Evaluation of residual stress based on indentation surface displacement analysis

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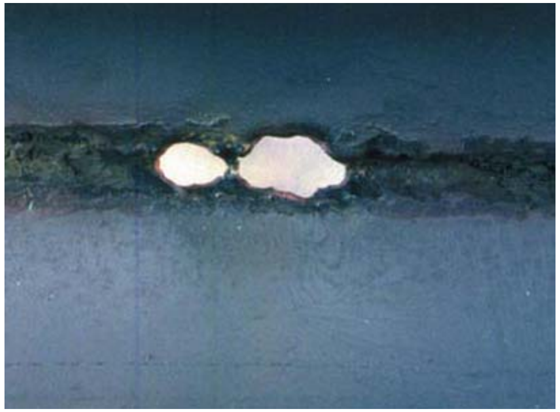
Material Reliability
& Forensic Safety Lab.

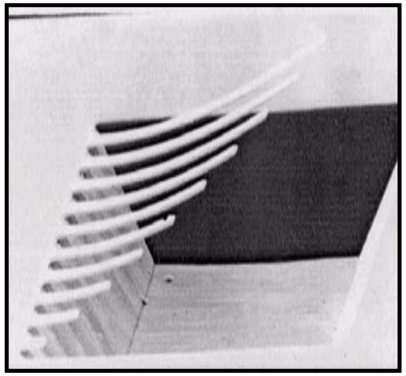
Effect of residual stress on reliability

► **Bulk material**

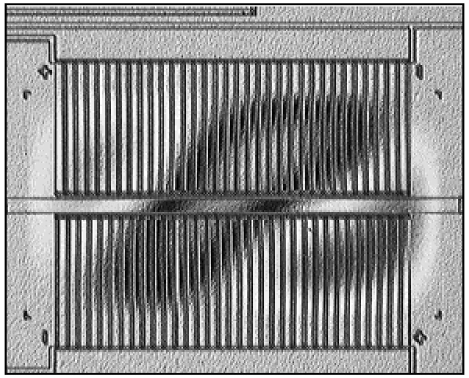


Crack initiation

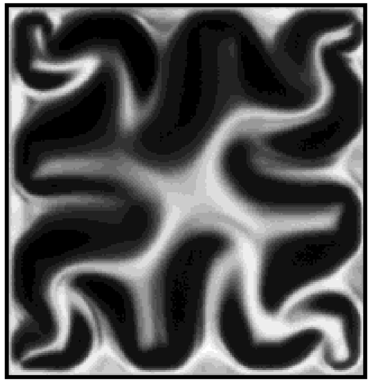
► **Thin film**



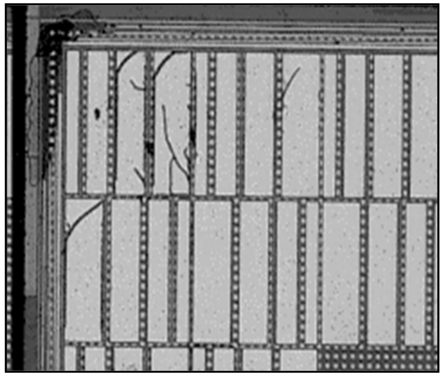
Bending



Twisting



Buckling

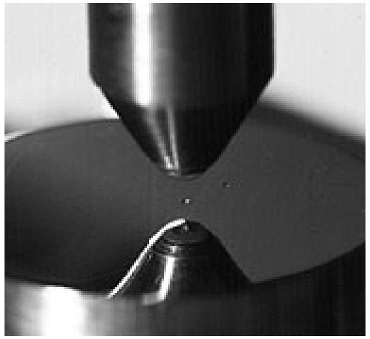


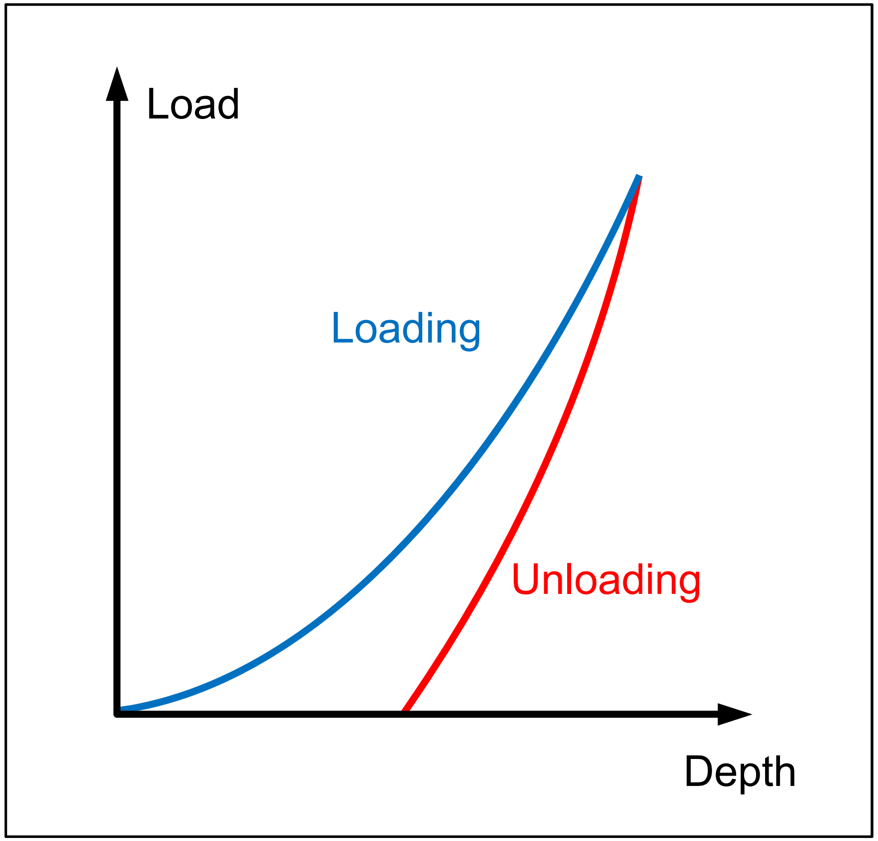
Cracking

Various Methods to Evaluate Residual Stress

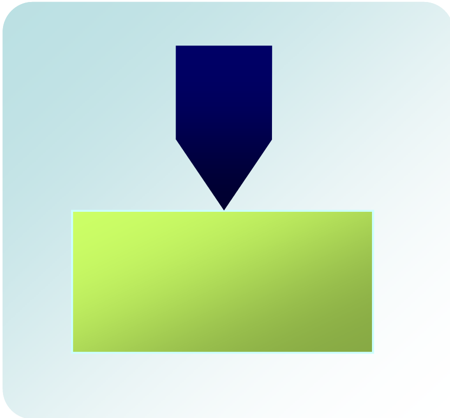
	Scale		Non-crystalline material	Local part	Non-destructive
	Bulk material	Thin film			
Hole-drilling method	O	O	O	O	X
Saw-cutting method	O	X	O	△	X
X-Ray Diffraction method	O	O	X	O	O
Curvature method	X	O	△	X	O
Instrumented Indentation Technique	O	O	O	O	O

Instrumented indentation testing (IIT)





Indentation load-depth curve

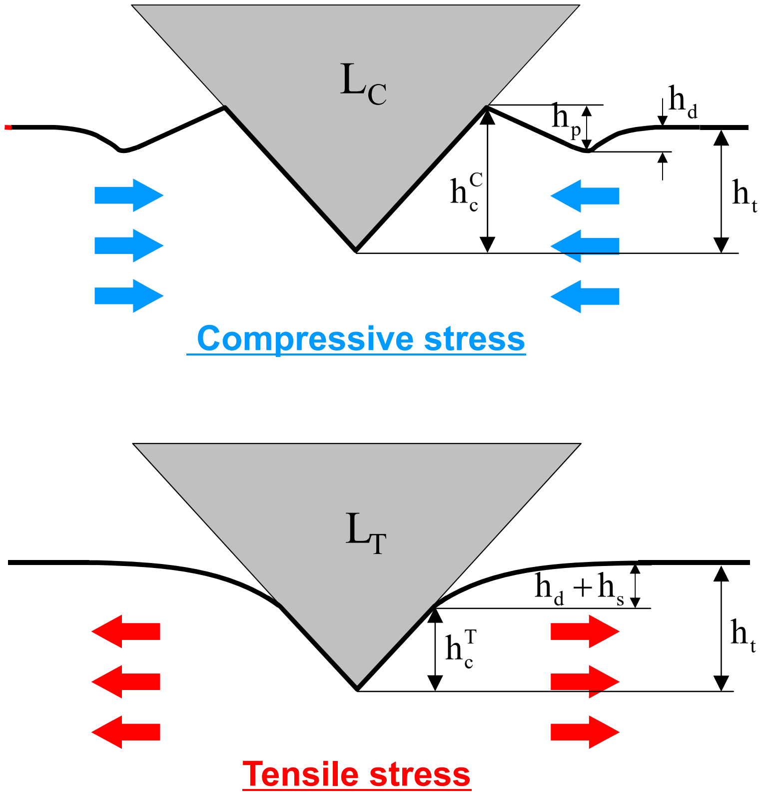


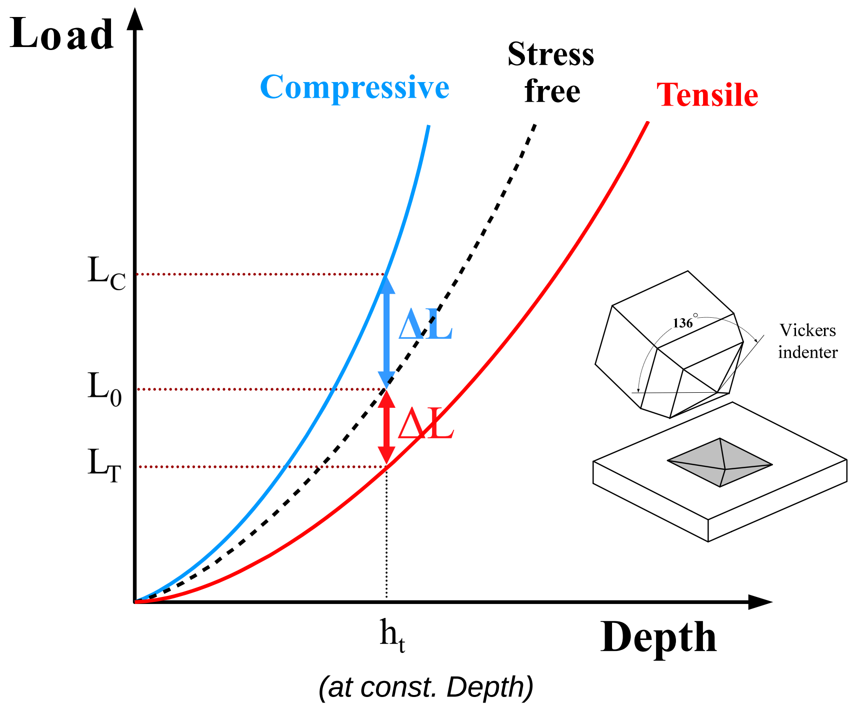
Hardness
Elastic modulus
Tensile properties
Fracture toughness
Residual stress
⋮

