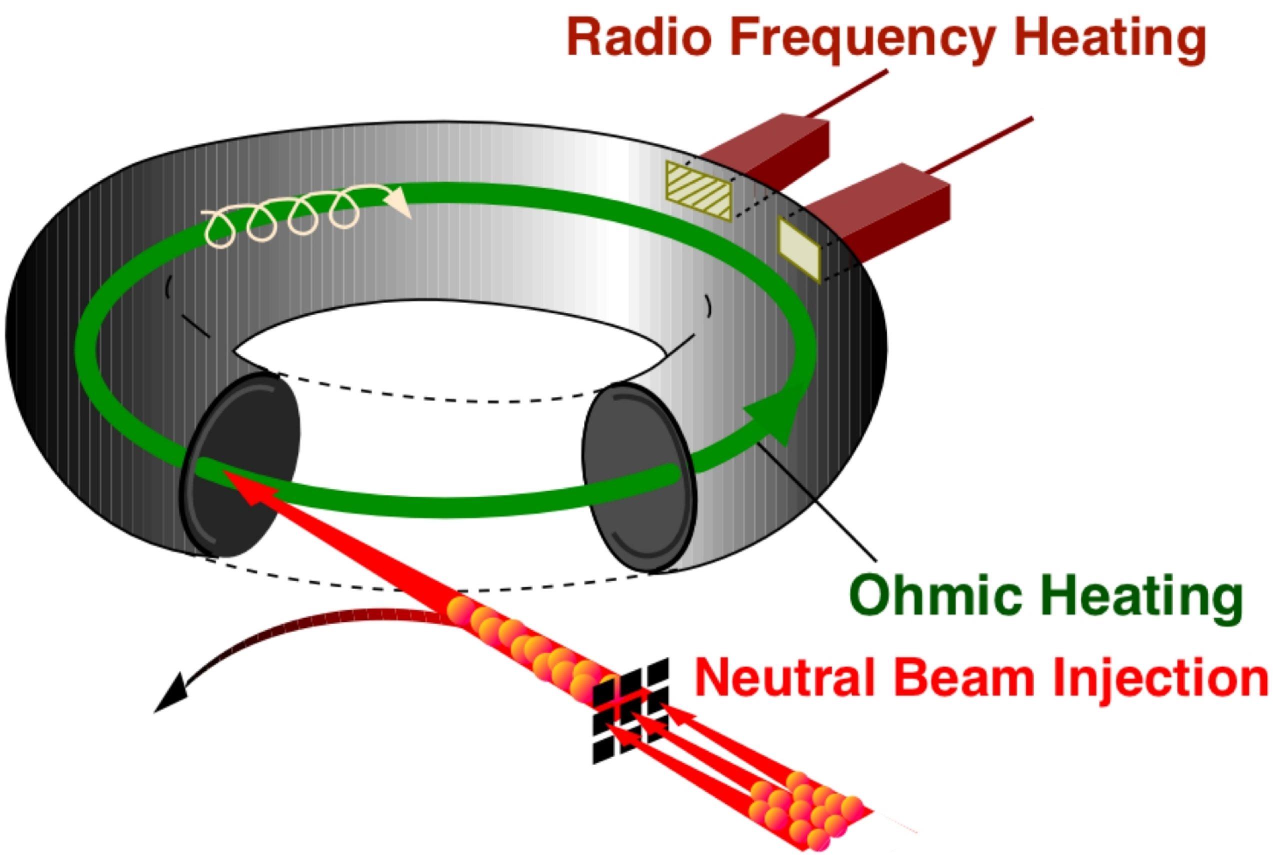
Introduction to Nuclear Fusion

Prof. Dr. Yong-Su Na

How to heat up a plasma?

Plasma Heating

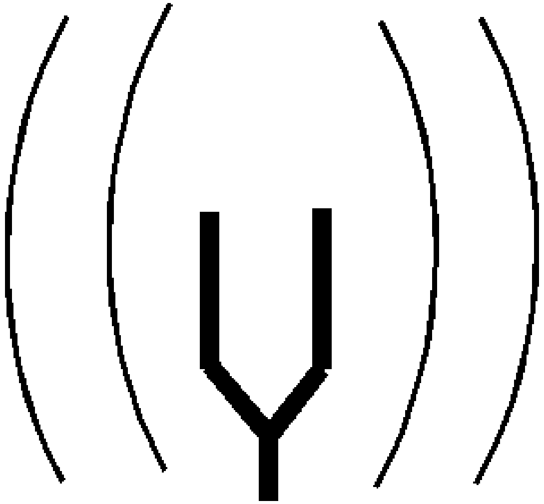


http://iter.rma.ac.be/en/img/Heating.jpg

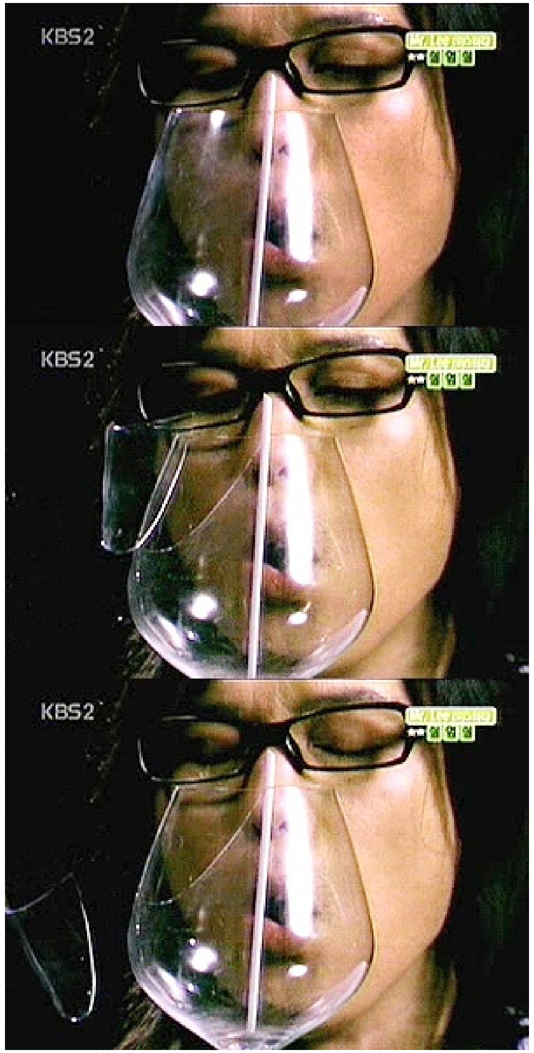
Wave heating by resonance

Electromagnetic Waves

Tuning fork



Resonance



KBS. 스펀지:목소리로 와인 잔 깨기. 2006.3.11 http://www.kbs.co.kr/end_program/2tv/enter/sponge/view/vod/1386311_1027.html

Electromagnetic Waves

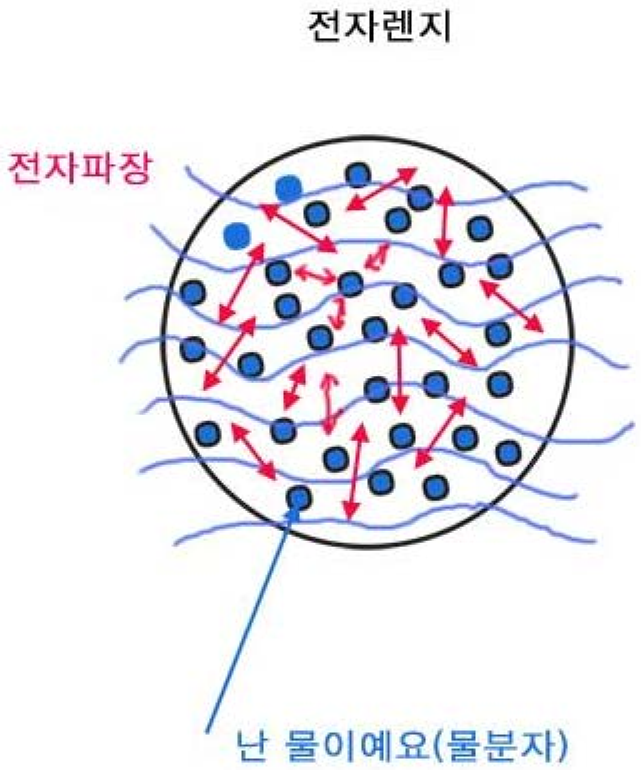
Tacoma Narrows Bridge
(1940. 11. 4)

