457.644 Advanced Bridge Engineering

Aerodynamic Design of Bridges

Part 1: Wind Characteristics in Boundary Layer

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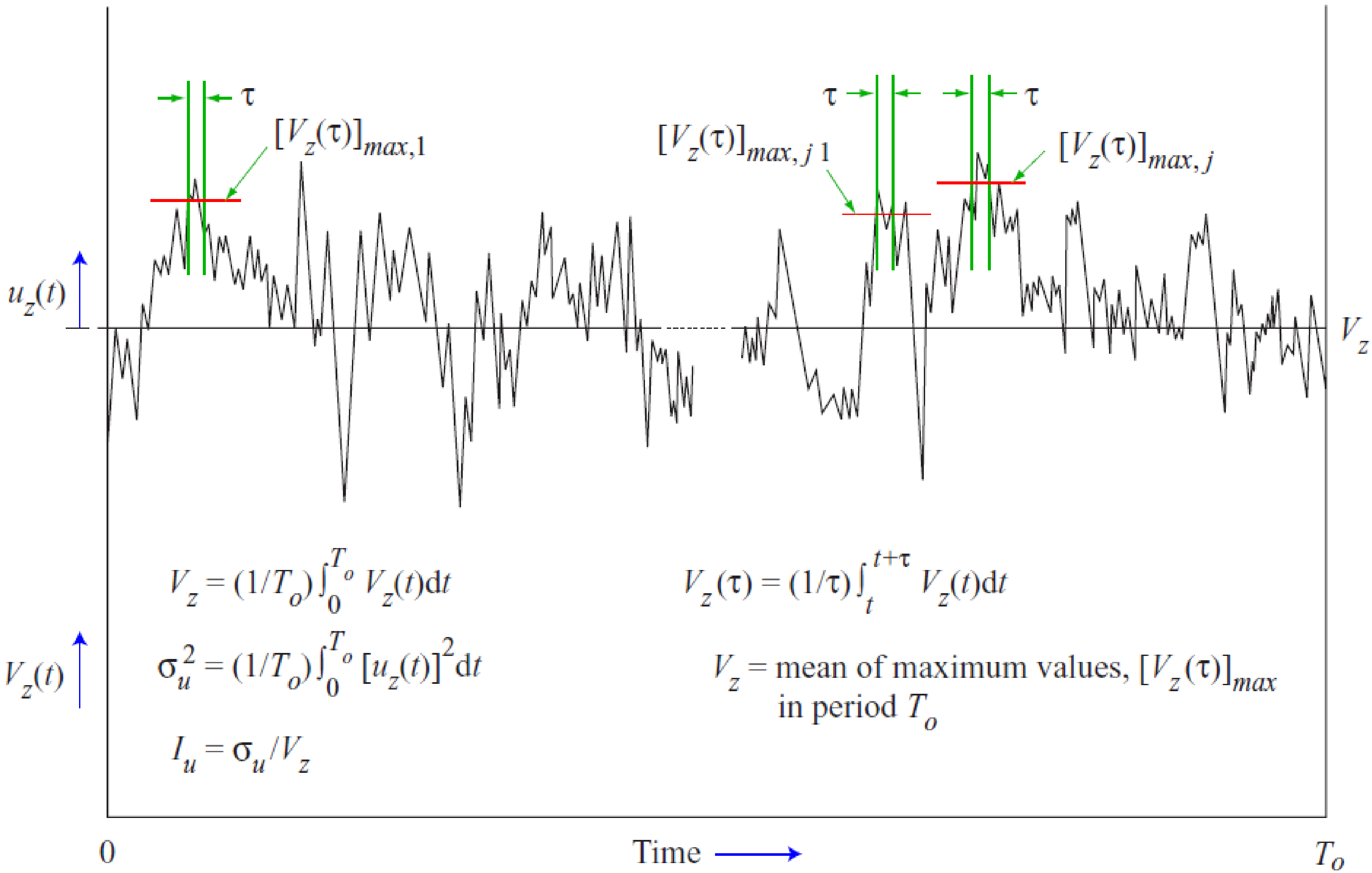
Definition of Wind Speed Characteristics



Wind Speeds and Averaging Times

- **If the flow were laminar, wind speeds would be the same for all averaging time. However, owing to turbulent fluctuations, the definition of wind speeds depends on averaging time.**
- **Mean wind speed**

$$\int_0^T u(t)\,dt \Big/ T$$, where T is averaging times

e.g.) 10min. averaging speed in KBDC, Japan, Eurocode, WMO

1hr. Averaging in National Building Code, Canada, ASCE7, ESDU

- **Instantaneous maximum wind speed**

$$\max\ \left[u(t)\right]_{t=0}^{t=T}$$

- **Peak 3-s gust speed, ASCE**

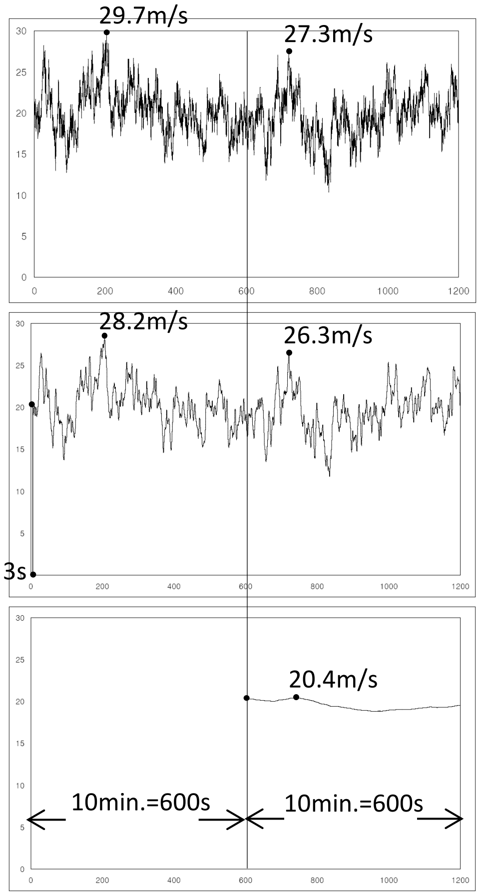
$$\max\left[\int_t^{t+3} u(t)dt \Big/ 3\right]_{t=0}^{t=T}$$

Wind Speeds and Averaging Times

- Gust factor = Max. wind speed / Mean wind speed

Raw data

(50Hz sampling)



→ Instantaneous max. speed=29.7m/s

e.g.1)

3sec-averaged

→ Peak 3-s gust speed=28.2m/s

e.g.2)

10min.-averaged

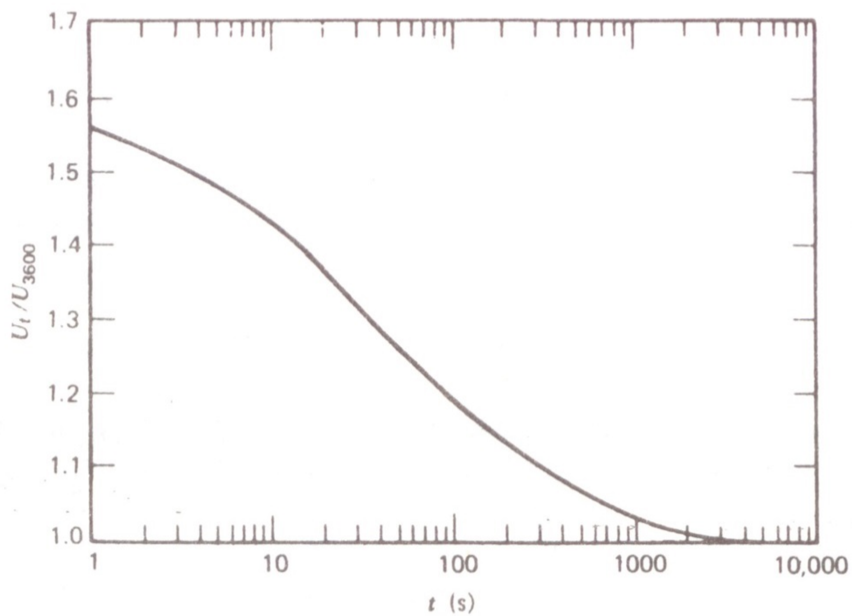
→ 10-min. averaged max. speed =20.4m/s

Wind Speeds and Averaging Times

Wind speeds with different averaging times for open terrain

- The approximate mean ratio *r* of the *t*-s speed to the hourly (3600-s) speed at 10m above ground in open terrain is listed for selected values of *t* as follows:

t (s)	3	5	40	60	600	3600
r	1.52	1.49	1.29	1.25	1.1	1.0



< Ratio of probable maximum speed averaged over period t to that averaged over one hour >

Wind Speeds and Averaging Times

Example 1

- For a peak 3-s gust speed at 10m over open terrain of 30m/s,
  - Corresponding hourly speed is 30/1.52=19.7m/s
  - Corresponding 10-min averaged speed is 19.7*1.1=21.7m/s

Example 2

- For the time history of wind speed in page 2
  - 10-min averaged max. wind speed=20.4m/s
  - Peak 3-s gust speed=28.2m/s

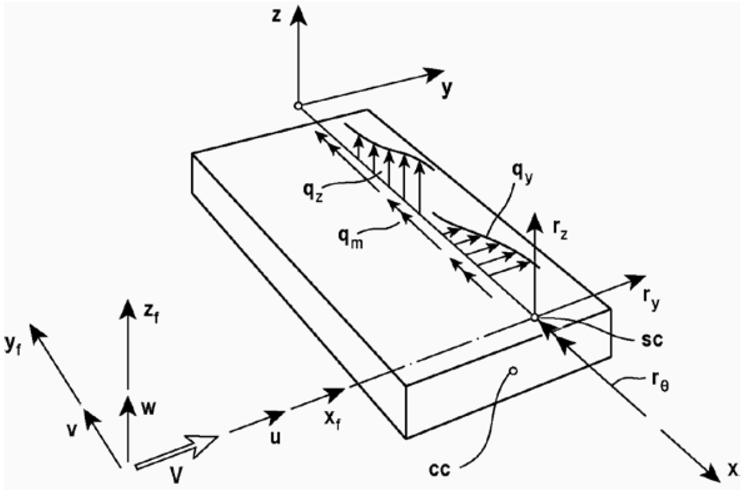
$$\frac{\text{peak } 3s \text{ gust speed}}{10-\text{min. averaged max. speed}} = 1.38 \approx \frac{1.52}{1.1} = 1.38$$

Example 3

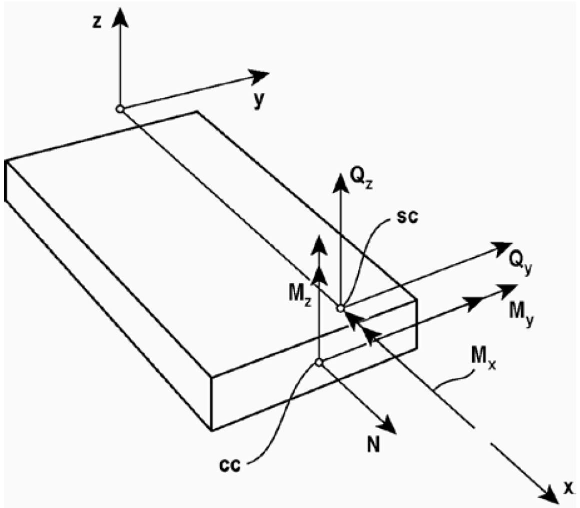
- For a fastest-mile wind speed at 10m over open terrain of 90mph
  - The averaging time is 3600/90=40s.
  - Corresponding hourly speed is 90/1.29=69.8mph
  - Corresponding peak 3-s gust is 69.8*1.52=106mph

Definition of Flow

Theory of Bridge Aerodynamics (Strommen)

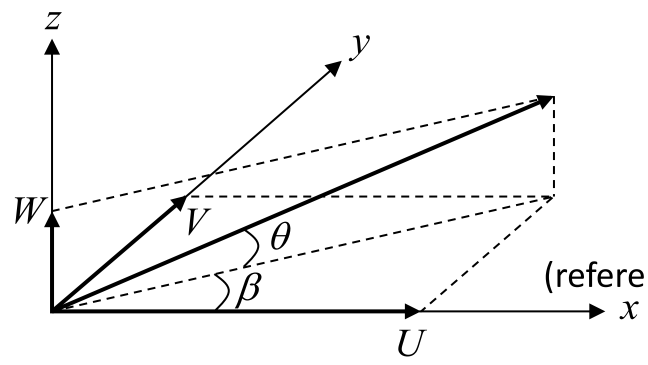


a) *Definition of flow and structural axes, displacements and loads*



b) *Definition of cross sectional forces (stress resultants)*

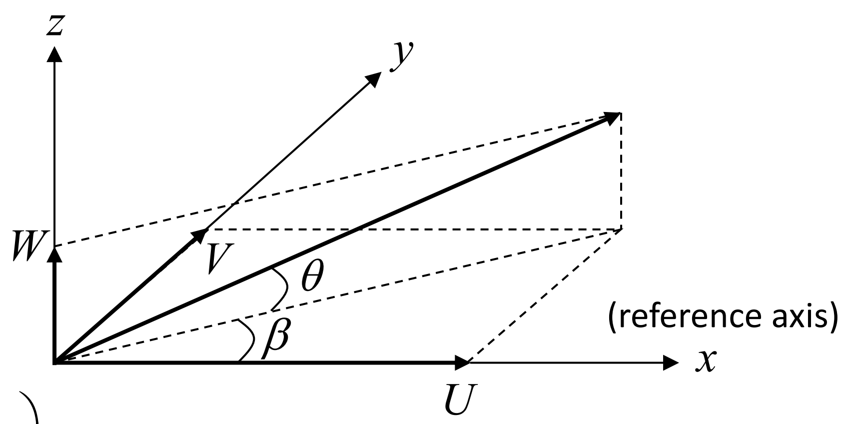
Wind Resistant Design of Bridges (Fujino et al.)



Components of Wind Speed

- Wind direction

$$\beta = \arctan\left(\frac{V}{U}\right)$$



- Incidence angle

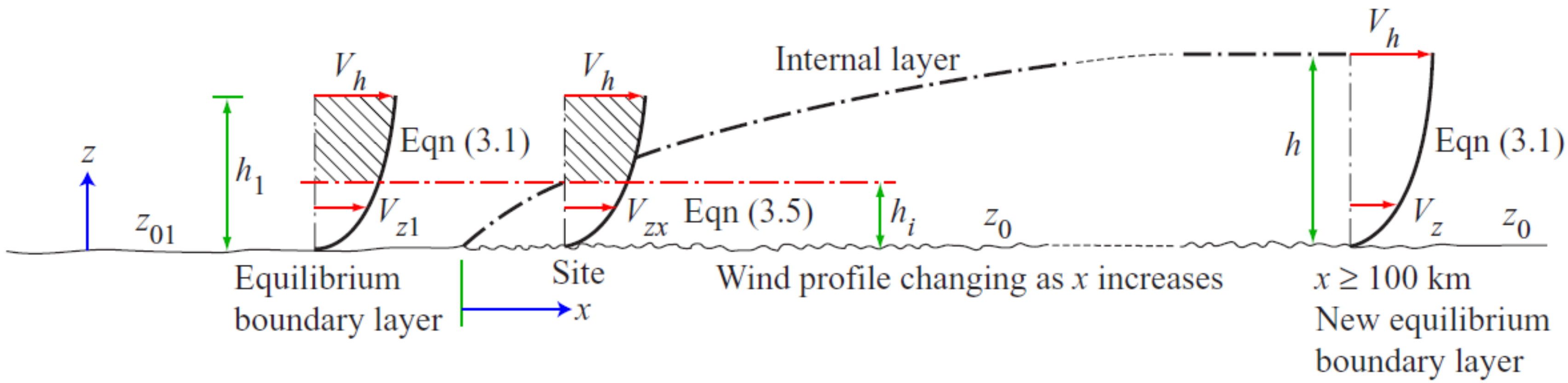
$$\theta = \arctan\left(\frac{W}{\sqrt{U^2+V^2}}\right)$$, where U, V, W=Mean wind speed according to x, y, z

Time averaging

- Wind direction $$\beta = \int_{t=0}^{T} \arctan\left(\frac{v(t)}{u(t)}\right) dt \Big/ T$$

- Incidence angle $$\theta = \int_{t=0}^{T} \arctan\left(\frac{w(t)}{\sqrt{u(t)^2+v(t)^2}}\right) dt \Big/ T$$

Developing Wind Profile



Logarithmic Law

Wind profile

$$\overline{U}(z) = 2.5\, u_* \ln \frac{z}{z_0}$$

where $\overline{U}(z)$: Mean wind speed at elevation z

u_* : friction velocity

z_0 : roughness length

Determine friction velocity u_*

- Let the mean wind speed 40m/s at the height of 10m
- Let roughness length $z_0 = 0.01\, m$

$$u_* = \frac{1}{2.5} \cdot \overline{U}(10) \Big/ \ln\left(\frac{10}{z_0}\right) = \frac{1}{2.5} \times 40 \Big/ \ln\left(\frac{10}{0.01}\right) = 2.316\, m/s$$