Easier and simpler to measure quantitative mechanical properties

Basic concept of evaluating residual stress using IIT

[Y.H. Lee, D. Kwon, *Acta Mater.* (2004)]





- L_C = Indentation load in compressive stress state
- L_0 = Indentation load in stress-free state
- L_T = Indentation load in tensile stress state
- h_t = Indentation depth (experimentally measured)
- h_d = Elastic deflection height
- h_p = Pile-up height
- h_s = Sink-in height
- h_c^C = Real contact depth in compressive stress
- h_c^T = Real contact depth in tensile stress

$$\sigma_{res} \propto \Delta L$$

Quantification of residual stress by stress tensor

[Y.H. Lee, D. Kwon, *Acta Mater.* (2004)]

$$\begin{pmatrix} \sigma_{res}^{x} & 0 & 0 \\ 0 & \sigma_{res}^{y} & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \sigma_{res}^{x} & 0 & 0 \\ 0 & p\sigma_{res}^{x} & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \left(p = \frac{\sigma_{res}^{y}}{\sigma_{res}^{x}} \right)$$

$$= \begin{pmatrix} \frac{(1+p)}{3}\sigma_{res}^{x} & 0 & 0 \\ 0 & \frac{(1+p)}{3}\sigma_{res}^{x} & 0 \\ 0 & 0 & \frac{(1+p)}{3}\sigma_{res}^{x} \end{pmatrix} + \begin{pmatrix} \frac{(2-p)}{3}\sigma_{res}^{x} & 0 & 0 \\ 0 & \frac{(2p-1)}{3}\sigma_{res}^{x} & 0 \\ 0 & 0 & -\frac{(1+p)}{3}\sigma_{res}^{x} \end{pmatrix}$$

Hydrostatic stress | **Deviatoric stress**

Change of indentation stress : | Deviatoric stress along z-direction :

$$\Delta\sigma_{ind} \left(= \frac{\Delta L}{A_c} \right) = \frac{(1+p)}{3}\sigma_{res}^{x}$$

$$\sigma_{res}^{x} = \frac{3}{(1+p)} \frac{\Delta L}{A_c}$$

A_c = contact area in stressed state

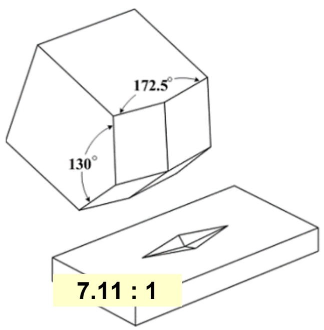
Necessity of Knoop indentation

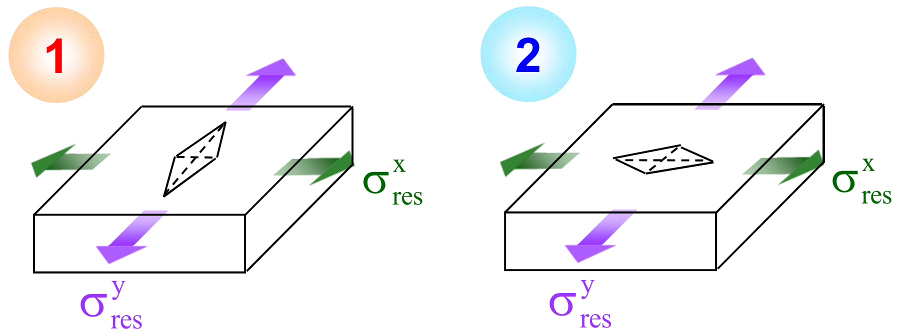
[J.H. Han, Thesis (2007)]

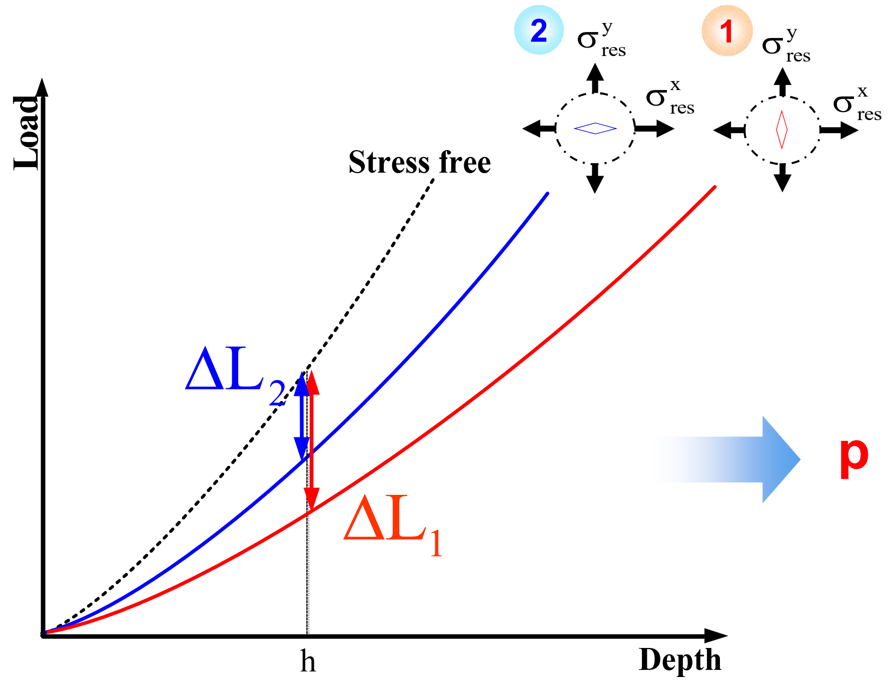
$$\sigma_{res}^{x} = \frac{3}{(1+p)}\frac{\Delta L}{A_c}$$

$$p = \frac{\sigma^{y}_{res}}{\sigma^{x}_{res}}$$ **(Stress Ratio)**

Knoop Indenter

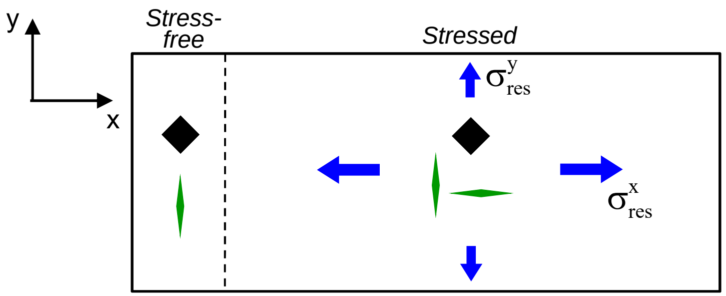
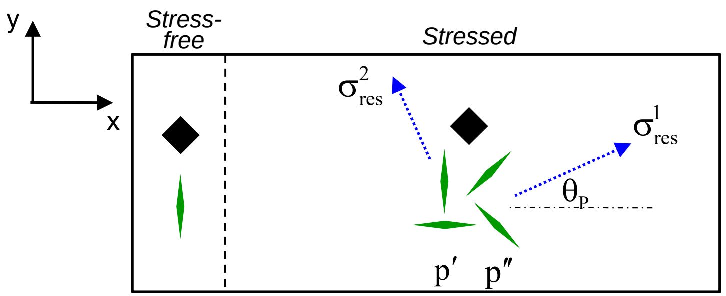
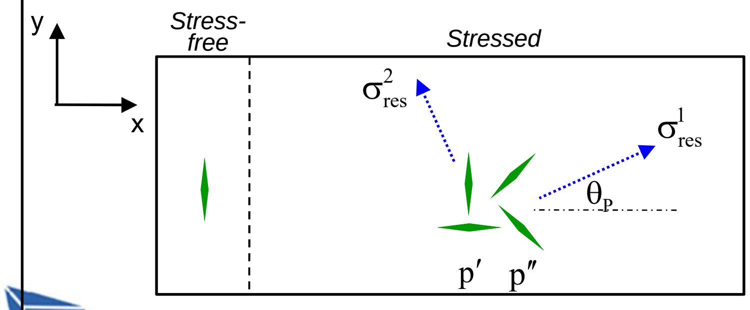






