Engineering Economic Analysis

2019 SPRING

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Chap. 2

BUDGET CONSTRAINT

Consumption Choice Sets

- A consumption choice set, **X,** is the collection of all consumption choices available to the consumer.

$$X = \mathbf{R}_+^n$$

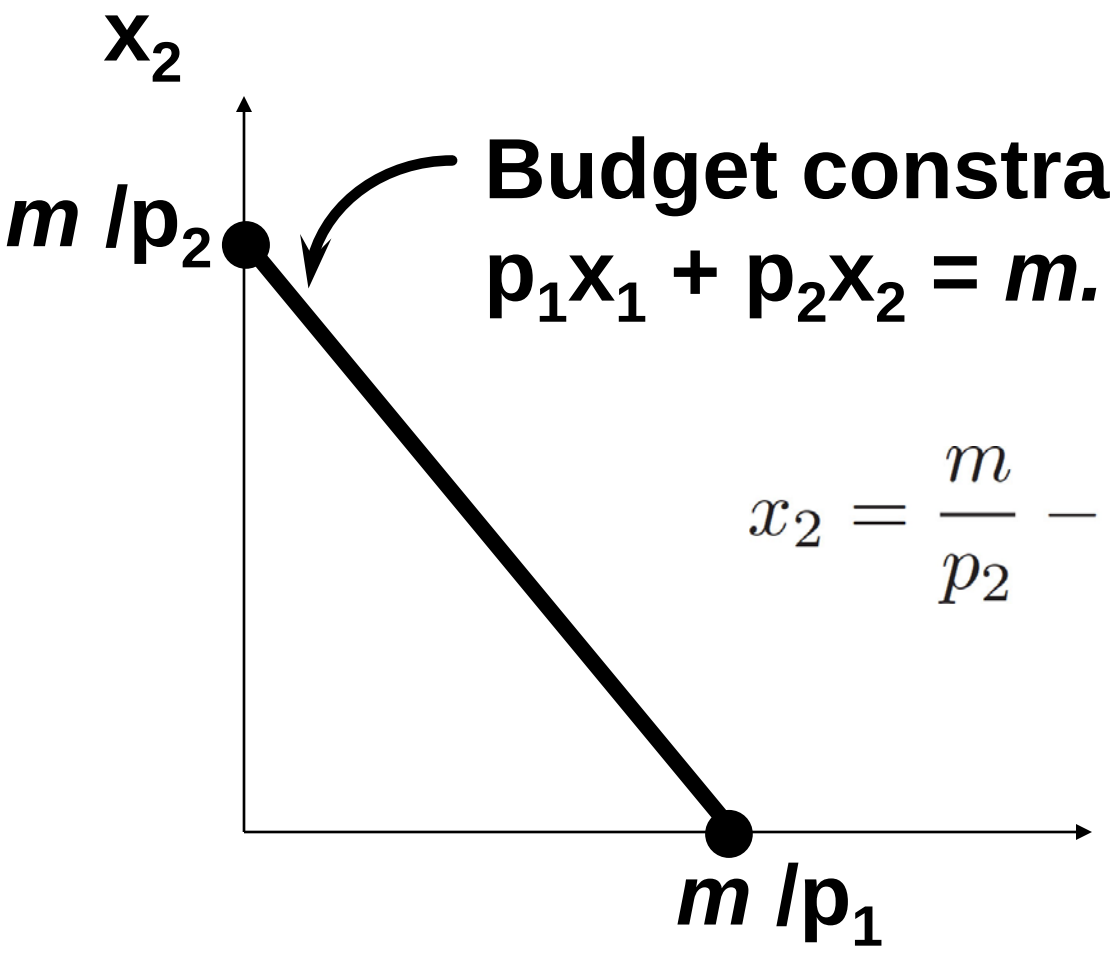
- A consumption bundle, $\tilde{x}$, containing x_1 units of commodity 1, x_2 units of commodity 2 and so on up to x_n units of commodity n is denoted by the vector $\tilde{x} = (x_1, ..., x_n) \in \mathbf{X}$

- Commodity price vector $\tilde{p} = (p_1, ..., p_n) \in \mathbf{R}_+^n$

Budget Constraints

- Q: When is a bundle $(x_1, \ldots, x_n)$ affordable at prices $p_1, \ldots, p_n$?
  - A: When $p_1x_1 + \ldots + p_nx_n \leq m$
    where m is the consumer's (disposable) income.
- The consumer's budget set is the set of all affordable bundles;
  $B(p_1, \ldots, p_n, m) = \{ (x_1, \ldots, x_n) \mid x_1 \geq 0, \ldots, x_n \geq 0$
  $\text{and } p_1x_1 + \ldots + p_nx_n \leq m \}$
- The budget constraint is the upper boundary of the budget set. $p_1x_1 + \ldots + p_nx_n = m$

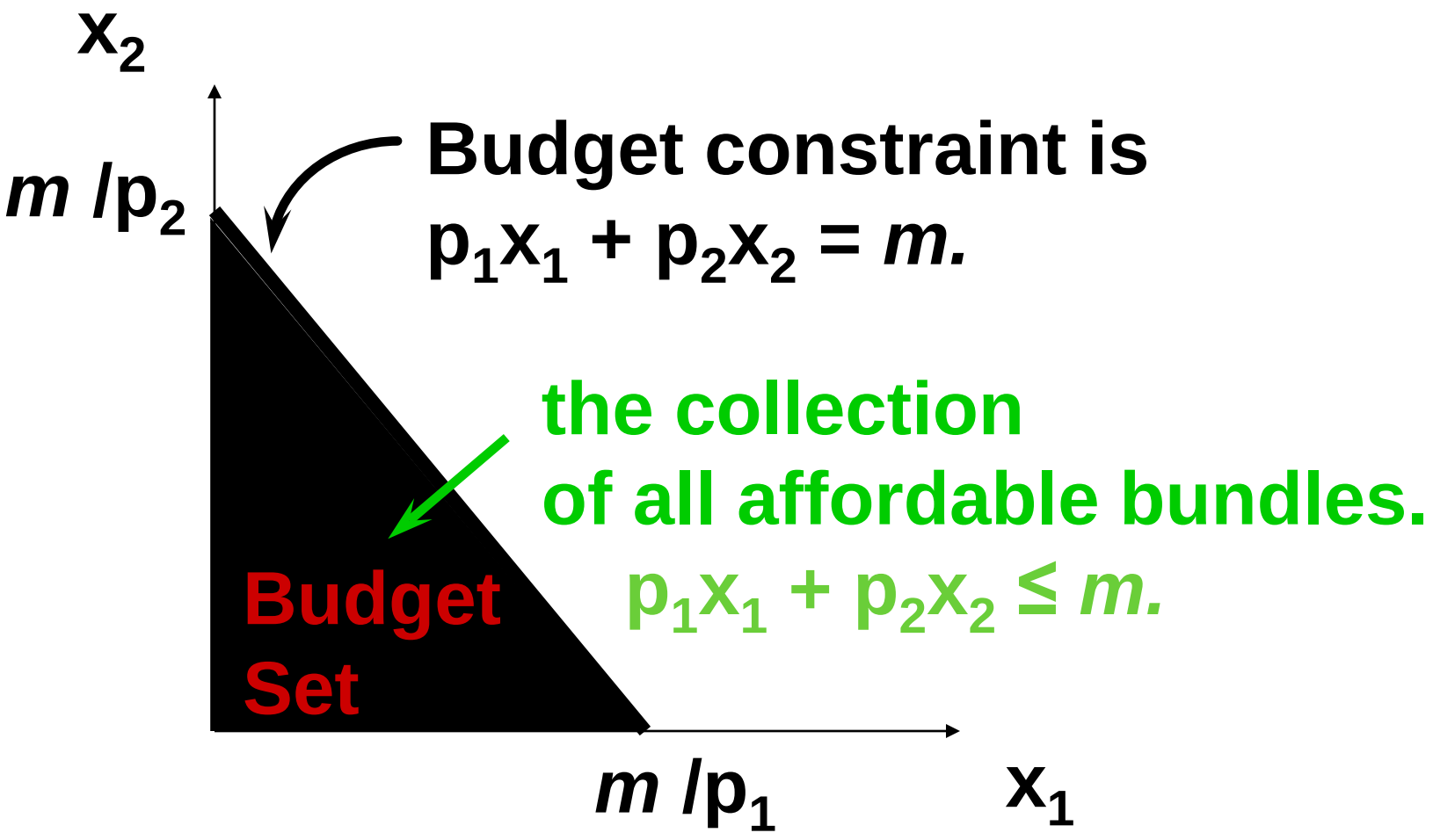
Budget Set and Constraint for Two Commodities



