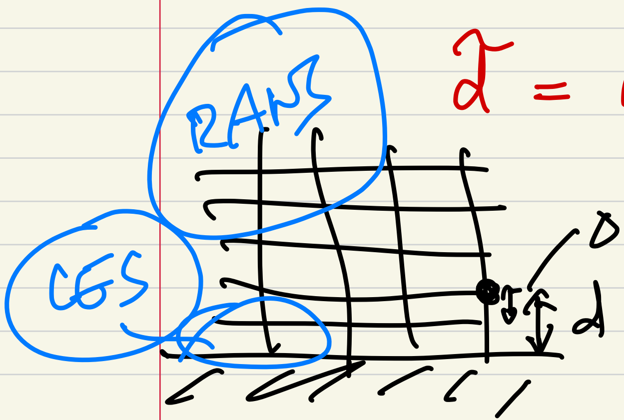
⊙ DES (detached eddy simulation) – hybrid RANS/LES

↳ single turbulence model acts as RANS and LES models for near-wall and detached regions, respectively, by adjusting the wall distance ft. (Spalart et al. 1997)

$$\nu_T = \tilde{\nu} f_{v_1}, \quad f_{v_1} = \frac{\chi^3}{\chi^3 + c_{v_1}^3}, \quad \chi = \frac{\tilde{\nu}}{\nu}$$

$$\frac{D\tilde{\nu}}{Dt} = \underbrace{c_{b_1}\tilde{S}\tilde{\nu}}_{\text{prod.}} + \underbrace{\frac{1}{\sigma}\left\{\nabla\cdot\left[(\nu+\tilde{\nu})\nabla\tilde{\nu}\right] + c_{b_2}(\nabla\tilde{\nu})^2\right\}}_{\text{diffusion}} \underbrace{- c_{w_1} f_w \left[\tilde{\nu}/\tilde{d}\right]^2}_{\text{destruction}}$$



$$\tilde{d} = \min(d_{RANS}, d_{LES})$$

$d_{RANS} = d$: wall distance

$d_{LES} = C_{DES}\Delta$

$\Delta = \max(\Delta x, \Delta y, \Delta z)$

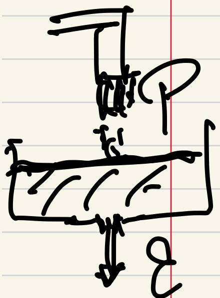
or $(\Delta x \Delta y \Delta z)^{1/3}$ or ...

$C_{DES} = 0.65$

problems → DDES → DDDES ... → DES + k-ω model

⑥ Numerical methods for spatial derivatives

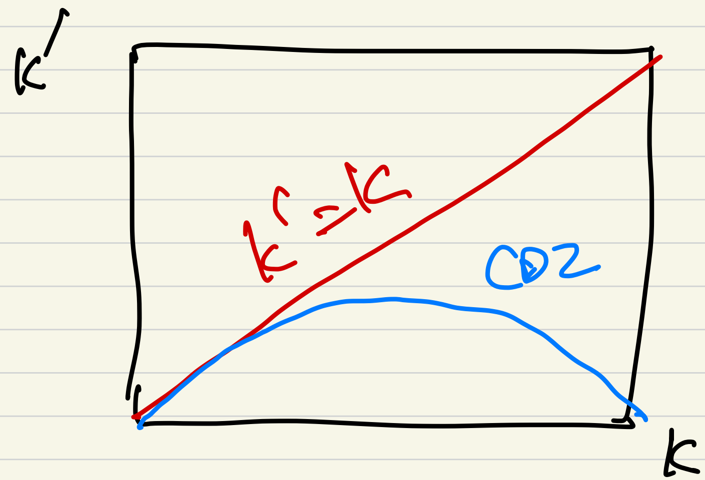
- 2nd-order central difference scheme → no dissipation error
  ↓
  successful results.