General form of logarithmic law

$$\overline{U}(z) = \overline{U}(z_{ref}) \frac{\ln(z/z_0)}{\ln(z_{ref}/z_0)}$$

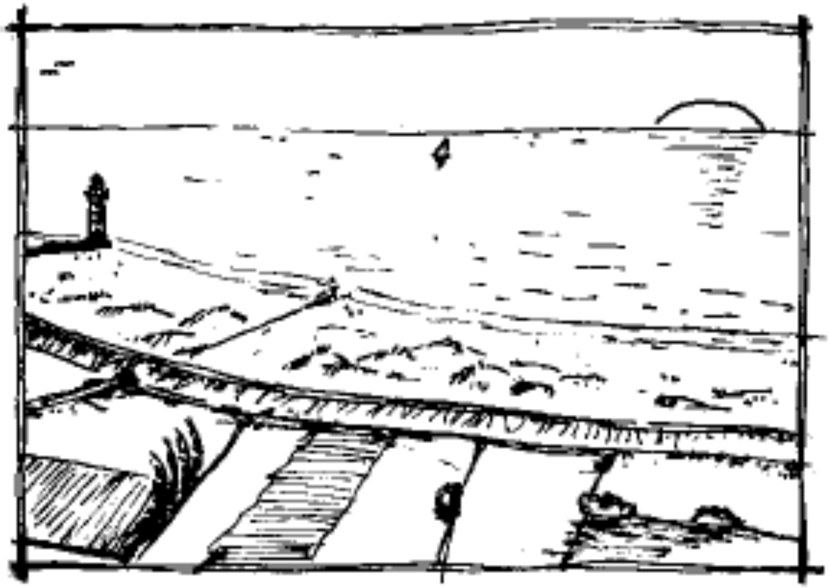
where z_{ref} : reference elevation

Roughness Length Z_0

e.g. Eurocode

Terrain category 0 (z_0=0.003m)

Sea, coastal area exposed to the open sea

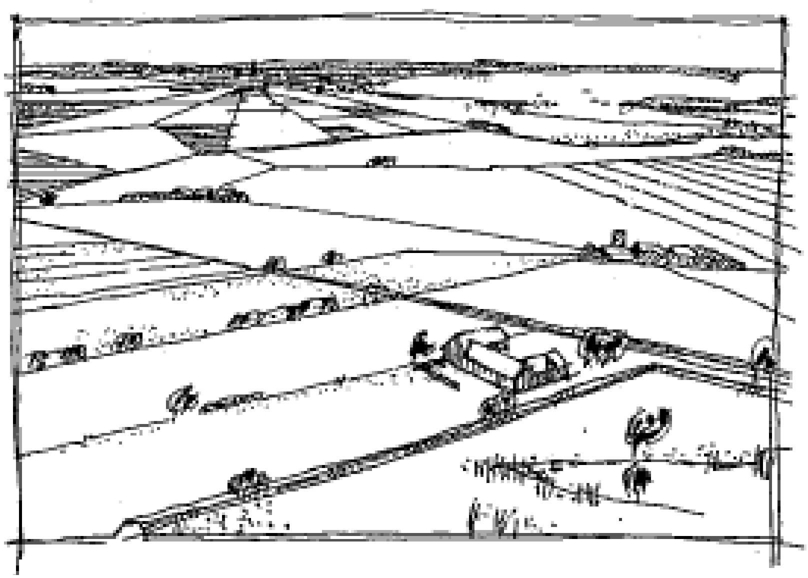


Roughness Length Z_0

e.g. Eurocode

Terrain category I (z_0=0.01m)

Lakes or area with negligible vegetation and without obstacles



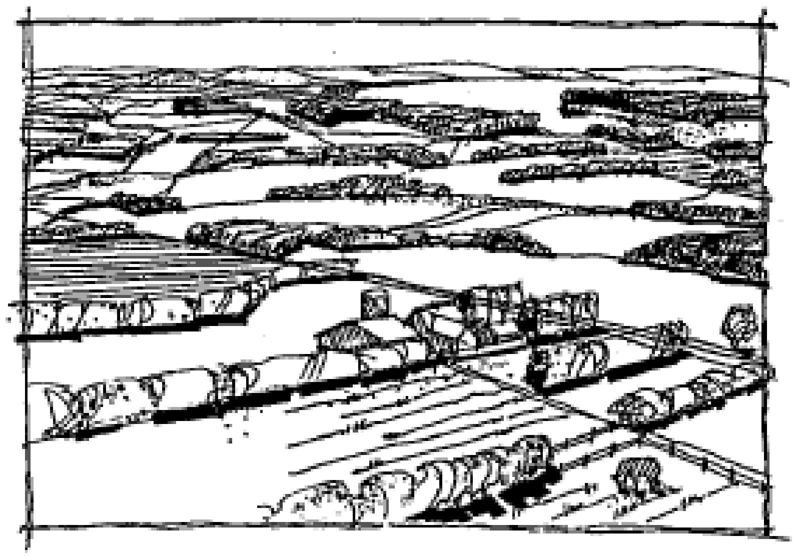
Roughness Length Z_0

e.g. Eurocode

Terrain category II (z_0=0.05m)

Area with low vegetation such as grass and isolated obstacles

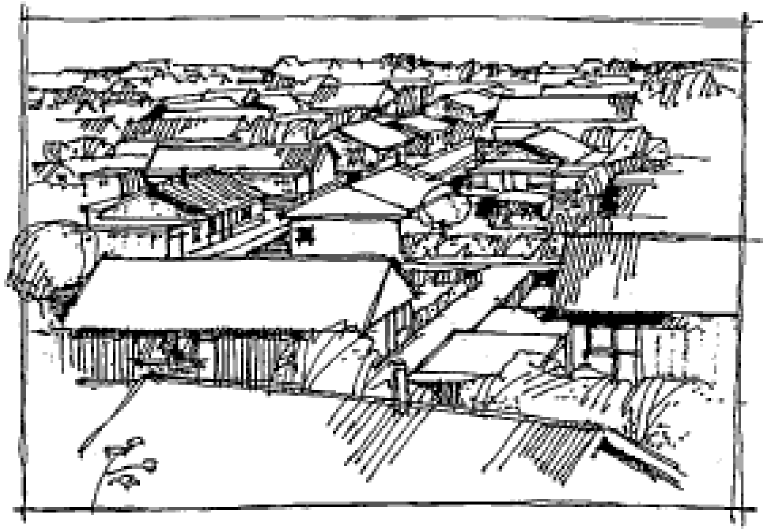
(trees, buildings) with separations of at least 20 obstacle heights



e.g. Eurocode

Terrain category III (z_0=0.3m)

Area with regular cover of vegetation or buildings or with isolated obstacles with separations of maximum 20 obstacle heights (such as villages, suburban terrain, permanent forest)



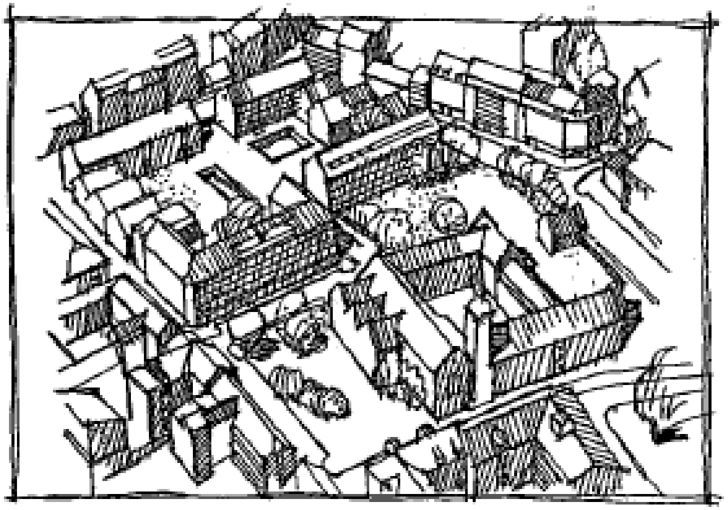
Roughness Length Z_0

e.g. Eurocode

Terrain category IV (z_0=1.0m)

Area in which at least 15% of the surface is covered with

bulidings and their average height exceeds 15m



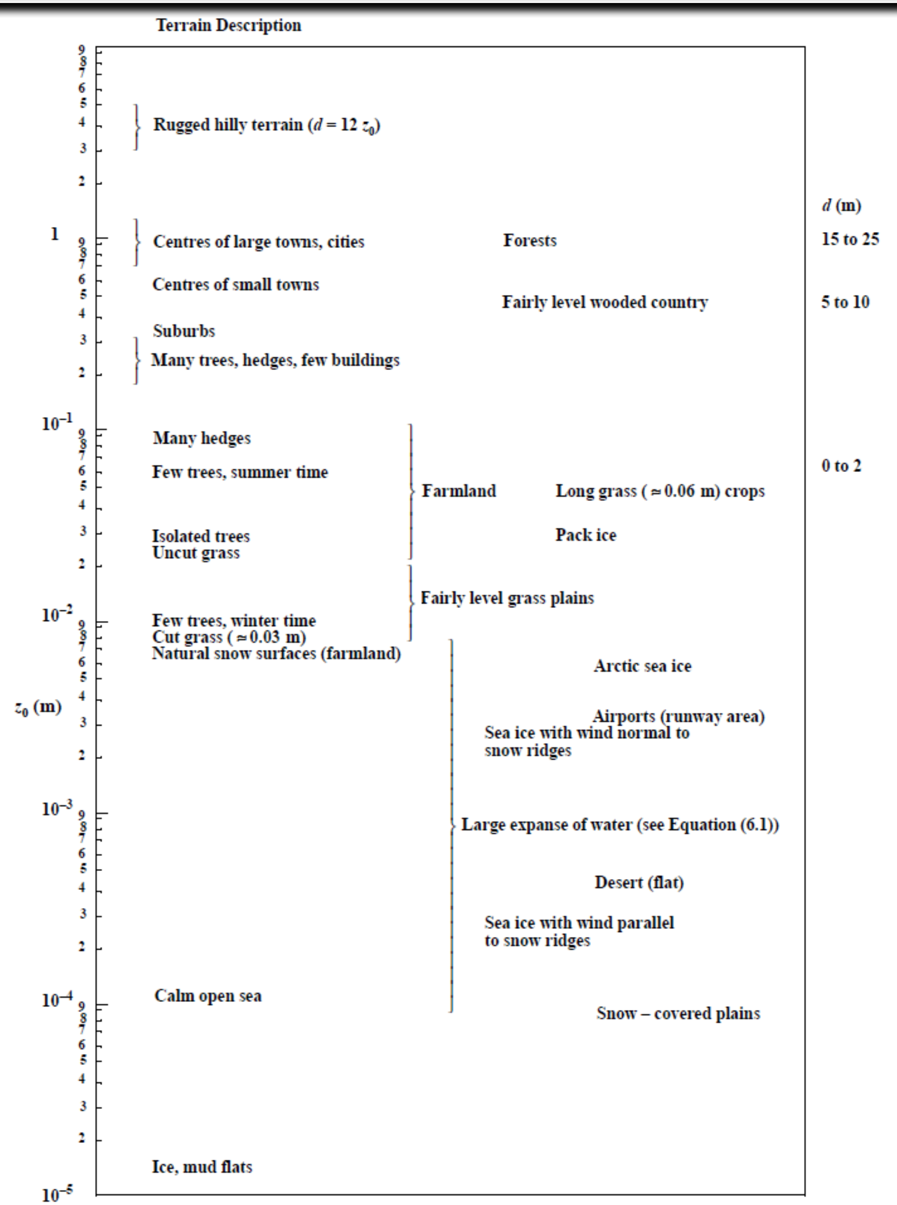
Roughness Length Z_0

e.g. ASCE 7-05 Commentary

Type of Surface	(m)
Water	0.005-0.01
Open terrain	0.015-0.15
Urban and suburban terrain, wooded areas	0.150-0.70

Roughness Length Z_0

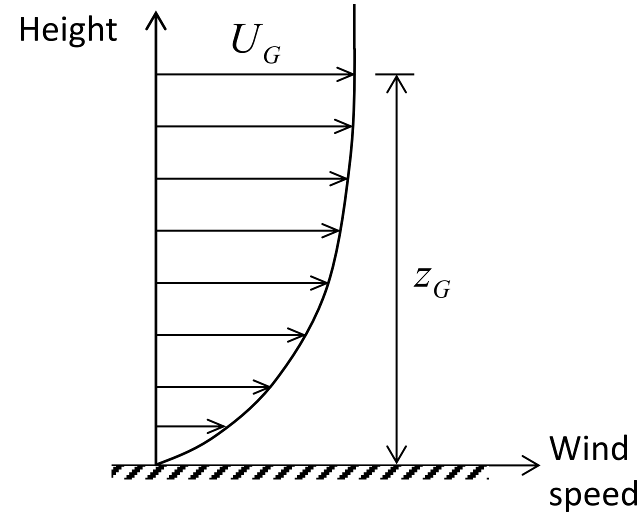
e.g. ESDU 82026



Power Law

Wind profile

$$\frac{\overline{U}(z)}{U_G} = \begin{cases} \left(\dfrac{z}{z_G}\right)^{\alpha} & (z \le z_G) \\ 1 & (z > z_G) \end{cases}$$



where $\overline{U}(z)$: Mean wind speed at height z

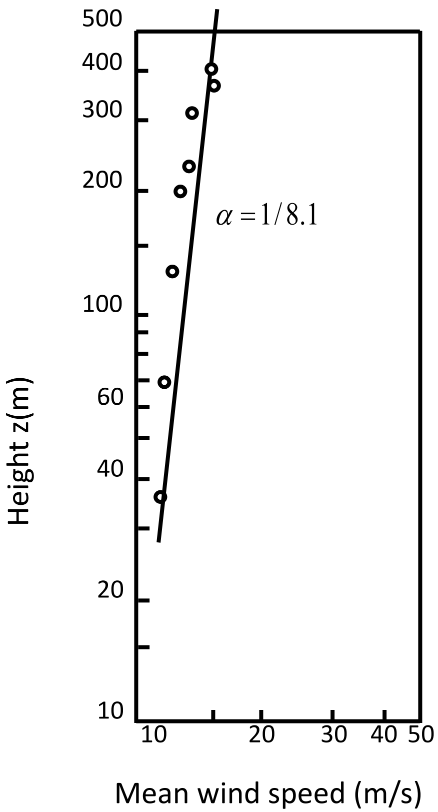
U_G : Gradient speed

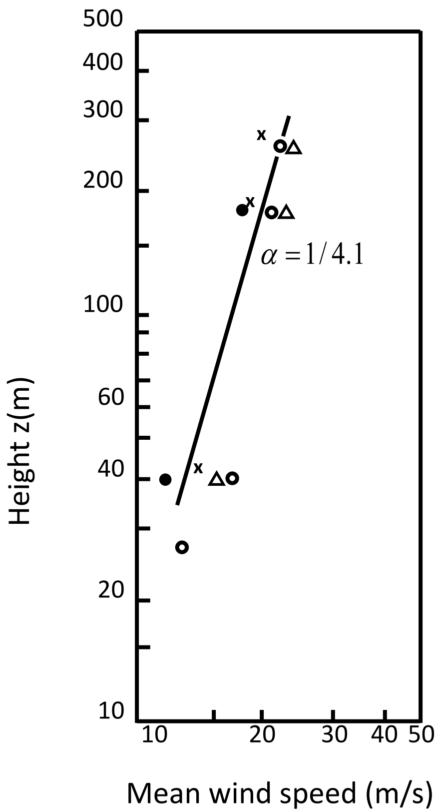
z_G : Gradient height

α : power law exponent

Power Law

- **Power law exponent** $(\alpha \approx \frac{1}{10} \sim \frac{1}{3})$





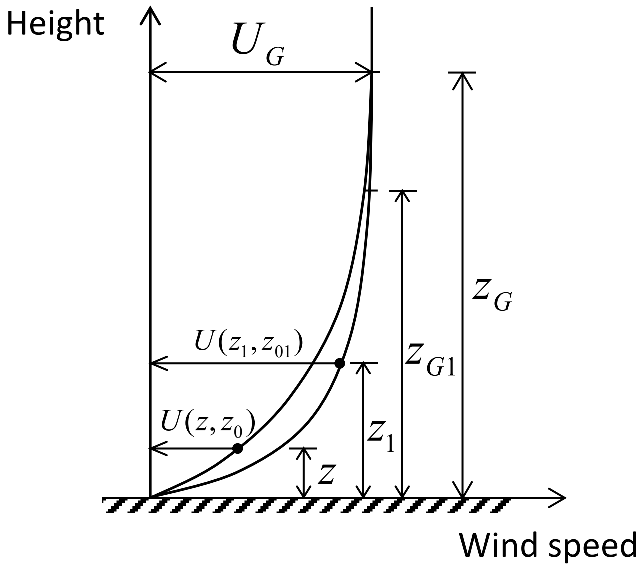
Wind Profiles for various terrain conditions

- If the profile of mean wind speed $U(z_1)$ at the height z_1 with roughness length z_{01} is known, the profile of mean wind speed at the height z with roughness length z_0 can be determined as follows.

at z $$\frac{U(z)}{U_G} = \left(\frac{z}{z_G}\right)^{\alpha(z)}$$

at z_1 $$\frac{U(z_1)}{U_G} = \left(\frac{z_1}{z_{G1}}\right)^{\alpha(z_1)}$$

$$\therefore U(z) = U(z_1)\left(\frac{z}{z_G}\right)^{\alpha(z)} \cdot \left(\frac{z_{G1}}{z_1}\right)^{\alpha(z_1)}$$



- If the basic wind speed is given for the terrain with roughness category II at the height of 10m in KBDC, then the basic wind speed for the other terrain is determined as

$$\left(\frac{z_{G1}}{z_1}\right)^{\alpha(z_1)} = \left(\frac{600}{10}\right)^{0.16} = 1.925 \qquad \therefore U(z) = 1.925\ U(10)\left(\frac{z}{z_G}\right)^{\alpha(z)}$$

8.2.2 설계기준풍속

설계기준풍속 V_D 는 대상 지역의 기본 풍속과 교량의 고도, 주변의 지형과 환경 등을 고려하여 합리적인 방법으로 결정한다.