Budget constraint is
$p_1x_1 + p_2x_2 = m$.

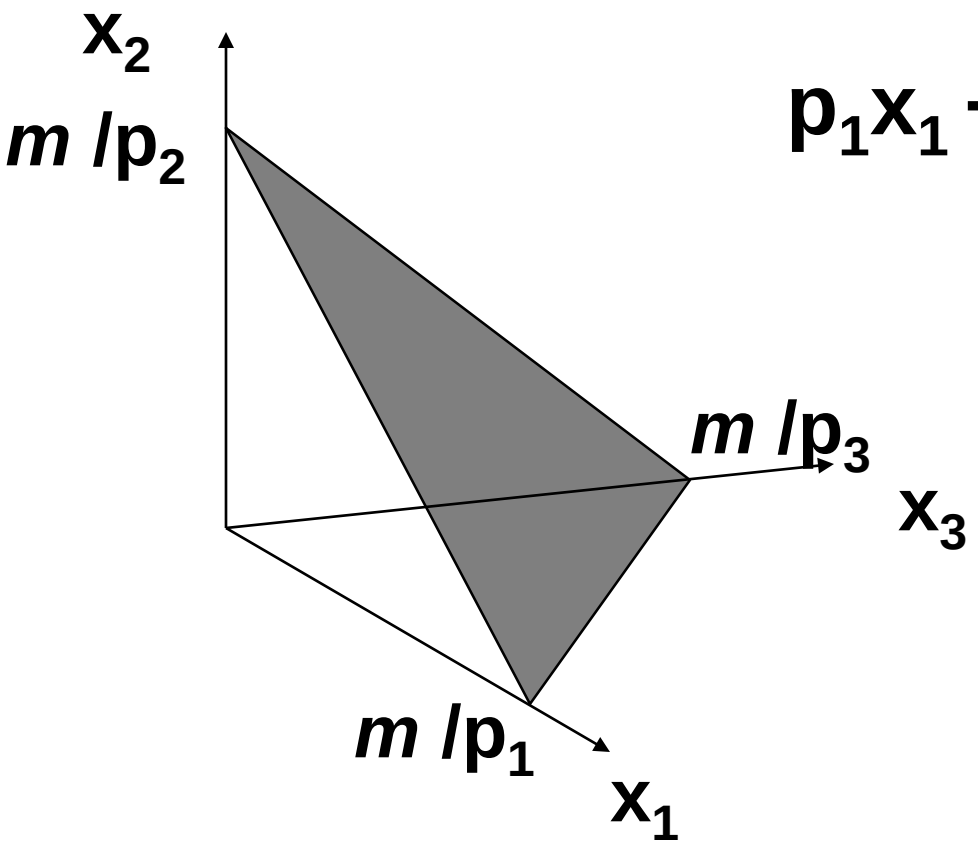
$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$$