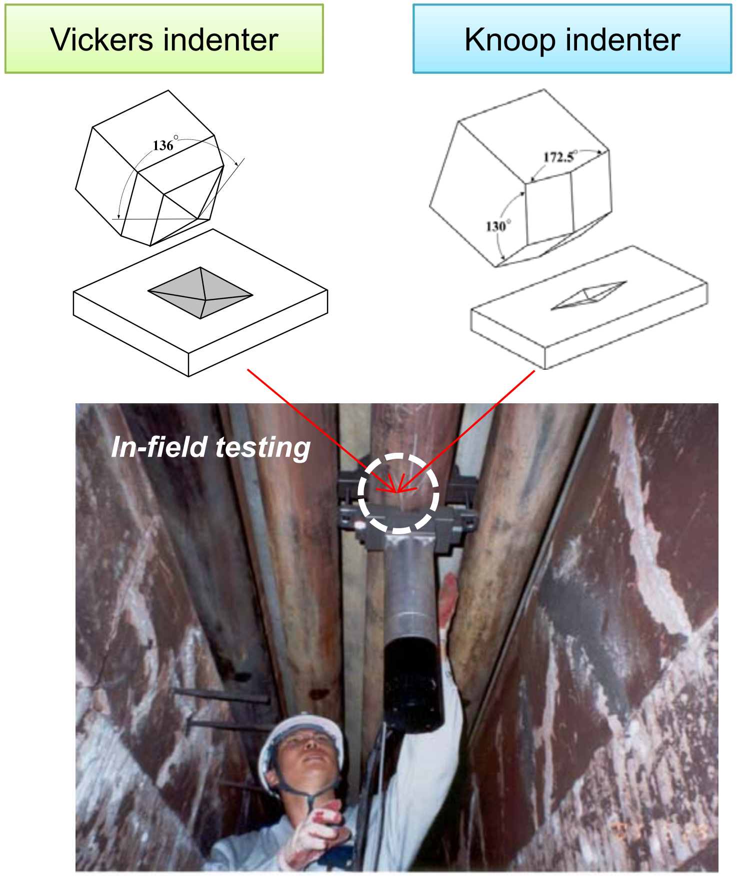
Review of previous researches

Assumption : bi-axial stress
: isotropic mechanical properties

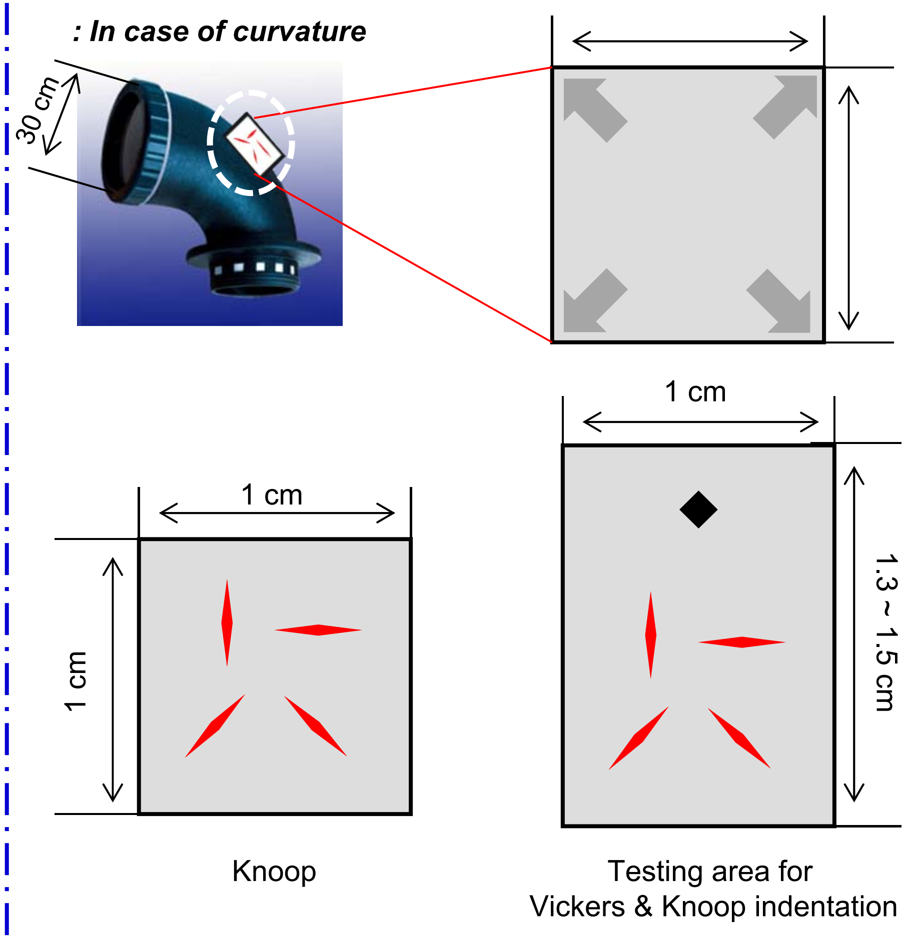
Schematic View	Results		
	Ave. Stress	Stress ratio	Principal direction
	σ_{res}^{x+y}	P	θ_P
 [Y.H. Lee, D. Kwon, *Acta Mater*. (2004)] $\sigma_{res}^{x} = \frac{3}{(1+p)}\frac{\Delta L}{A_c}$ [J.H. Han, Thesis (2007)] $\frac{\Delta L_2}{\Delta L_1} = \frac{\frac{\alpha_{//}}{\alpha_{\perp}} + p}{1 + \frac{\alpha_{//}}{\alpha_{\perp}} p}$	O	O	X
 [M.J. Choi, Thesis (2012)] $\tan 2\theta_P = \frac{(1+p')(1-p'')}{(1-p')(1+p'')}$ $\left(\sigma_{res}^{1} = \frac{3}{(1+P)}\frac{\Delta L}{A_c}\right)$ $P = \frac{\sigma^2_{res}}{\sigma^1_{res}} = \frac{(1+p')\cos 2\theta_P - (1-p')}{(1+p')\cos 2\theta_P + (1-p')}$	O	O	O
 [Y.C. Kim, Thesis (2013)] $\tan 2\theta_P = -\frac{(\Delta L_3 - \Delta L_4)}{(\Delta L_1 - \Delta L_2)}$ $\sigma_I = \frac{3}{\psi}\frac{1}{2A_c}\left\{(\Delta L_1 + \Delta L_2) + \frac{k+1}{k-1}\frac{\Delta L_1 - \Delta L_2}{\cos 2\theta_P}\right\}$ $\sigma_{II} = \frac{3}{\psi}\frac{1}{2A_c}\left\{(\Delta L_1 + \Delta L_2) - \frac{k+1}{k-1}\frac{\Delta L_1 - \Delta L_2}{\cos 2\theta_P}\right\}$	O	O	O

Limitations

Applicability to in-field testing



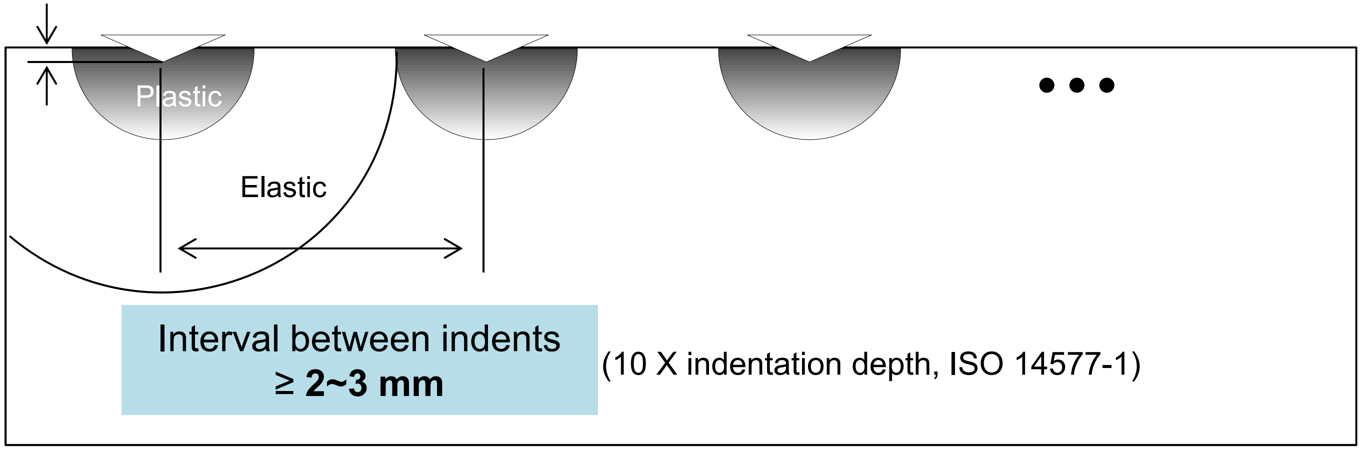
- Inconvenience(or impossible) to change indenter tip
- Time consuming procedure for two kind of indentations



- Restriction on testing area in case of some structures

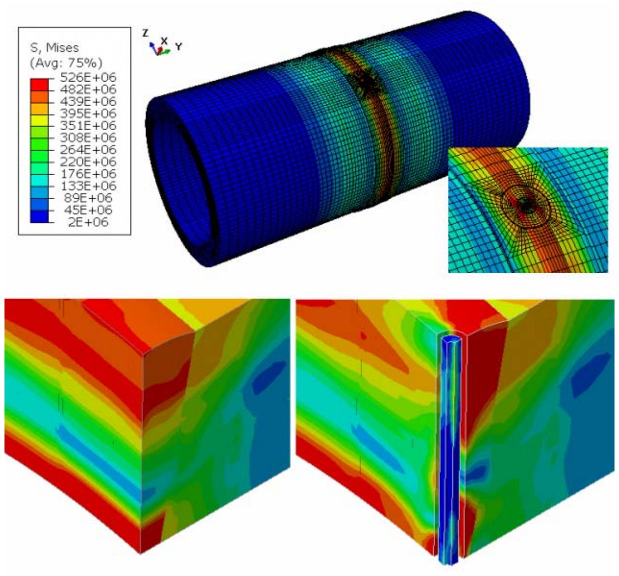
Limitations

Testing area for indentation



Interval between indents
≥ **2~3 mm**

(10 X indentation depth, ISO 14577-1)

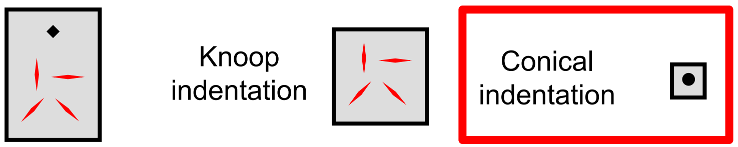


- ***Residual stress = f(distance)***
- ***Testing area*** ↓ ⇨ ***Measuring resolution*** ↑

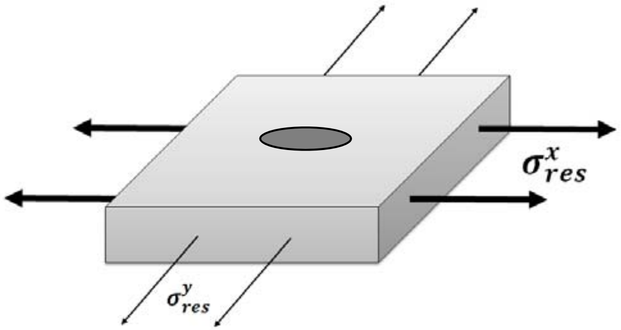
Vickes + Knoop indentation

Knoop indentation

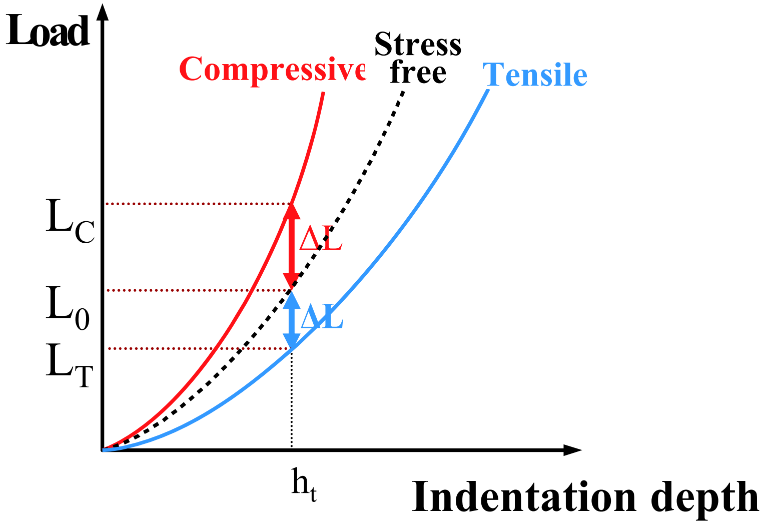
Conical indentation



Indentation Results Depending on Residual Stress



(1) Indentation Load-depth Curve



(2) Surface Displacement due to Indentation

Tensile stress

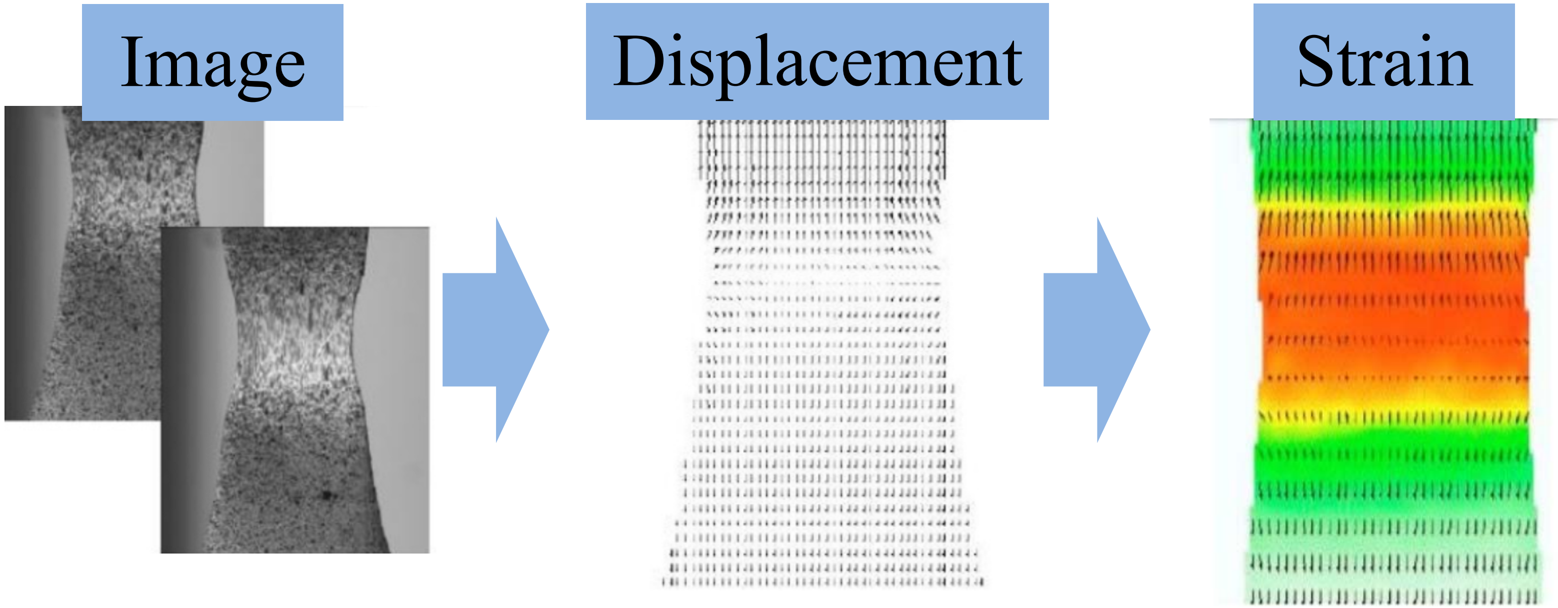
Stress-free

Compressive stress

Digital Image Correlation (DIC)

