457.562 Special Issue on River Mechanics (Sediment Transport)

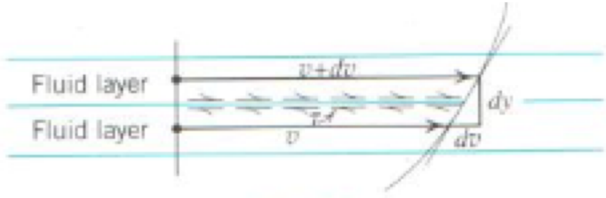
.02 Review of Fluid Mechanics



1. Flow velocity distribution : law of the wall

- Shear stress between adjacent layers in a simple parallel flow is det ermined by the viscosity and the velocity gradient

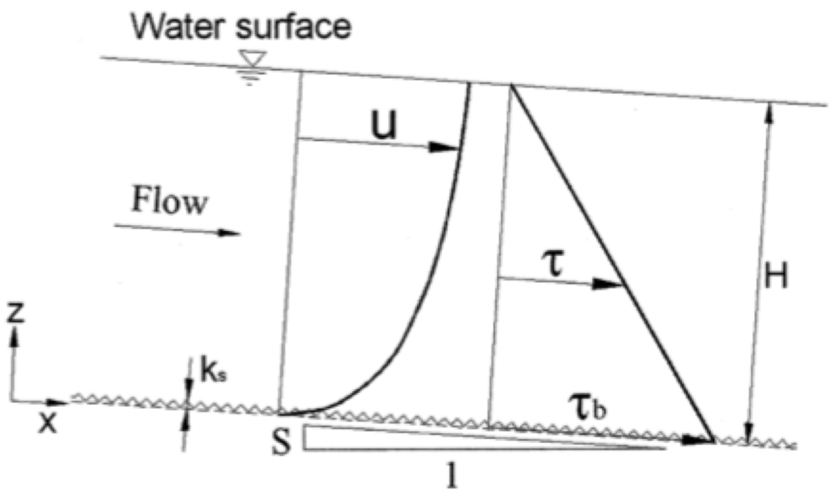
$$\tau = \mu \frac{dv}{dy}$$



- If laminar flow is disturbed by an obstacle or roughness, then disturb ances must be damped by the molecular viscosity (stable)
- However, in turbulent flow, inertia overcomes the viscous force and disturbance grows and becomes random motion.
- Therefore, in turbulent motion rapid mixing of fluid particles during fl ow cause significant diffusion.
- What is the actual meaning of viscosity? Is there any other name of viscosity?

1. Flow velocity distribution : law of the wall

- Consider a steady, turbulent, uniform, open-channel flow having
  - H : a mean depth
  - U : a mean flow depth
  - B : a mean width (much greater than depth)
  - S : a mean slope
  - k_s : effective height
  - In a wide channel (B/H >>1)



1. Flow velocity distribution : law of the wall

- The roughness height (or effective height) will be proportional to a m ean size or diameter *D.*
- The bed shear stress

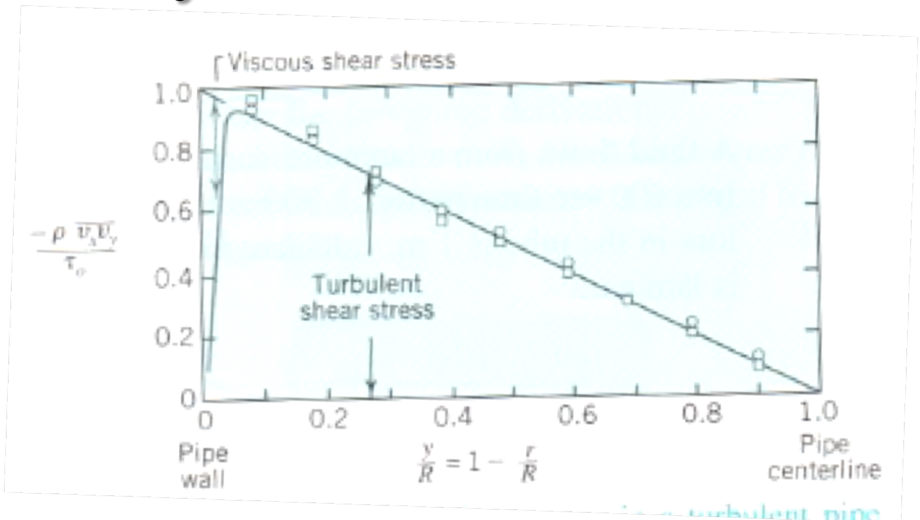
$$\tau_b = \rho g H S \qquad (\rho = \text{water density}, \; g\text{=gravity})$$