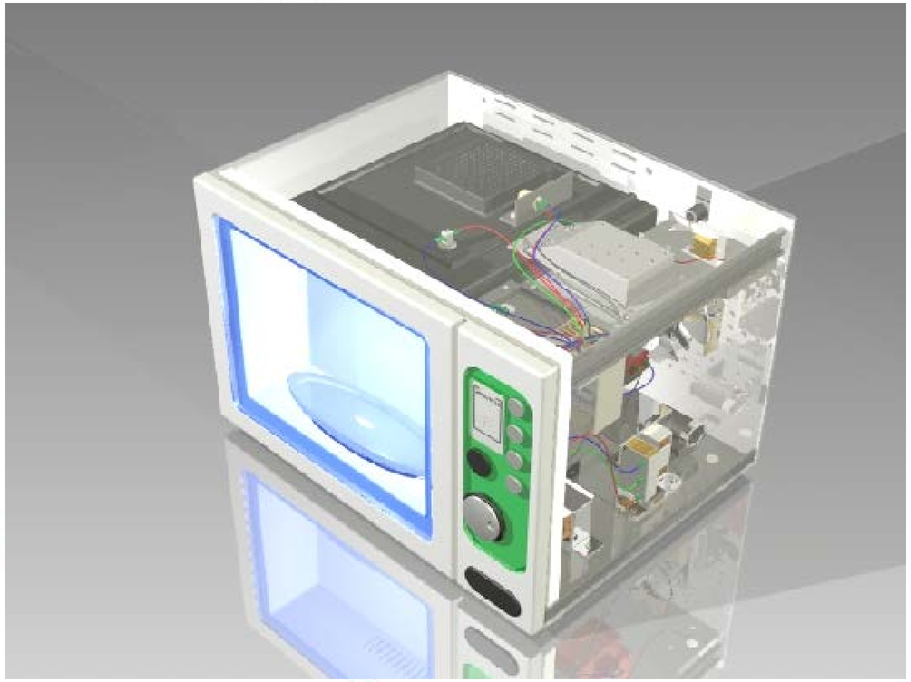
Electromagnetic Waves

Broughton Suspension Bridge

http://www.joysf.com/?mid=forum_sf&document_srl=4265290&page=5

Electromagnetic Waves



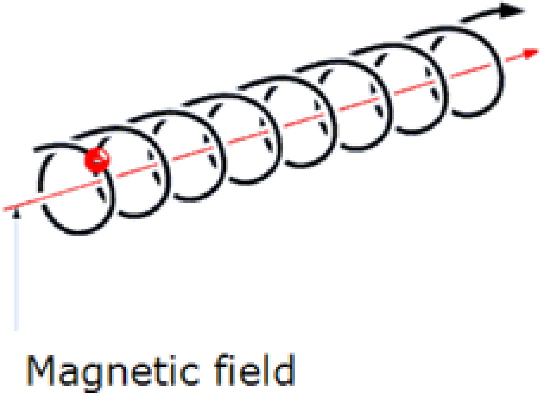


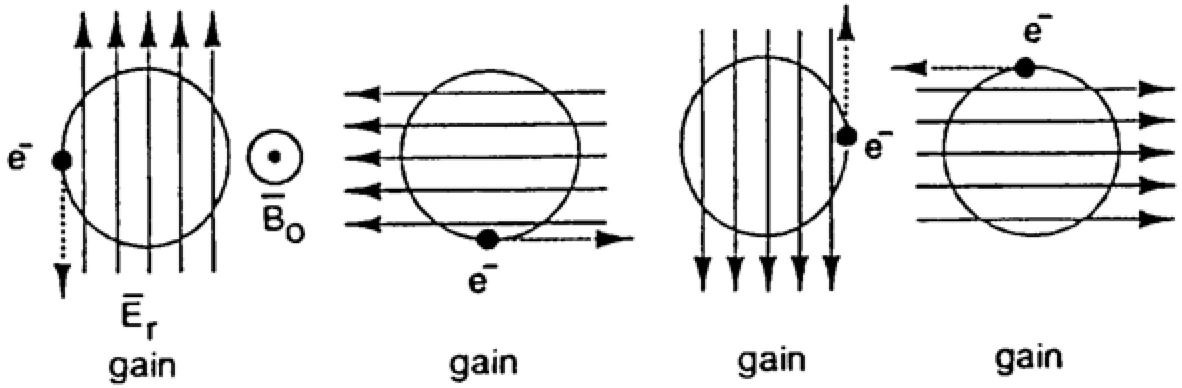
Microwave oven (2.45 GHz)

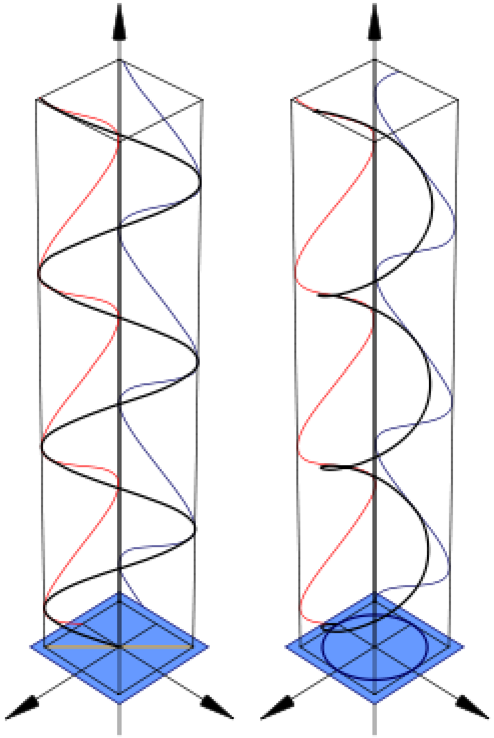
http://blog.naver.com/rlhyuny27?Redirect=Log&logNo=30029307561
http://cafe.naver.com/nadobaker.cafe?iframe_url=/ArticleRead.nhn%3Farticleid=82