Budget Set and Constraint for Two Commodities



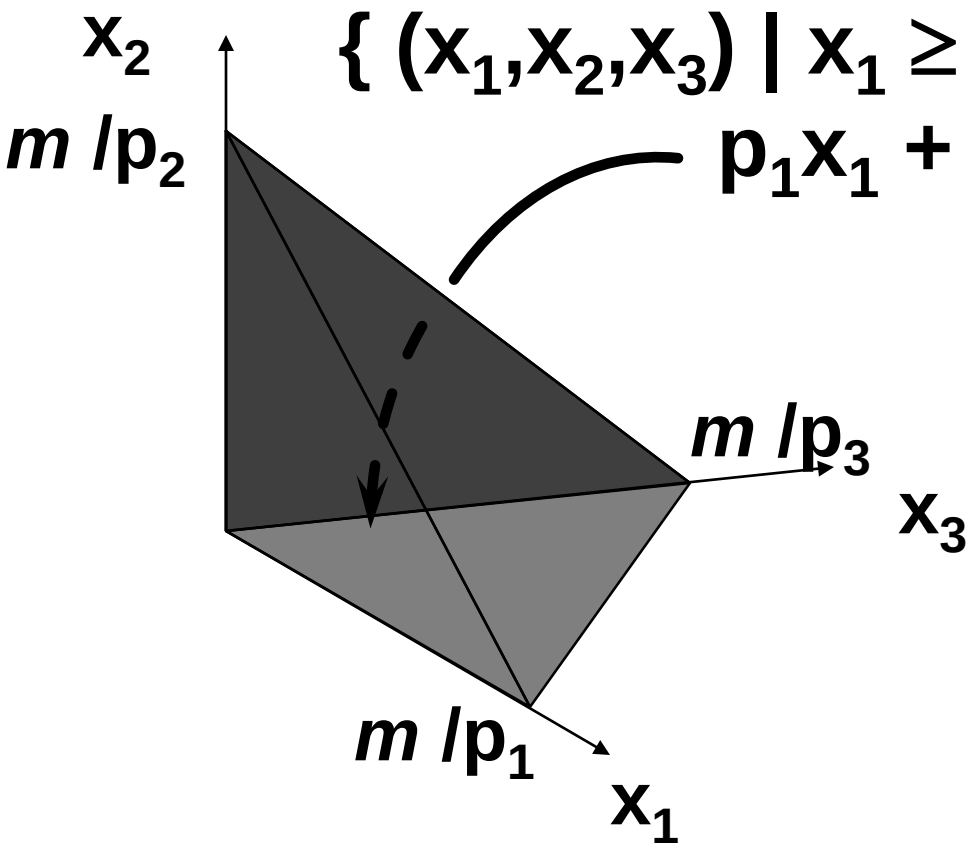
Budget Constraint for Three Commodities

- If n = 3



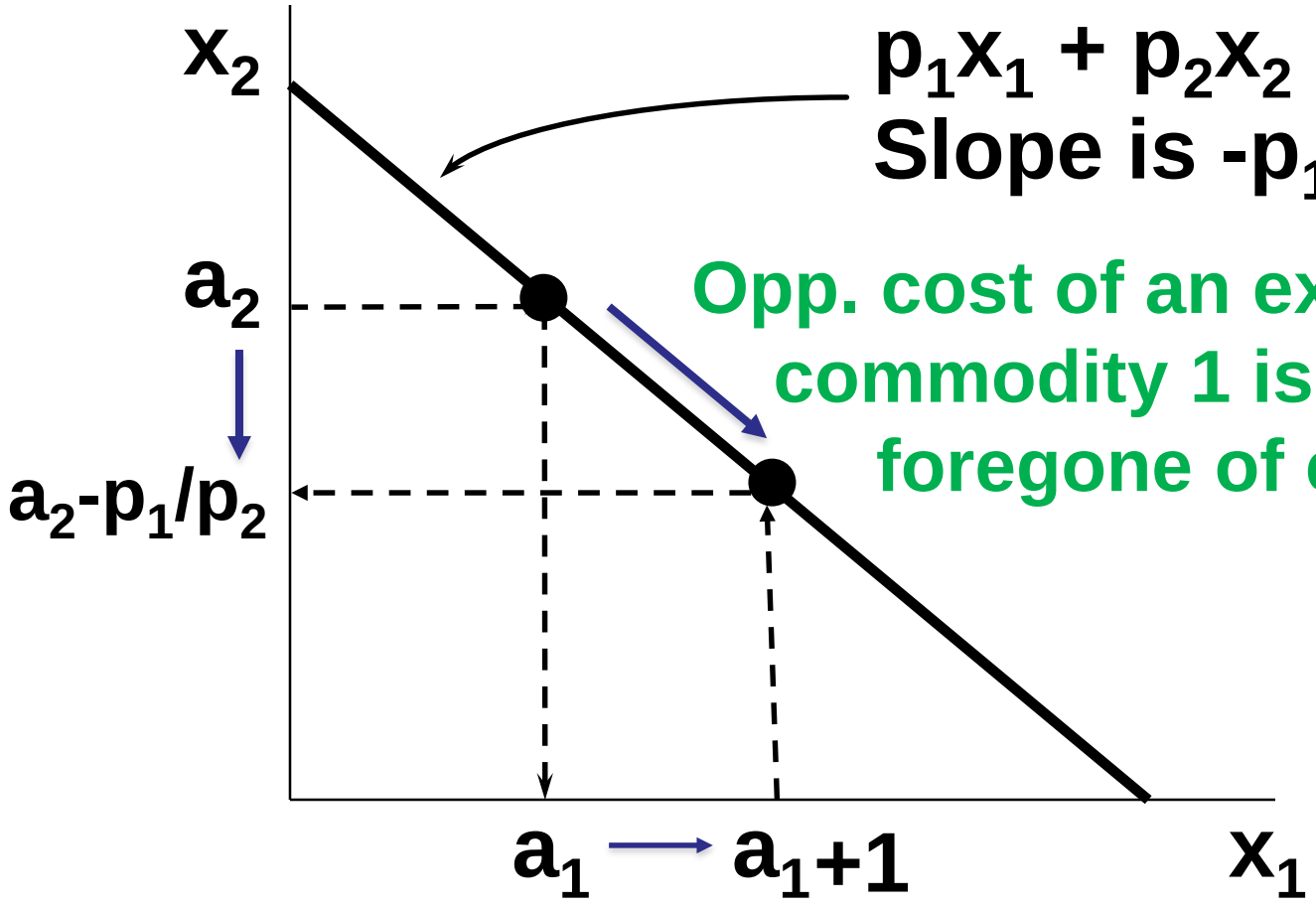
$$p_1x_1 + p_2x_2 + p_3x_3 = m$$

Budget Set for Three Commodities



$$\{ (x_1,x_2,x_3) \mid x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0 \text{ and } p_1x_1 + p_2x_2 + p_3x_3 \leq m \}$$

Opportunity cost in Budget Constraints



Opportunity cost in Budget Constraints (M)

- Suppose that the consumer changes her consumption while satisfying budget constraint

$$p_1(x_1 + \Delta x_1) + p_2(x_2 + \Delta x_2) = m$$

- Since,

$$p_1 x_1 + p_2 x_2 = m$$

- Then,

$$p_1 \Delta x_1 + p_2 \Delta x_2 = 0$$

- Thus,

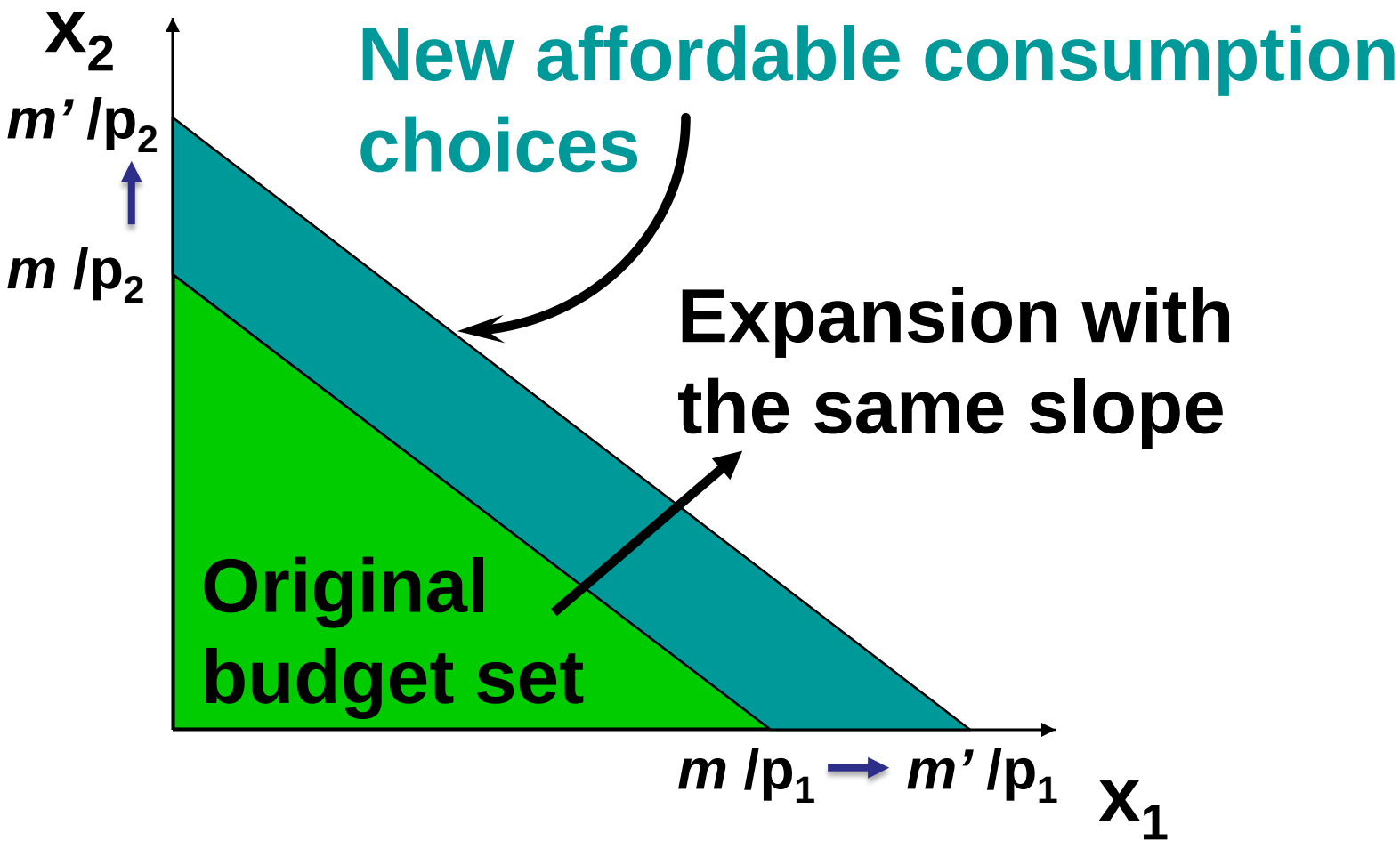
$$\frac{\Delta x_2}{\Delta x_1} = -\frac{p_1}{p_2}$$