- The above equation is the one-dimensional momentum conservatio n equation
- Using the boundary shear stress, the shear velocity can be defined as

$$u_* = \sqrt{\tau_b / \rho}$$

- The sear velocity (the boundary shear stress)
  - provides a direct measure of the flow intensity
  - able to entrain and transport sediment particles.

1. Flow velocity distribution : law of the wall



- The above figure shows typical variation of total shear stress in a tur bulent pipe flow.
- Based on the linear relationship of shear profile (in the previous clas s), and the Prandtl relationship,

$$\tau = \tau_b \left(1 - \frac{z}{H}\right) = \rho l^2 \left(\frac{du}{dz}\right)^2$$

1. Flow velocity distribution : law of the wall

- Near wall, *l=kz*, and mixing length in a pipe flow

$$l = kz\left(1 - \frac{z}{H}\right)^{1/2}$$

- Then the previous equation can be written

$$\frac{\tau_b}{\rho\kappa^2 z^2} = \left(\frac{du}{dz}\right)^2$$

$$\frac{du}{dz} = \frac{\sqrt{\tau_b / \rho}}{\kappa z} = \frac{u_*}{\kappa z}$$

Only in a relatively thin layer (z/H=0.2)

$$\frac{u}{u_*} = \frac{1}{\kappa}\ln\left(\frac{z}{z_0}\right) \qquad (1)$$

u = time -averaged flow velocity at a distance z above the bed

z_0 = bed roughness length (distance above the bed where $u(z_0) = 0$)

$\kappa =$ is von Karman's constant ~0.41.

1. Flow velocity distribution : law of the wall

- Through intensive experiment works,

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{u_* z}{\nu} + 5.5 \qquad (2)$$

- When the effective roughness height becomes by (1) and (2)