Electromagnetic Waves

- **Electron Cyclotron Resonance Heating (ECRH)**







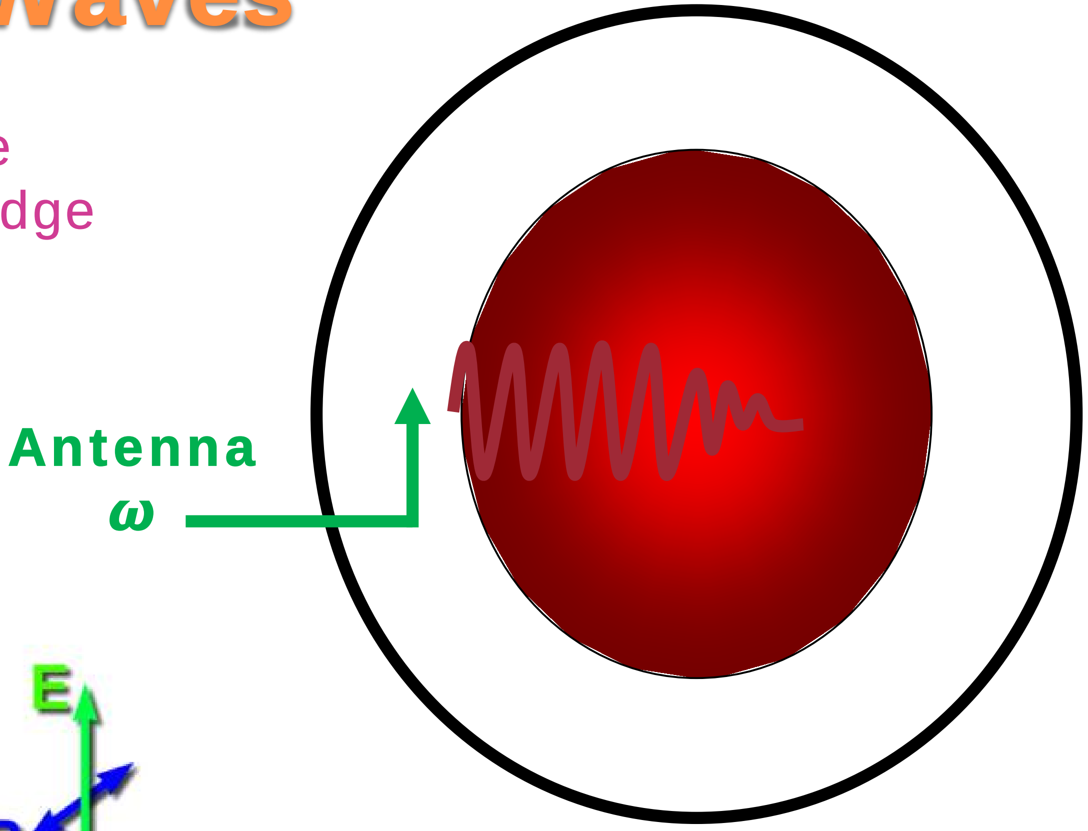
Linear / Circular
polarisation

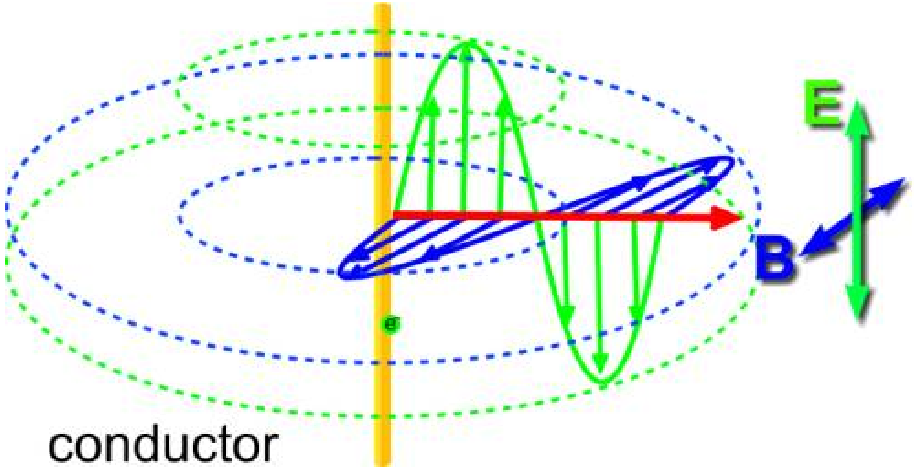
Electromagnetic Waves

Excitation of plasma wave
(frequency ω) near plasma edge

$\Downarrow$

wave transports power
into the plasma center





http://www.asc-csa.gc.ca/images/radarsat2_figure2_l_h.jpg

Electromagnetic Waves

Excitation of plasma wave
(frequency ω) near plasma edge

$\Downarrow$

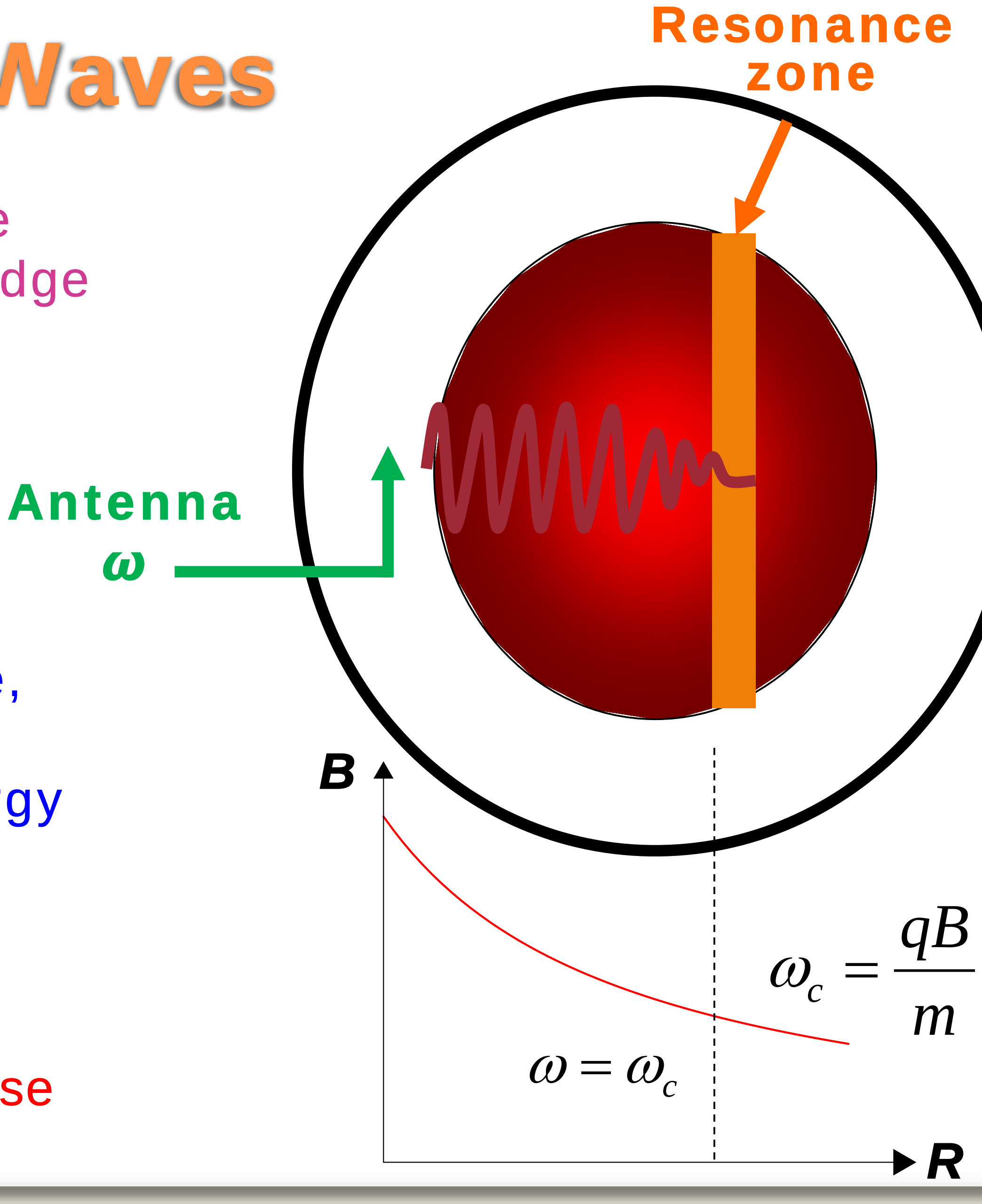
wave transports power
into the plasma center

$\Downarrow$

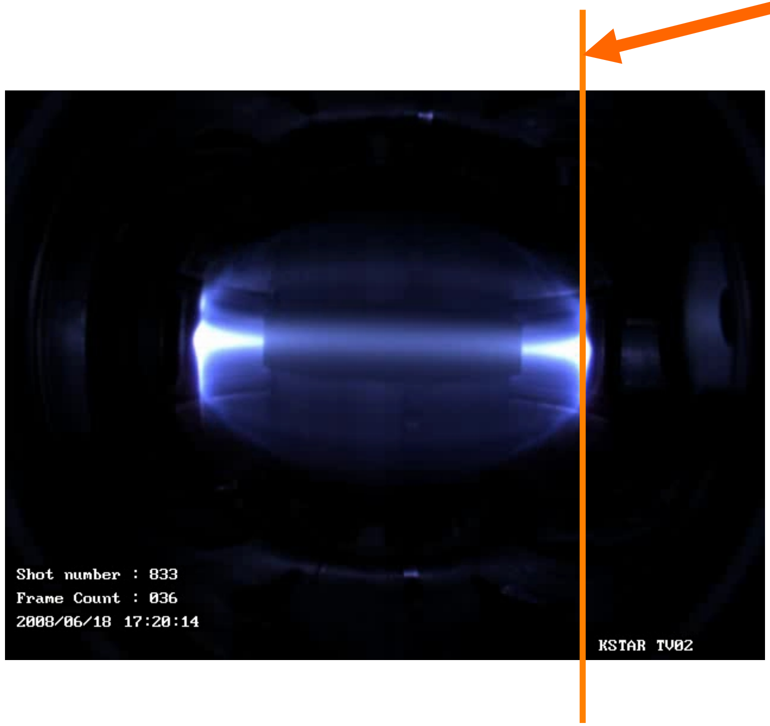
absorption near resonance,
e.g. $\omega \approx \omega_c$,
i.e. conversion of wave energy
into kinetic energy of
resonant particles

