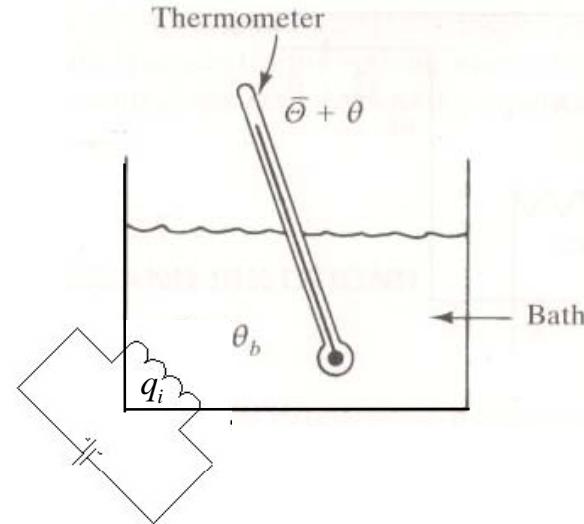
$$\left\{ \begin{array}{l} \frac{d\theta_b}{dt} = \frac{1}{C_b}(q_i - q) \\ q = \frac{1}{R}(\theta_b - \theta) \\ \frac{d\theta}{dt} = \frac{1}{C}q \end{array} \right.$$

$$\Rightarrow \frac{d\theta_b}{dt} = -\frac{1}{RC_b}\theta_b + \frac{1}{RC_b}\theta + \frac{1}{C_b}q_i, \quad \frac{d\theta}{dt} = -\frac{1}{RC}\theta + \frac{1}{RC}\theta_b$$

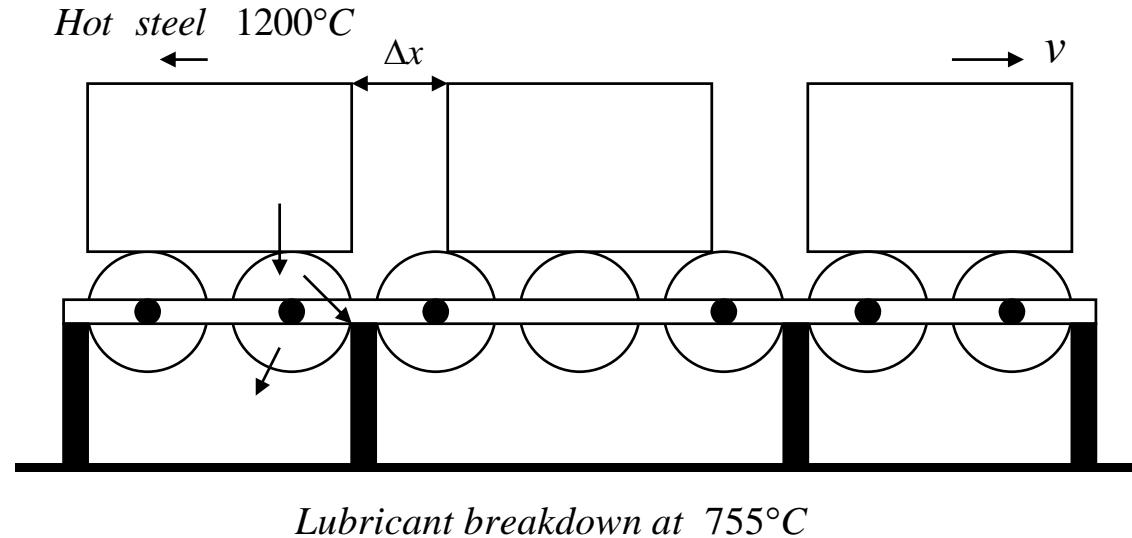
$$(RC_b s + 1)\Theta_b(s) = \Theta(s) + RQ_i(s), \quad (RCs + 1)\Theta(s) = \Theta_b(s)$$

$$\Rightarrow (R^2 C_b C s^2 + RCs + RC_b s)\Theta(s) = RQ_i(s)$$

$$\therefore \frac{\Theta(s)}{Q_i(s)} = \frac{1}{s(RC_b C s + C_b + C)} \approx \frac{1}{sC_b} \quad (C \ll C_b, \quad RC \text{ is small})$$



Thermal System : A Steel Processing Plant



- Large slabs of red-hot steel
- $T_{steel} = 1600^{\circ}F$: almost constant
- $T_A = 100^{\circ}F$ (ambient temperature)



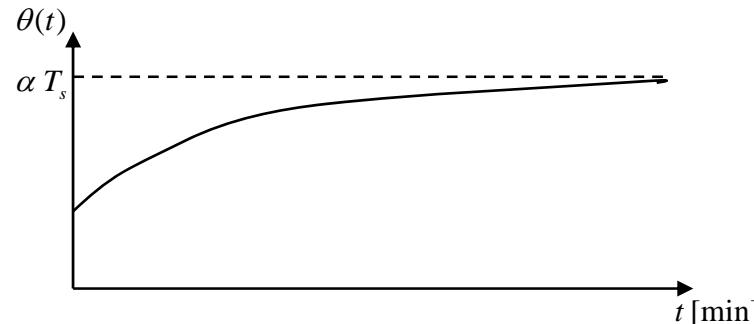
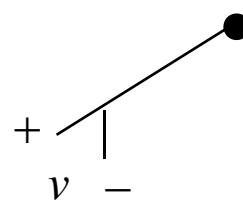
Thermal System : A Steel Processing Plant