$$z_b = \nu / 9u_*$$ → Hydraulic smooth condition of flow

- Very near the smooth wall (viscous sublayer), velocity is linear with depth

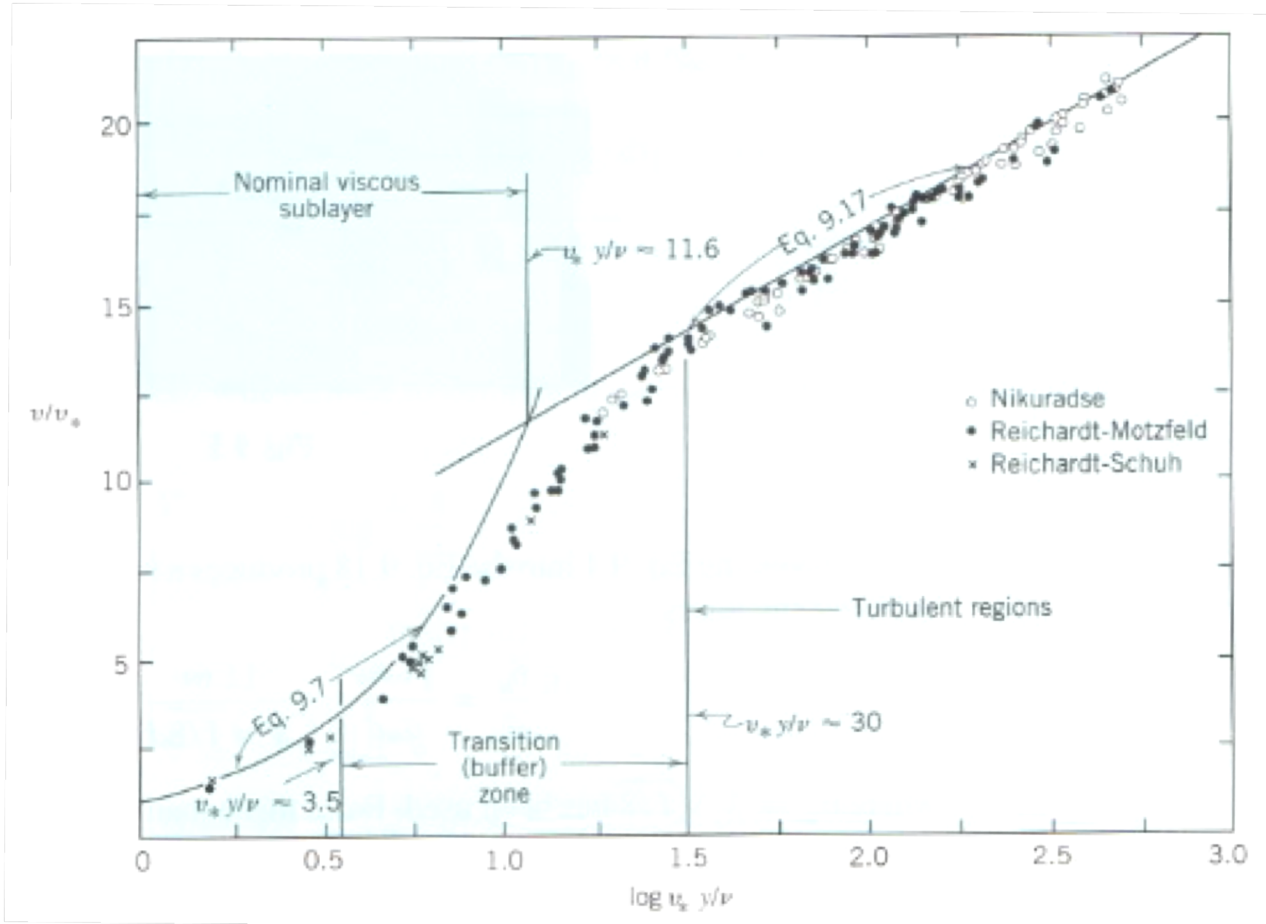
$$\frac{u}{u_*} = \frac{u_* z}{\nu} \qquad (\text{until } z = \delta_\nu)$$

- Then two equations above need to match

$$\frac{u}{u_*} = \frac{u_* \delta_\nu}{\nu} = \frac{1}{\kappa} \ln \frac{u_* \delta_\nu}{\nu} + 5.5 \qquad (at\ z = \delta_\nu)$$

$$\frac{u_* \delta_\nu}{\nu} = 11.6 \qquad \delta_\nu = 11.6 \frac{\nu}{u_*}$$

1. Flow velocity distribution : law of the wall



- Velocity distribution near a smooth wall

1. Flow velocity distribution : law of the wall

- But, most boundaries in river flow are rough. Therefore we need to c onsider roughness height.
- If $k_s/\delta_v>1$, then viscous sublayer will not exist.
- The corresponding logarithmic velocity profile is given by

$$\frac{u}{u_*}=\frac{1}{\kappa}\ln\left(\frac{z}{k_s}\right)+8.5=\frac{1}{\kappa}\ln\left(30\frac{z}{k_s}\right)$$

- It follows that

$$z_0=k_s/30 \Rightarrow$$ Hydraulically rough flow

- Then what happens in the intermediate condition?