$\Downarrow$

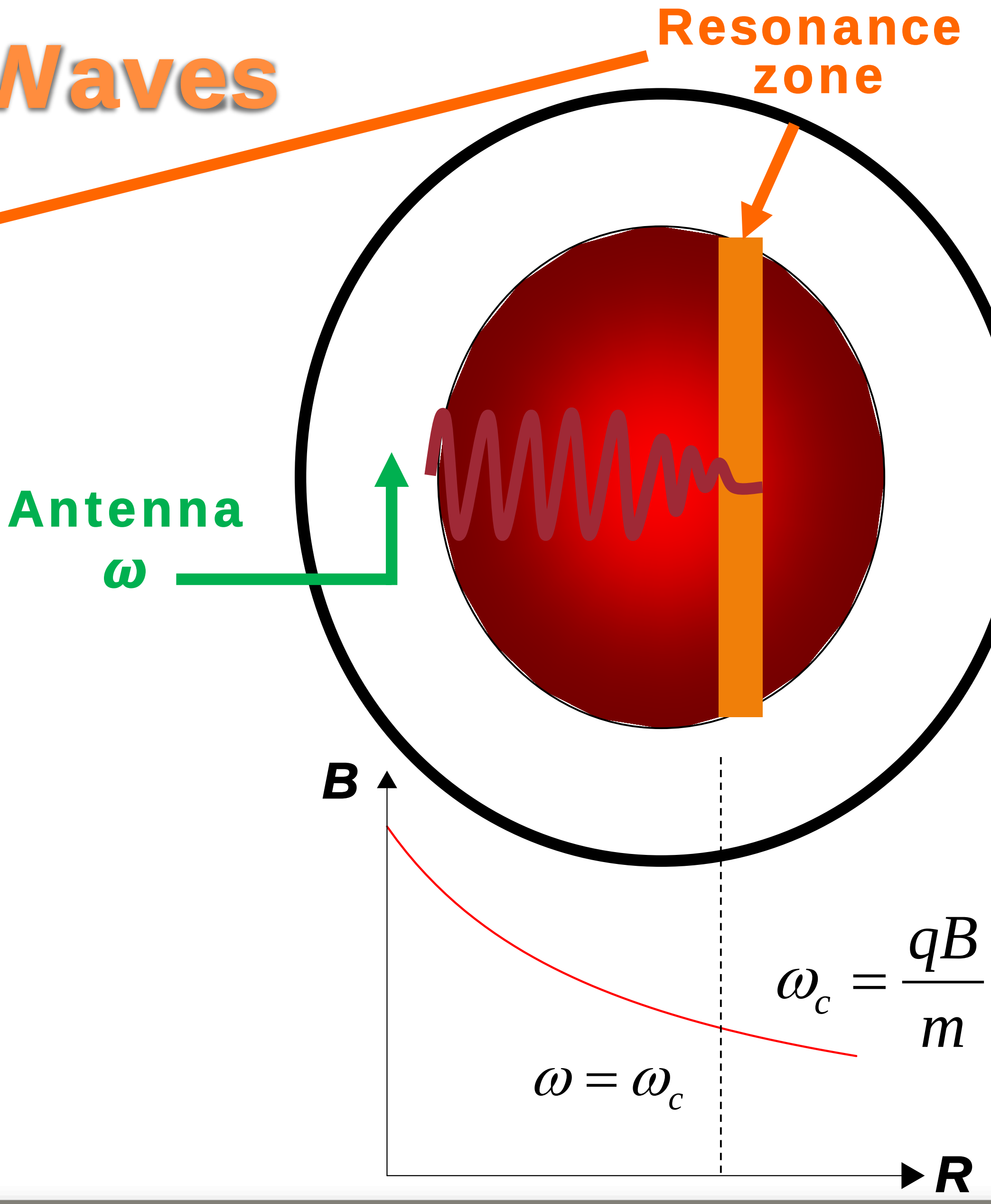
Resonant particles thermalise



Electromagnetic Waves



KSTAR first plasma



Waves in a plasma

Plasma Waves

- Considering externally driven perturbations in the magnetic and electric fields and in the current, relative to an equilibrium condition for a cold plasma w/o external magnetic fields

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

Maxwell equation

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\nabla \times \left(\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right) = -\mu_0 \boxed{\frac{\partial \vec{j}}{\partial t}} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

$$j \equiv \sum_i n_i q_i u_i$$

$$m_i n_i \left(\frac{\partial \vec{u}_i}{\partial t} + (\vec{u}_i \cdot \nabla)\vec{u}_i\right) = n_i q_i (\vec{E} + \vec{u}_i \times \vec{B}) - \nabla p_i$$

Equations of motion: isotropic pressure assumed

$$\Rightarrow \quad \frac{\partial \vec{u}_i}{\partial t} = \frac{q_i}{m_i}\vec{E} \qquad \frac{\partial \vec{j}}{\partial t} = \sum_i n_i q_i \frac{\partial \vec{u}_i}{\partial t} = \sum_i \frac{n_i q_i^2}{m_i}\vec{E}$$

Plasma Waves

$$\nabla(\nabla\cdot\vec{E})-\nabla^2\vec{E}=-\mu_0\frac{\partial\vec{j}}{\partial t}-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\frac{\partial\vec{j}}{\partial t}=\sum_i n_i q_i\frac{\partial\vec{u}_i}{\partial t}=\sum_i\frac{n_i q_i^2}{m_i}\vec{E}$$

$$\nabla(\nabla\cdot\vec{E})-\nabla^2\vec{E}=-\mu_0\left(\sum_i\frac{n_i q_i^2}{m_i}\right)\vec{E}-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

Plasma Waves

$$\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\mu_0\left(\sum_i \frac{n_i q_i^2}{m_i}\right)\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

$$-\vec{k}(\vec{k}\cdot\vec{E}) + k^2\vec{E} = -\mu_0\left(\sum_i \frac{n_i q_i^2}{m_i}\right)\vec{E} + \frac{\omega^2}{c^2}\vec{E}$$

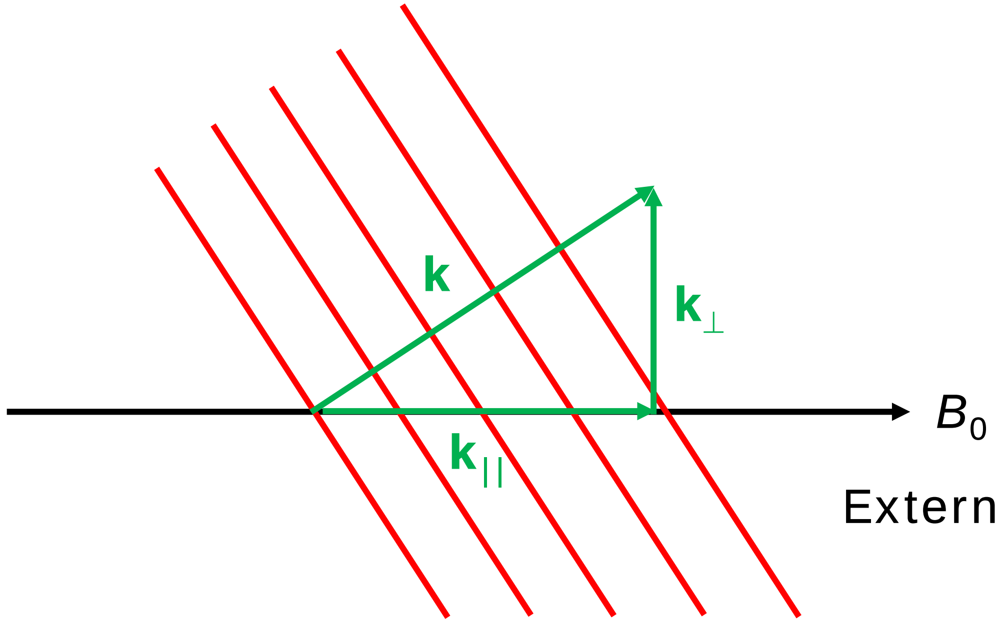
Plane waves with space and time dependences

$$\exp\left[i(\vec{k}\cdot\vec{r} - \omega t)\right]$$

$$\nabla = i\vec{k}, \quad \frac{\partial}{\partial t} = -i\omega$$

$\vec{k}$: wave vector

ω : frequency



External magnetic field

Plasma Waves

$$\nabla(\nabla\cdot\vec{E})-\nabla^2\vec{E}=-\mu_0\left(\sum_i\frac{n_i q_i^2}{m_i}\right)\vec{E}-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

$$-\vec{k}(\vec{k}\cdot\vec{E})+k^2\vec{E}=-\mu_0\left(\sum_i\frac{n_i q_i^2}{m_i}\right)\vec{E}+\frac{\omega^2}{c^2}\vec{E}$$

$$-\vec{k}(\vec{k}\cdot\vec{E})+k^2\vec{E}+\mu_0\left(\sum_i\frac{n_i q_i^2}{m_i}\right)\vec{E}-\frac{\omega^2}{c^2}\vec{E}=0$$

$$\rightarrow\ (\ldots)\vec{E}=0$$

Plane waves with space and time dependences

$$\exp\left[i(\vec{k}\cdot\vec{r}-\omega t)\right]$$

$\vec{k}$: wave vector

ω : frequency

Determinant (...)= 0

→ Disperison relation $D(w, k) = 0$

Plasma Waves

$$\nabla(\nabla\cdot\vec{E})-\nabla^2\vec{E}=-\mu_0\left(\sum_i\frac{n_i q_i^2}{m_i}\right)\vec{E}-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

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Plane waves with space and time dependences

$$\exp\left[i(\vec{k}\cdot\vec{r}-\omega t)\right]$$

$\vec{k}$: wave vector

ω : frequency

- $\vec{k}\parallel\vec{E}$

$$\omega^2=\frac{1}{\varepsilon_0}\sum_i\frac{n_i q_i^2}{m_i}\equiv\omega_p^2$$

Plasma frequency

- $\vec{k}\perp\vec{E}$

$$\omega^2=c^2k^2+\omega_p^2$$

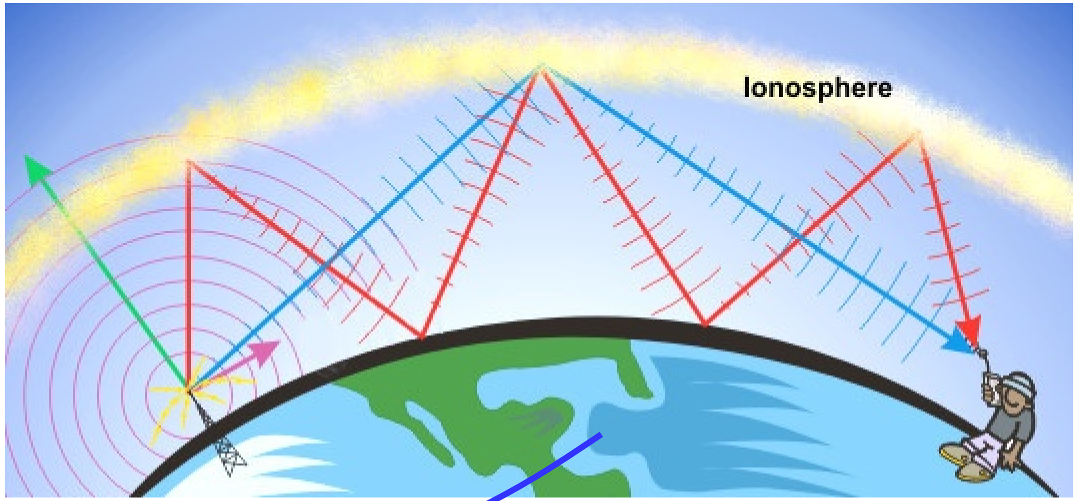
Plasma wave

Dispersion Relation

- **Plasma waves are solutions of disperison relation D(ω, k)=0.**

Generally:
ω given by generator
$k_{||}$ given by antenna
where $k_{||} > k_{||,\text{Vacuum}}$