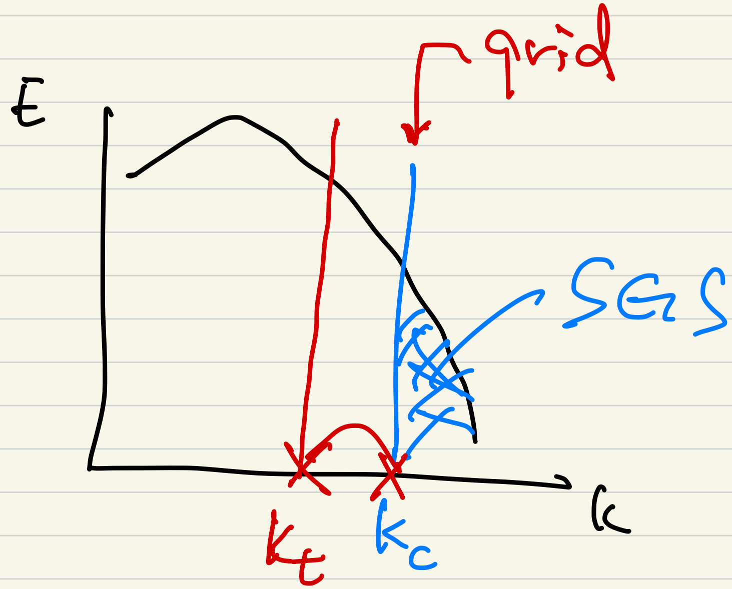


- upwind-type schemes are not recommended
  → numerical dissipation >> turbulent dissipation produced by SGS model

  Park et al. (JCP, 2004)

- modified wavenumber





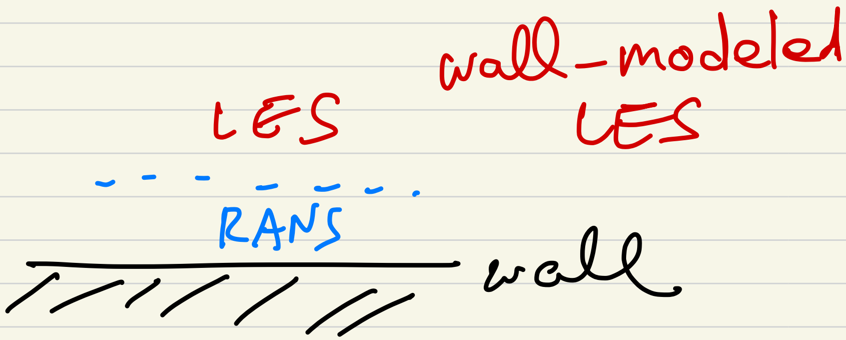
- Number of grid points requirement for LES (Chapman 1979)

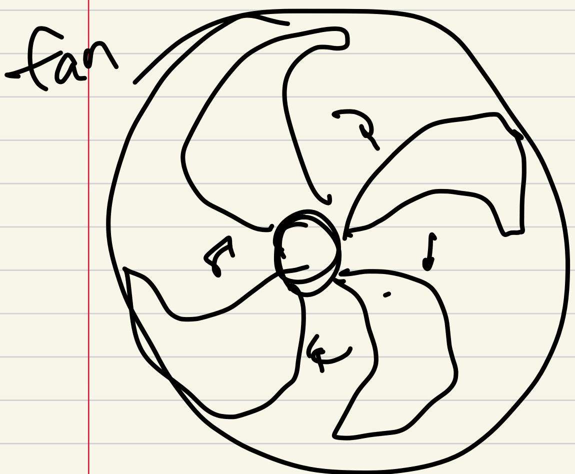
revisited by Choi & Moin (2012) for boundary layer flow

$$N_{DNS} \sim Re^{37/14}$$

$$N_{LES} \sim Re^{13/7} \Leftarrow \text{very high}$$

$$N_{LES\ \underline{w/\ wall\ modeling}} \sim Re^{1} \quad \text{much better}$$





$Re = 547{,}000$

$N = 800M$

2048 processors

MPI

LES → 11 hrs per rotation

① RANS (Reynolds-averaged N-S eqs)

book : Turbulence models and their application in hydraulics
by Wolfgang Rodi (1980)

$$\begin{cases} \dfrac{\partial \tilde{u}_i}{\partial x_i} = 0 \\[2ex] \dfrac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \dfrac{\partial \tilde{u}_i}{\partial x_j} = -\dfrac{1}{\rho_r}\dfrac{\partial \tilde{p}}{\partial x_i} + \nu \nabla^2 \tilde{u}_i + g_i \dfrac{\tilde{\rho} - \rho_r}{\rho_r} \\[2ex] \dfrac{\partial \tilde{\phi}}{\partial t} + \tilde{u}_j \dfrac{\partial \tilde{\phi}}{\partial x_j} = \lambda \nabla^2 \tilde{\phi} + S_\phi \end{cases}$$