- 대상지역의 풍속자료가 가용치 못한 경우에 지표조도와 고도가 다른 타 지역 풍속 V_1으로부터 현장 풍속 V_2를 구하기 위하여 식 (해설 8.2.1)을 사용할 수 있다. 이때 지표조도계수 α, 경고도 z_G, 최소높이 z_b 그리고 조도길이 z_0는 해설 표 8.2.1의 값을 사용하고, 지표조도구분은 도로교설계기준의 것을 따른다.

$$V_2 = C_t \cdot V_1 \cdot \left(\frac{z_2}{z_{G2}}\right)^{\alpha 2}, \quad z_2 \geq z_b$$
$$= C_t \cdot V_1 \cdot \left(\frac{z_b}{z_{G2}}\right)^{\alpha 2}, \quad z_2 < z_b \qquad \text{(해설 8.2.1)}$$

여기서 C_t는 고도 및 조도 보정계수로 V_1 지역에 해당하는 값을 입력한다.

$$C_t = \left(\frac{z_{G1}}{z_1}\right)^{\alpha 1} \qquad \text{(해설 8.2.2)}$$

해설 표 8.2.1 지표조도구분에 따른 계수값

지표조도구분	I	II	III	IV
α	0.12	0.16	0.22	0.29
z_G (m)	500	600	700	700
z_b (m)	5	10	15	30
z_0 (m)	0.01	0.05	0.3	1.0

- 식 (해설 8.2.2)에 지표조도구분 II에 해당하는 값을 대입하고 식 (해설 8.2.1)에 $V_1 = V_{10}$과 교량 현장의 z_2, z_{G2}, α_2을 대입하면, 기본풍속을 사용하여 설계기준풍속을 구할 수 있다. 이때 고도 및 조도 보정계수 C_t는 1.925가 된다.
- 설계기준풍속을 산정하기 위한 기준고도로 주형은 중앙경간의 평균고도를 사용하고, 행어나 사장재는 주형 높이와 주탑 높이의 중간을 사용한다.

Wind Speeds v.s. Averaged Time Intervals for Any Type of Surface

- We considered the relation between wind speeds averaged over various time intervals for the case of open terrain.
- The following approximate relation may be used for any type of surface.

$$V_t(z) = \bar{V}(z)\left[1 + \frac{\eta c(t)}{2.5 ln(z/z_0)}\right]$$

where $V_t(z)$=speed averaged over t seconds

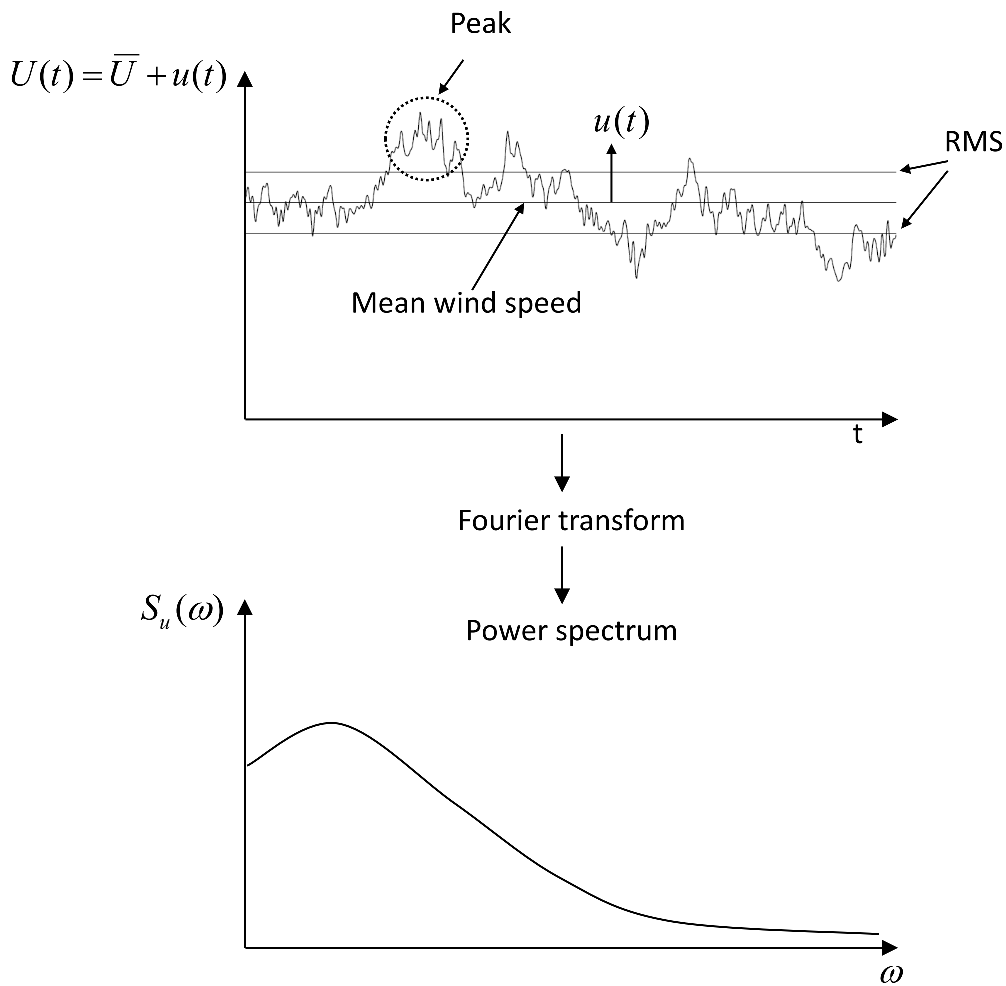
$V(z)$ =speed averaged over 1 h for terrain with surface roughness z_0

TABLE 2.3.3. Factors $\eta(z_0)$ and $c(t)$

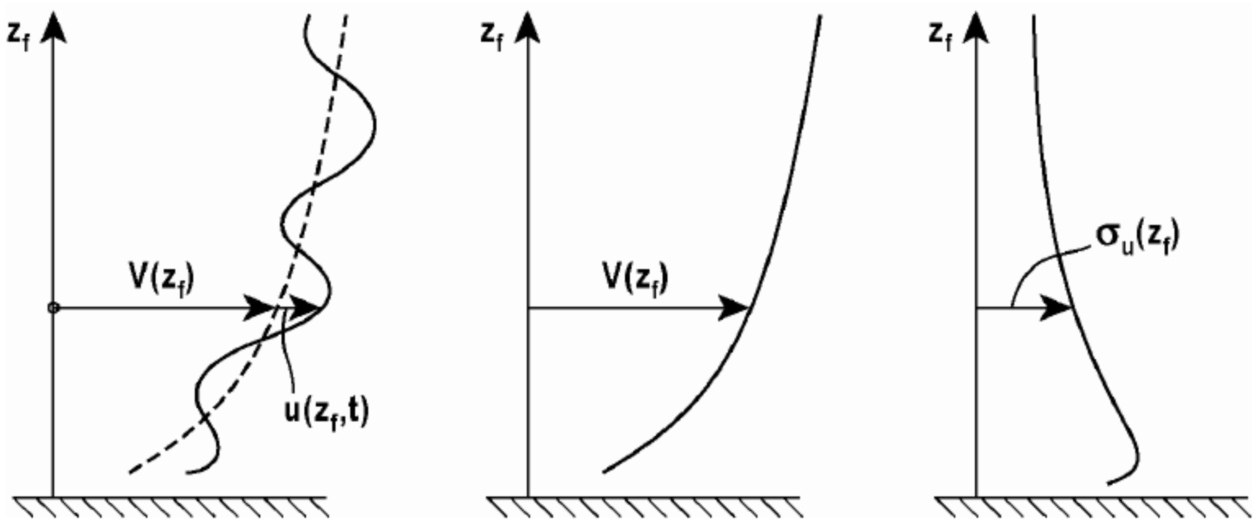
(a)	z_0 (m)			0.005		0.03		0.30			1.00
	$\eta(z_0)$			2.55		2.45		2.30			2.20
(b)	t	1	10	20	30	50	100	200	300	600	1000	3600
	$c(t)$	3.00	2.32	2.00	1.73	1.35	1.02	0.70	0.54	0.36	0.16	0.00

Wind Velocity Fluctuations (Atmospheric Turbulence)

- **The wind flow is not larminar (smooth). Rather, it is *turbulent* – it fluctuates in time and space.**



Wind Velocity Fluctuations (Atmospheric Turbulence)



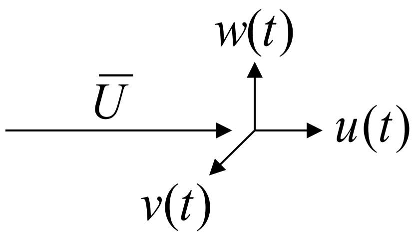
- **Why is atmospheric flow turbulence of interest?**
  - The turbulence can influence significantly the wind flow around a structure and therefore the wind-induced forces.
  - The flow fluctuations produce dynamic effects in flexible structures.
- **Useful descriptors of atmospheric turbulence**
  - Turbulence intensities
  - Integral lengths
  - Turbulence spectra and co-spectra

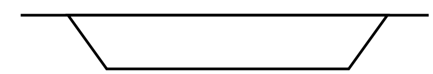
Turbulence Intensities

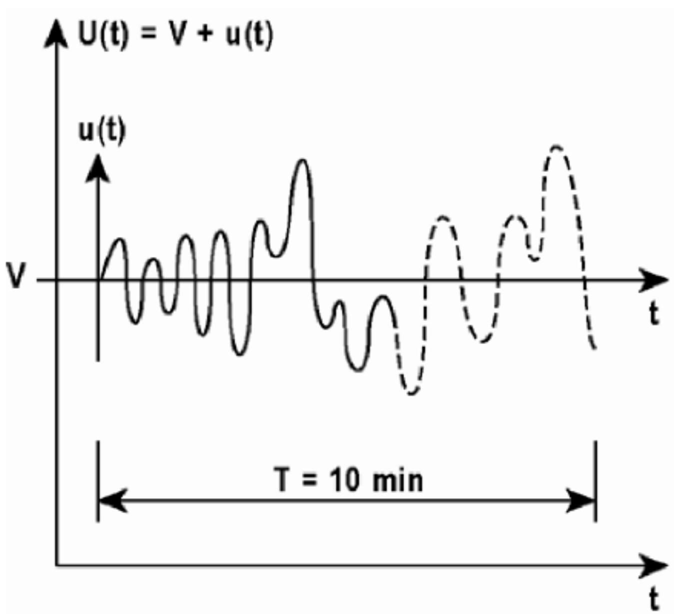
Definition

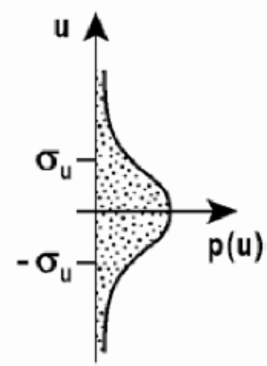
$$I_u = \frac{\sqrt{\sigma_u^2}}{\overline{U}} \quad , \quad I_v = \frac{\sqrt{\sigma_v^2}}{\overline{U}} \quad , \quad I_w = \frac{\sqrt{\sigma_w^2}}{\overline{U}}$$

$\overline{U}$ =Mean wind velocity along-wind direction   u, v, w=3 components of wind fluctuation









Turbulence Intensities

- $\sigma_u^2 = \beta u_*^2$

< Values of β Corresponding to Various Roughness Lengths >

z_0	0.005	0.07	0.30	1.00	2.50
β	6.5[a]	6.0	5.25	4.85	4.00

- $$I_u(z) = \frac{\sqrt{\beta}u_*}{\bar{U}} = \frac{\sqrt{\beta}}{\bar{U}}\frac{\bar{U}(z)}{2.5\ln(z/z_0)} = \frac{\sqrt{\beta}}{2.5\ln(z/z_0)} \approx \frac{\eta(z_0)}{2.5\ln(z/z_0)}$$

$$\therefore \eta = \sqrt{\beta}$$

- **e.g. z_0=0.03m, z=20m then $\eta(z_0)$=2.45 and Iu(z)=0.15**

Turbulence Intensities in other references

Design Guidelines for Steel Cable-Supported Bridges (KSCE 2006)

- 고도에 따른 기류방향 난류강도 I_u는 식 (해설 8.2.5)를 사용하여 산정할 수 있다. 이때 지표 조도계수 α 최소높이 z_b 그리고 조도길이 z_0는 해설 표 8.2.1의 값을 사용한다.

$$I_u = \frac{1}{\ln(30/z_0)} \cdot \left(\frac{30}{z}\right)^{\alpha}, \qquad z_b < z < 100m$$
$$= \frac{1}{\ln(30/z_0)} \cdot \left(\frac{30}{z_b}\right)^{\alpha}, \qquad z \le z_b \qquad \text{(해설 8.2.5)}$$

- 수평방향(v) 및 수직방향(w)의 난류강도는 각각 식 (해설 8.2.6)의 값을 사용할 수 있다.

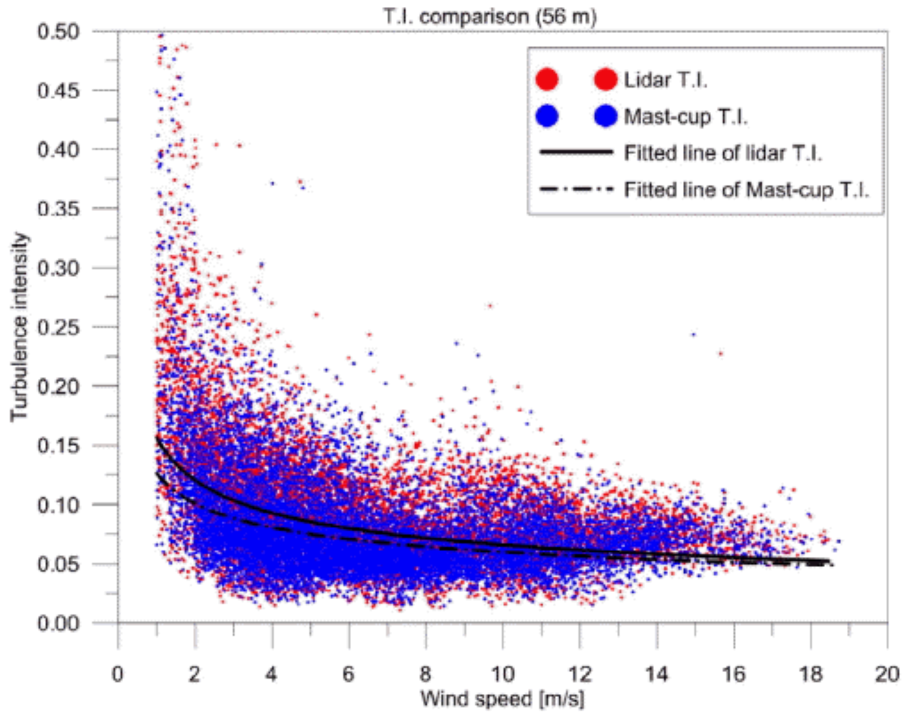
$$I_v = 0.80 \cdot I_u$$
$$I_w = 0.50 \cdot I_u \qquad \text{(해설 8.2.6)}$$

- e.g. z_0=0.03m, z=20m then α≈0.14 and Iu(z)=0.153

Theory of Bridge Aerodynamics (Strømmen 2010)

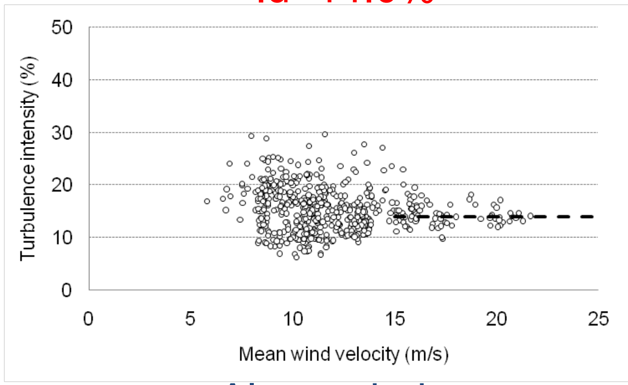
- $I_u = 1/\ln(z/z_0)$ when $z>z_{min}$
- $I_v = 3/4\, I_u \qquad I_w = 1/2\, I_u$
- e.g. z_0=0.03m, z=20m then α≈0.14 and Iu(z)=0.154