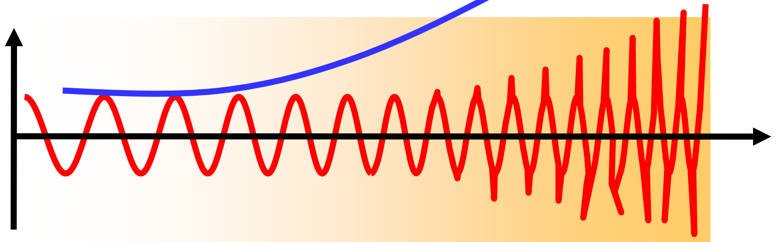
solution: $k_\perp = k_\perp(\omega, k_{||})$



Special cases:

1. $k_\perp \to 0$ „cutoff“

2. $k_\perp \to \infty$ „resonance“



http://www.srh.noaa.gov/srh/jetstream/atmos/ionosphere_max.htm

Dispersion Relation

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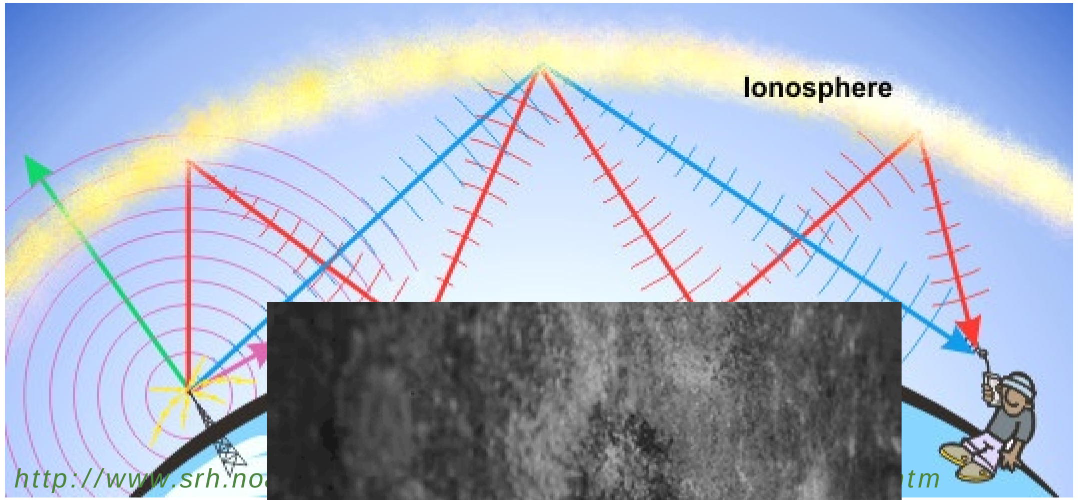

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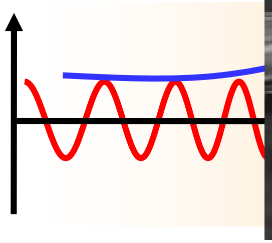
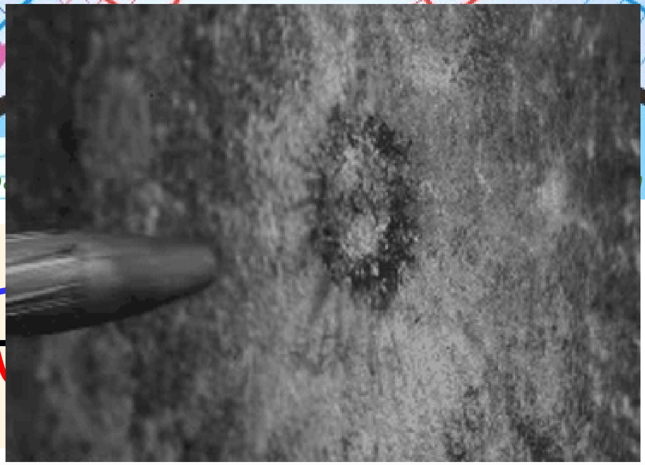




http://star.mk.co.kr/new/view.php?sc=40500009&year=2010&no=391063&mc=PT&mc=PT

Wave heating in a tokamak

Electromagnetic Wave

- **Ion Cyclotron Resonance Heating (ICRH):**

  occurring only when two or more ion species are present

  $\omega \sim \omega_{ci}$, 30 MHz – 120 MHz ($\lambda \sim$ 10 m)

  $$\omega_{ii}^2 = \frac{\omega_{c1}\omega_{c2}(1+n_2m_2/n_1m_1)}{(m_2Z_1/m_1Z_2+n_2Z_2/n_1Z_1)}, \quad \omega_{ci} = \frac{z_i eB}{m_i}$$

  : Ion-ion resonance frequency

- **Lower Hybrid (LH) Resonance Heating:**

  $\omega_{ci} < \omega < \omega_{ce}$, 1 GHz – 8 GHz ($\lambda \sim$ 10 cm)

  $$\omega_{LH}^2 \approx \omega_{pi}^2 /(1+\omega_{pi}^2/\omega_{ce}^2), \quad \omega_{pi}^2 >> \omega_{ci}^2$$

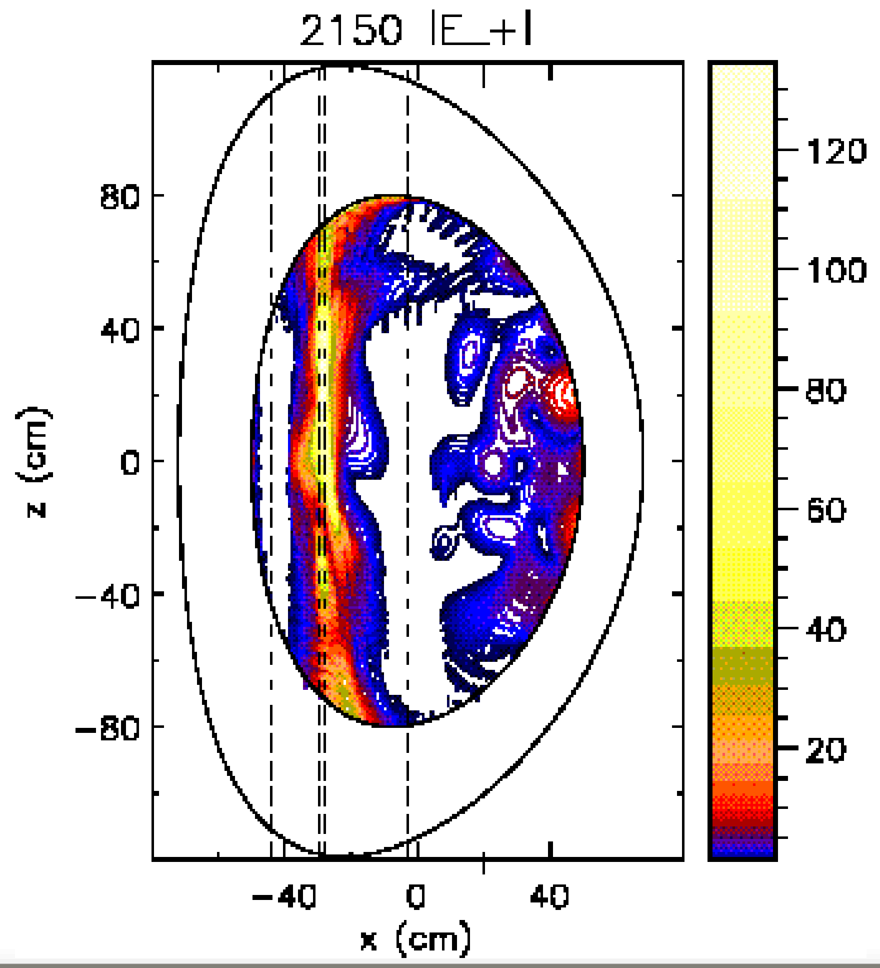
- **Electron Cyclotron Resonance Heating (ECRH):**

  $\omega \sim \omega_{ce}$, 100 GHz – 200 GHz ($\lambda \sim$ mm)

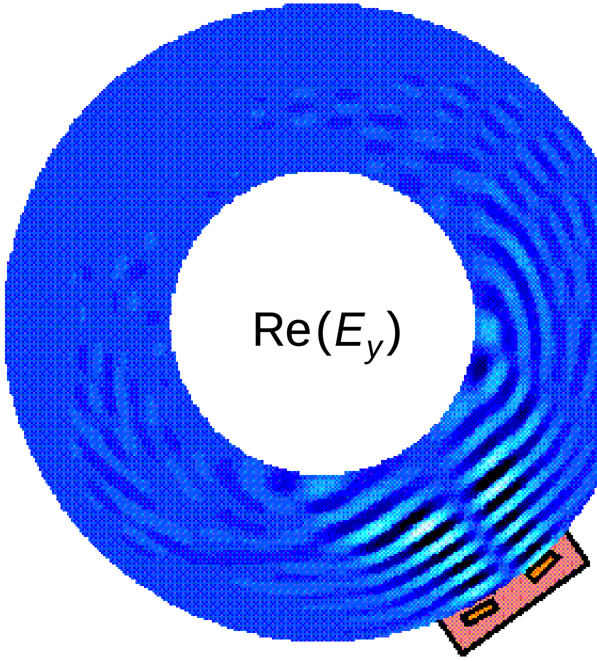
  $$\omega_{UH}^2 \approx \omega_{pe}^2 + \omega_{ce}^2$$