1. Flow velocity distribution : law of the wall

- Hydraulically rough flow $\frac{k_s}{\delta_v} \gg 1, \mathrm{Re}_* \gg 11.6$
- Hydraulically smooth flow $\frac{k_s}{\delta_v} \ll 1, \quad \mathrm{Re}_* \ll 11.6$
- Where " the Roghness Reynolds number"

$$\mathrm{Re}_* = \frac{u_* k_s}{\nu}, \qquad (\mathrm{R}_{*C} = 11.6)$$

- Yalin (1992) proposed a way to represent both ranges and intermedi ate ranges as

$$\frac{\bar{u}}{u_*} = \frac{1}{\kappa} \ln\left(\frac{z}{k_s}\right) + B_s$$

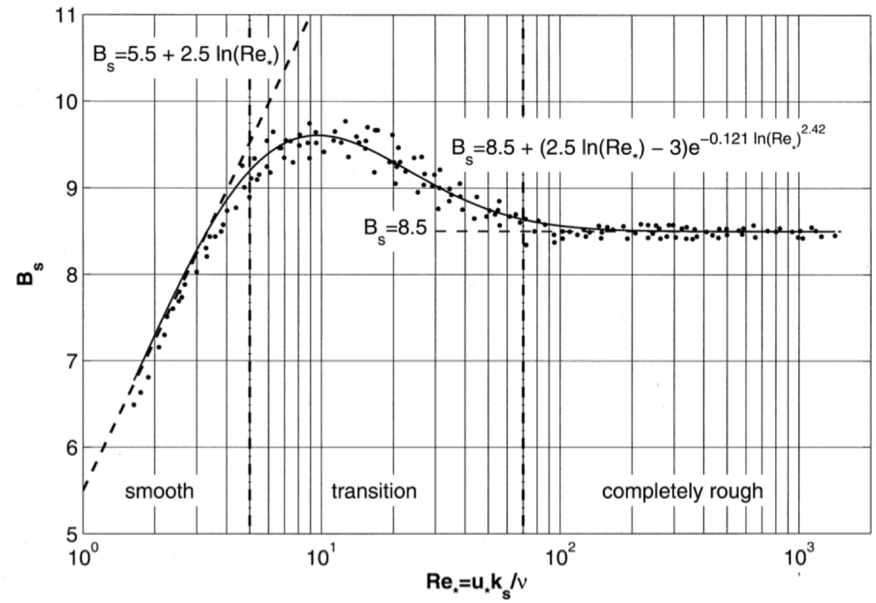
$$B_s = 8.5 + \left[2.5 \ln\left(\mathrm{Re}_*\right) - 3\right] e^{-0.121[\ln \mathrm{Re}_*]^{2.42}}$$

1. Flow velocity distribution : law of the wall

- Alternative way of writing,

$$\frac{\bar{u}}{u_*} = \frac{1}{\kappa}\ln\left(A_s \frac{z}{k_s}\right)$$

$$A_s = e^{\kappa B_s}$$



2. Velocity-Defect and Log-Wakes laws

- Equation (1) requires some knowledge of the bed roughness charact eristics.
- An alternative formulation can be obtained if the flow depth H is intro duced as the relevant length scale.
- Assuming that the maximum flow velocity u_{max} takes places at the w ater surface z=H, then equation (1) can be manipulated to obtain the velocity-defect law (outer low of the wall).

$$\frac{u_{max} - u}{u_*} = -\frac{1}{\kappa}\ln\frac{z}{H} \qquad (3)$$

- But, there were so many arguments and defects or log law working only in z/H<0.2. Nezu and Nakagawa added a wake function.

$$\frac{u}{u_*} = \frac{1}{\kappa}\ln\left(\frac{u_* z}{\nu}\right) + 5.5 + w\left(\frac{z}{H}\right)$$