Dependency of Turbulence Intensity on Wind Speed



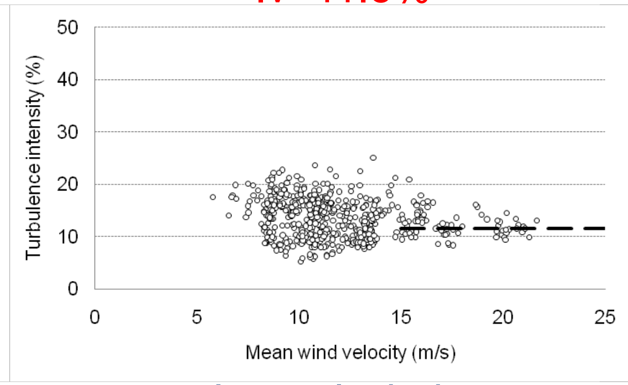
Turbulence Intensities in the Mokpo Bridge Site

Iu=14.0%



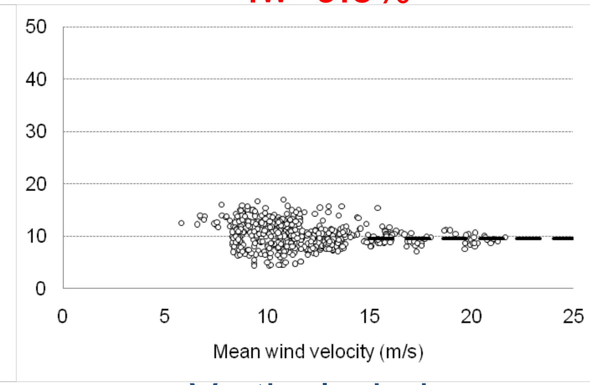
Along wind

Iv=11.5%



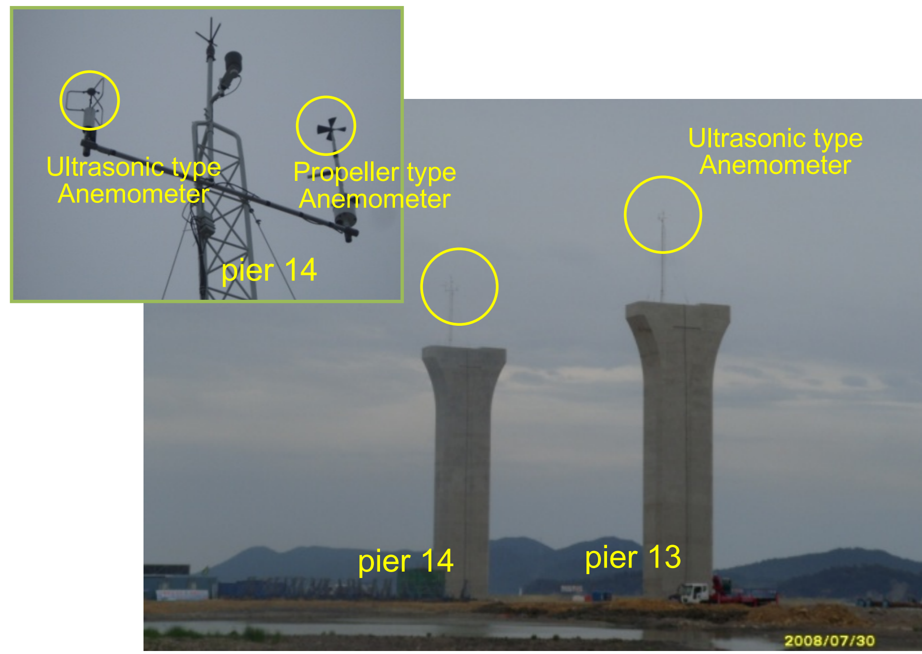
Lateral wind

Iw=9.5%



Vertical wind

Turbulence intensity	Roughness Category Ⅰ	Roughness Category Ⅱ
Iu	11.6 %	14.1 %
Iv	9.3 %	11.3 %
Iw	5.8 %	7.1 %



Temporal Statistics and Ensemble Statistics

Temporal Statistics

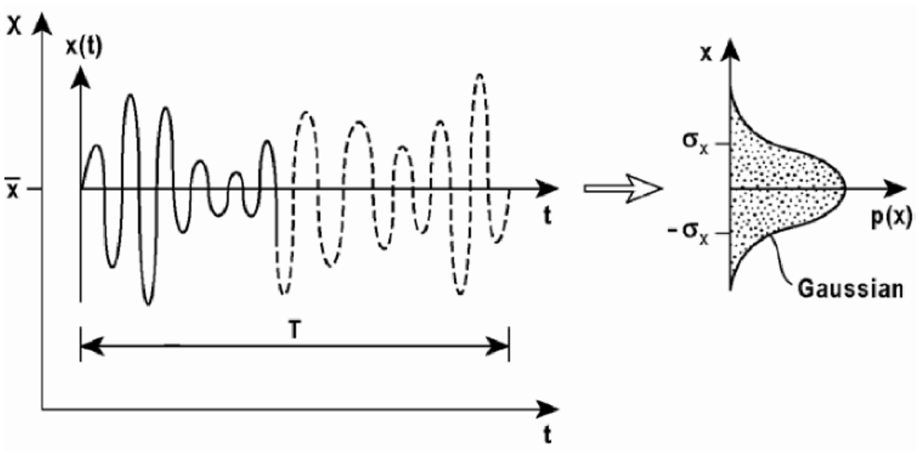
- Any temporal statistics are based on a continuous or discrete time variable *X*, which is stationary and homogenous (i.e. have constant statistical properties) such that

$$X = \bar{x} + x(t)$$

- Its mean value and variance are then given by

$$\bar{x} = \lim_{T\to\infty} \frac{1}{T}\int_0^T X dt$$

$$\sigma_x^{\;2} = \lim_{T\to\infty} \frac{1}{T}\int_0^T [x(t)]^2 dt$$



Ensemble Statistics

- **(e.g.) N simultaneous realizations of the along wind velocity in space**
  - $X_k(t), k = 1, \cdots, N$
  - Type of statistics that provides a stochastic description of the wind field distribution in space

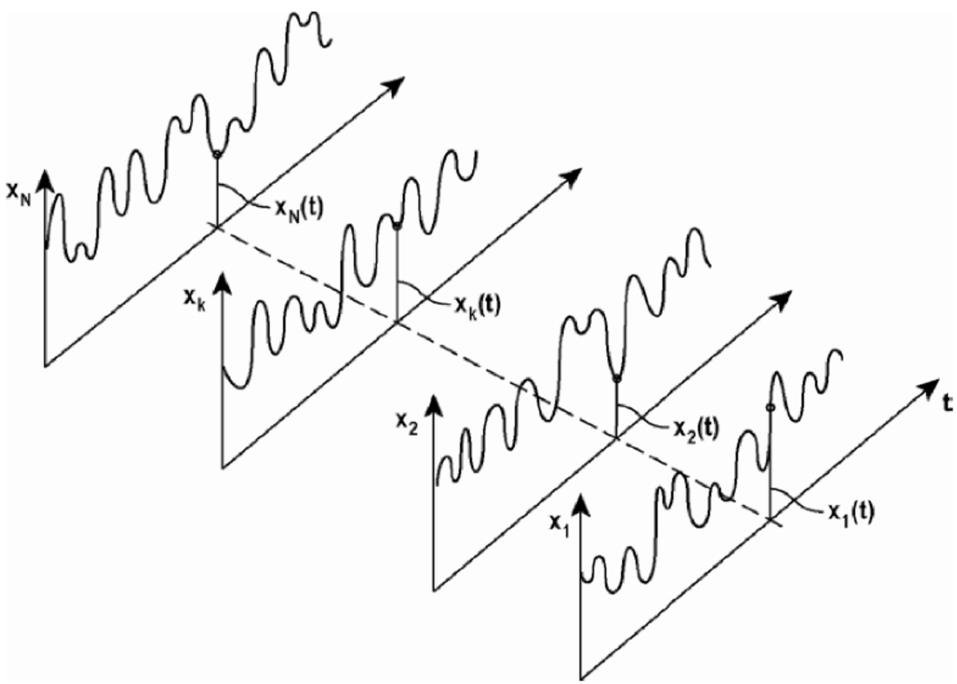
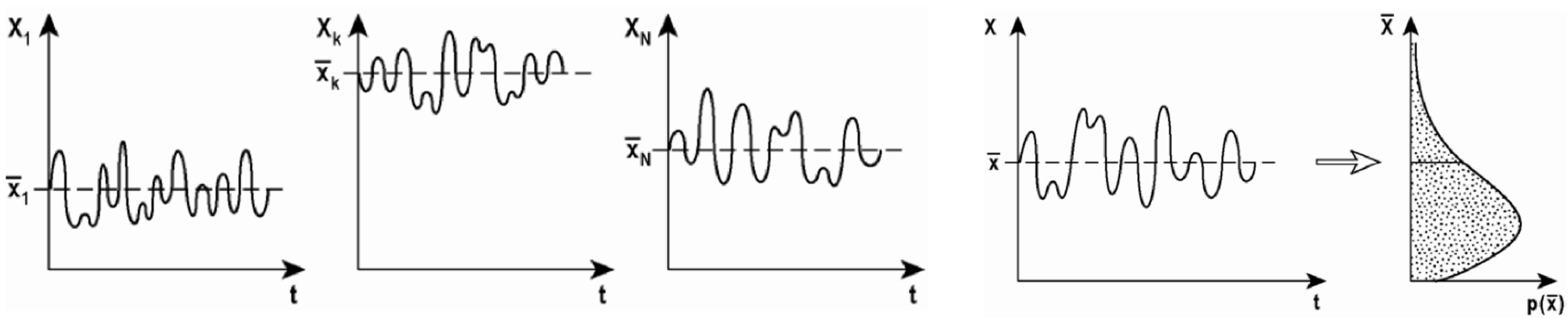


Fig. 2.3 *Ensemble statistics of simultaneous events*

Ensemble Statistics

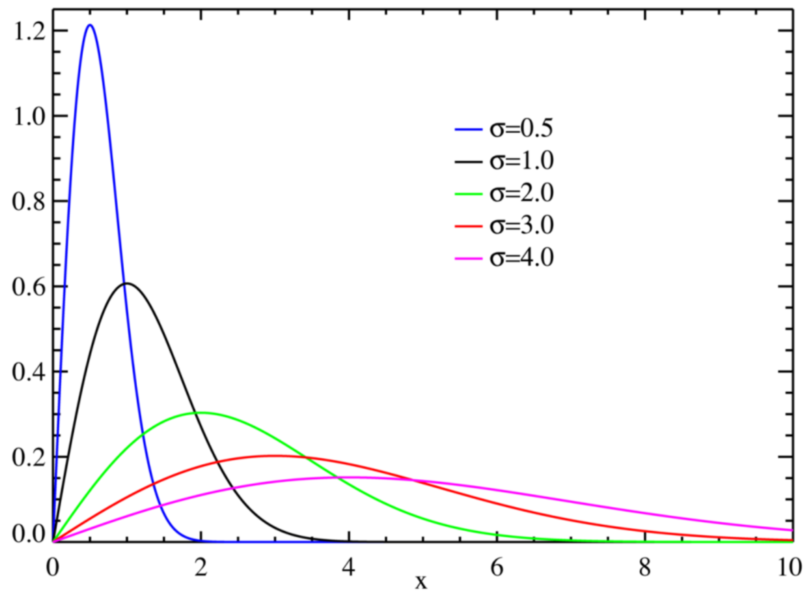
- **(e.g.) N different observations of a stochastic process have been recorded, each taken within a certain time window not necessarily at the same time**
  - An illustration of the situation when a number of time series have been recorded of the wind velocity at a certain point in space, each taken during different weather conditions.
  - The statistical properties of the data set of extracted mean values will then represent an example of long term ensemble statics.
  - Typically, PDF of a data set of mean values may attain a shape of Weibull or a Rayleigh distribution.

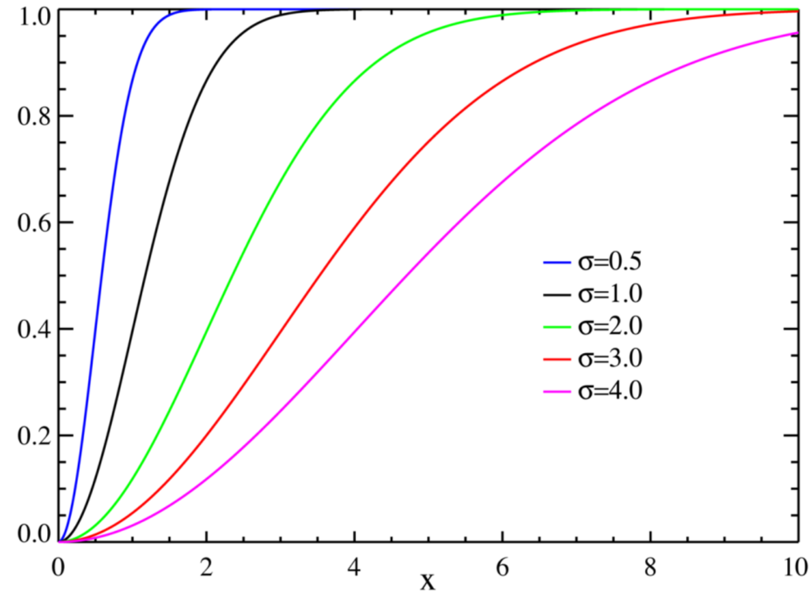


a) *N independent short term realisations* b) *The probability of mean values*

Fig. 2.4 *Ensemble statistics of mean value recordings*

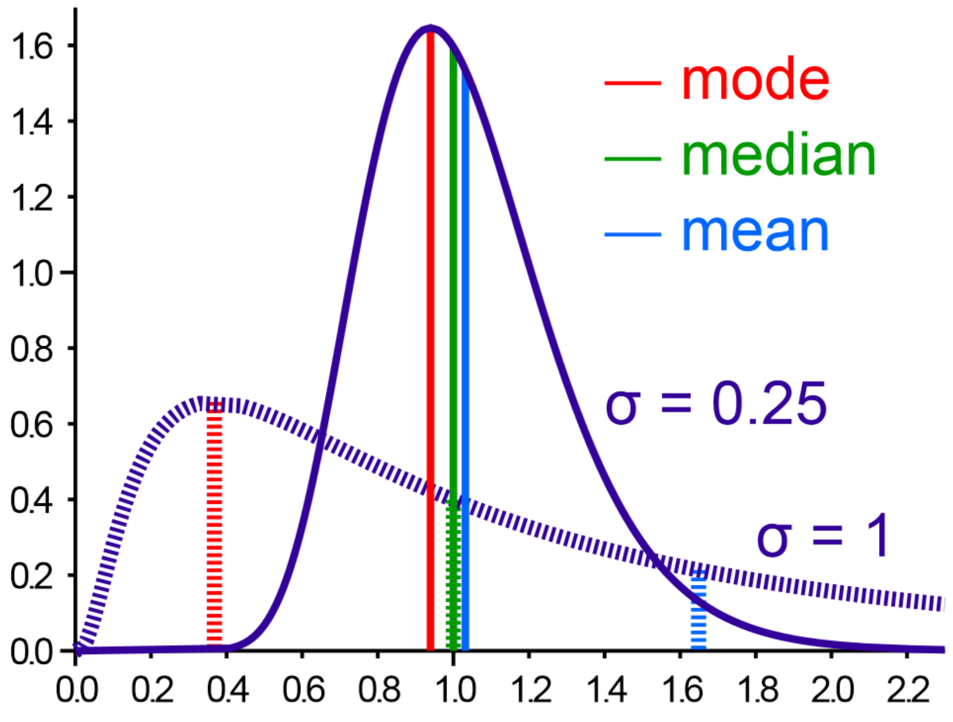
- **A Rayleigh distribution is often observed when the overall magnitude of a vector is related to its directional components.**
  - PDF $f(x;\sigma) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}, \quad x \geq 0$ for parameter $\sigma (= mode) > 0$
  - CDF $F(x) = 1 - e^{-x^2/2\sigma^2}$, for $x \in [0, \infty)$





Mean, Median, and Mode

- The expected value (mean, or the first moment) of a random variable is the weighted average of all possible values that this random variable can take on.
- The median is described as the numerical value separating the higher half of a sample, a population, or a probability distribution, from the lower half.
- The mode is the number that appears most often in a set of numbers.