ICRH – Wave Propagation

ASDEX Upgrade

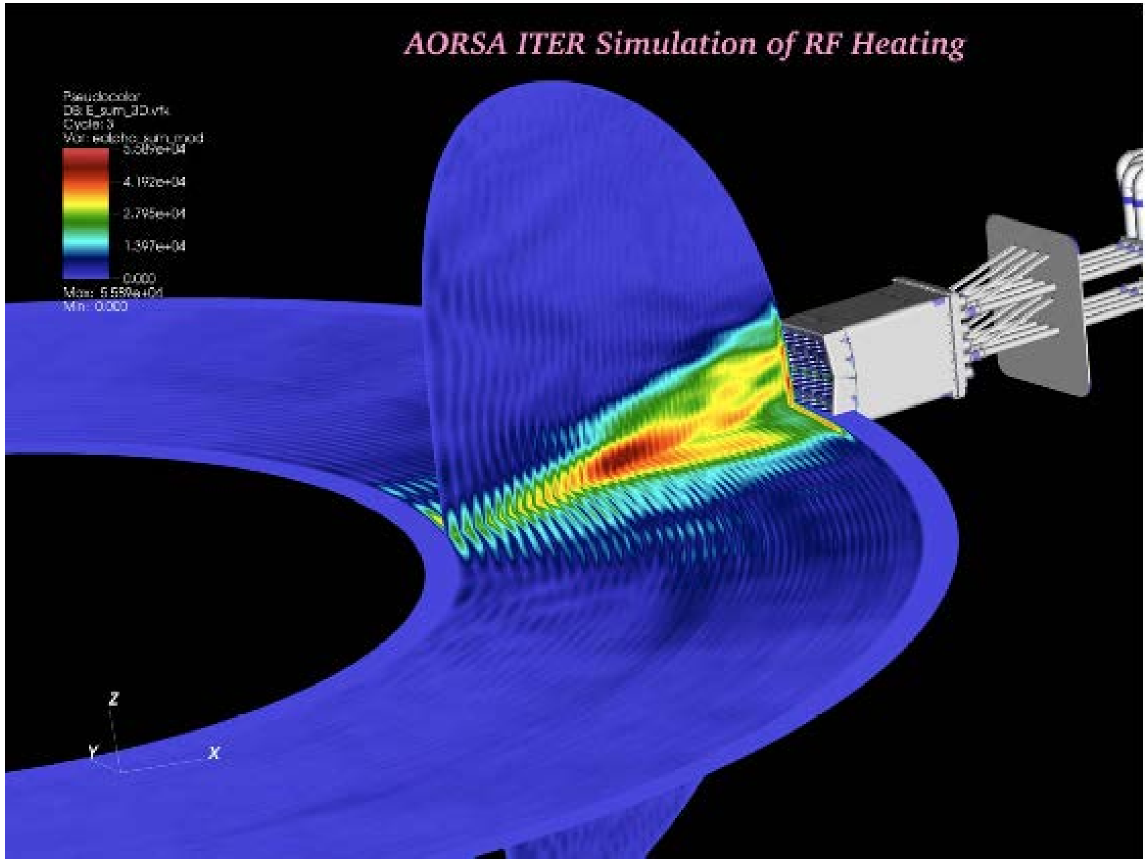


Alcator C-Mod



Multiple current straps

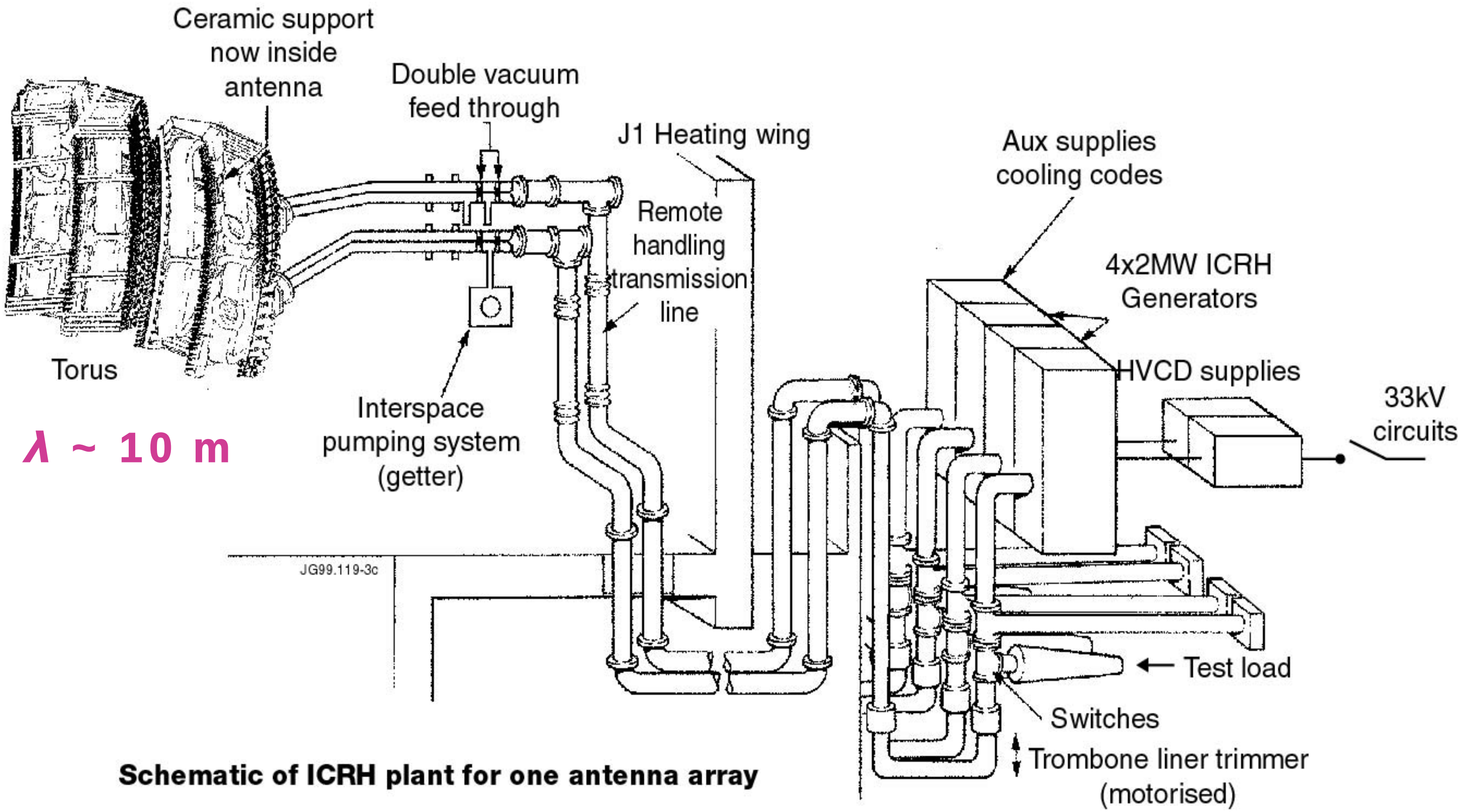
ICRH – Wave Propagation



https://www.burningplasma.org/newsandevents/?article=enews&issue=081507

Ion Cyclotron Resonance Heating

- **JET ICRH System**



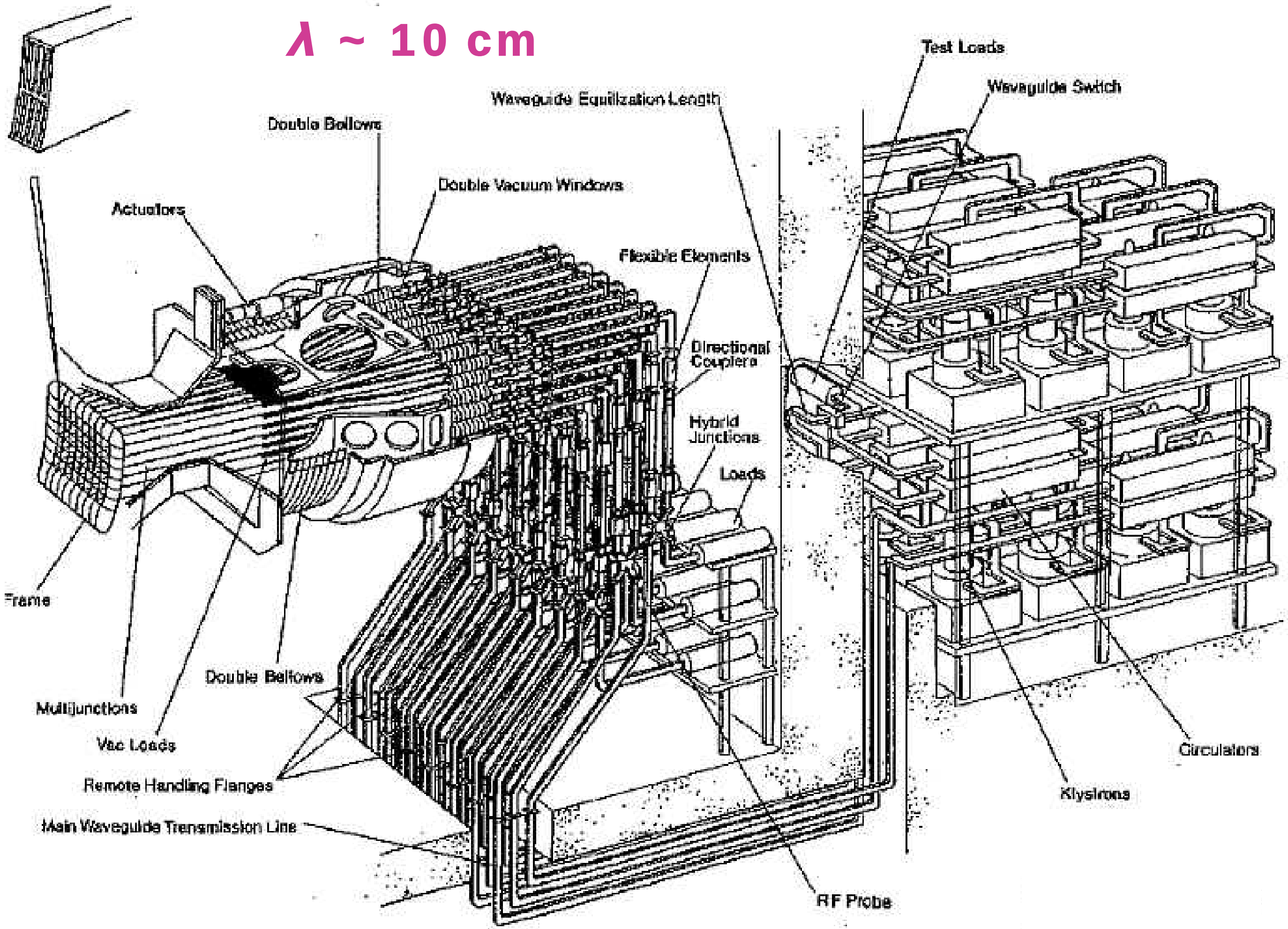
λ ~ 10 m

Schematic of ICRH plant for one antenna array

Lower Hybrid Heating

- **JET LH System**

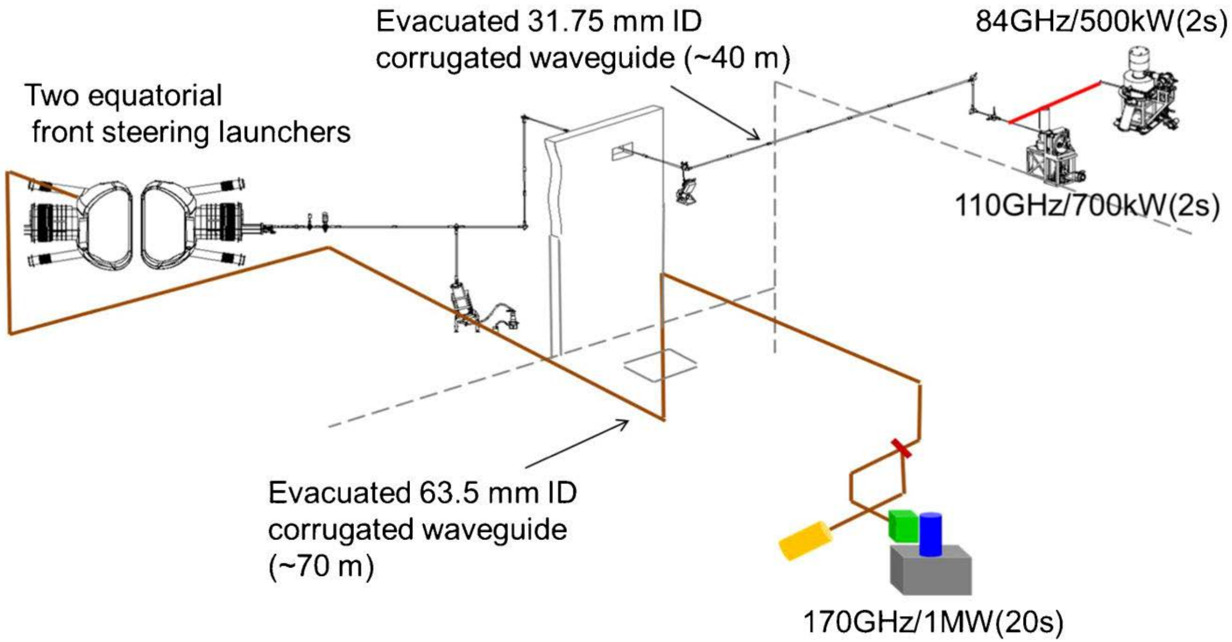