Budget changes As Income and Price Changes

- The budget constraint and budget set depend upon prices and income. What happens as prices or income change?

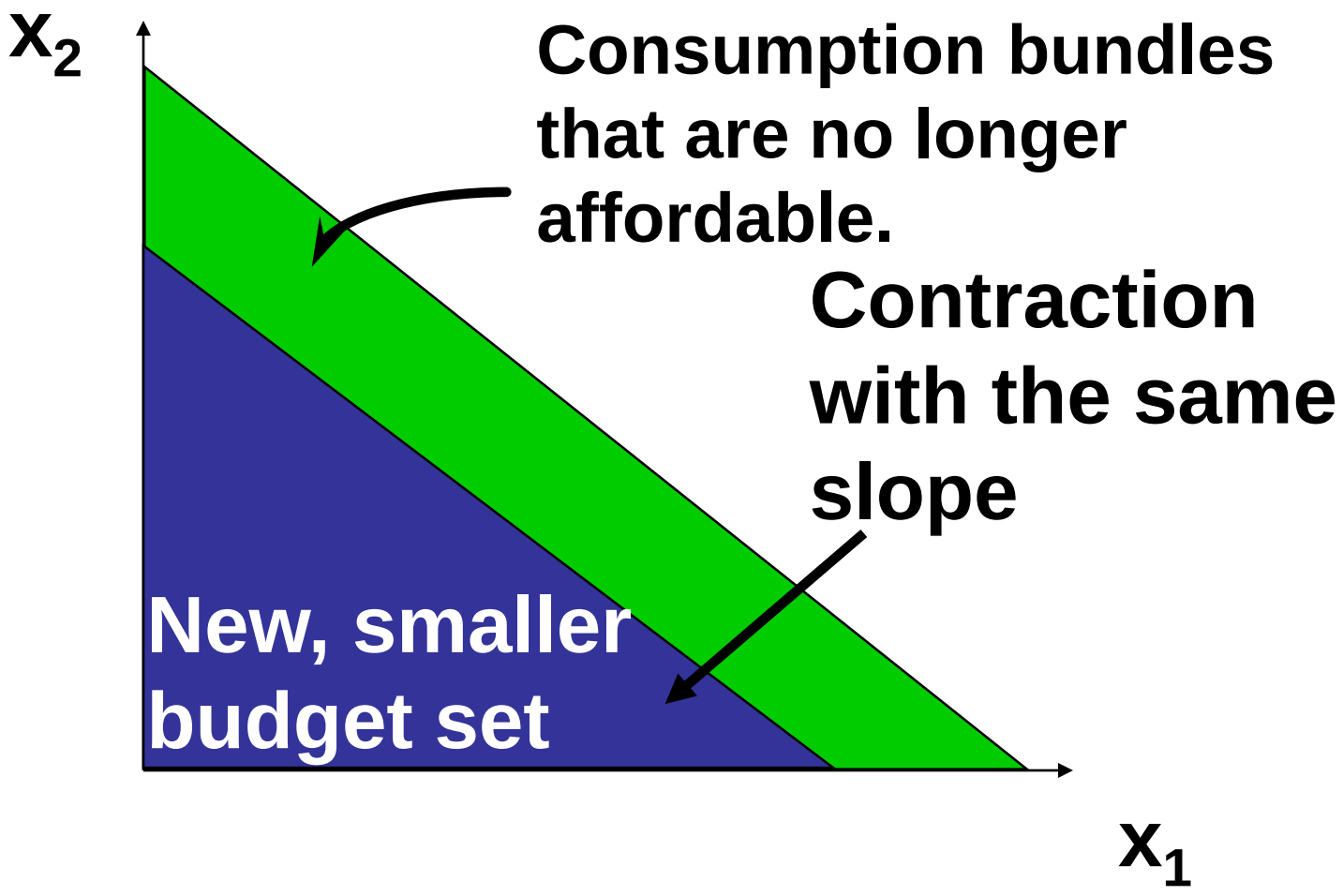
Budget Constraints - Income Changes

- Change(increase) in income: m → m'>m



Budget Constraints - Income Changes

- Change(decrease) in income: m → m'<m

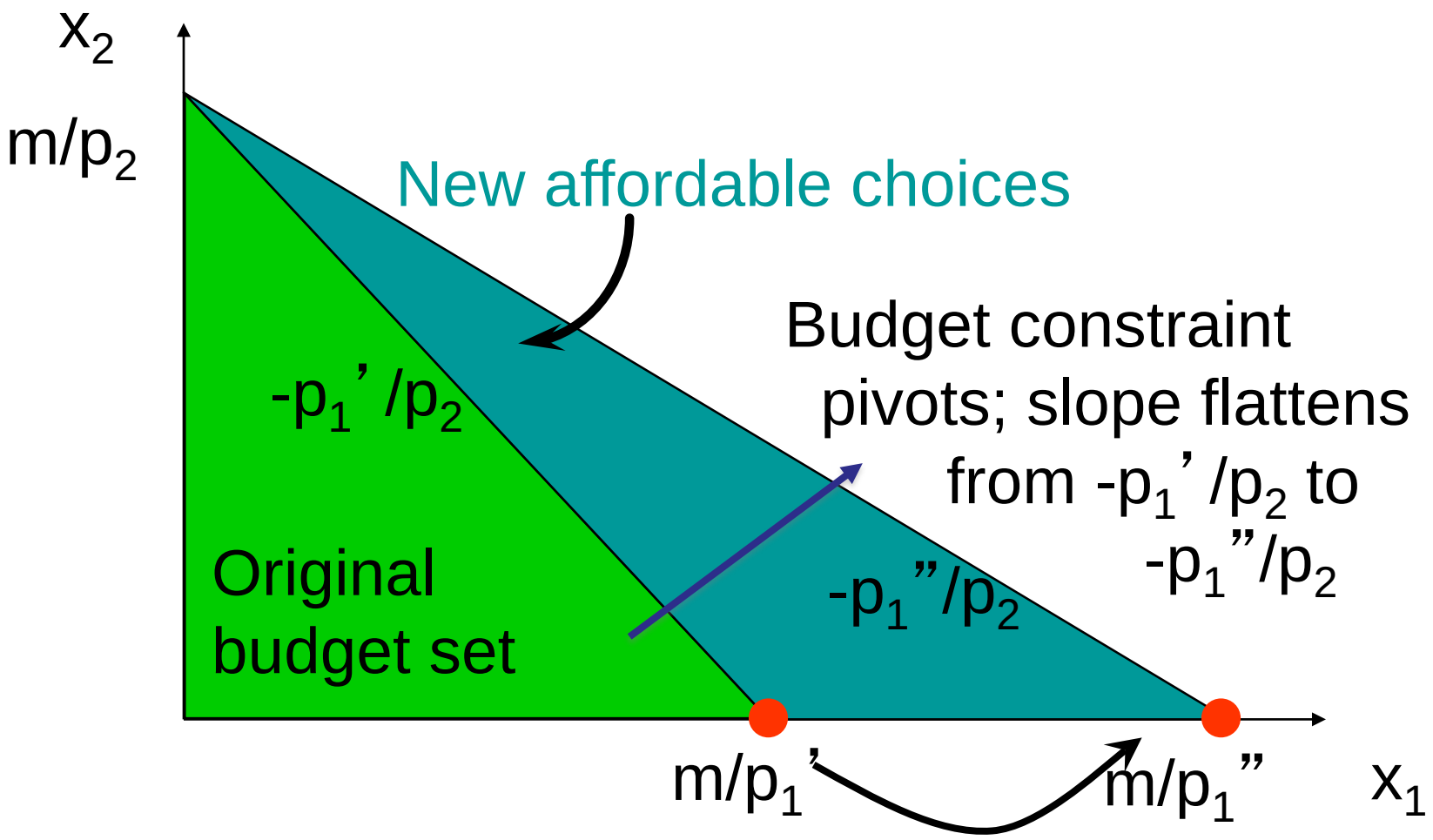


Budget Constraints - Income Changes

- Increases in income *m* shift the constraint outward in a parallel manner, thereby enlarging the budget set and improving choice.
- Decreases in income *m* shift the constraint inward in a parallel manner, thereby shrinking the budget set and reducing choice. Thus income decrease will make the consumer worse off.