$\tilde{u}_i$: inst. velocity

↑ buoyancy term

↑ source term

$\tilde{\phi}$: inst. temperature
or " concentration → passive scalar

$$\tilde{u}_i = U_i + u_i$$
$$\tilde{p} = P + p$$
$$\tilde{\phi} = \Phi + \phi$$

$$\rightarrow \begin{cases} \dfrac{\partial U_i}{\partial x_i} = 0 \\ \dfrac{\partial U_i}{\partial t} + U_j \dfrac{\partial U_i}{\partial x_j} = \boxed{-\dfrac{1}{\rho_r}\dfrac{\partial P}{\partial x_i}} + \dfrac{\partial}{\partial x_j}\left(\nu \dfrac{\partial U_i}{\partial x_j} - \overline{u_i u_j}\right) + g_i \dfrac{\rho - \rho_r}{\rho_r} \\ \dfrac{\partial \Phi}{\partial t} + U_j \dfrac{\partial \Phi}{\partial x_j} = \dfrac{\partial}{\partial x_j}\left(\kappa \dfrac{\partial \Phi}{\partial x_j} - \overline{u_j \phi}\right) + S_\phi \end{cases}$$

closure problem

- eddy-viscosity concept by Boussinesq (1877)

$$\boxed{-\overline{u_i u_j} = \nu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3} k \delta_{ij}}$$

k : kinetic energy

ν_t ↑ turb. viscosity
eddy 〃

$j \to i$: $-\overline{u_i u_i} = \nu_t \left(\cancel{\dfrac{\partial U_i}{\partial x_i} + \dfrac{\partial U_i}{\partial x_i}}\right)^{0} - \dfrac{2}{3} k \delta_{ii} = -2k$ $\quad$ ($\delta_{ii} = 3$)

$k = \frac{1}{2}\overline{u_i u_i}$

$$\frac{\partial}{\partial x_j}\left(-\overline{u_i u_j}\right) = \frac{\partial}{\partial x_j}\left[\nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3} k \delta_{ij}\right] \rightarrow \boxed{\frac{\partial}{\partial x_i}\left(-\frac{2}{3}k\right)}$$

$$\Rightarrow -\frac{\partial}{\partial x_i}\left(\frac{p}{\rho_r}+\frac{2}{3}k\right) = -\frac{\partial}{\partial x_i}\left(\frac{P}{\rho_r}\right)$$

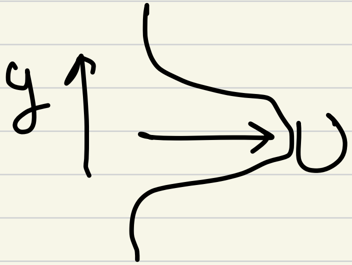
$\therefore$ the appearance of k does not neccessitate the determination of k.

$$\nu_t \propto \tilde{V}\, L$$

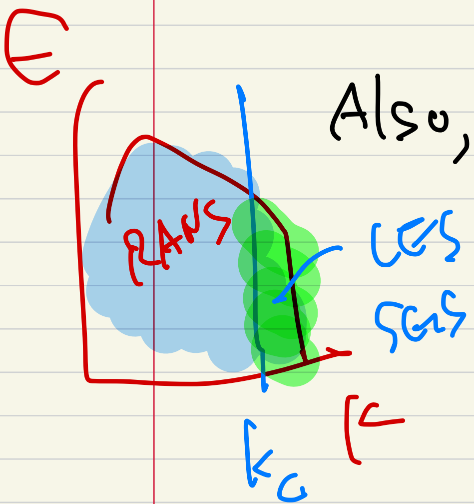
vel. scale, length. scale

eddy viscosity concept breaks down.

2D thin shear layer $\frac{\partial U}{\partial y}$ $\qquad \tau = \rho \nu_t \frac{\partial U}{\partial y}$



in some cases, $\tau \cdot \frac{\partial U}{\partial y} < 0 \rightarrow \nu_t < 0$



Also, $-\overline{u_i u_j} = \nu_t\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right) + \cdots$

tensor $\qquad$ scalar $\qquad$ tensor

$\rightarrow$ isotropic

but not true in reality

In spite of all its shortcomings, the eddy viscosity concept has proven successful in many practical calculations and is still the basis of most turb. models in use today

- eddy diffusivity $\quad -\overline{u_j \phi} = \Gamma \frac{\partial \Phi}{\partial x_j}$ $\quad \Gamma$: turb. diffusivity
  eddy "

$\Gamma = \nu_t / \sigma_t$ $\quad \sigma_t$: turb. Prandtl number
" Schmidt "

exp. results : σ_t varies only little across any flow and also little from flow to flow

→ Many models assume σ_t as a constant,
Buoyancy and streamline curvature affect the value of σ_t.