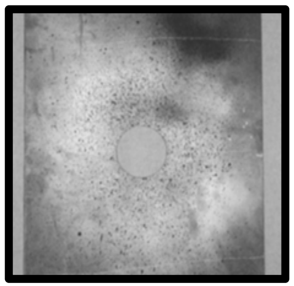
What is DIC?

- Using random speckle patterns
- Searching the same points between reference and target images

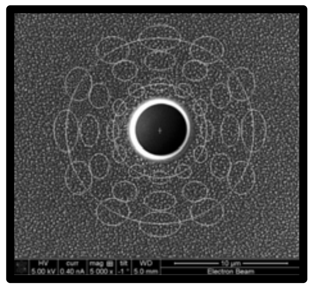


Why DIC?

- Full field optical technique (lots of information)
- Applicable to multiscale from macro to nano
- Simple equipment



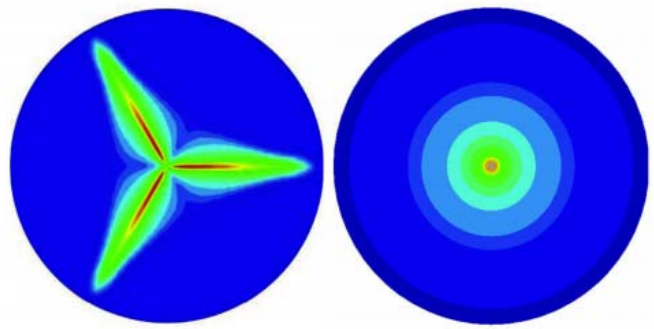
[McGinnis et al, 2005]



[Schajer et al., 2012]

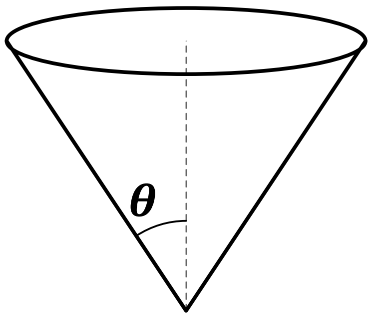
Characteristic of Conical indenter

[Z. Shi, et al., Int. J. Plasticity, 2010]



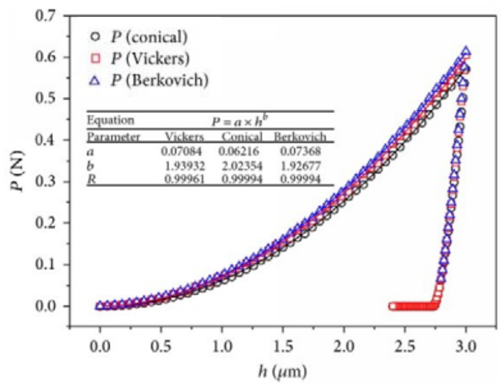
Berkovich indenter **Conical indenter**

- **Deformation of axisymmetry**
  → specimen for unknown direction of principal stresses



$\theta = 70.3°$
for same projected contact area with Vickers

[Z. Tang, et al., Adv. Mater. Sci. Eng., 2015]



- **Similar mechanical response as sharp indenter**
  → using Lee's model for evaluating average of principal residual stresses

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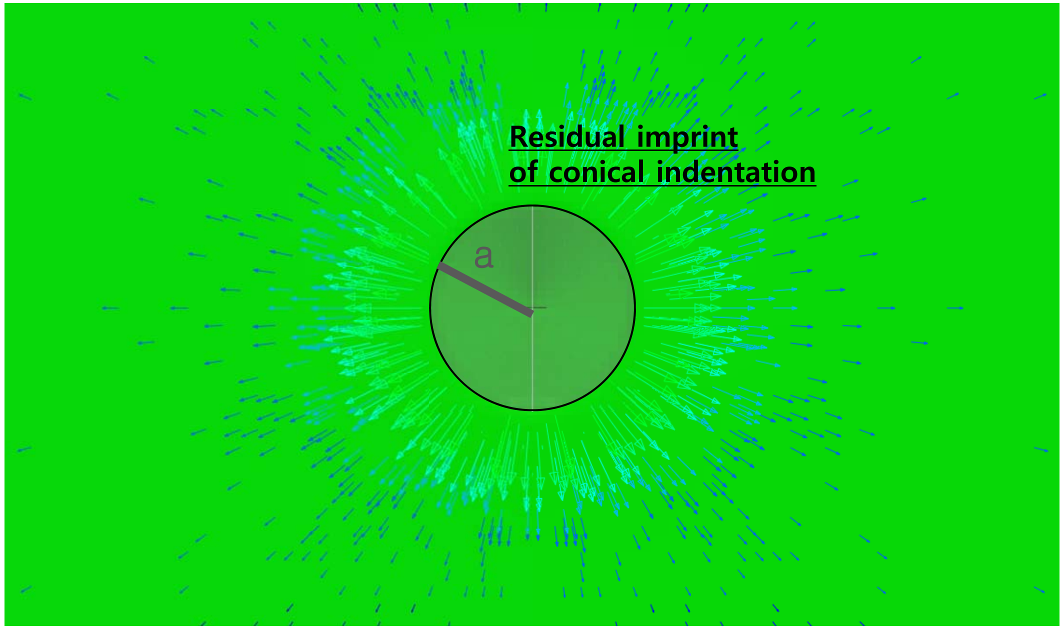
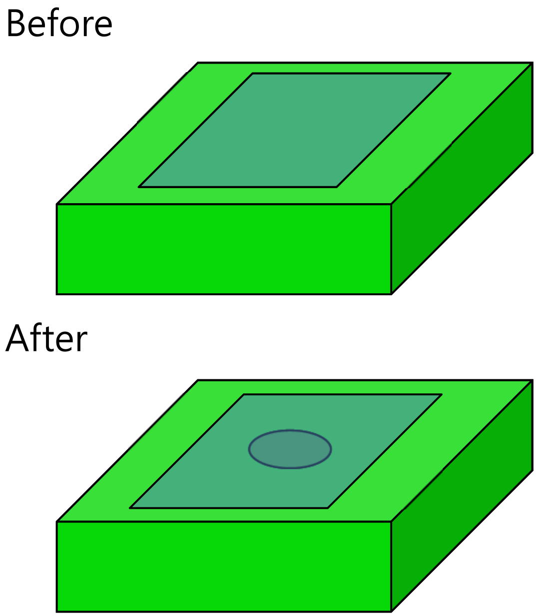
Indentation & Digital Image Correlation

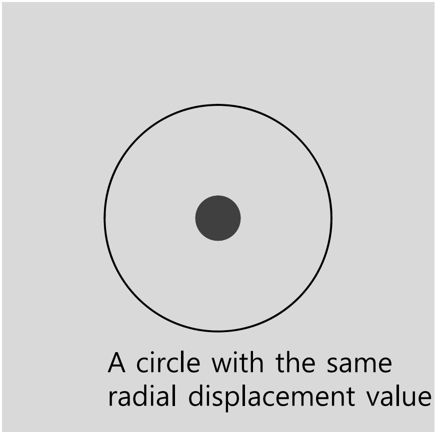


DIC analysis area

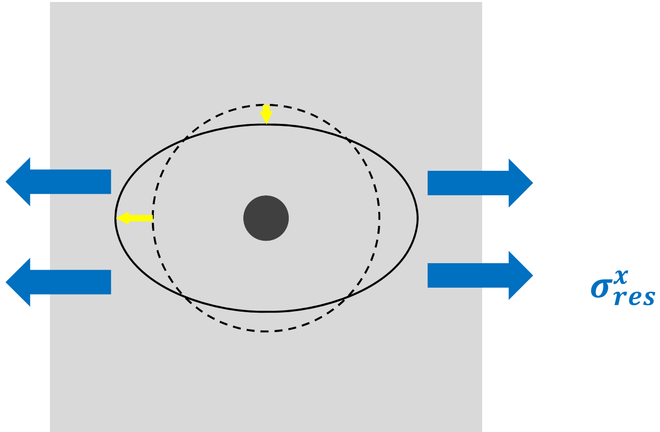


- **Evaluating surface displacement caused by indentation using DIC method.**
- **Confirming the dependence of surface displacement on residual stress.**

Basic Concept



Stress-free state



Stressed state

- Distribution of indentation surface displacement depends on stress state of specimen

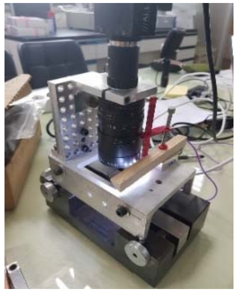
Relation between "indentation surface displacement" and "residual stress"

→ **Principal direction (θ_P), Stress ratio, p** $\left(p = \frac{\sigma_{res}^{y}}{\sigma_{res}^{x}} \right)$

Experimental Procedures

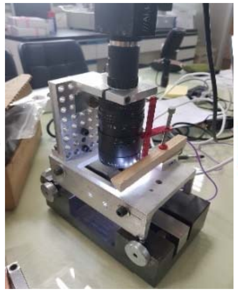
1) Apply uniaxial stress state to the steel beam
2) Select the testing area
3) Capture the reference image using DIC
4) Perform a single indentation (conical indenter / $h_{max} = 200\mu m$)
5) Capture the target image using DIC
6) Analyze the displacement change of testing area

1), 2)

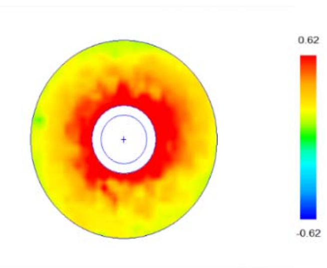


3)

4)