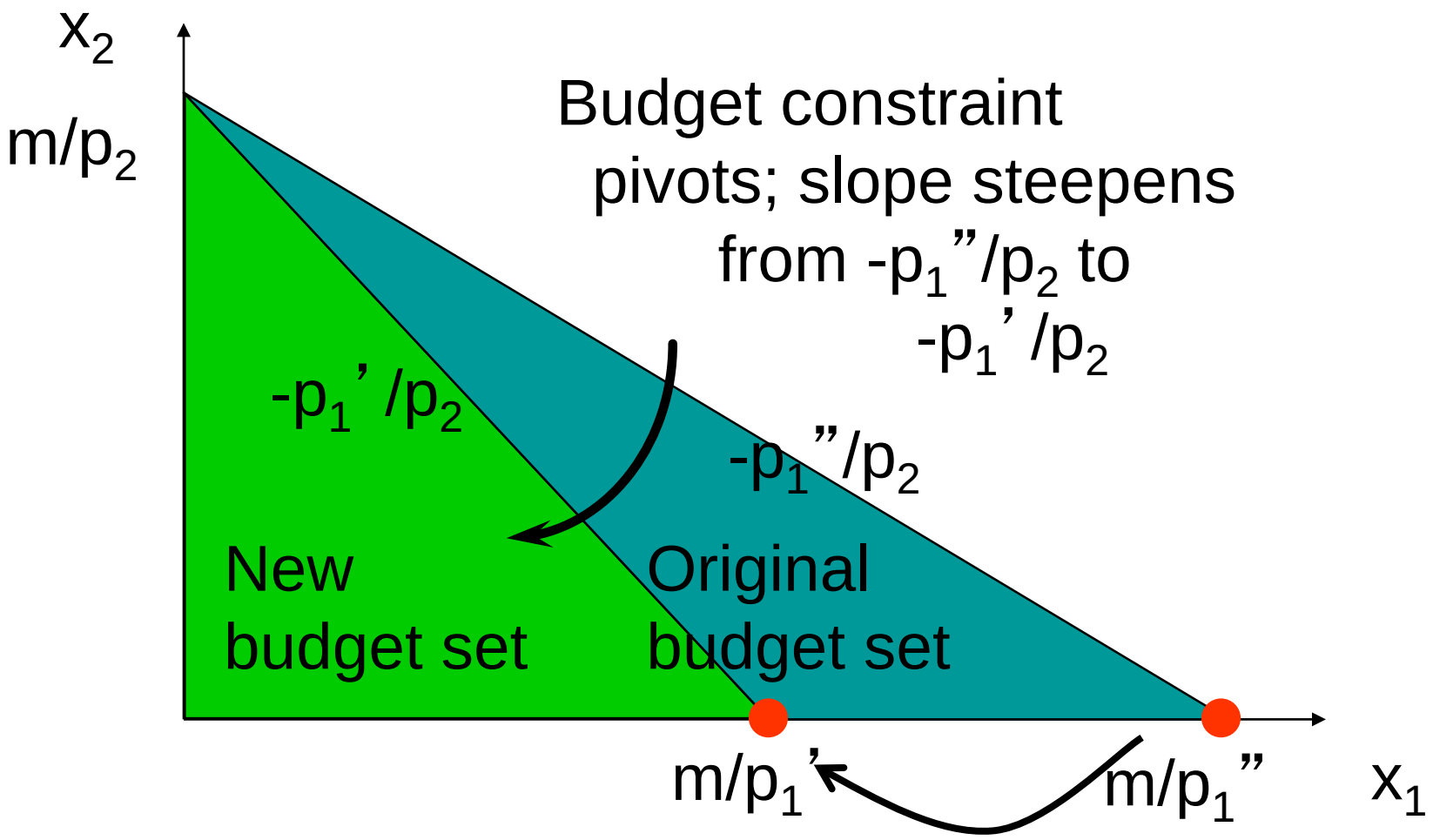
Budget Constraints - Price Changes

- Suppose p_1 decreases.($p_1' \rightarrow p_1'' < p_1'$)



Budget Constraints - Price Changes

- Suppose p_1 increases. (p_1" → p_1' >p_1")



Budget Constraints - Price Changes

- Reducing the price of one commodity pivots the constraint outward. No old choice is lost and new choices are added, so reducing one price can make the consumer better off.
- Similarly, increasing one price pivots the constraint inwards, reduces choice and may (typically will) make the consumer worse off.