T_A : ambient temperature

R_1 : thermal resistance between the slab and the rollers (conduction)

R_2 : thermal resistance between the rollers and the ambient air (convection)
and the bearing support (conduction)

Thermocouple : Produce voltage proportional to its temperature

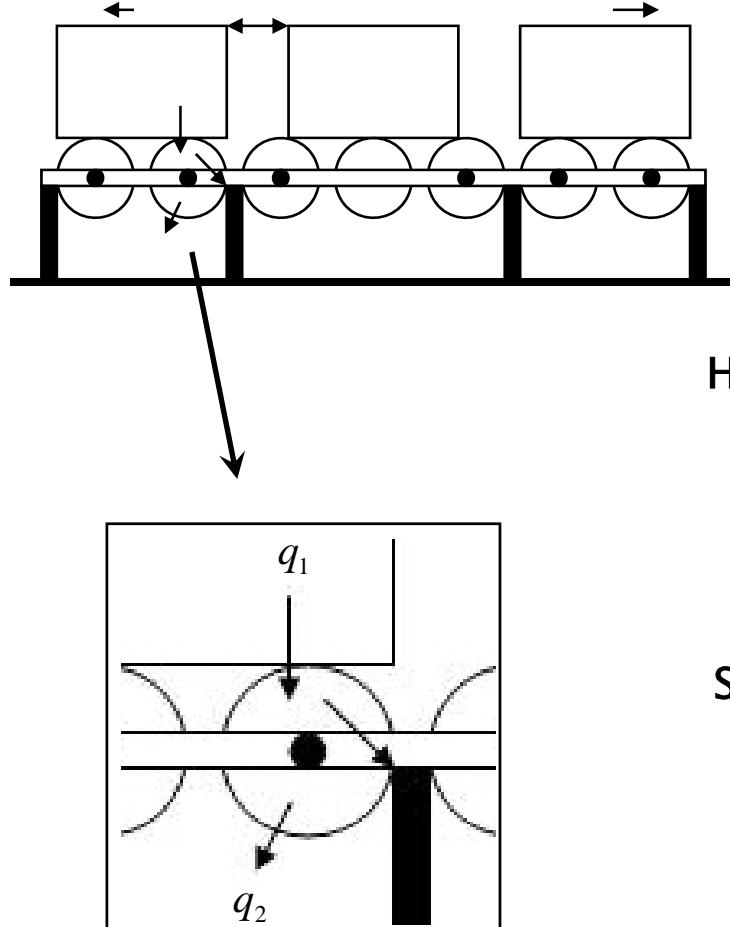


roller : mass = m , specific heat capacitance = C_p [$kcal / kg^\circ C$]

$$\text{heat capacitance} = C = m \cdot C_p$$



Thermal System : A Steel Processing Plant



$$q_1 = \frac{1}{R_1} (T_s - T_r)$$

$$q_2 = \frac{1}{R_2} (T_r - T_A)$$

Heat balance : $m \cdot C_p \cdot \frac{dT_r}{dt} = q_1 - q_2$

$$= -\left(\frac{1}{R_1} + \frac{1}{R_2} \right) T_r + \frac{1}{R_1} T_s + \frac{1}{R_2} T_A$$

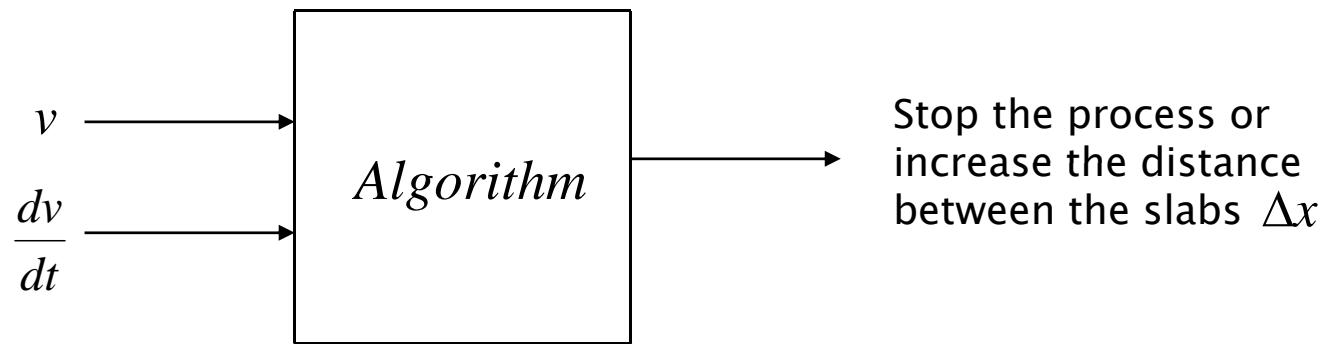
Sensor : $\frac{v(s)}{T_r(s)} = \frac{\alpha}{\tau s + 1}, \quad \tau \dot{v} = -v + \alpha T_r$
 $x = v + \beta \dot{v} \quad (\tau : \text{time constant})$

if, $x > x_{cr} \Rightarrow stop$

$x < x_{cr} \Rightarrow restart$



Thermal System : A Steel Processing Plant



$$x = v + \beta \dot{v}$$

if, $x > x_{cr}$ stop