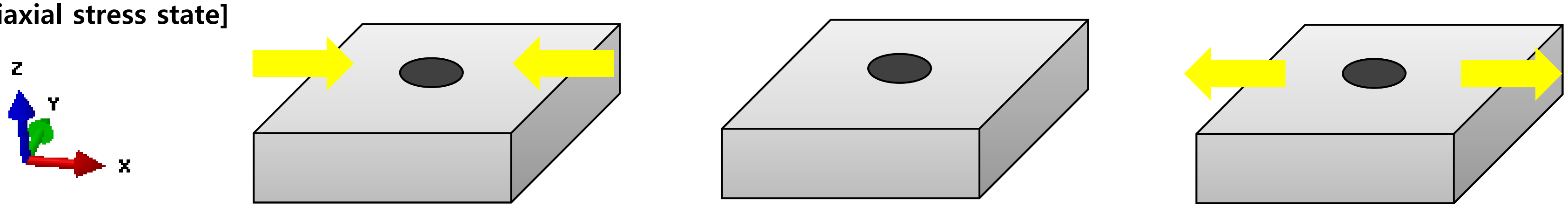
5)



6)

Experimental Result (SUS316L)

[Uniaxial stress state]



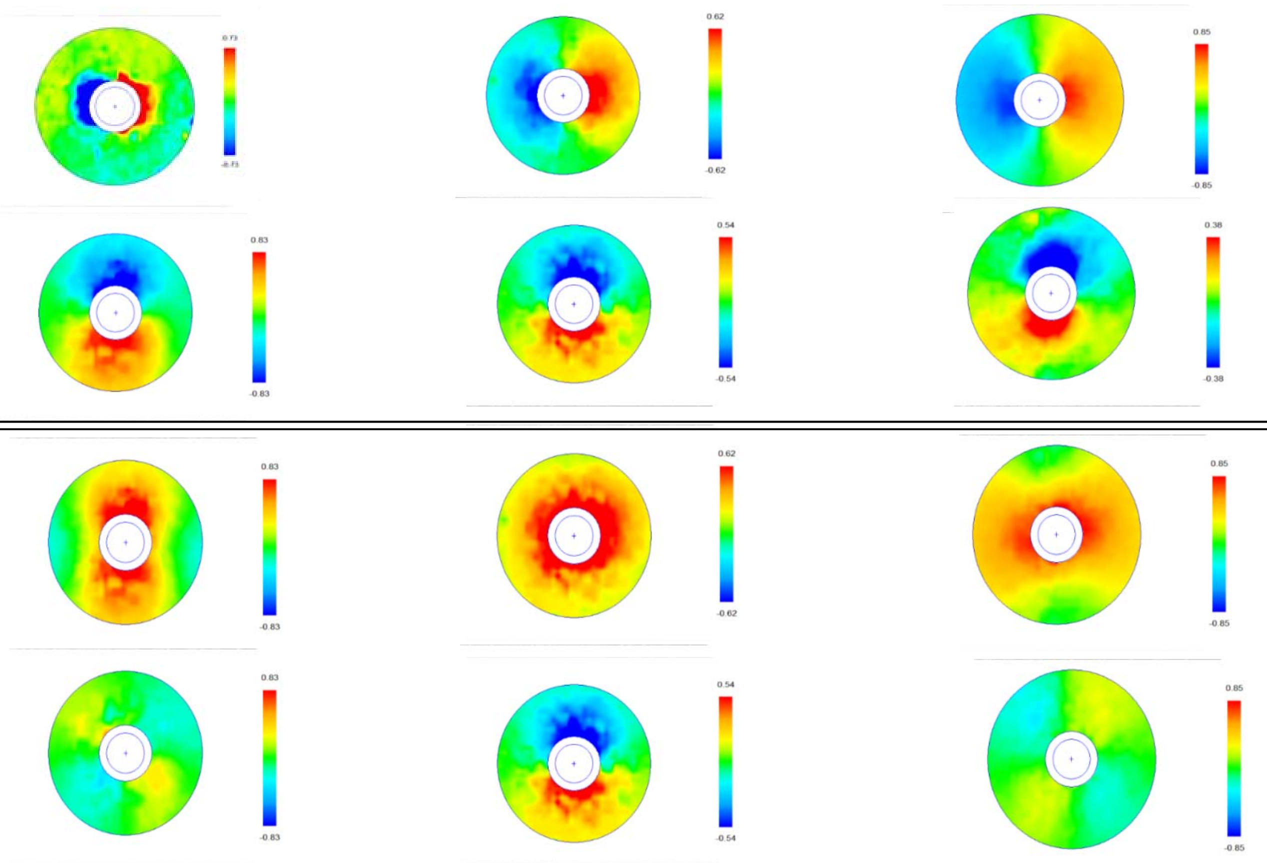
Compressive | Stress-free | Tensile

X displacement

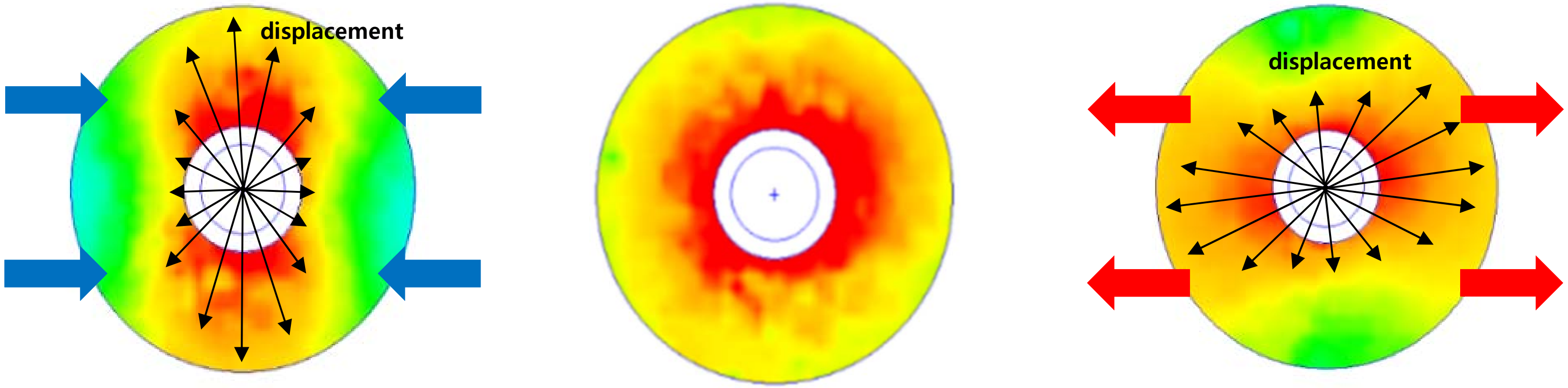
Y displacement

Radial displacement

Hoop displacement



Relation between stress and displacement

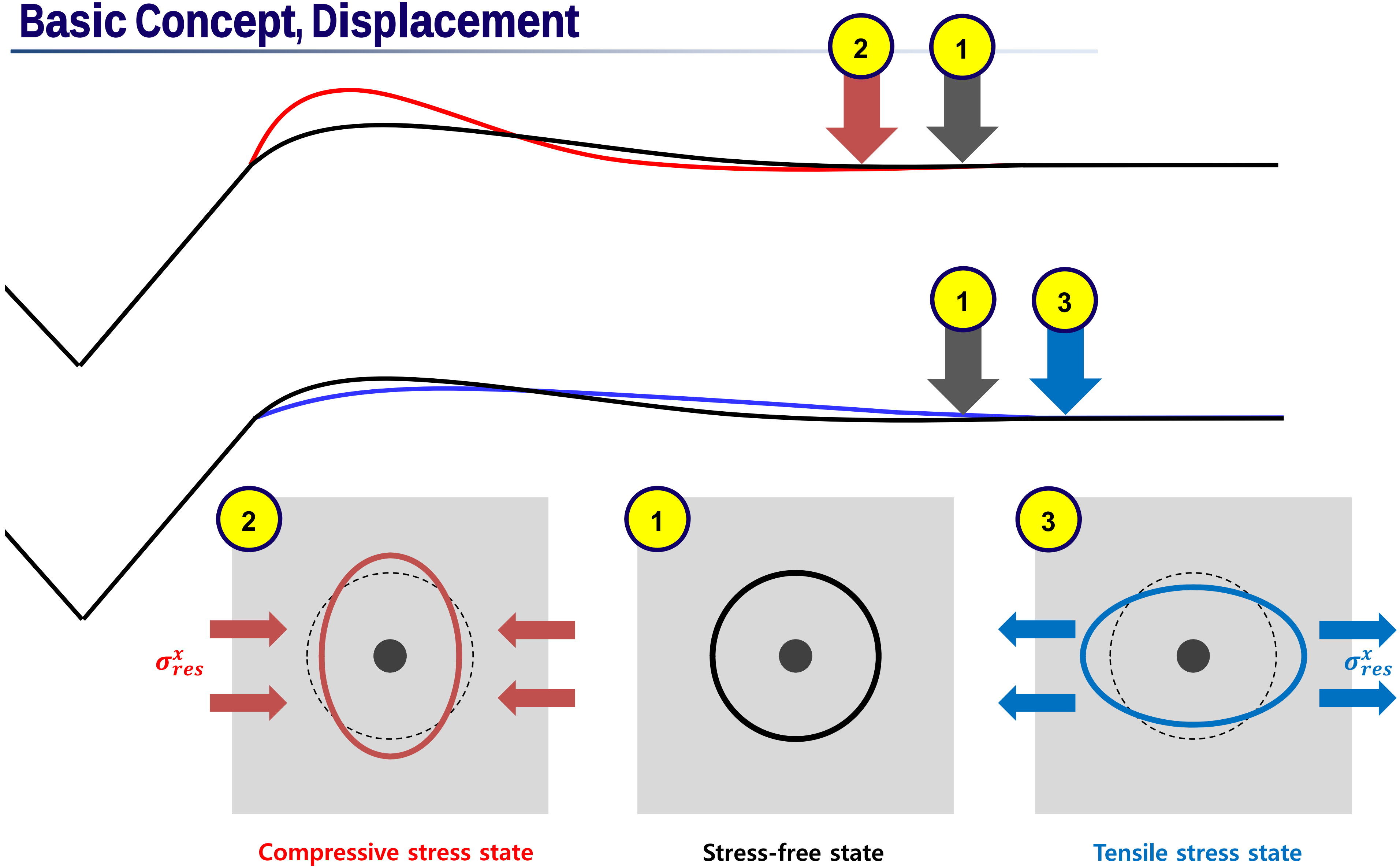


Compressive stress | Stress free | Tensile stress

<u>Residual stresses influence the indentation surface displacement</u>

- Compressive stress
  - **promotes** displacement in a vertical direction
  - **disturbs** displacement in a parallel direction
- Tensile stress
  - **promotes** displacement in a parallel direction
  - **disturbs** displacement in a vertical direction

Basic Concept, Displacement



- Distribution of indentation surface displacement depends on stress state of specimen

Basic Concept, Tensor

- **Stress-free state**

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- **Uniaxial stress state**

$$\begin{pmatrix} \sigma_{res}^{x} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \frac{\sigma_{res}^{x}}{3} & 0 & 0 \\ 0 & \frac{\sigma_{res}^{x}}{3} & 0 \\ 0 & 0 & \frac{\sigma_{res}^{x}}{3} \end{pmatrix} + \begin{pmatrix} \frac{2\sigma_{res}^{x}}{3} & 0 & 0 \\ 0 & -\frac{\sigma_{res}^{x}}{3} & 0 \\ 0 & 0 & -\frac{\sigma_{res}^{x}}{3} \end{pmatrix}$$

matched with ΔL

$$\sigma_{res}^{x} = \frac{3}{1+p}\frac{\Delta L}{A_c}$$

[Lee and Kwon, Acta Mater., 2004]