Rayleigh Distribution

- **One example where the Rayleigh distribution naturally arises is when wind speed is analyzed into its orthogonal 2-dimensional vector components. Assuming that the magnitude of each component is uncorrelated and normally distributed with equal variance, then the overall wind speed (vector magnitude) will be characterized by a Rayleigh distribution.**
- **A second example of the distribution arises in the case of random complex numbers whose real and imaginary components are i.i.d. (independently and identically distributed) Gaussian. In that case, the absolute value of the complex number is Rayleigh-distributed.**
- $R \sim Rayleigh(\sigma)$ **is Rayleigh distributed if** $R = \sqrt{X^2 + Y^2}$**, where** $X \sim N(0, \sigma^2)$ **and** $Y \sim N(0, \sigma^2)$ **are independent normal random variables. (This gives motivation to the use of the symbol σ in the above parameterization of the Rayleigh density.)**

Correlation and Covariance

- **Given two realizations $\boldsymbol{X_1(t) = \overline{x_1} + x_1(t)}$ and $\boldsymbol{X_2(t) = \overline{x_2} + x_2(t)}$**
- **Correlation**

$$R_{x_1 x_2} = E[X_1(t) \cdot X_2(t)] = \lim_{T\to\infty} \frac{1}{T} \int_0^T X_1(t) \cdot X_2(t) dt$$

- **Covariance**

$$Cov_{x_1 x_2} = E[x_1(t) \cdot x_2(t)] = \lim_{T\to\infty} \frac{1}{T} \int_0^T x_1(t) \cdot x_2(t) dt$$

- **Ensemble correlation**

$$R_{x_1 x_2}(\tau) = E[X_1 X_2] = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} X_{1_k} \cdot X_{2_k}$$

- **Ensemble covariance**

$$Cov_{x_1 x_2}(\tau) = E[(X_1 - \overline{x_1}) \cdot (X_2 - \overline{x_2})] =$$

$$\lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} (X_{1_k} - \overline{x_1}) \cdot (X_{2_k} - \overline{x_2})$$

- **taken on the process variable itself**

- **Auto correlation**
  - $R_x(\tau) = E[X(t)X(t+\tau)]$
    $= \lim_{T\to\infty} \frac{1}{T}\int_0^T X(t)\cdot X(t+\tau)dt$

- **Auto covariance**
  - $Cov_x(\tau) = E[x(t)x(t+\tau)]$
    $= \lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)\cdot x(t+\tau)dt$

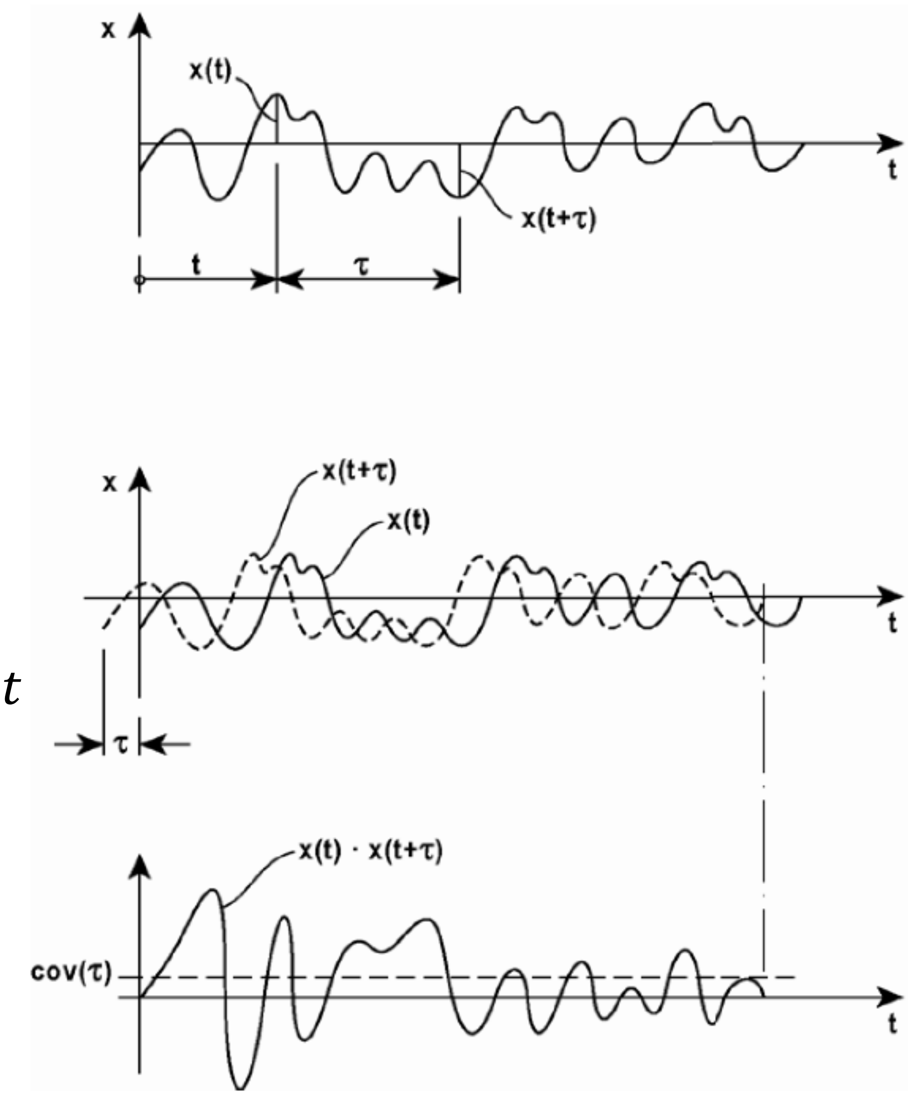


Fig. 2.5 *The auto covariance function*

Auto Correlation and Auto Covariance

- **As long as τ is considerably smaller than *T***

$$E[X(t)] = E[X(t+\tau)] = \bar{x}$$

- **Relationship between R_x and Cov_x**

$$Cov_x(\tau) = E[\{X(t)-\bar{x}\}\cdot\{X(t+\tau)-\bar{x}\}] = R_x(\tau) - \bar{x}^2$$

- **Symmetry**

$$E[x(t)\cdot x(t-\tau)] = E[x(t-\tau)\cdot x(t)] = E[x(t-\tau)\cdot x(t-\tau+\tau)]$$

$$Cov_x(\tau) = Cov_x(-\tau)$$

- **Calculation with discrete data (*j* must be considerably smaller than *N*.)**

$$Cov_x(\tau = j\cdot\Delta t) = E[x(t)\cdot x(t+\tau)] = \frac{1}{N-j}\sum_{k=1}^{N-j} x_{k+j}\cdot x_k$$

- **Auto covariance coefficient**

$$\boldsymbol{\rho_x(\tau) = \frac{Cov_x(\tau)}{{\sigma_x}^2}}$$

$$\boldsymbol{\rho_x(\tau = 0) = 1}$$

Cross Correlation and Cross Covariance

- **Given two realizations $X_1(t) = \overline{x_1} + x_1(t)$ and $X_2(t) = \overline{x_2} + x_2(t)$**
- **Cross correlation**

$$R_{X_1X_2}(\tau) = E[X_1(t) \cdot X_2(t+\tau)] = \lim_{T\to\infty} \frac{1}{T}\int_0^T X_1(t) \cdot X_2(t+\tau)dt$$

- **Cross covariance**

$$Cov_{x_1x_2}(\tau) = E[x_1(t) \cdot x_2(t+\tau)] = \lim_{T\to\infty} \frac{1}{T}\int_0^T x_1(t) \cdot x_2(t+\tau)dt$$

- **Cross covariance coefficient**

$$\rho_{x_1x_2}(\tau) = \frac{Cov_{x_1x_2}(\tau)}{\sigma_{x_1}\sigma_{x_2}}$$

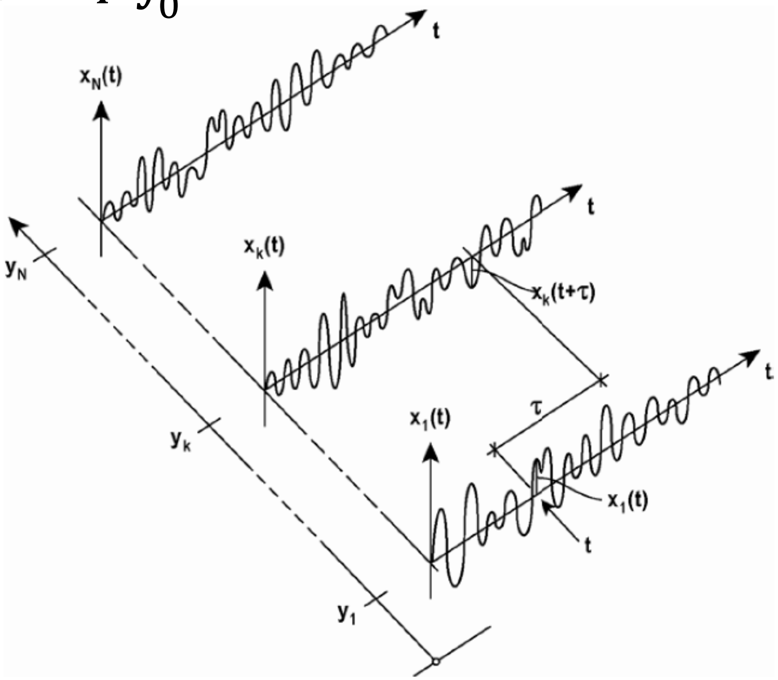
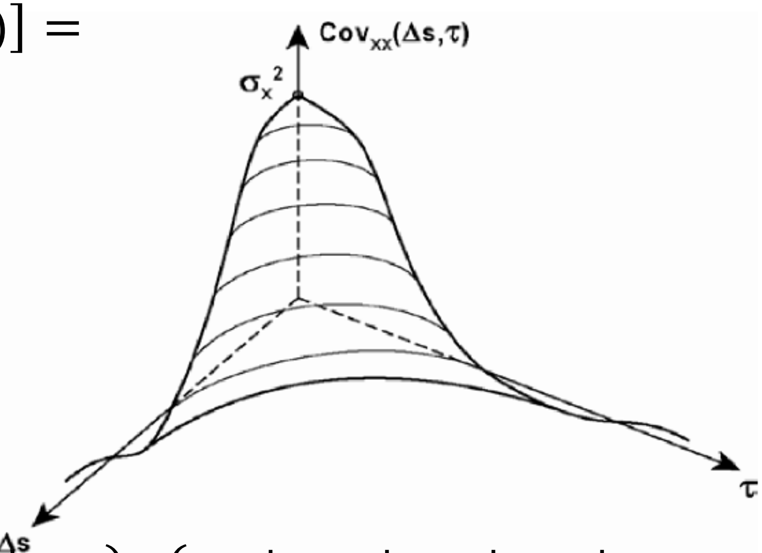


Fig. 2.6 *Cross covariance of time series at positions y_k ($k = 1,2,\ldots,N$)*

Spatial Separation and Time Lag Covariance Function

- $Cov_{xx}(\Delta y, \tau) = E[x(y,t) \cdot x(y+\Delta y, t+\tau)] = \lim_{T\to\infty} \frac{1}{T}\int_0^T x\,(y,t) \cdot x(y+\Delta y, t+\tau)dt$



Extended to sum of variables $x_1(t)$ and $x_2(t)$

- $Var(x_1+x_2) = E[(x_1+x_2)\cdot(x_1+x_2)]$
  $= Var(x_1) + Var(x_2) + 2\cdot Cov(x_1 \cdot x_2)$
- $Var\left(\sum_{i=1}^{N} x_i\right) = E\left[(x_1+x_2+\cdots+x_i+\cdots+x_N)\cdot(x_1+x_2+\cdots+x_j+\cdots+x_N)\right]$
  $\Rightarrow Var\left(\sum_{i=1}^{N} x_i\right) = \sum_{i=1}^{N}\sum_{j=1}^{N} Cov(x_i \cdot x_j) = \sum_{i=1}^{N}\sum_{j=1}^{N} \rho(x_i \cdot x_j)\cdot\sigma_i\sigma_j$
- If $x_i(t)$ are independent (i.e. uncorrelated)

  then $Cov\left(x_i \cdot x_j\right) = \begin{cases} {\sigma_{x_i}}^2 \ when\ i=j \\ 0\ when\ i \neq j \end{cases}$ then $Var\left(\sum_{i=1}^{N} x_i\right) = \sum_{i=1}^{N} {\sigma_{x_i}}^2$

Auto Covariance for Wind Velocity Fluctuations

Auto Covariance Functions

- $$\begin{bmatrix} Cov_u(\tau) \\ Cov_v(\tau) \\ Cov_w(\tau) \end{bmatrix} = \begin{bmatrix} E[u(t)\cdot u(t+\tau)] \\ E[v(t)\cdot v(t+\tau)] \\ E[w(t)\cdot w(t+\tau)] \end{bmatrix} = \frac{1}{T}\int_0^T \begin{bmatrix} u(t)\cdot u(t+\tau) \\ v(t)\cdot v(t+\tau) \\ w(t)\cdot w(t+\tau) \end{bmatrix}$$

Auto Covariance Coefficients

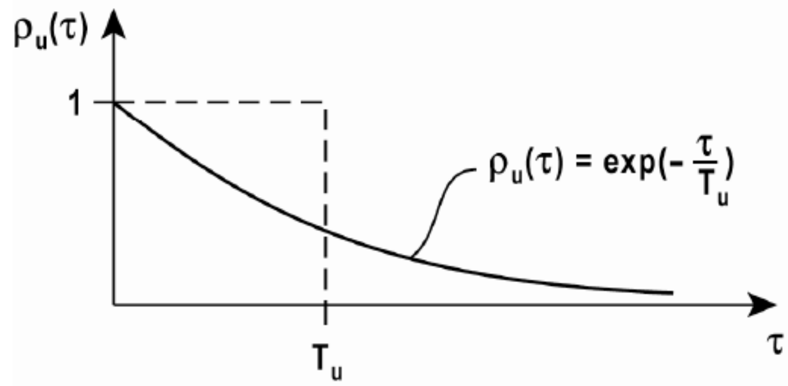
- $$\rho_n(\tau) = \frac{Cov_n(\tau)}{\sigma_n^2}$$

Properties

- $\rho_n(\tau = 0) = 1 \qquad where\ \ n = u, v, w$
- $\lim_{\tau\to\infty} \rho_n(\tau) = 0 \qquad where\ \ n = u, v, w$

Integral Turbulence Scales

- **Time Scale**
  - $\boldsymbol{T_n = \int_0^{\infty} \rho_n(\tau)\, d\tau}$ $\quad where\ \ n = u, v, w$
  - $\rho_n(\tau) = \exp(-\tau/T_n)$ $\ where\ \ n = u, v, w$



- **In homogeneous terrain, at heights below 100m (Strømmen 2010)**
  - T_u=5~20s, T_v=2~5s, T_w=0~2s

- **Turbulence convection in the main flow direction takes place with the mean wind velocity (i.e. that flow disturbances travel with the average velocity V),**
  - ${}^{x_f}L_n = V \cdot T = V \cdot \int_0^{\infty} \rho_n(\tau) d\tau$ $\qquad where\ \ n = u, v, w$
  - These turbulence length scales may be interpreted as the average eddy size of the *u, v,* and *w* components in the direction of the main flow.