Numeraire

- “Numeraire” means “unit of account”.
- Budget set with relative price

$$\frac{p_1}{p_2}x_1 + x_2 = \frac{m}{p_2}$$

- Setting p_2=1 makes commodity 2 the numeraire and defines all prices relative to p_2
- Can consider only one price without changing the original budget set
- Any commodity can be chosen as the numeraire without changing the budget set or the budget constraint.
- Composite good: Two goods are often enough

Taxes, Subsidies, and Rationing

- Quantity tax
  - A certain amount of tax should be paid to the government for each unit of the good purchased
  - t: quantity tax per unit of good i
  - $p_i \rightarrow p_i + t$ (higher price)

- Value tax (Ad Valorem tax)
  - Tax on the value (i.e. price) of the good
  - τ: tax rate
  - $p_i \rightarrow p_i (1+ \tau)$ (higher price)

Taxes, Subsidies, and Rationing

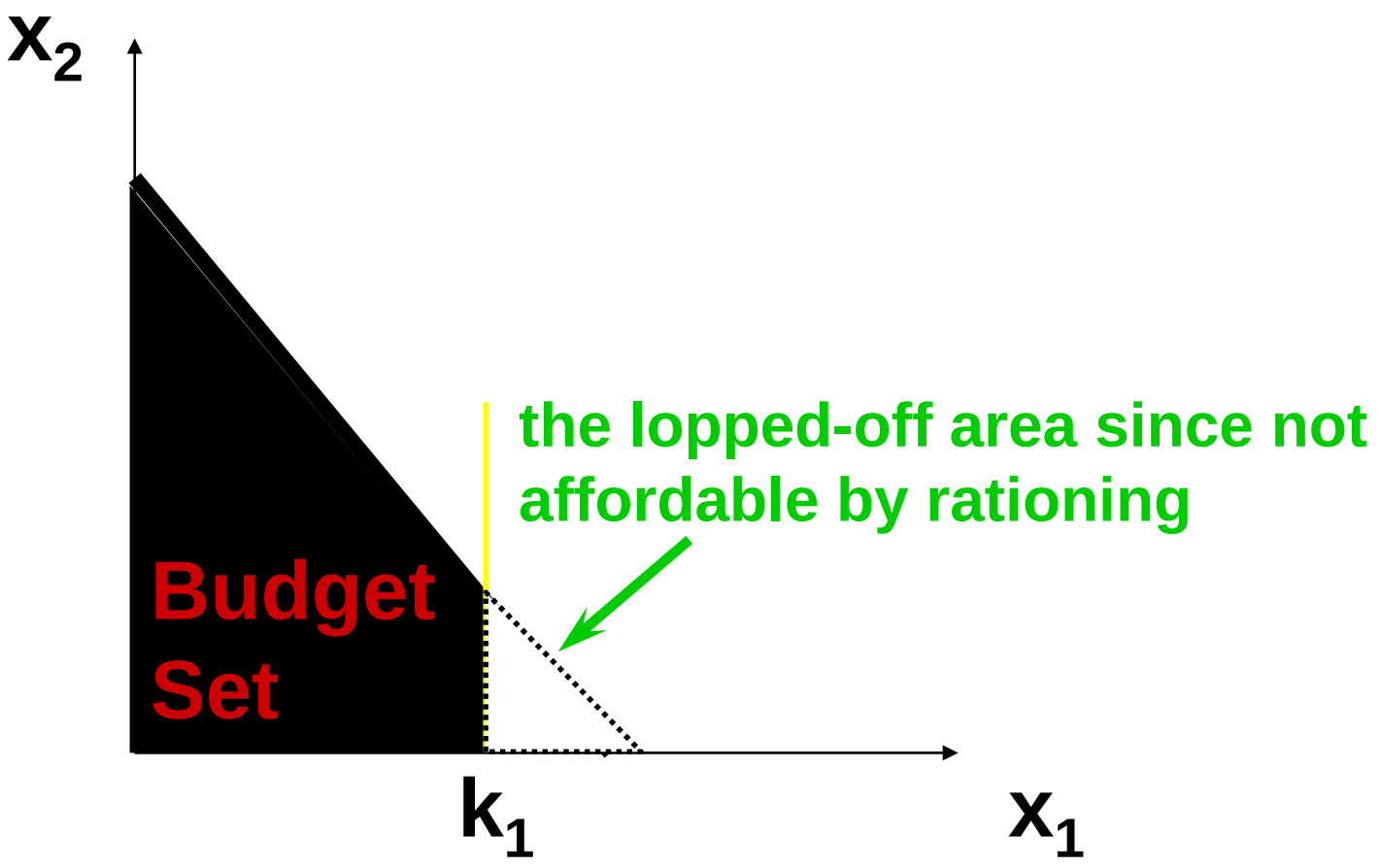
- Subsidy
  - Government gives an amount of money to the consumer that depends on the amount of the good purchased
  - the opposite of a tax
  - s: quantity subsidy per unit of good i
  - $p_i \rightarrow p_i - s$ (lower price)
  - σ: subsidy rate
  - $p_i \rightarrow p_i (1- \sigma)$ (lower price)

Taxes, Subsidies, and Rationing

- Lump-sum tax or subsidy
  - Government takes away (or gives) some fixed amount of money, regardless of individual's behavior
  - Impact on the income
  - t: lump-sum tax
  - $m \rightarrow m - t$ (lower income)
  - s: lump-sum subsidy
  - $m \rightarrow m + s$ (higher income)

Rationing

- The level of consumption of some good is fixed to be no larger than some amount, say k_1



Combinations

- Sometimes taxes, subsidies, and rationing can be combined
- Suppose that the quantity tax with tax t for good 1 is levied in excess consumption of k_1