2. Velocity-Defect and Log-Wakes laws

- W(z/H) is the wake function first proposed by Coles (1956) for turbul ent boundary-layer flows,

$$w\left(\frac{z}{H}\right)=\frac{2W_0}{\kappa}\sin^2\left(\frac{\pi}{2}\frac{z}{H}\right)$$

- W_0 is the Coles wake parameter, expressing the strength of the wak e function.
- Equation (3) can also be written in log-wake form

$$\frac{u_{max}-u}{u_*}=-\frac{1}{\kappa}\ln\left(\frac{z}{H}\right)+\frac{2W_0}{\kappa}\cos^2\left(\frac{\pi z}{2H}\right)$$

- Coles wake parameter varies depending on the condition and incras es with increasing sediment concentration raging from 0.191 to 0.86 1.

2. Relations for channel resistance

- Most river flows are indeed hydraulically rough.
- The previous log law is used to obtain an appropriate expression for depth-averaged velocity U.

$$U = \frac{1}{H}\int_0^H u\,dz$$

$$\frac{U}{u_*} = \frac{1}{H}\int_{k_s}^{H}\left[\frac{1}{\kappa}\ln\left(\frac{z}{k_s}\right) + 8.5\right]dz$$

(Avoiding z=0 : singular point)

$$\frac{U}{u_*} = \frac{1}{\kappa}\ln\left(\frac{H}{k_s}\right) + 6 = \frac{1}{\kappa}\ln\left(11\frac{H}{k_s}\right)$$

- This relation is known as Keulegan’s resistance law for rough flow.

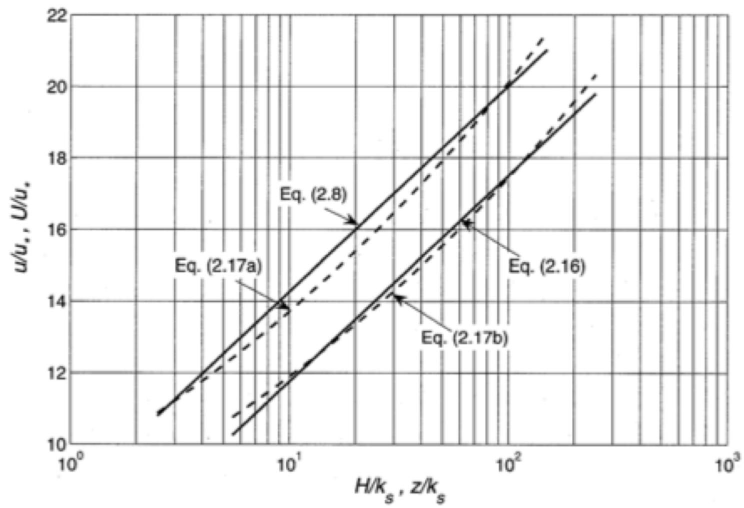
2. Relations for channel resistance

- Manning-Strickler form.

$$\frac{u}{u_*} = \frac{1}{\kappa}\ln\left(30\frac{z}{k_s}\right) \cong 9.34\left(\frac{z}{k_s}\right)^{1/6}$$

$$\frac{U}{u_*} = \frac{1}{\kappa}\ln\left(11\frac{H}{k_s}\right) \cong 8.1\left(\frac{H}{k_s}\right)^{1/6}$$

- Log - > power : was first presented by Keulegan.



2. Relations for channel resistance

- Bed shear stress can be found as

$$\tau_b = \rho u_*^2 = \rho C_f U^2$$

$$C_f = \left[\frac{1}{\kappa} \ln\left(11 \frac{H}{k_s} \right) \right]^{-2} \quad or \quad \left[8.1 \left(\frac{H}{k_s} \right)^{1/6} \right]^{-2}$$

- In two-dimensional case

$$\left(\tau_{bs}, \tau_{bn} \right) = \rho C_f \left[U^2 + V^2 \right]^{1/2} (U, V)$$