

# 10.6 Applications to trusses and beams

## 10.6.1 Application to trusses

- 3-bar, hyperstatic truss (Fig. 10.16)

- bar length :  $L_1 = L_3 = \frac{L}{\cos \theta}$ ,  $L_2 = L$

- bar elongations : Eq.(9.27),  $e_1 = u_1 \cos \theta + u_2 \sin \theta$ ,  $e_2 = u_2$ ,

$$e_3 = -u_1 \cos \theta + u_2 \sin \theta$$

- bar strain E :  $A = \frac{1}{2} k e^2$ , Eq.(10.29),  $k = \frac{EA}{L}$  (bar stiffness)

$$\begin{aligned} A &= \frac{1}{2} \left( \frac{EA \cos \theta}{L} e_1^2 + \frac{EA}{L} e_2^2 + \frac{EA \cos \theta}{L} e_3^2 \right) \\ &= \frac{1}{2} \frac{EA}{L} [(u_1 \cos \theta + u_2 \sin \theta)^2 \cos \theta + u_2^2 \\ &\quad + (-u_1 \cos \theta + u_2 \sin \theta)^2 \cos \theta] \\ &= \frac{1}{2} \frac{EA}{L} [2u_1^2 \cos^3 \theta + (1 + 2\sin^2 \theta \cos \theta)u_2^2] \end{aligned}$$

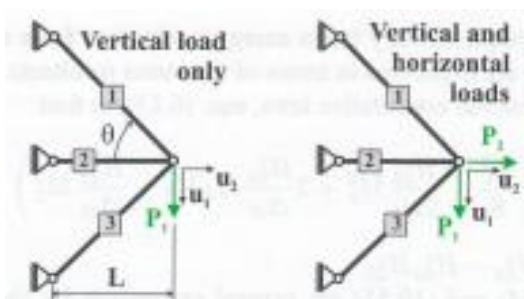


Fig. 10.16. Simple 3-bar truss

# 10.6 Applications to trusses and beams

- potential of externally applied load,  $P_1 \rightarrow \Phi = -P_1 u_1$

total potential  $\Pi = A + \Phi = A - P_1 u_1$

- 2 D.O.F.'s, PMTPE, Eq.(10.17)  $\rightarrow$

$$\frac{\partial \Pi}{\partial u_1} = \frac{EA}{L} 2u_1 \cos^3 \theta - P_1 = 0$$

$$\frac{\partial \Pi}{\partial u_2} = \frac{EA}{L} (1 + 2 \sin^2 \theta \cos \theta) u_2 = 0$$

- Matrix form ... two linear eqn.s for the 2 generalized coord.

$$\begin{bmatrix} z \cos^3 \theta & 0 \\ 0 & 1 + 2 \sin^2 \theta \cos \theta \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \frac{L}{EA} \begin{Bmatrix} P_1 \\ 0 \end{Bmatrix}$$

$$\rightarrow u_1 = \frac{P_1 L}{2EA \cos^3 \theta}, u_2 = 0$$

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## 10.6.3 Applications to beams

- beam under a distributed transverse load,  $p_2(x_1)$ , Fig. 5.14
- Potential of the externally applied loads

$$\Phi = - \int_0^L p_2(x_1) \bar{u}_2(x_1) dx_1 \quad (10.58)$$

- Total Potential  $\Pi$  of the beam .....from Eq.(10.9)

$$\Pi = A + \Phi = \frac{1}{2} \int_0^L H_{33}^c \left( \frac{d^2 \bar{u}_2}{dx_1^2} \right)^2 dx_1 - \int_0^L p_2 \bar{u}_2 dx_1$$

Eq.(10.40)

.... now  $\Pi = \Pi(\bar{u}_2(x_1))$ , a function of another function → "functional"

- ⇒ Beam problems are *infinite dimensional or continuous problems* since determination of the transverse displacement field,  $\bar{u}_2(x_1)$
- ↔ planar truss w/  $2N$  unknowns, " finite dimensional, discrete"

# 10.6 Applications to Truss and Beams

- Minimization of the TPE of finite dimension → standard calculus functional → calculus of variations
- Reduction of infinite # of D.O.F → finite # .....by choosing specific functions for  $u_2(x_1)$  → Chap.11

3-D beam under complex loading condition

distributed loads  $p_1(x_1), p_2(x_1), p_3(x_1)$

concentrated loads  $P_1, P_2, P_3$

distributed moment  $q_1(x_1), q_2(x_1), q_3(x_1)$

concentrated moment  $Q_1, Q_2, Q_3$

$$\begin{aligned} \rightarrow \Phi = & -\int_0^L p_1 \bar{u}_1 dx_1 - P_1 \bar{u}_1(\alpha L) - \int_0^L q_1 \Phi_1 dx_1 - Q_1 \Phi_1(\alpha L) \\ & - \int_0^L p_2 \bar{u}_2 dx_1 - P_2 \bar{u}_2(\alpha L) + \int_0^L q_2 \frac{d\bar{u}_3}{dx_1} dx_1 + Q_2 \frac{d\bar{u}_3}{dx_1}(\alpha L) \\ & - \int_0^L p_3 \bar{u}_3 dx_1 - P_3 \bar{u}_3(\alpha L) - \int_0^L q_3 \frac{d\bar{u}_2}{dx_1} dx_1 - Q_2 \frac{d\bar{u}_2}{dx_1}(\alpha L) \end{aligned} \quad (10.59)$$

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Euler-Bernoulli assumption  $\Phi_3 = \frac{d\bar{u}_2}{dx_1}, -Q_3\Phi_3(\alpha L) \rightarrow -Q_3 \frac{d\bar{u}_2}{dx_1}(\alpha L)$

$$\Phi_2 = -\frac{d\bar{u}_3}{dx_1}, -Q_2\Phi_2(\alpha L) \rightarrow Q_2 \frac{d\bar{u}_3}{dx_1}(\alpha L)$$