λ ~ 10 cm

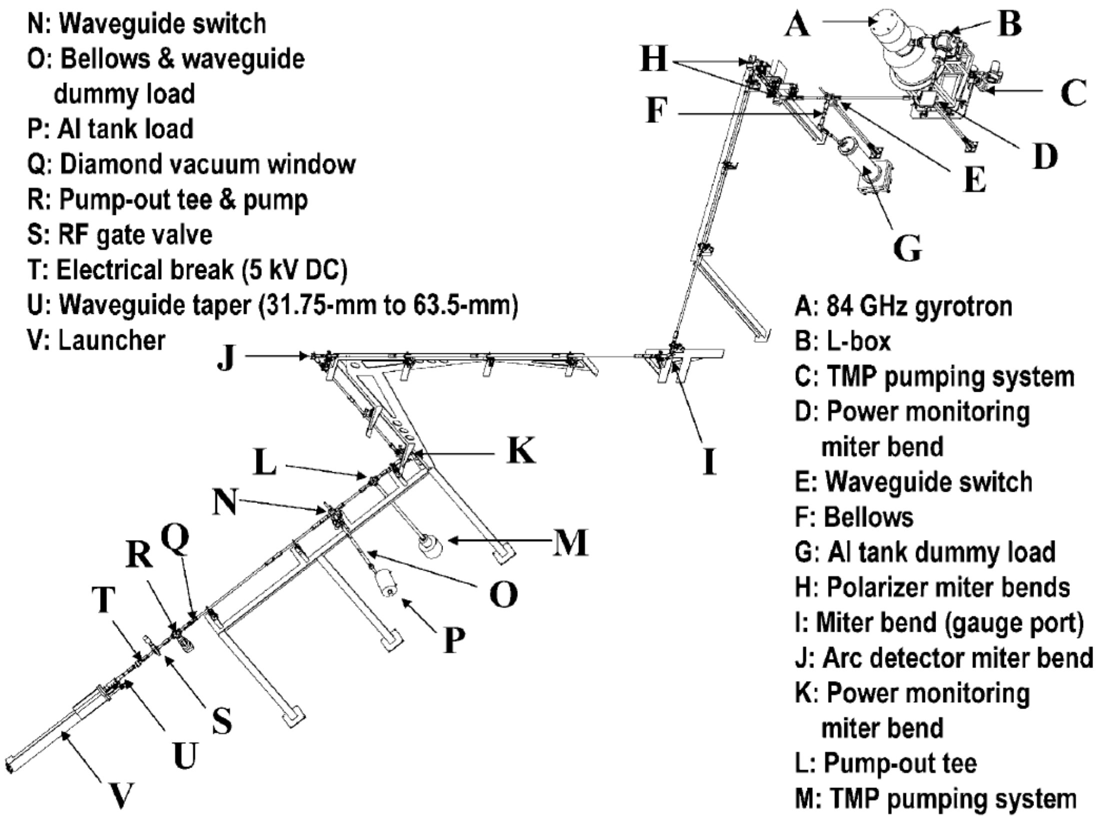


Electron Cyclotron Heating

- **KSTAR ECH System**

λ ~ mm





Y. S. Bae et al, Fusion Science and Technology 52 321 (2007), 65 88 (2014)

Alpha particle heating

α-Particle Heating

- Intrinsic self-heating by Coulomb collision of fusion α particles with plasma particles in D-T reactions

$$D + T \rightarrow n\,(14.1\,MeV) + He^4(3.5\,MeV)$$

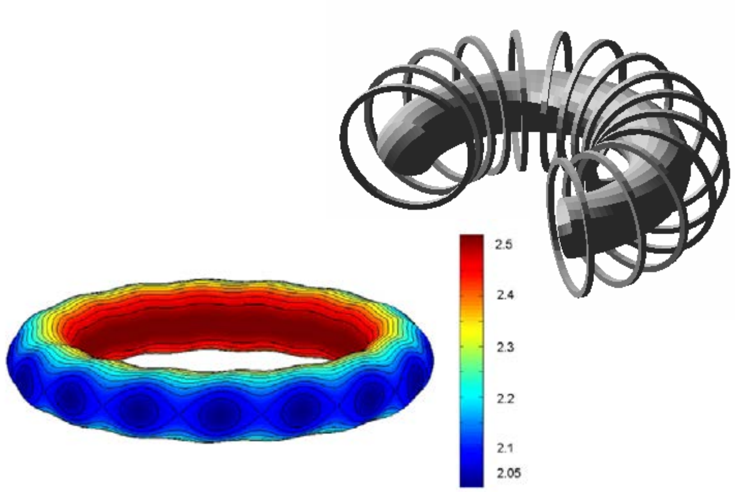
$n\,(14.1\,MeV)$ $\downarrow$ leaves plasma

