Capacitor (cont).

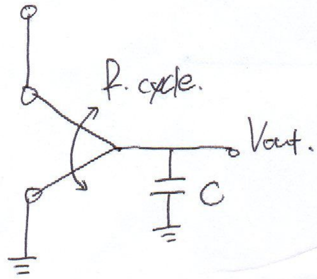
1) Current Source input.

$V = \frac{1}{C}\int i\,dt$

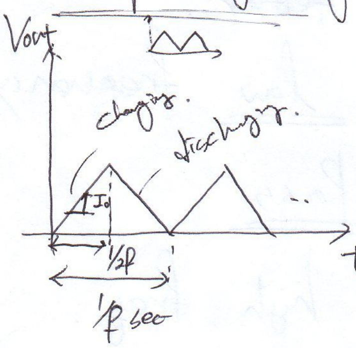
$= \frac{1}{C}\int I_0\,dt$

$= \frac{1}{C} I t$

$\hookrightarrow i = I_0 = \text{Const}$



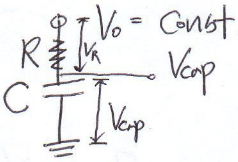
"Ramp Signal generator" ⇒ oscilloscope.



$H = \frac{I_0}{C}\frac{1}{2f}$ [Volt]

= Amplitude.

2) Voltage Source input.



$V_0 = V_R + V_{cap}$ ← Kirchhoff.

$V_R = i \cdot R$

$V_{cap} = \frac{1}{C}\int_0^t i\,dt$

$V_0 = i \cdot R + \frac{1}{C}\int_0^t i\,dt$

$0 = R \cdot \frac{di}{dt} + \frac{1}{C} i$

$\therefore i(t) = I_0 e^{-t/RC}$

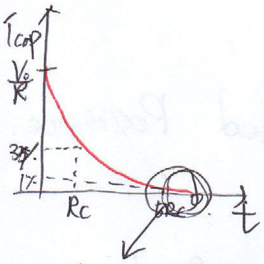
I_0 = initial current (Capacitor still doesn't work)

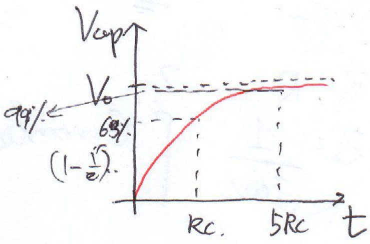
$= i(0)$

$= \frac{V_0 - 0}{R} = \frac{V_0}{R}$

$V_{cap} = \frac{1}{C}\int_0^t i\,dt$

$= V_0 (1 - e^{-t/RC})$





high Resistance (∞)

= "disconnected"

R	C	R·C
kΩ	μf	10^{-3} sec
MΩ	μf	1 sec

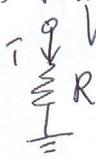
typically

if V_0 = A.C input ?

$V(t) = \sum V_i e^{j\omega_i t}$, $i = 0, 1, 2, \cdots \infty$

⇒ general form of Voltage

1) Resistor

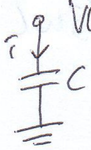


$V(t) = A.C.$, $i = \frac{V(t)}{R} = \frac{V}{R}$

→ AC or DC? 상관 X. No matter

$\frac{V}{i}$ = generalized Resistance = R (Ω)

2) Capacitor



$V(t) = Ve^{j\omega t}$, $\omega > 0$

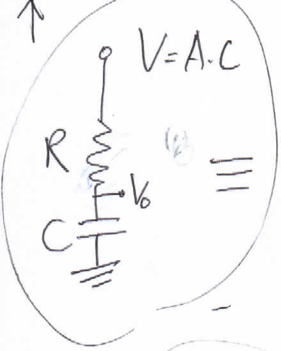
$i \overset{\text{def}}{=} \frac{d}{dt}[V_{cap}]$.

$= j\omega C V e^{j\omega t} = j\omega C \cdot V(t)$

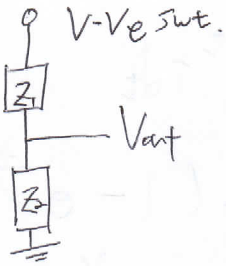
$\frac{V}{i}$ = Reactance = Generalized Resistance.

$= \frac{1}{j\omega C}$

*



$\equiv$



$Z_1 = R$
$Z_2 = \frac{1}{j\omega C}$ } Generalized Resistance.

$\frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1+Z_2}$ = Generalized Voltage Divider.

if $\omega = 0$?

$V(t) = V_0 e^{j\omega t} \rightarrow$ D.C., $\frac{V}{I} \rightarrow \infty$

if $\omega = \infty$? $\rightarrow \frac{V}{I} = 0.$

↳ frequency dependent
→ Capacitor behavior.

∴ filtering, Blocking, Smoothing are possible !!

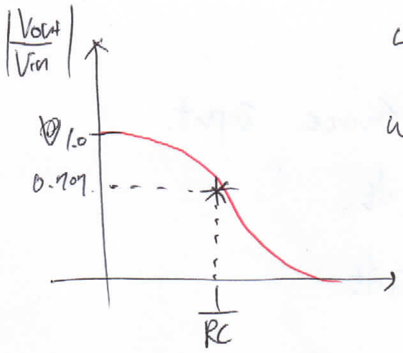
$\frac{V_{out}}{V_{in}}$ = Transfer fct.