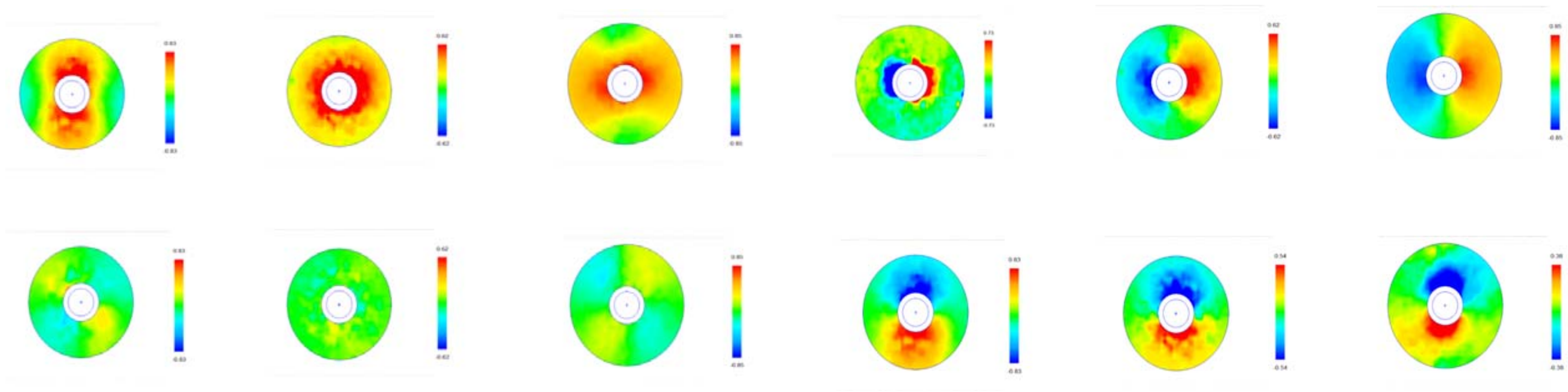
Effect on the plastic deformation in x-direction

$$\begin{matrix} \frac{2\sigma_{res}^{x}}{3} & 0 \\ 0 & -\frac{\sigma_{res}^{x}}{3} \end{matrix}$$

Effect on the plastic deformation in y-direction

β=-0.5

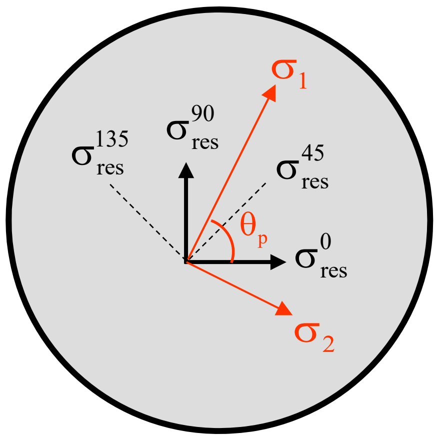
Directional Information



How ?

▶ **Principal Direction**

θ_P



▶ **Stress Ratio**

$$p = \frac{\sigma_2}{\sigma_1}$$

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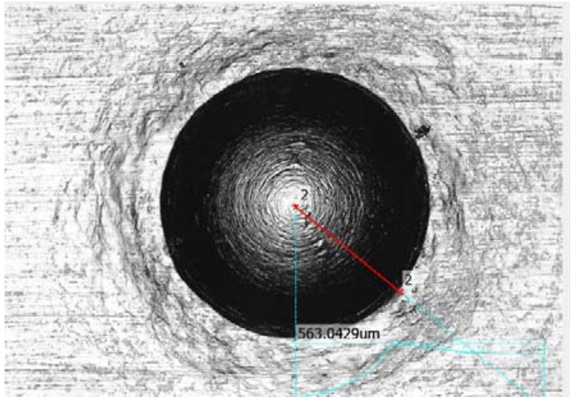
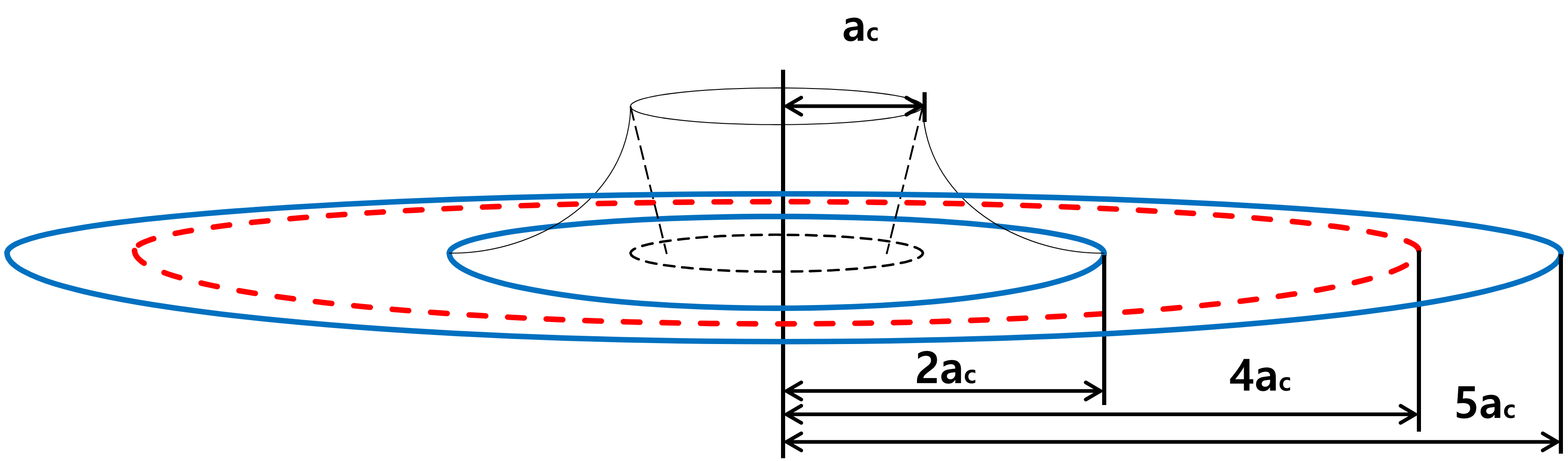
Analysis area

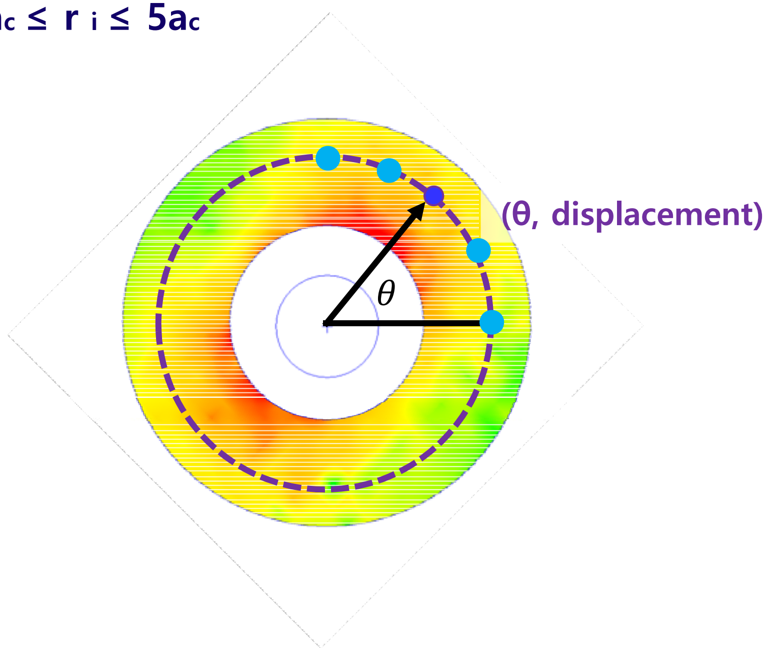


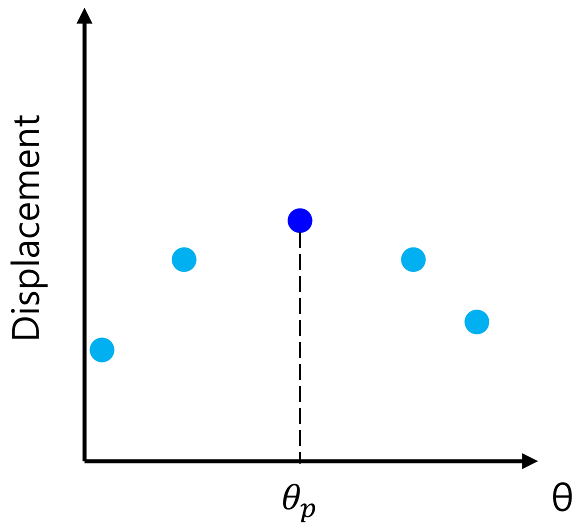
- Large plasticity around residual imprint : ~$2a_c$
- Pileup/sink-in (deformation in direction of indenting axis) causes error in 2D digital image correlation analysis
- The area between $2a_c$ ~ $5a_c$ is considered suitable for digital image correlation analysis



Determination of principal direction

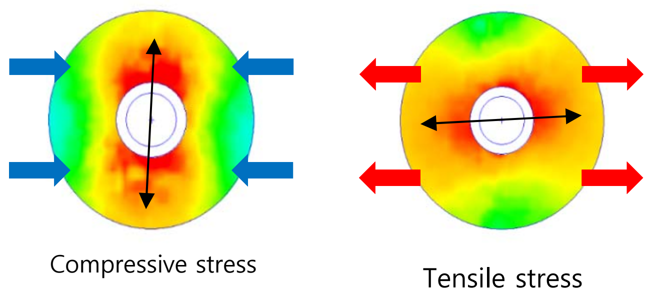
$2a_c \leq r_i \leq 5a_c$





Premise: "Maximum radial displacement occurs in principal directions"

1. Obtain x, y coordinates of maximum radial displacement in each circumference.
2. Display the θ, D coordinates.
3. Identify the direction of principal stresses.