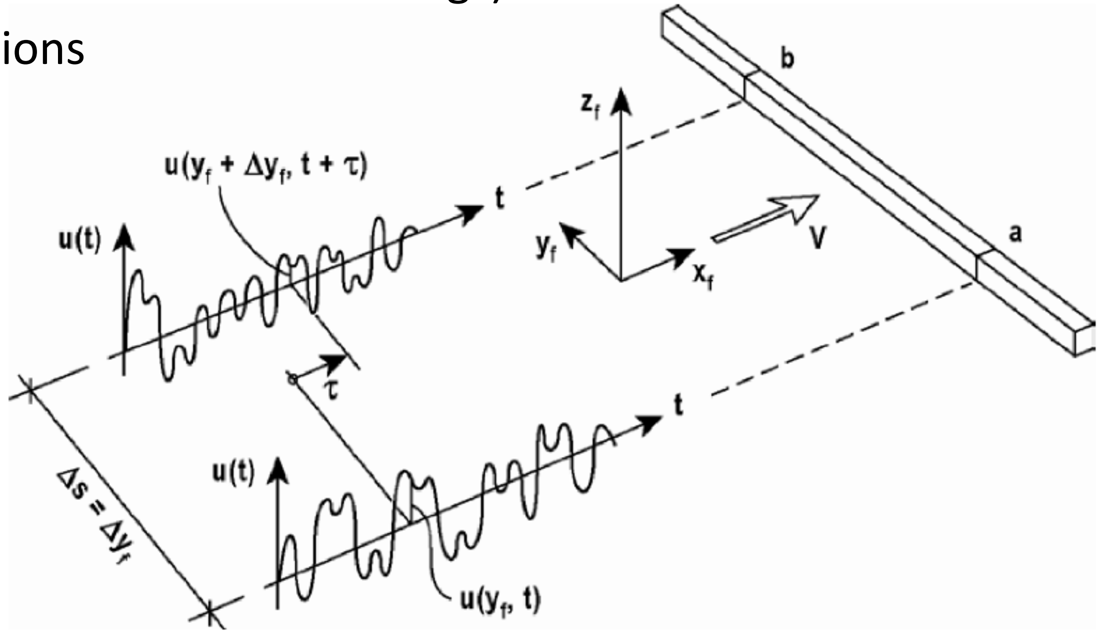
Spatial Properties of Wind Turbulence

- **The flow is assumed to be homogeneous in space as well as stationary in time.**
- **Simultaneous two point recordings**
  - $\boldsymbol{u}_a = \begin{bmatrix} u(s,t) \\ v(s,t) \\ w(s,t) \end{bmatrix}$ and $\boldsymbol{u}_b = \begin{bmatrix} u(s+\Delta s, t+\tau) \\ v(s+\Delta s, t+\tau) \\ w(s+\Delta s, t+\tau) \end{bmatrix}$

    $s = x_f, y_f \; or \; z_f$

    $\tau =$ a time lag that theoretically can take any value within $\pm T$

    $\Delta s =$ arbitrary separation (between the two recordings) in the $x_f, y_f \; or \; z_f$ directions



3×3 Covariance Functions

- **Three by Three Covariance Matrix (27 possible covariance functions)**

  - $$\boldsymbol{Cov}(\Delta s,\tau)=\begin{bmatrix} Cov_{uu} & Cov_{uv} & Cov_{uw} \\ Cov_{vu} & Cov_{vv} & Cov_{vw} \\ Cov_{wu} & Cov_{wv} & Cov_{ww} \end{bmatrix}=E\left[\boldsymbol{u}_a\cdot\boldsymbol{u}_b{}^T\right]=\frac{1}{T}\int_0^T\left(\boldsymbol{u}_a\cdot\boldsymbol{u}_b{}^T\right)dt$$
  - $$Cov_{mn}(\Delta s,\tau)\qquad \begin{cases} m,n=u,v,w \\ \Delta s=\Delta x_f,\Delta y_f \text{ or } \Delta z_f \end{cases}$$
  - Covariance Coefficients

$$\rho_{mn}(\Delta s,\tau)=\frac{Cov_{mn}(\Delta s,\tau)}{\sigma_m\cdot\sigma_n}\qquad \begin{cases} m,n=u,v,w \\ \Delta s=\Delta x_f,\Delta y_f,\Delta z_f \end{cases}$$

- **Cross covariance between two different turbulence components may be neglected. Then, the number of possible covariance estimates is reduced to nine:**

  - $$\begin{bmatrix} Cov_{uu}(\Delta s,\tau) \\ Cov_{vv}(\Delta s,\tau) \\ Cov_{ww}(\Delta s,\tau) \end{bmatrix}=E\begin{bmatrix} u(s,t)\cdot u(s+\Delta s,t+\tau) \\ v(s,t)\cdot v(s+\Delta s,t+\tau) \\ w(s,t)\cdot w(s+\Delta s,t+\tau) \end{bmatrix}=\frac{1}{T}\int_0^T\begin{bmatrix} u(s,t)\cdot u(s+\Delta s,t+\tau) \\ v(s,t)\cdot v(s+\Delta s,t+\tau) \\ w(s,t)\cdot w(s+\Delta s,t+\tau) \end{bmatrix}dt$$

    where $s=x_f,y_f$ or z_f
  - Covariance Coefficients

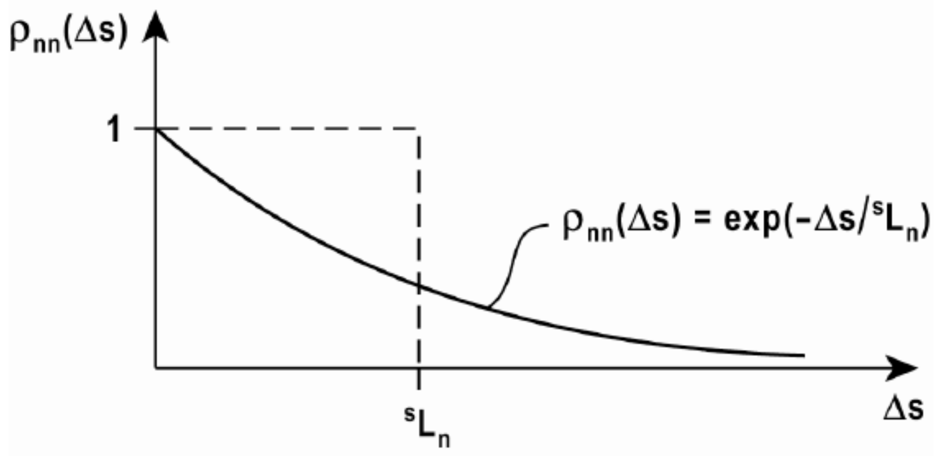
$$\rho_{nn}(\Delta s,\tau)=\frac{Cov_{nn}(\Delta s,\tau)}{\sigma_n{}^2}\qquad \begin{cases} n=u,v,w \\ \Delta s=\Delta x_f,\Delta y_f,\Delta z_f \end{cases}$$

Length Scale for the Spatially Distributed Wind Turbulences

- **The situation at $\tau = 0$ is particularly interesting because**

$${}^{s}L_n = \int_0^{\infty} \rho_{nn}(\Delta s, \tau = 0) d(\Delta s)$$

is a characteristic length scale that may be interpreted as the average eddy size of component *n* in the direction *s*.

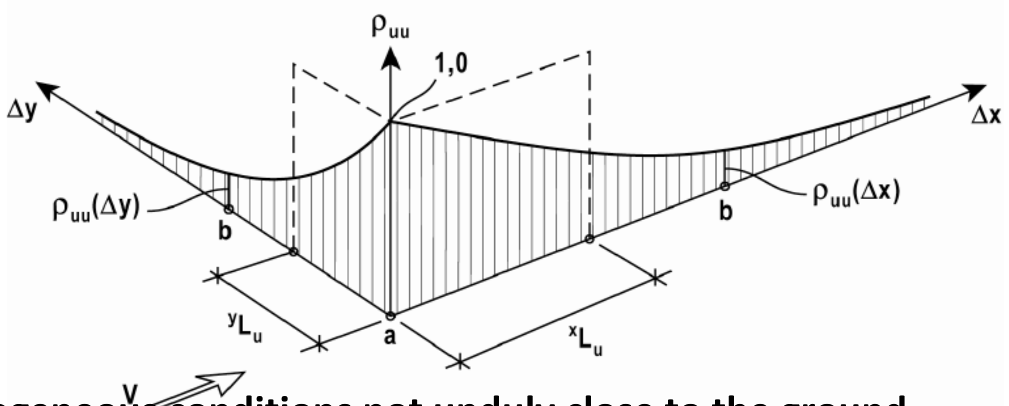


- (e.g.) ${}^{x_f}L_u, {}^{x_f}L_v, {}^{x_f}L_w$ are quantities representing the average eddy size of *u, v, w* components in the direction of the main flow.
- (e.g.) $L_u{}^x (= {}^{x_f}L_u)$is an indicator of the extent to which a longitudinal wind speed fluctuation *u* will engulf a structure in the along-wind direction and will thus affect its windward and leeward sides simultaneously.
- (e.g.) $L_u{}^y (= {}^{y_f}L_u)$and $L_u{}^z (= {}^{z_f}L_u)$are measures of the transverse and vertical spatial extent of the longitudinal fluctuation *u*.
- (e.g.) $L_w{}^x (= {}^{x_f}L_w)$is a measure of the longitudinal spatial extent of the vertical wind speed fluctuation *w*. If the mean wind is normal to a bridge span and $L_w{}^x (= {}^{x_f}L_w)$is large, a vertical speed gust will act on the entire width of the bridge deck.

Turbulence Scale

Length Scale for the Spatially Distributed Wind Turbulences

- **Full scale recording in sites**
  - $\rho_{nn}(\Delta s, \tau = 0) \approx \exp(-\Delta s / {}^{s}L_n)$
    $$\begin{cases} n = u, v, w \\ s = x_f, y_f, z_f \end{cases}$$



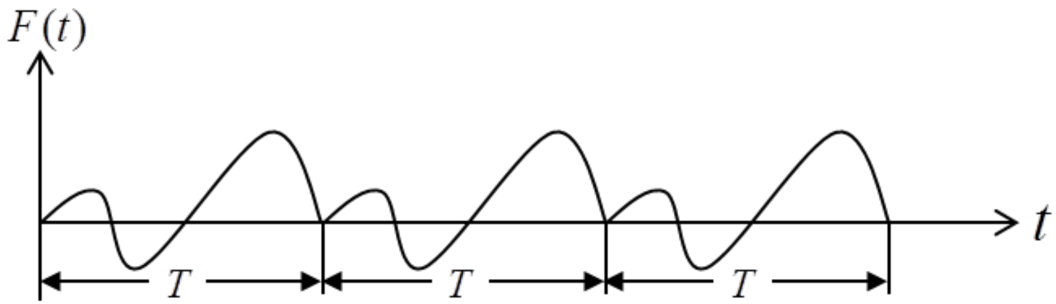
- **Approximation of the length scale for homogeneous conditions not unduly close to the ground (Strømmen 2010)**

$$\begin{bmatrix} {}^{y_f}L_u \\ {}^{z_f}L_u \\ {}^{x_f}L_v \\ {}^{y_f}L_v \\ {}^{z_f}L_v \\ {}^{x_f}L_w \\ {}^{y_f}L_w \\ {}^{z_f}L_w \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/4 \\ 1/4 \\ 1/4 \\ 1/12 \\ 1/12 \\ 1/16 \\ 1/16 \end{bmatrix} \cdot {}^{x_f}L_u$$

where:

$$\begin{cases} \dfrac{{}^{x_f}L_u(z_f)}{{}^{x_f}L_u(z_{f0})} \approx \left(\dfrac{z_f}{z_{f0}}\right)^{0.3} \\ z_f \geq z_{f0} = 10\ m \\ {}^{x_f}L_u(z_{f0}) = 100\ m \end{cases}$$

- A periodic function F(t) can be expressed as the infinite sum of sine and cosine functions.



- $$F(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\bar{\omega}t + \sum_{n=1}^{\infty} b_n \sin n\bar{\omega}t \qquad (1)$$

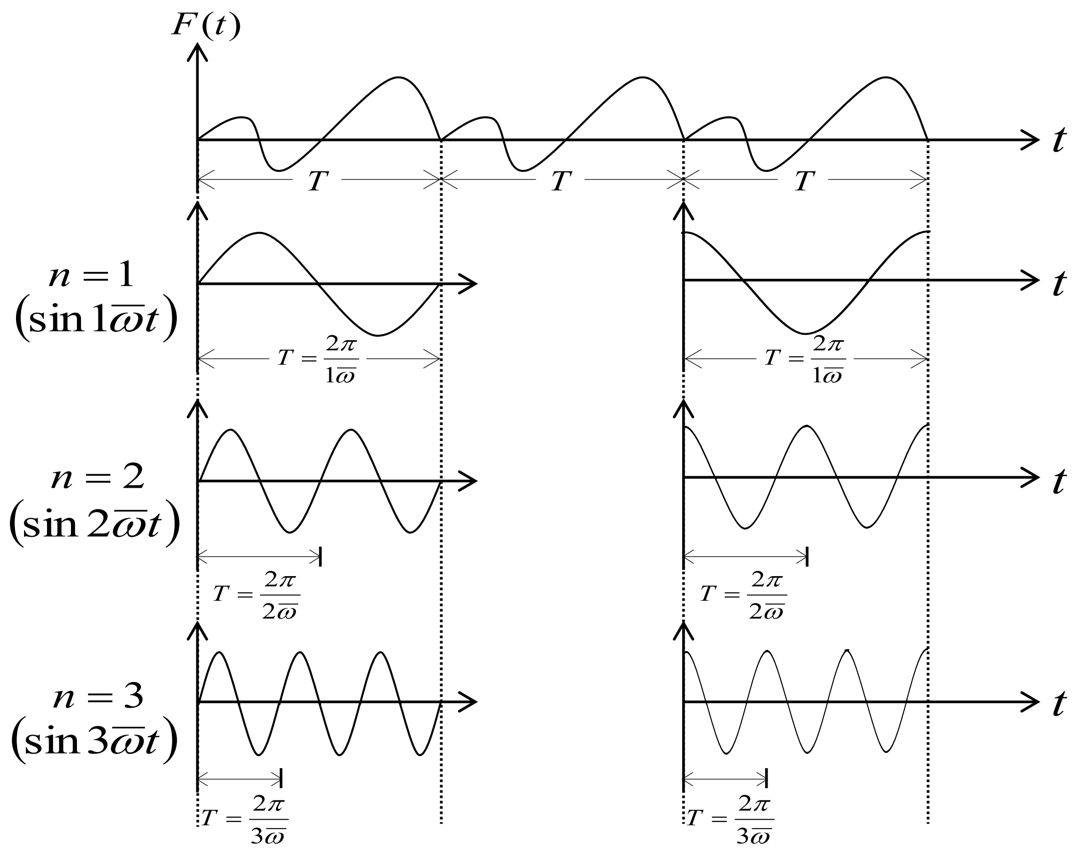
where, $$a_0 = \frac{1}{T}\int_0^T F(t)dt \quad , \quad \bar{\omega} = \frac{2\pi}{T} \qquad (2)$$

$$a_n = \frac{2}{T}\int_0^T F(t)\cos n\bar{\omega}t\, dt \qquad (3)$$

$$b_n = \frac{2}{T}\int_0^T F(t)\sin n\bar{\omega}t\, dt \qquad (4)$$

Fourier Series

- $$F(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\overline{\omega}t + \sum_{n=1}^{\infty} b_n \sin n\overline{\omega}t$$



<Sine term> <Cosine term>

Complex Fourier Series

- Euler eq. $e^{ix} = \cos x + i\sin x$, $e^{-ix} = \cos x - i\sin x$ (1)

$$\sin n\bar{\omega}t = \frac{e^{in\bar{\omega}t} - e^{-in\bar{\omega}t}}{2i} \quad , \quad \cos n\bar{\omega}t = \frac{e^{in\bar{\omega}t} + e^{-in\bar{\omega}t}}{2} \tag{2}$$

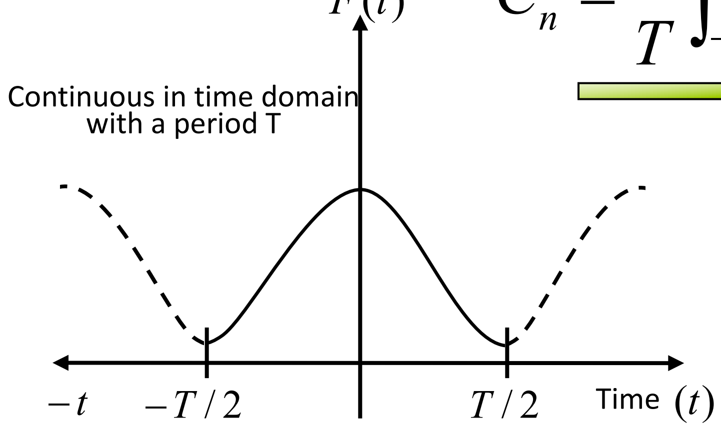
$$F(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\bar{\omega}t + \sum_{n=1}^{\infty} b_n \sin n\bar{\omega}t \tag{3}$$

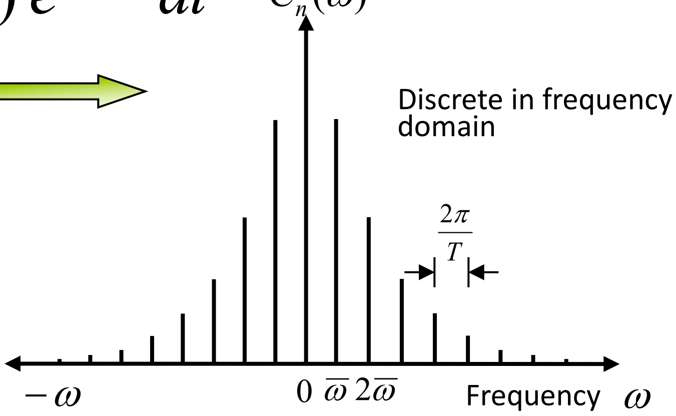
$$F(t) = \sum_{n=-\infty}^{\infty} C_n e^{in\bar{\omega}t} \tag{4}$$

where, $C_n = \dfrac{1}{T}\displaystyle\int_0^T F(t)\, e^{-in\bar{\omega}t}\, dt$ (5)

Complex Fourier Series

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} F(t)\, e^{-in\overline{\omega}t}\, dt \qquad (1)$$