$= \frac{Z_2}{Z_1+Z_2} = \frac{1/j\omega C}{R + 1/j\omega C}$

$= \frac{1}{1+j\omega RC}$

$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+(j\omega RC)^2}}$

$\angle \frac{V_{out}}{V_{in}} = \tan^{-1}(\omega RC).$

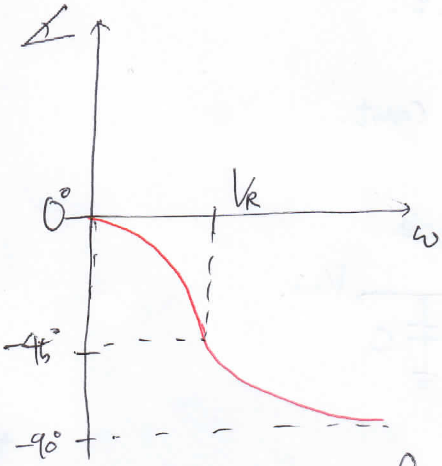


$\omega = 0$ $\left|\frac{V_{out}}{V_{in}}\right| = 1, \angle = 0^\circ$

$\omega = \infty$ $\left|\frac{V_{out}}{V_{in}}\right| \Rightarrow 0$ $\angle = -\frac{\pi}{2}$ rad

$\omega = \frac{1}{RC}$, $\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{2}} = 0.707$

$\angle = -45^\circ = -\frac{\pi}{4}$ rad



* Low Pass filter.
: Signals of low frequency Can Pass
Signals of high freq. Cannot Pass.

At) $\omega = 1/RC$,

$\left|\frac{V_{out}}{V_{in}}\right| = 0.707 = 70.7\%.$
(Engineer Compromise)

$-20 \cdot \log_{10}\left|\frac{V_{out}}{V_{in}}\right| \overset{def.}{\equiv} dB.$

$-10 \cdot \log_{10}\left|\frac{P_{out}}{P_{in}}\right| \overset{def}{\equiv} dB.$

When $\left|\frac{V_{out}}{V_{in}}\right| = 0.707 = \frac{1}{\sqrt{2}}$

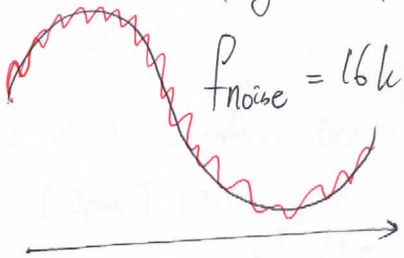
$-20\log_{10}\left|\frac{V_{out}}{V_{in}}\right| = 3dB$ ∴ $\frac{1}{RC}$ is Called $\omega_{3dB} = \frac{1}{RC}$

ex) Design a LPF

$f_{signal} = 1kHz$ (expect)

$f_{noise} = 16kHz$. (to remove, filter out)

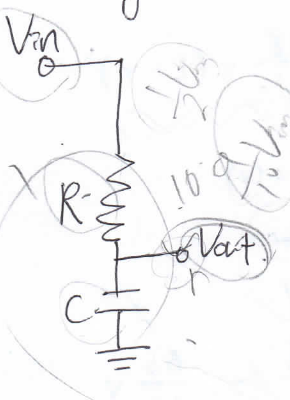
→ "LPF."



1ms.

1. Freq identification.

$f_{signal} = 1kHz$, $f_{noise} = 16kHz$.



$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\angle = -\tan^{-1}(\omega RC)$$

2. $f_{signal} \equiv f_{3dB}$.

$1000Hz = 2\pi(1000)$ [rad/sec].

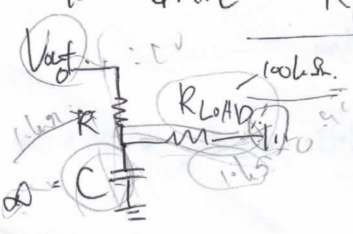
$= \omega_{signal}$.

$\omega_{3dB} = 2\pi \cdot f_{3dB} = \omega_{signal}$

$= \frac{1}{RC}$ by LPF.

$= 2\pi(1000) = 6280$ ···①.

To drive $R_{LOAD} = 100k\Omega$ by spec or Measurement.



10X Rule of Design

R to Rload = 1:10

∴ $R = 10k\Omega$.

$C = 1.59 \times 10^{-8}F$ (commercially available)

slightly distort.

$\simeq 2.0 \times 10^{-8}F$.

1.0×10^{-8} (buy)

3. Verification.

$f_{noise} = 16kHz \Rightarrow \omega_{noise} = 2\pi f_{noise}$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}} \simeq 0.06.$$

6%.

$$\angle = -\tan^{-1}(\omega RC) = -87.1^\circ$$

$f_{signal} = 1kHz$.

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}} = 0.623.$$

$(\neq 0.707)$ why?

" because we choose $2.0 \times 10^{-8}F$ Capacitor."

$$\angle = -\tan^{-1}(\omega RC) = -51.4^\circ \ (\neq -45^\circ)$$

4. Signal to Noise (S/N) Ratio.

Before LPF	After LPF
S = 1.0	S = 0.623.
N = 1.0	N = 0.06.
S/N = 1	S/N = 10.3

∴ S/N ratio is 10.3 times improved.