$He^4(3.5\,MeV)$ $\downarrow$ heats plasma if sufficiently long confined

- Heating power density: $N_D N_T \langle \sigma v \rangle \dfrac{Q_{DT}}{5}$

  where $\langle \sigma v \rangle \propto T_i$

  $\Rightarrow$ peaked heating profile



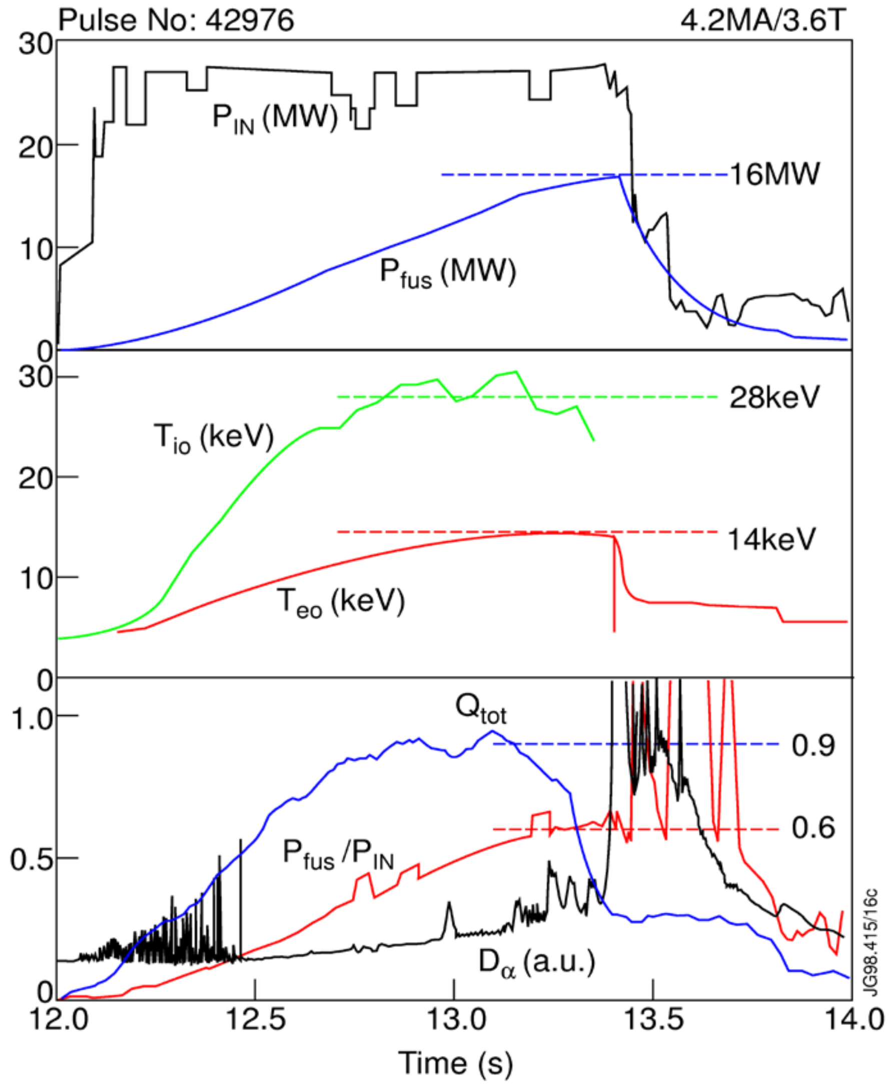
- α-particle loss mechanisms: field ripples
  MHD events e.g. Alfvèn Eigenmode (AE)

A.V. Melnikov et al, NF **53** 093019 (2013)

α-Particle Heating

- DT-Experiments only in
  - JET
  - TFTR

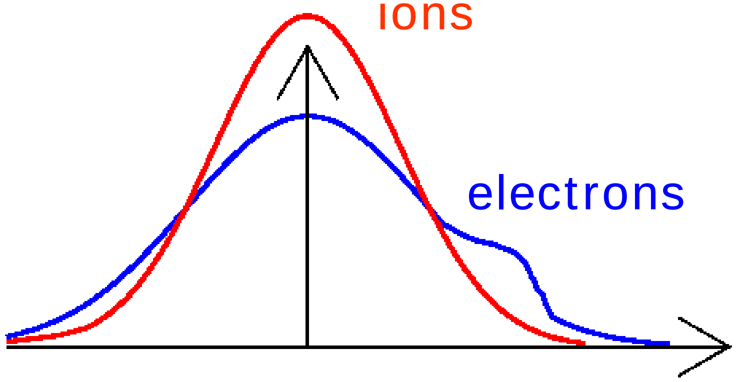
- with world records in JET:
  - P_{fusion} = 16 MW
  - Q = 0.64



Current drive

Non-inductive Current Drive

- Asymmetric velocity distribution can be a side effect of plasma heating.

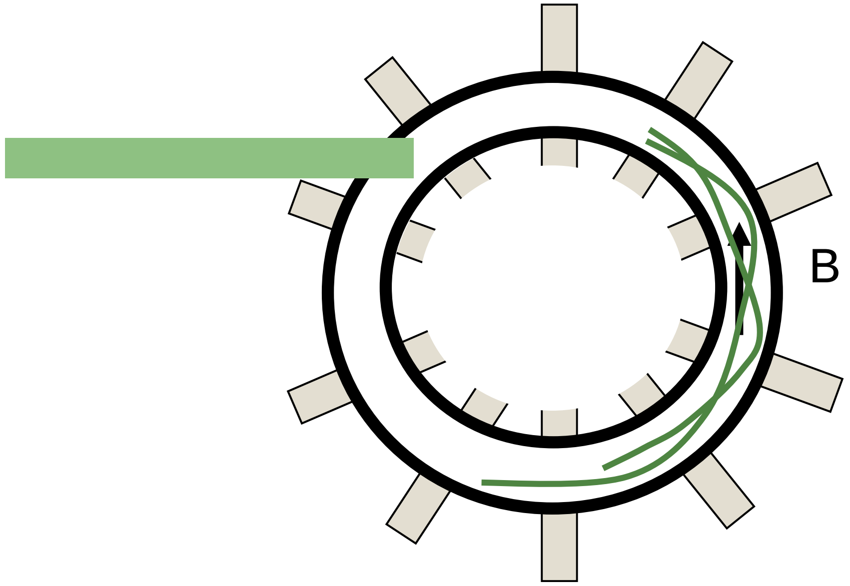


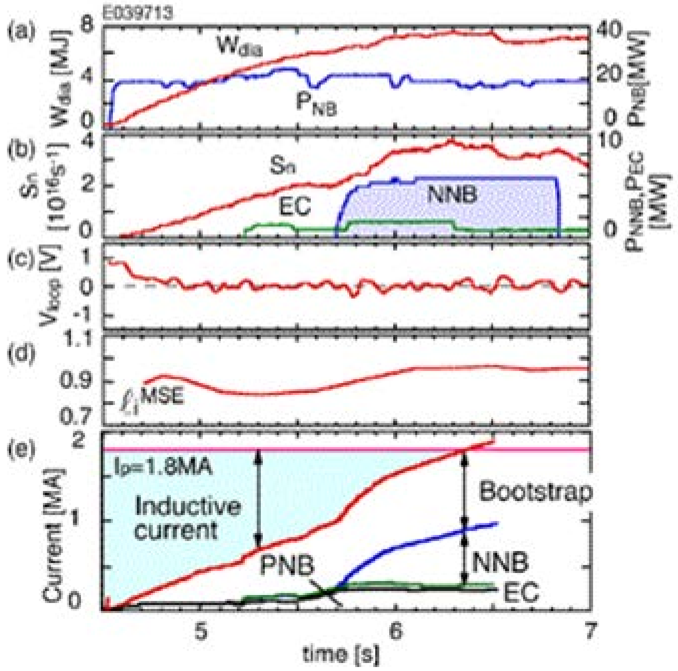
$$j = \sum_s q_s n_s \int v_\parallel f\left(v_\parallel\right) dv$$

- **Needed for:** Steady-state tokamak
  current profile control in tokamaks
  bootstrap current compensation in stellarators

Neutral Beam Current Drive

Tangential injection





JT-60U high β_p ELMy H-mode

Heating and Current Drive

Heating Scheme	Advantages	Disadvantages
OH	Efficient	Cannot reach ignition
NBI	Reliable	Close to torus, Negative ion source necessary
LH	Efficient current drive (CD)	Antenna close to plasma, off-axis CD
ECRH	Reliable, Flexible Localised CD	Electron heating
ICRH	Ion heating Central heating	Antenna close to plasma, Antenna coupling