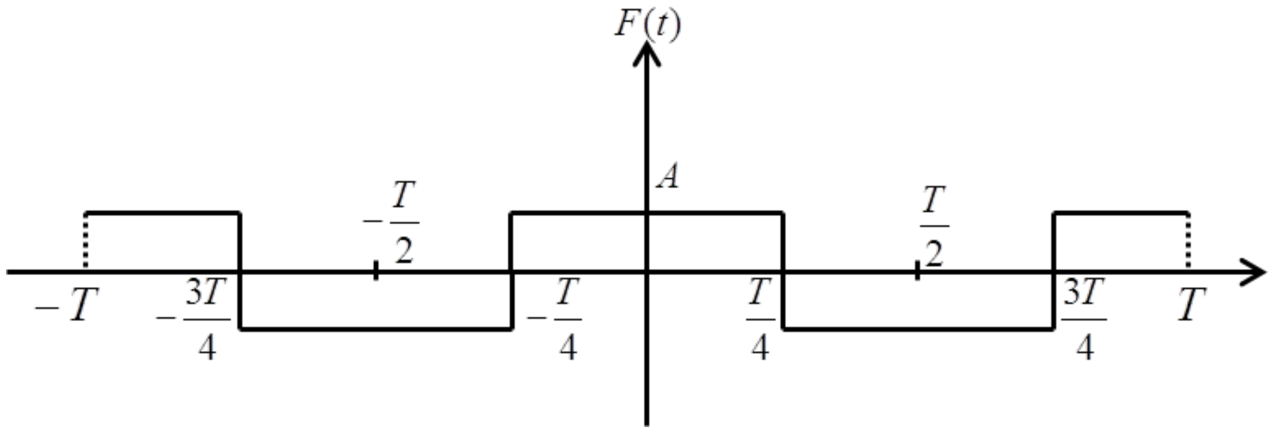




$$F(t) = \sum_{n=-\infty}^{\infty} C_n\, e^{in\overline{\omega}t} \qquad (2)$$

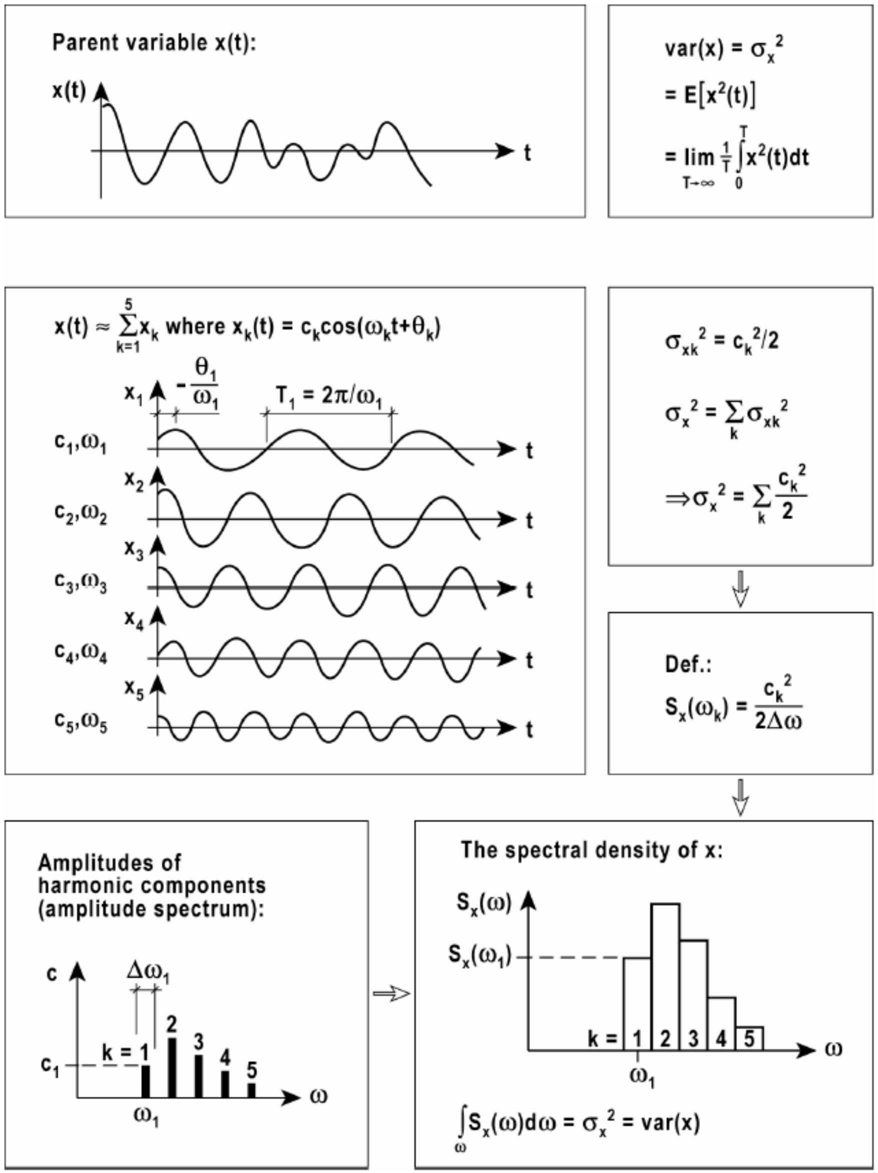
Homework

- Express the given periodic function F(t) with the complex Fourier series expansion. (Due to next class)



Auto Spectral Density from a Fourier Decomposition

- **The auto spectral density is the frequency domain counterpart to the concept of variance.**
- **Given zero mean time variable x(t),**
- $x(t) = \lim_{N\to\infty} \sum_{k=1}^{N} X_k(\omega_k, t)$ where $\begin{cases} \omega_k = k \cdot \Delta\omega \\ \Delta\omega = 2\pi/T \end{cases}$
  - $X_k(\omega_k, t) = c_k \cdot cos(\omega_k t + \varphi_k)$
  - $c_k = \sqrt{{a_k}^2 + {b_k}^2}\,, \varphi_k = \arctan(b_k/a_k)$
  - $\begin{bmatrix} a_k \\ b_k \end{bmatrix} = \frac{2}{T}\int_0^T x(t) \begin{bmatrix} cos\omega_k t \\ sin\omega_k t \end{bmatrix} dt$
- $S_x(\omega_k) = \frac{E[{X_k}^2]}{\Delta\omega} = \frac{{\sigma_{X_k}}^2}{\Delta\omega}$
- $S_x(\omega_k) = \lim_{T\to\infty} \frac{1}{\Delta\omega} \cdot \frac{1}{T}\int_0^T [c_k\cos(\omega_k t + \varphi_k)]^2 dt$
- $S_x(\omega_k) = \lim_{n\to\infty} \frac{1}{\Delta\omega} \cdot \frac{1}{n \cdot T_k} \cdot n \cdot \int_0^{T_k} \left[c_k \cos\left(\frac{2\pi}{T_k} t + \varphi_k\right)\right]^2 dt = \frac{{c_k}^2}{2\Delta\omega}$
- $\sigma_x{}^2 = \lim_{N\to\infty} \sum_{k=1}^{N} \sigma_{X_k}{}^2 = \lim_{N\to\infty} \sum_{k=1}^{N} \frac{{c_k}^2}{2}$
  $= \lim_{N\to\infty} \sum_{k=1}^{N} S_x(\omega_k) \cdot \Delta\omega$
- $S_x(\omega) = \lim_{T\to\infty} \lim_{N\to\infty} \frac{E[X^2(\omega,t)]}{\Delta\omega}$
- $\sigma_x{}^2 = \int_0^\infty S_x(\omega) d\omega$



Spectral Density Functions

Spectral Densities

- Represents **the frequency properties** for the turbulence components

Kaimal Spectra

$$\frac{f \cdot S_n\{f\}}{\sigma_n^2} = \frac{A_n \cdot \hat{f}_n}{\left(1 + 1.5 \cdot A_n \cdot \hat{f}_n\right)^{5/3}} \qquad where \quad n = u, v, w$$

- $\hat{f}_n = f \cdot {}^{x_f}L_n / V$ and ${}^{x_f}L_n$ is the integral length scale of the turbulence components. An is defined as follows: $A_u = 6.8, \quad A_v = A_w = 9.4$

Von Karman Spectra

- It contains only the length scale that require fitting to the relevant data

$$\frac{f \cdot S_n\{f\}}{\sigma_n^2} = \frac{4 \cdot \hat{f}_n}{\left(1 + 70.8 \cdot \hat{f}_n^{\,2}\right)^{5/6}} \qquad where \quad n = u$$

$$\frac{f \cdot S_n\{f\}}{\sigma_n^2} = \frac{4 \cdot \hat{f}_n \cdot \left(1 + 755.2 \cdot \hat{f}_n^{\,2}\right)}{\left(1 + 283.2 \cdot \hat{f}_n^{\,2}\right)^{11/6}} \qquad where \quad n = v, w$$

Example of Turbulence Spectrum

Given condition

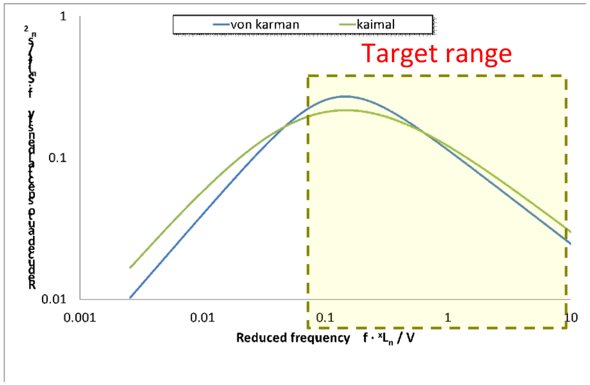
- $U = 43.8 m/s$
- $z = 37.965m$, $z_0 = 0.01m$, $\alpha = 0.12$
- $I_u = 12.1\%$, $I_v = 9.71\%$, $I_w = 6.07\%$
- $L_u = 112.49m$, $L_v = 28.124m$, $L_w = 9.375m$
- $\sigma_u = 5.3182m/s$, $\sigma_v = 4.2546m/s$, $\sigma_w = 2.6591m/s$

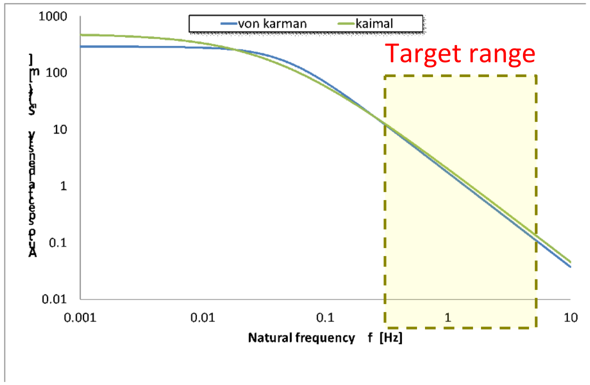
Target ranges for bridge aerodynamics

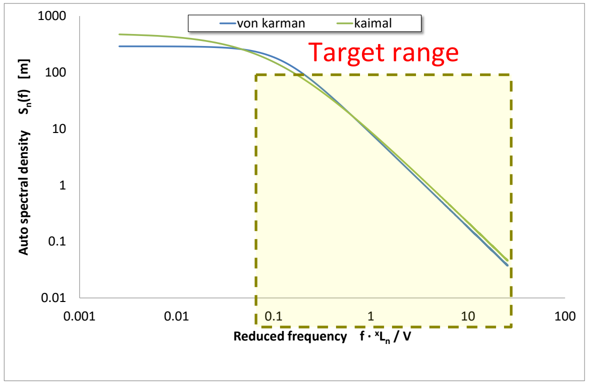
- Structural frequency: 0.05Hz – 5Hz
- 10 mean wind velocity: 10m/s – 80m/s
- Reduced frequency ($f \cdot {}^{x_f}L_n / V$)
  - Longitudinal: 0.07 – 28.12
  - Vertical: 0.006 – 2.344

Examples of Turbulence Spectrum

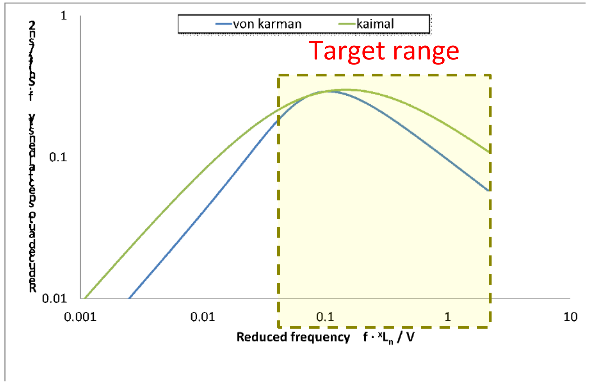
Longitudinal Turbulence

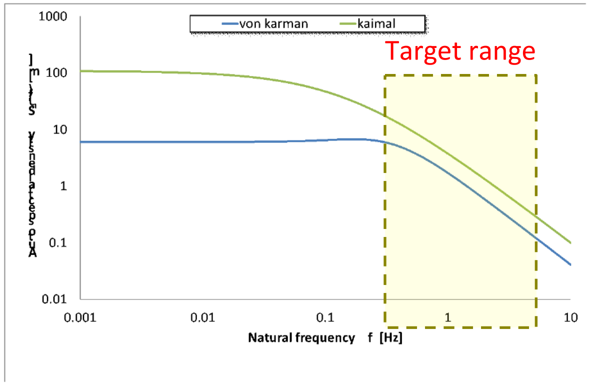


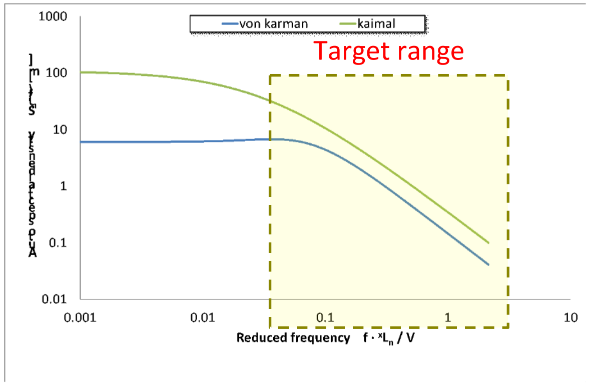




Vertical Turbulence







[Normalized]
: x, y axis

[Un - normalized]
: none of axis

[Semi – normalized]
: x axis

Auto Spectral Density in a Complex Format

- **Adopting a frequency axis**
  - spanning the entire range of both of positive and negative (imaginary) values, introducing the Euler formulae

$$\begin{bmatrix} e^{i\omega t} \\ e^{-i\omega t} \end{bmatrix} = \begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix} \begin{bmatrix} cos\omega t \\ sin\omega t \end{bmatrix}$$

and defining the complex Fourier amplitude

$$d_k = \frac{1}{2}(a_k - i \cdot b_k)$$

$$x(t) = \sum_{-\infty}^{\infty} X_k(\omega_k, t) = \sum_{-\infty}^{\infty} d_k(\omega_k) \cdot e^{i \cdot \omega_k t}$$

Taking the variance of the complex Fourier components and dividing by $\Delta\omega$

$$\frac{E\left[X_k{}^* \cdot X_k\right]}{\Delta\omega} = \frac{1}{T}\int_0^T \frac{\left(d_k{}^* e^{-i\omega_k t}\right)\left(d_k e^{i\omega_k t}\right)}{\Delta\omega} dt = \frac{d_k{}^* d_k}{\Delta\omega}$$

$$\frac{E\left[X_k{}^* \cdot X_k\right]}{\Delta\omega} = \frac{1}{4}\frac{(a_k + i \cdot b_k)(a_k - i \cdot b_k)}{\Delta\omega} = \frac{c_k{}^2}{4\Delta\omega}$$

Auto Spectral Density in a Complex Format

A symmetric double-sided auto spectrum

- Associated with $-\omega_k$ as well as $+\omega_k$
- The complex Fourier component is used to extend the frequency axis from minus infinity to plus infinity

$$S_x(\pm\omega_k) = \frac{E\left[{X_k}^* \cdot X_k\right]}{\Delta\omega} = \frac{{d_k}^* d_k}{\Delta\omega} = \frac{{c_k}^2}{4\Delta\omega}$$

- In the limit of T and N → ∞, becomes the continuous function $S_x(\pm\omega_k)$
- The variance of the process may be obtain by $\int_{-\infty}^{+\infty} S_x(\pm\omega_k)d\omega$
- Thus, the connection between double- and single-sided spectra is simply that$S_x(\omega) = 2 \cdot S_x(\pm\omega)$.
- Assuming that the process is stationary and of infinite length, such that the position of the time axis for integration purposes is arbitrary, then the non-normalized amplitude is defined as a Fourier constant.

$$a_k(\omega_k) = \int_0^T x(t) \cdot e^{-i\cdot\omega_k t} dt = T \cdot d_k$$

Auto Spectral Density in a Complex Format

in which case the double-sided auto-spectral density defined by

$$S_x(\pm\omega_k) = \frac{{d_k}^* d_k}{\Delta\omega} = \frac{({a_k}^*/T)(a_k/T)}{2\pi/T} = \frac{1}{2\pi T} \cdot {a_k}^* a_k$$

In the limit of T and N → ∞ this may be written on the following continuous form

$$S_x(\pm\omega) = \lim_{T\to\infty} \lim_{N\to\infty} \frac{1}{2\pi T} \cdot a^*(\omega) \cdot a(\omega)$$

and accordingly, the single sided version is given by

$$S_x(\omega) = \lim_{T\to\infty} \frac{1}{\pi T} \cdot a^*(\omega) \cdot a(\omega)$$

- **Application of the frequency *f* (hz) for the wind engineering,**

$$S_x(f) \cdot \Delta f = S_x(\omega) \cdot \Delta\omega = S_x(\omega) \cdot (2\pi \cdot \Delta f)$$
$$\Rightarrow S_x(f) = 2\pi \cdot S_x(\omega) = \lim_{T\to\infty} \lim_{N\to\infty} \frac{1}{2\pi T} \cdot a^*(f) \cdot a(f)$$

Cross spectral density

- **Given two stationary time variable functions *x(t)* and *y(t)*, both with length T and zero mean value**

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \lim_{n\to\infty} \sum_{-N}^{N} \begin{bmatrix} X_k(\omega_k, t) \\ Y_k(\omega_k, t) \end{bmatrix}$$

where,

$$\begin{bmatrix} X_k(\omega_k, t) \\ Y_k(\omega_k, t) \end{bmatrix} = \frac{1}{T} \begin{bmatrix} a_{X_k}(\omega_k) \\ a_{Y_k}(\omega_k) \end{bmatrix} \cdot e^{i\cdot\omega_k t}, \quad \begin{bmatrix} a_{X_k}(\omega_k) \\ a_{Y_k}(\omega_k) \end{bmatrix} = \lim_{n\to\infty} \int_{-T/2}^{T/2} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \cdot e^{-i\cdot\omega_k t} dt$$

and where $\omega_k = k \cdot \Delta\omega$ and $\Delta\omega = 2\pi/T$.