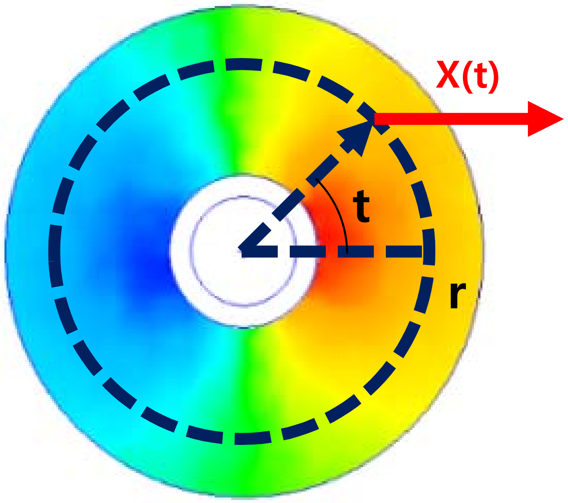


Quantification of Indentation Surface Displacement

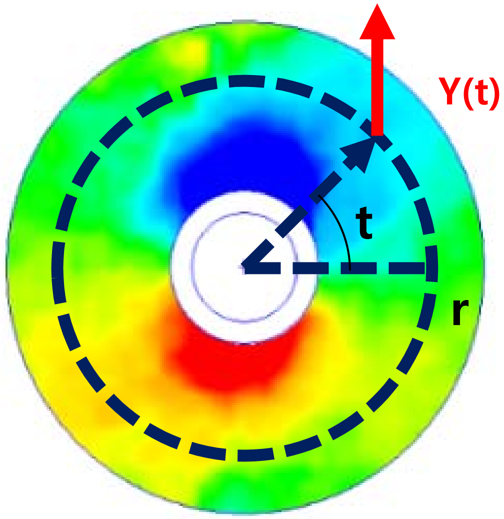
(1) Rotating the results by principal direction (setting principal direction to x, y-axis)

(2) Reanalyze the images to obtain x- and y-displacements

$X(t) = X\ displacement$ at radius of r



$Y(t) = Y\ displacement$ at radius of r



(3) Calculate D^x and D^y using X(t) and Y(t)

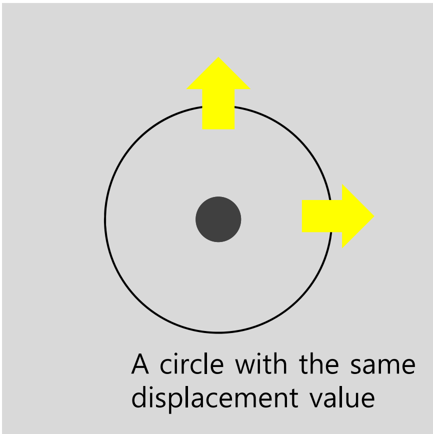
$$D^x = 4 \cdot \int_0^{\frac{\pi}{2}} X(t) \cdot dt$$

$$D^y = 4 \cdot \int_0^{\frac{\pi}{2}} Y(t) \cdot dt$$

$r = 4 \times a_c$
(a_c : contact radius)

Displacement Change in Uniaxial Stress State

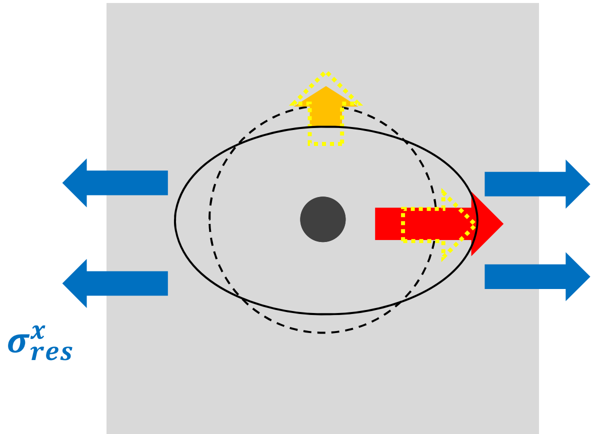
[Uniaxial stress state]



Stress-free state

$$D^{x,0}$$

$$D^{y,0}$$



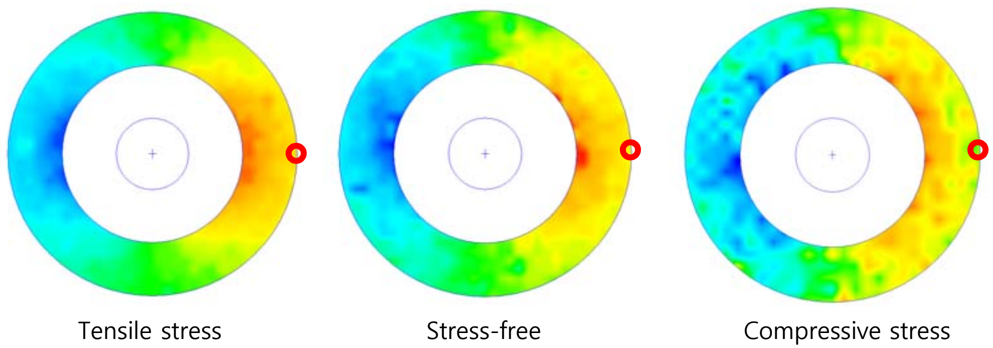
Stressed state

$$D^{x,s} = D^{x,0} + \Delta D_{res}^{x}$$

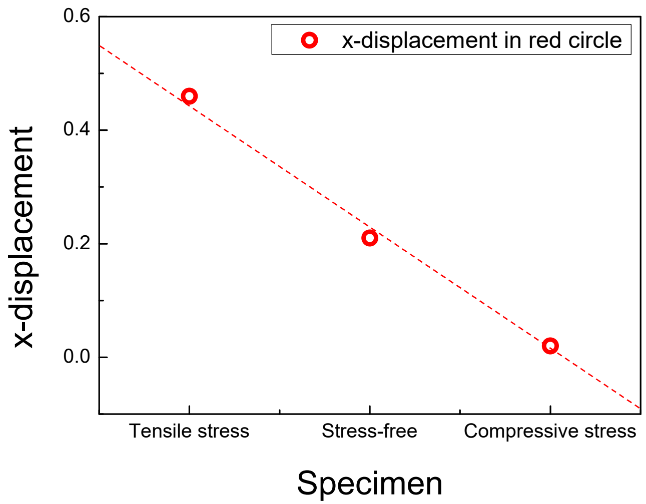
$$D^{y,s} = D^{y,0} + \Delta D_{res}^{y}$$

$$(\Delta D_{res}^{y} = \beta \cdot \Delta D_{res}^{x} = -0.5 \cdot \Delta D_{res}^{x})$$

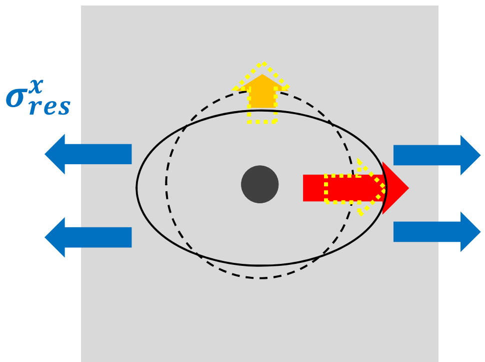
Parameter k



< x-displacement in uniaxial stress state >



x-displacement at red circle linearly depends on stress state



$$D^{x,s} = D^{x,0} + \Delta D_{res}^{x}$$

$$\Delta D_{res}^{x} \propto \sigma_{res}^{x}$$

$$\Delta D_{res}^{x} = k \cdot \sigma_{res}^{x}$$

k is a proportional factor

$$D^{x,s} = D^{x,0} + k \cdot \sigma_{res}^{x}$$

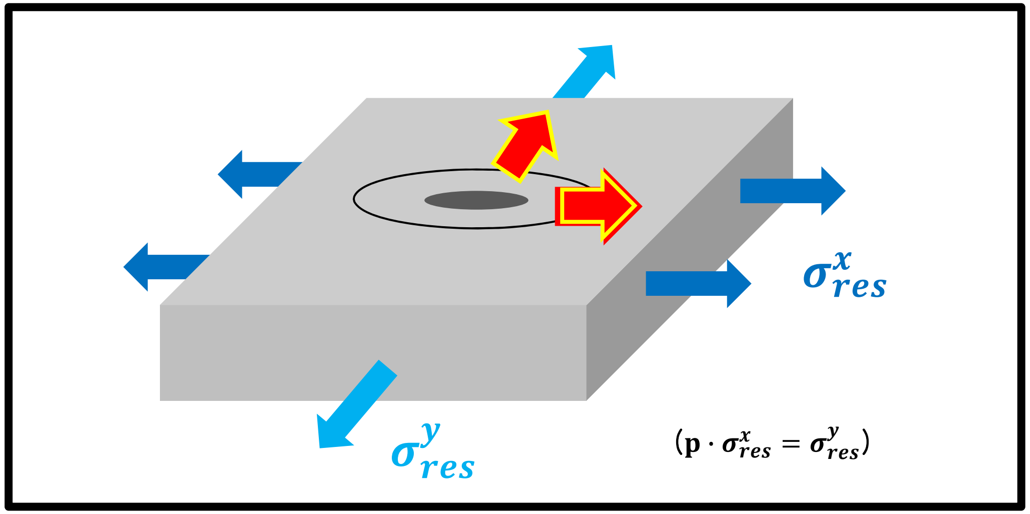
$$D^{y,s} = D^{y,0} + \Delta D_{res}^{y}$$

$$(\Delta D_{res}^{y} = -0.5 \cdot \Delta D_{res}^{x})$$

$$= D^{y,0} - 0.5 \cdot k \cdot \sigma_{res}^{x}$$

Displacement Change in Biaxial Stress State

[Biaxial stress state]



$$D^{x,s} = D^{x,0} + \Delta D_{res}^{x} + \beta \cdot \Delta D_{res}^{y} = D^{x,0} + k \cdot \sigma_{res}^{x} - 0.5 \cdot k \cdot \sigma_{res}^{y} = D^{x,0} + (1 - 0.5 \cdot p) \cdot k \cdot \sigma_{res}^{x}$$

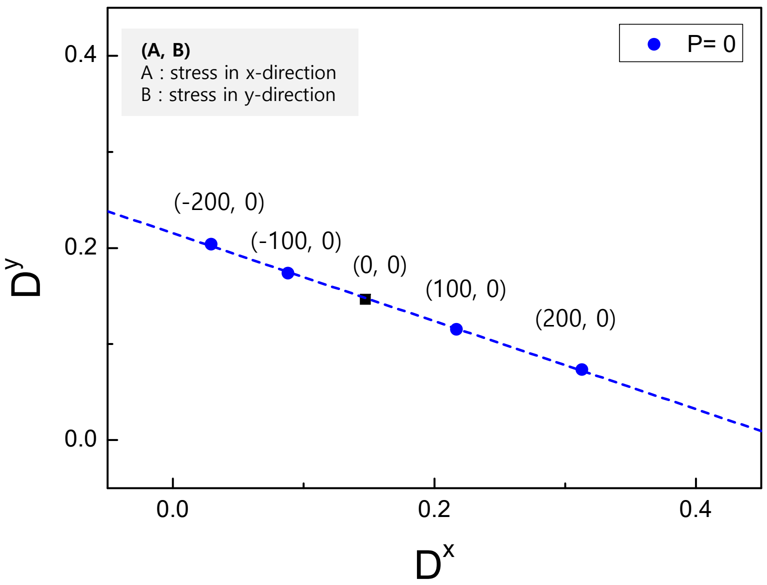
$$D^{y,s} = D^{y,0} + \Delta D_{res}^{y} + \beta \cdot \Delta D_{res}^{x} = D^{y,0} + k \cdot \sigma_{res}^{y} - 0.5 \cdot k \cdot \sigma_{res}^{x} = D^{y,0} + (-0.5 + p) \cdot k \cdot \sigma_{res}^{x}$$