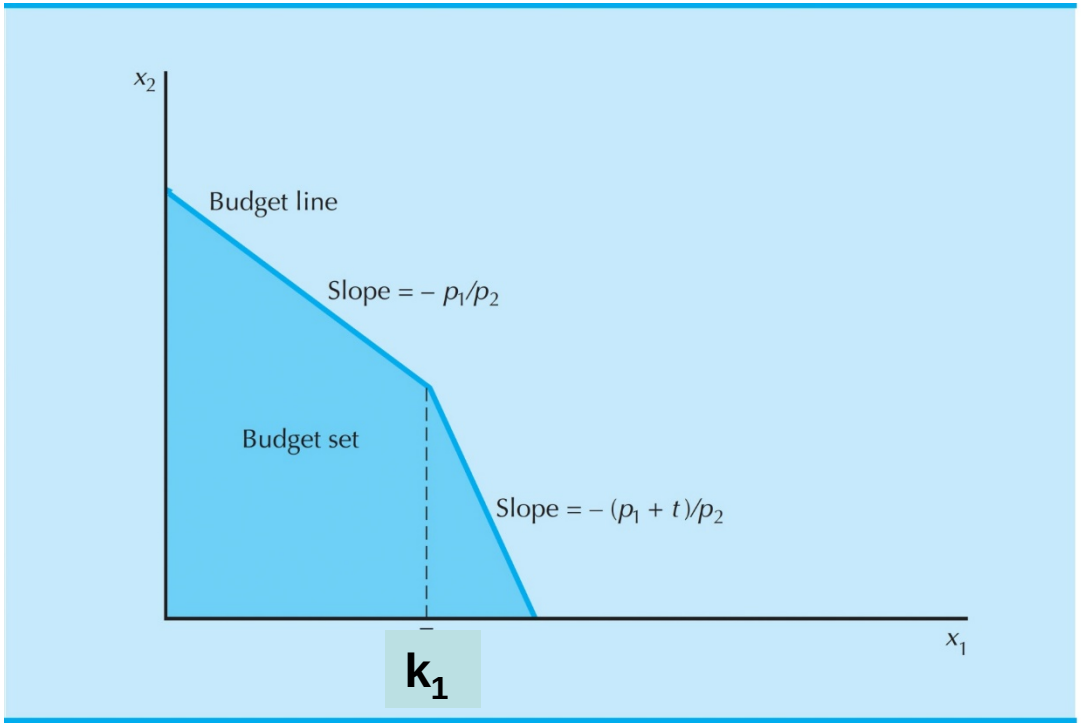


Figure 2.5

The Food Stamp Program

- Food stamps are coupons that can be legally exchanged only for food.

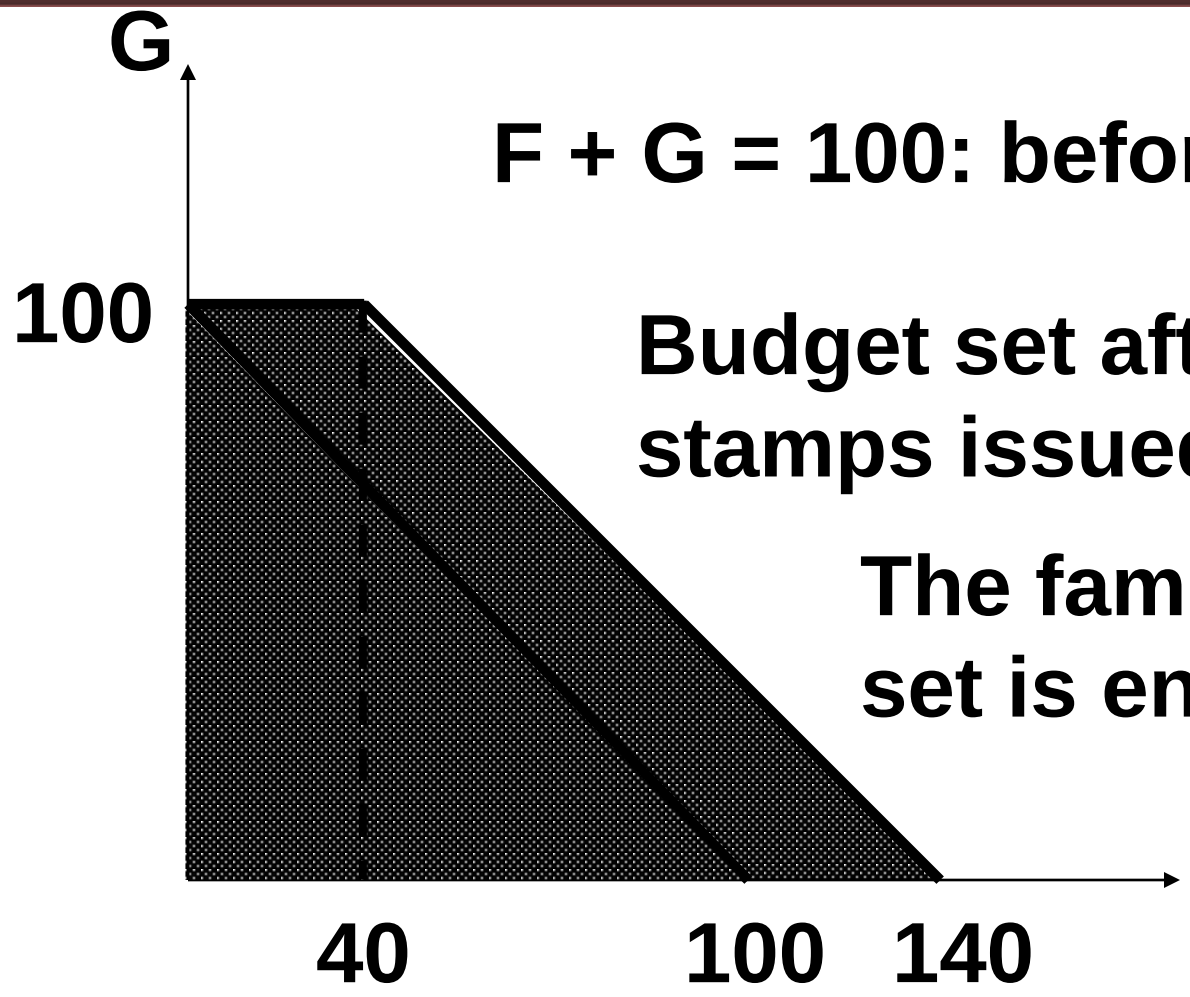
- How does a commodity-specific gift such as a food stamp alter a family's budget constraint?

The Food Stamp Program

- Suppose m = $100
- Let X = the amount of money spent for ‘Food(F)’, Y= the amount of money for ‘other goods(G)’
- Then, p_F = $1 and the price of “other goods” is p_G = $1.
- The budget constraint is then

$$F + G = 100.$$

The Food Stamp Program



F + G = 100: before stamps.