- **The definition of the double-sided cross-spectral density S_{xy} associated with the frequency ω_k**

$$S_{xy}(\pm\omega_k) = \frac{E[X_k^* \cdot Y_k]}{\Delta\omega} = \frac{1}{2\pi T} a_{X_k}^* \cdot a_{Y_k}$$

Cross spectral density

- **Orthogonality of the Fourier components**

$$E\left[X_i(\omega_i, t) \cdot Y_j(\omega_j, t)\right] = \begin{cases} S_{xy}(\omega_k, t) \cdot \Delta\omega \ when \ i = j = k \\ 0 \ when \ i \neq j \end{cases}$$

- **Covariance between *x(t)* and *y(t)* are given by**

$$Cov_{xy} = E[x(t) \cdot y(t)] = \lim_{N\to\infty} E\left[\sum_{-N}^{N} X_i \cdot \sum_{-N}^{N} Y_j\right] = \lim_{N\to\infty} \sum_{-N}^{N} (E[X_k \cdot Y_k])$$

$$\Rightarrow Cov_{xy} = \lim_{N\to\infty} \sum_{-N}^{N} S_{xy}(\pm\omega_k) \cdot \Delta\omega$$

- **Double-sided cross spectral density in a continuous format**

$$S_{xy}(\pm\omega) = \lim_{T\to\infty} \lim_{N\to\infty} \frac{E[X^*(\omega, t) \cdot Y(\omega, t)]}{\Delta\omega} = \lim_{T\to\infty} \lim_{N\to\infty} \frac{1}{2\pi T} a_X{}^*(\omega) \cdot a_Y(\omega)$$

- **The single sided cross spectral density**

$$S_{xy}(\omega) = 2 \cdot S_{xy}(\pm\omega) = \lim_{T\to\infty} \lim_{N\to\infty} \frac{1}{\pi T} a_X{}^*(\omega) \cdot a_Y(\omega)$$

Cross spectrum

- **Single-sided version using frequency *f* (Hz)**

$$S_{xy}(f) = 2\pi \cdot S_{xy}(\omega) = \lim_{T\to\infty} \lim_{N\to\infty} \frac{2}{T} \cdot a_x{}^*(f) \cdot a_y(f)$$

- **Covariance between the two processes**

$$Cov_{xy} = \int_{-\infty}^{+\infty} S_{xy}(\pm\omega)d\omega = \int_{0}^{+\infty} S_{xy}(\omega)d\omega = \int_{0}^{+\infty} S_{xy}(f)df$$

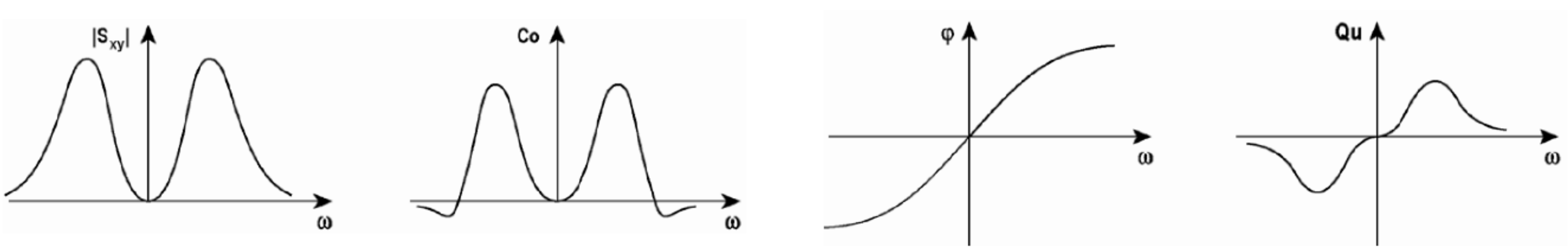
- **Cross-spectrum**

$$S_{xy}(\omega) = Co_{xy}(\omega) - i \cdot Qu_{xy}(\omega)$$

- **Alternatively, the cross-spectrum may be expressed by its modulus and phase**

$$S_{xy}(\omega) = \left|S_{xy}(\omega)\right| \cdot e^{i\cdot\varphi_{xy}(\omega)}$$

- **where the phase spectrum** $\varphi_{xy}(\omega) = arctan\left[Qu_{xy}(\omega)/Co_{xy}(\omega)\right]$



- **Auto-spectra can also be calculate from the auto covariance function.**

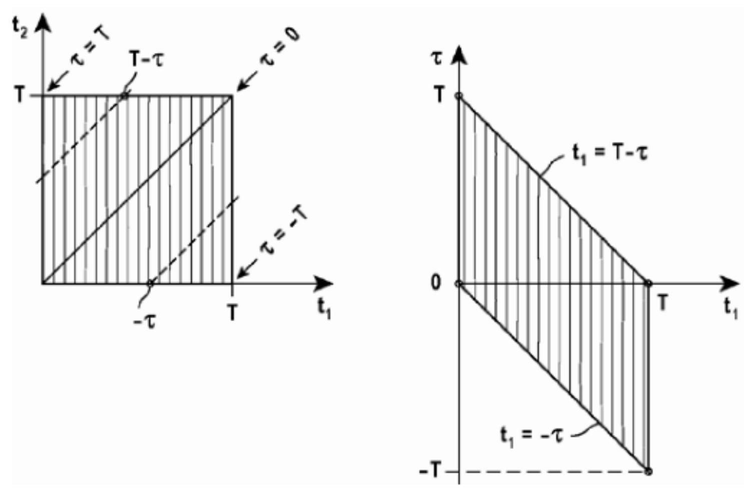
$$S_x(\omega) = \lim_{T\to\infty} \frac{E\left[X_k^{\,*}\ X_k\right]}{\Delta\omega} = \lim_{T\to\infty} \frac{E\left[\left(\frac{1}{T}\int_0^T x(t)e^{i\omega t}dt\right)\cdot\left(\frac{1}{T}\int_0^T x(t)e^{-i\omega t}dt\right)\right]}{2\pi/T}$$

$$= \lim_{T\to\infty} \frac{1}{2\pi T}\int_0^T\int_0^T E[x(t_1)\cdot x(t_2)]\cdot e^{-i\omega(t_2-t_1)}dt_1dt_2$$

$$\Rightarrow S_x(\omega) = \lim_{T\to\infty}\frac{1}{2\pi T}\int_0^T\int_0^T Cov_x(t_2-t_1)\cdot e^{-i\omega(t_2-t_1)}dt_1dt_2$$

- **Replacing t_2 with $t_1+\tau$, the integration limit changes accordingly**

$$\int_0^T\int_0^T dt_1dt_2 = \int_{-T}^{0}\int_{-\tau}^{T} dt_1d\tau + \int_0^T\int_0^{T-\tau} dt_1d\tau \quad\Rightarrow$$



- **Replacing t_2 with $t_1 + d\tau$**

$$S_x(\omega) = \lim_{T\to\infty} \frac{1}{2\pi T}\left[\int_{-T}^{0}\int_{-\tau}^{T} Cov_x(\tau)\cdot e^{-i\omega\tau} dt_1\, d\tau + \int_{0}^{T}\int_{0}^{T-\tau} Cov_x(\tau)\cdot e^{-i\omega\tau} dt_1\, d\tau\right]$$

$$= \lim_{T\to\infty} \frac{1}{2\pi}\left[\int_{-T}^{0}\left(1+\frac{\tau}{T}\right) Cov_x(\tau)\cdot e^{-i\omega\tau} d\tau + \int_{0}^{T}\left(1-\frac{\tau}{T}\right) Cov_x(\tau)\cdot e^{-i\omega\tau} d\tau\right]$$

$$\Rightarrow S_x(\omega) = \lim_{T\to\infty} \frac{1}{2\pi}\int_{-T}^{0}\left(1-\frac{|\tau|}{T}\right) Cov_x(\tau)\cdot e^{-i\omega\tau} d\tau$$

As a result of the limit of $T\to\infty$,

$$S_x(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} Cov_x(\tau)\cdot e^{-i\omega\tau} d\tau$$

Wiener-Khintchine Relationship

- **This shows that the auto spectral density is the Fourier transform of the auto covariance function.**

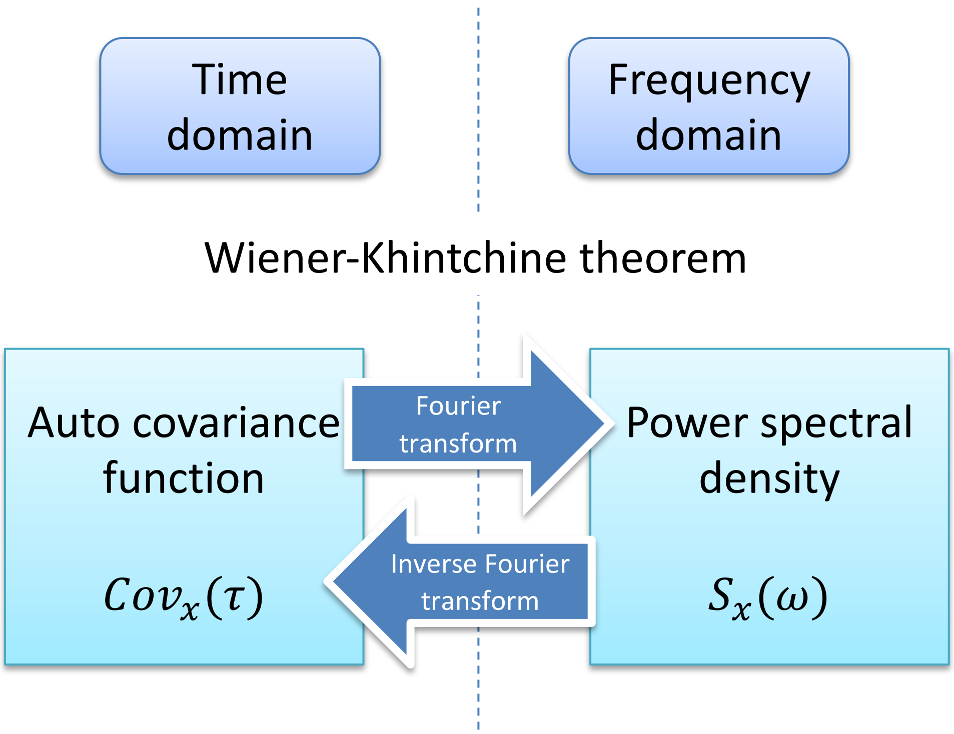
$$S_x(\omega) = \int_{-\infty}^{+\infty} Cov_x(\tau) \cdot e^{-i\omega\tau} d\tau$$

$$Cov_x(\tau) = \int_{-\infty}^{+\infty} S_x(\omega) \cdot e^{i\omega\tau} d\omega$$

- **Similarly, the cross spectral density is the Fourier trans form of the cross covariance function.**

$$S_{xy}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} Cov_{xy}(\tau) \cdot e^{-i\omega\tau} d\tau$$

$$Cov_{xy}(\tau) = \int_{-\infty}^{+\infty} S_{xy}(\omega) \cdot e^{i\omega\tau} d\omega$$



Coherence Function and Normalized Co-spectrum

Coherence function

$$Coh_{xy}(\omega) = \frac{|S_{xy}(\omega)|^2}{S_x(\omega) \cdot S_y(\omega)}$$

- If $\boldsymbol{x}(\boldsymbol{t})$ and $\boldsymbol{y}(\boldsymbol{t})$ are realizations of the same process

$$S_{xx}(\omega) = S_x(\omega) \cdot \sqrt{Coh_{xx}(\omega)} \cdot e^{i\varphi_{xx}(\omega)}$$

$\sqrt{Coh_{xx}(\omega)}$ **: root-coherence function**

φ_{xx} **: phase spectrum**

A normalized co-spectrum

$$\hat{C}o_{xx}(\omega) = \frac{Re[S_{xy}(\omega)]}{\sqrt{S_x(\omega) \cdot S_y(\omega)}}$$

- If $\boldsymbol{x}(\boldsymbol{t})$ and $\boldsymbol{y}(\boldsymbol{t})$ are realizations of the same stationary and ergodic process

$$Re[S_{xy}(\omega)] = S_x(\omega) \cdot \hat{C}o_{xx}(\omega)$$

Spectra and Coherence Functions for Wind Turbulences

- **While cross covariance functions represent the time and space domain properties of the turbulence components, it is a auto and cross spectral densities that describe the frequency-space domain properties.**

$$S_{nn}(\Delta s, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} Cov_{nn}(\Delta s, \tau) \cdot e^{-i\omega\tau} d\tau$$

- **Application of the frequency *f* (hz) for the wind engineering,**

$$S_{nn}(\Delta s, f) = \int_{-\infty}^{+\infty} Cov_{nn}(\Delta s, \tau) \cdot e^{-2\pi f\tau} d\tau$$

- **Expression with single point spectra, coherence function and phase spectra.**

$$S_{nn}(\Delta s, f) = S_n(f) \cdot \sqrt{Coh_{nn}(\Delta s, f)} \cdot \exp[i\varphi]$$

where $n = u, v, w$ **and** $\Delta s = \Delta x_f, \Delta y_f, \Delta z_f$**.**

Normalized Co-spectrum

The normalized co-spectrum with single point spectrum

$$\hat{C}o_{nn}(\Delta s, f) = \frac{\mathrm{Re}[S_{nn}(\Delta s, f)]}{S_n(f)}$$

- Since the wind field is usually assumed homogeneous and perpendicular to the span of the (line-like) structure, phase spectra may be neglected.
- In structural response calculations, spatial averaging takes place along the span of the structures, and then all imaginary parts cancel out and only a double set of real parts remain.

where $n = u, v, w$ **and** $\Delta s = \Delta x_f, \Delta y_f, \Delta z_f$**. That is necessary to give special attention to in wind engineering.**

Normalized Co-spectrum (Root Coherence)

- **Some simple expressions occurs in the literature**
  - For a first approximation
  - Under homogeneous conditions

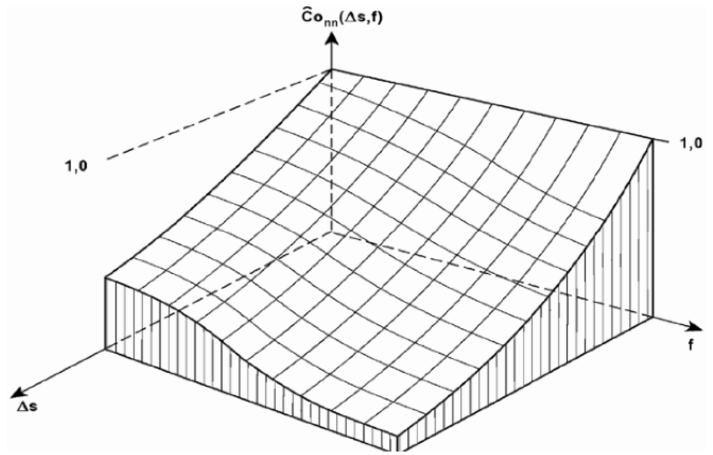
$$\hat{C}o_{nn}(\Delta s, f) = \exp\left(-c_{ns} \cdot \frac{f \cdot \Delta s}{V(z_f)}\right)$$

where $n = u, v, w$**,** $s = x_f, y_f, z_f$**,** $\Delta s = \Delta x_f, \Delta y_f, \Delta z_f$ **and**

$$c_{ns} = \begin{cases} c_{uy_f} = c_{uz_f} = 9 \\ c_{vy_f} = c_{vz_f} = c_{wy_f} = 6 \\ c_{wz_f} = 3 \end{cases}$$

Caution should be exercised as the variation in c_{ns} **value is considerable.**

- **The simple expressions for normalized co-spectrum has the obvious disadvantages:**
  - the spectrum value become unity at all Δs when $f = 0$ while typical reduced-co spectrum will decay at any value of f
  - It is positive in entire range of Δs and it is in conflict with the definition of zero mean turbulence components.



Typical reduced-co spectrum

- **Krenk's derivation applicable for the along-wind *u* component**
  - Under the assumption of isotropic condition

$$\hat{C}o_{nn}(\Delta s, f) = \left(1 - \frac{\kappa \cdot \Delta s}{2}\right) \cdot \exp(-\kappa \cdot \Delta s)$$

where

$$\kappa = \left[\left(\frac{2\pi f}{V}\right)^2 + \left(\frac{2\pi f}{1.34 \cdot {}^{x}L_f}\right)^2\right]^{1/2}$$

THANK YOU

for your attention!