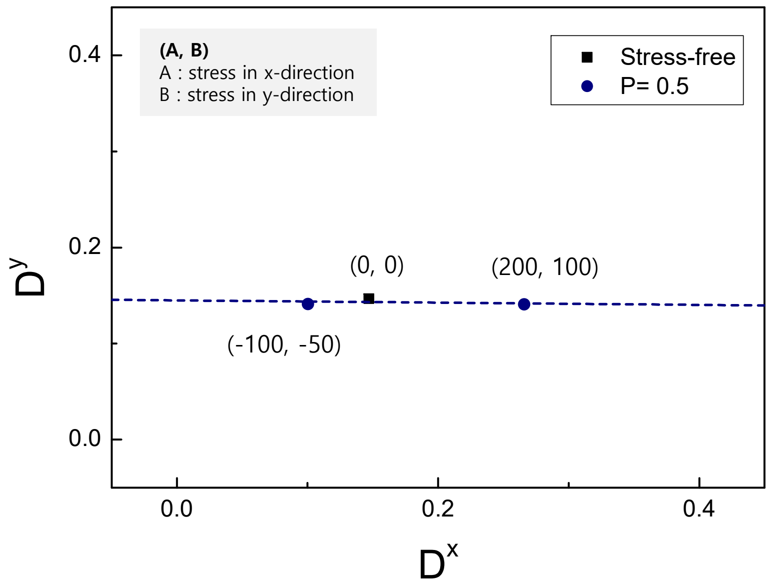
$$D^{x,s} - D^{x,0} = (1 - 0.5 \cdot p) \cdot k \cdot \sigma_{res}^{x}$$

$$D^{y,s} - D^{y,0} = (-0.5 + p) \cdot k \cdot \sigma_{res}^{x}$$

$$\rho = \frac{D^{y,s} - D^{y,0}}{D^{x,s} - D^{x,0}} = \frac{(-0.5 + p) \cdot k \cdot \sigma_{res}^{x}}{(1 - 0.5 \cdot p) \cdot k \cdot \sigma_{res}^{x}} = \frac{-0.5 + p}{1 - 0.5 \cdot p}$$

Same Stress Ratio (p)

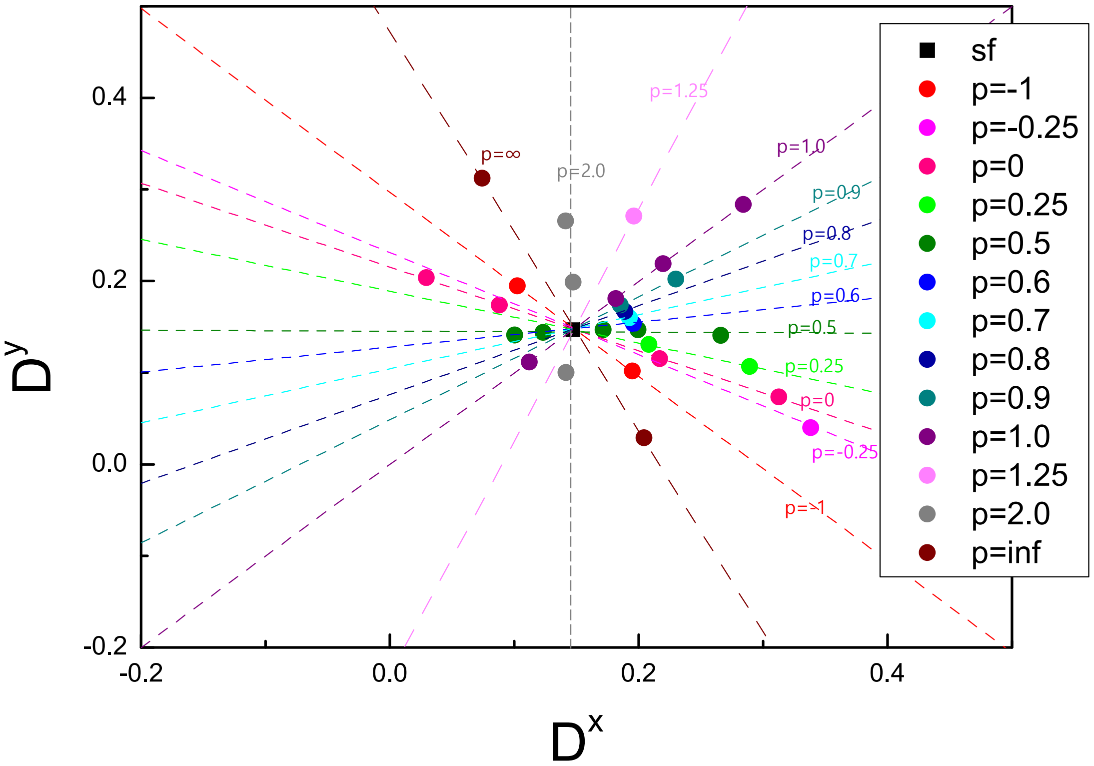


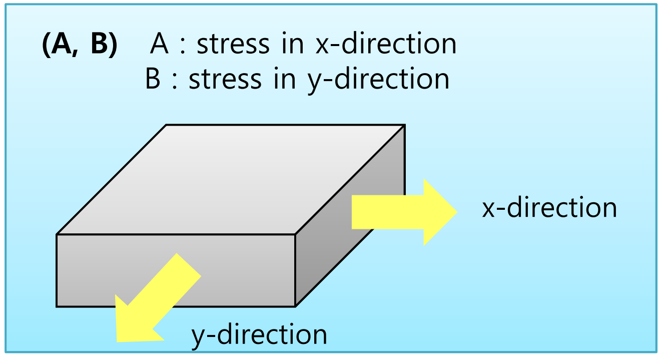


$$\rho = \frac{D^{y,s} - D^{y,0}}{D^{x,s} - D^{x,0}}$$

Same stress ratio (p) → ***Same slope (ρ)***

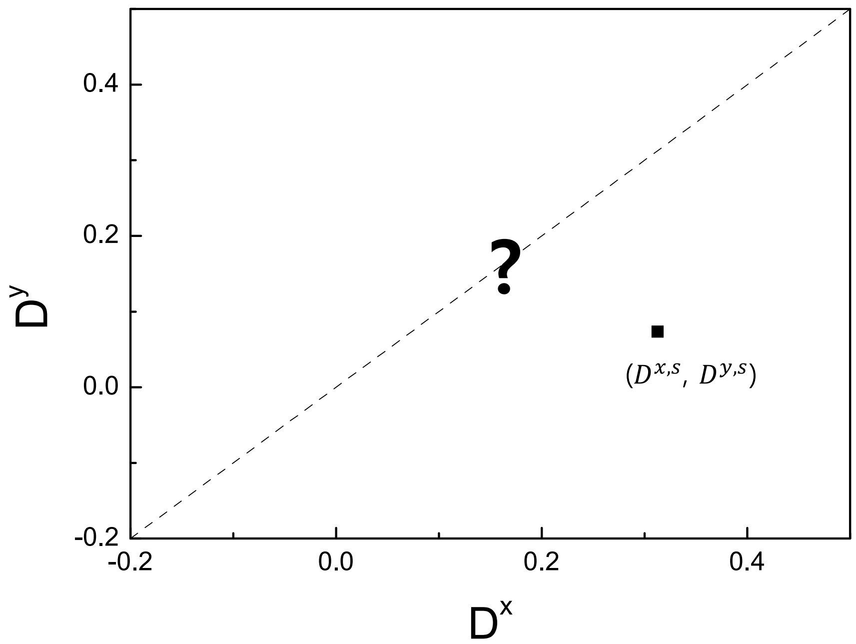
FEA Results





The slope (ρ) is a value determined by the ratio between principal stresses and rule of β.

Stress Free Point



$$D^{y,s} = \rho \cdot (D^{x,s} - D^{x,0}) + D^{y,0}$$

$(D^{x,0},\ D^{y,0})$ **?**

$$\rho = \frac{D^{y,s} - D^{y,0}}{D^{x,s} - D^{x,0}} = \frac{\boldsymbol{(-0.5 + p) \cdot k \cdot \sigma_{res}^{x}}}{\boldsymbol{(1 - 0.5 \cdot p) \cdot k \cdot \sigma_{res}^{x}}} = \frac{\boldsymbol{-0.5 + p}}{\boldsymbol{1 - 0.5 \cdot p}}$$

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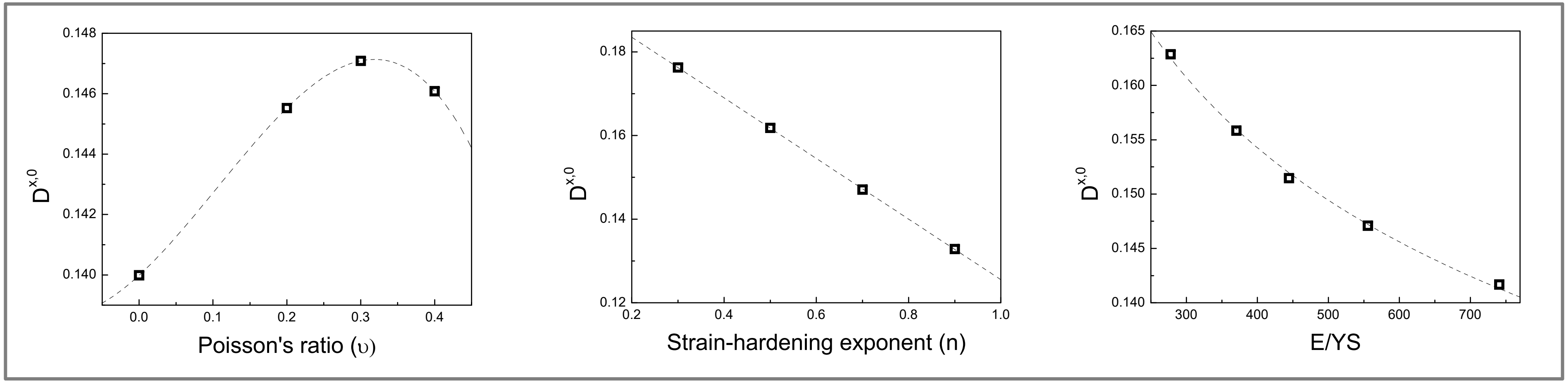
Mechanical Property Dependency – Stress Free Point

[from ABAQUS]



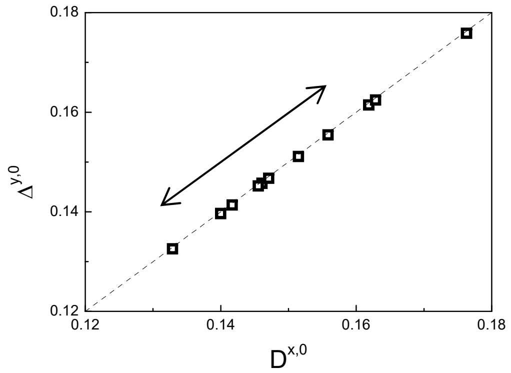
Effect of Strain-hardening exponent

$$D^{x,0} = 0.146746 \times (0.95174 + 0.15274 \cdot \nu + 0.47714 \cdot \nu^2 - 1.50022 \cdot \nu^3) \times (1.34625 - 0.49286 \cdot n) \times 2.46324 \cdot \left(\frac{E}{\sigma_y}\right)^{-0.1425}$$

($\Delta^{x,0} = \Delta^{y,0}$ for conical indentation)

Effect of Poisson's ratio

Effect of Yield strain



- For conical indentation, $D^{x,0} = D^{y,0}$
- Stress-free point depends on mechanical properties (Poisson's ratio, Strain-hardening exponent, E/YS)

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Thank you!