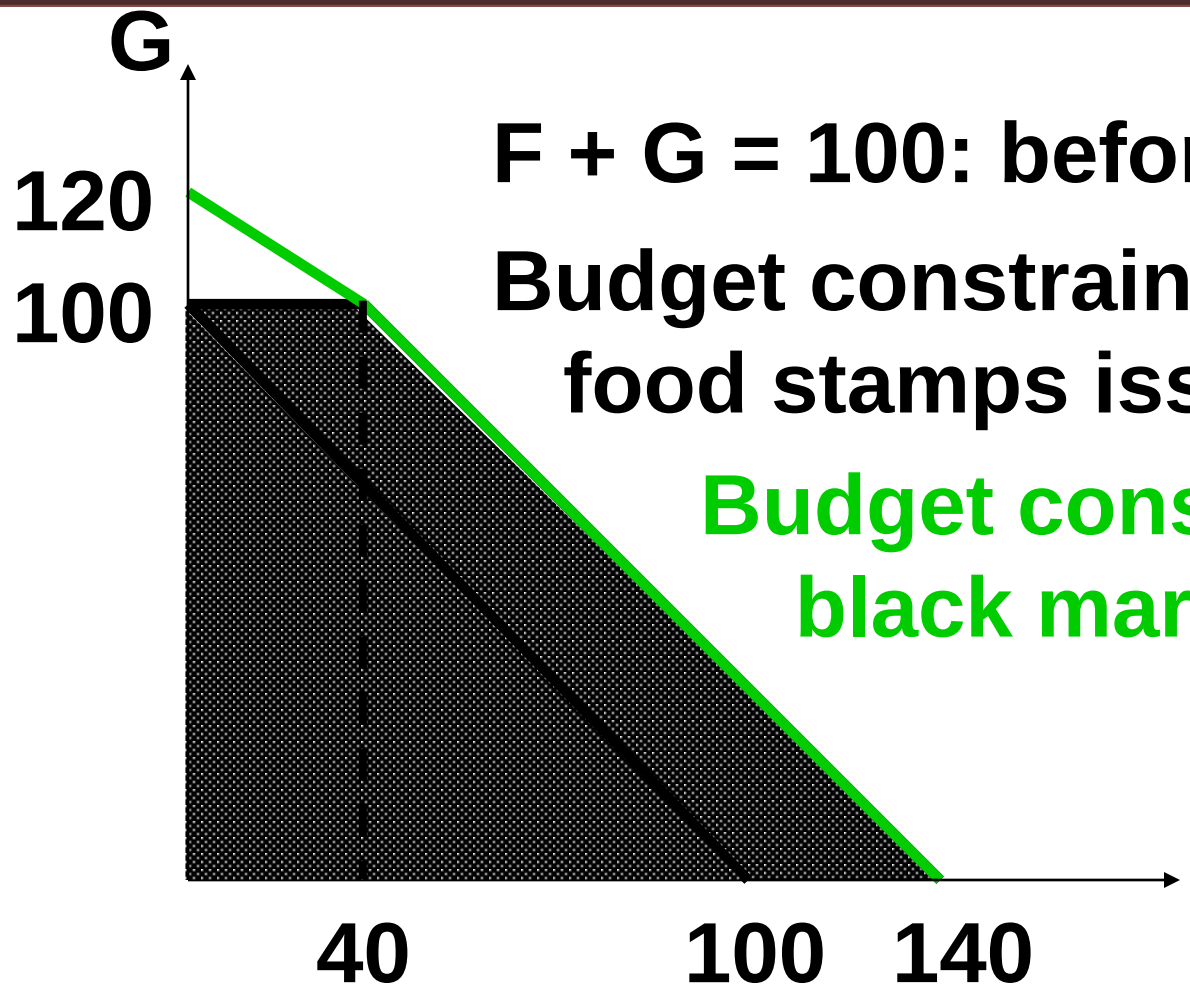
Budget set after $40 food stamps issued.

The family's budget set is enlarged.

The Food Stamp Program

- What if food stamps of $1 can be traded on a black market for $0.50 each?
- If the consumer wants to sell all $40 food stamps, her cash income increases to $120
  - Thus (0, 120) is just affordable
- If the consumer wants to use all $40 food stamps for food, her cash income remains as $100
  - Thus (40, 100) is just affordable
- If the consumer wants to sell half of $40 food stamps, her cash income increases to $110
  - Thus (20, 110) is just affordable

The Food Stamp Program

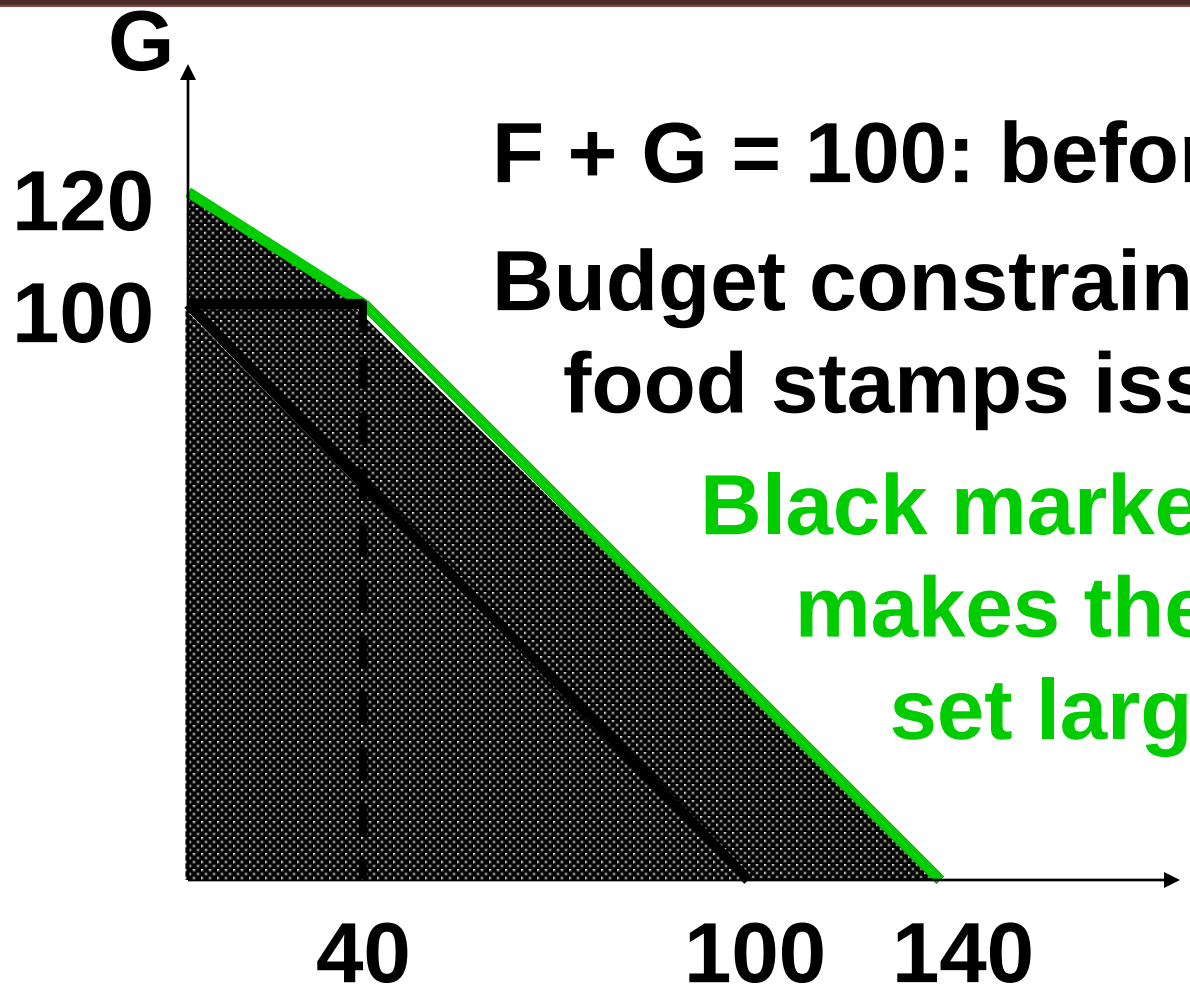


F + G = 100: before stamps.

Budget constraint after 40 food stamps issued.

Budget constraint with black market trading.

The Food Stamp Program



F + G = 100: before stamps.

Budget constraint after 40 food stamps issued.

Black market trading makes the budget set larger again.