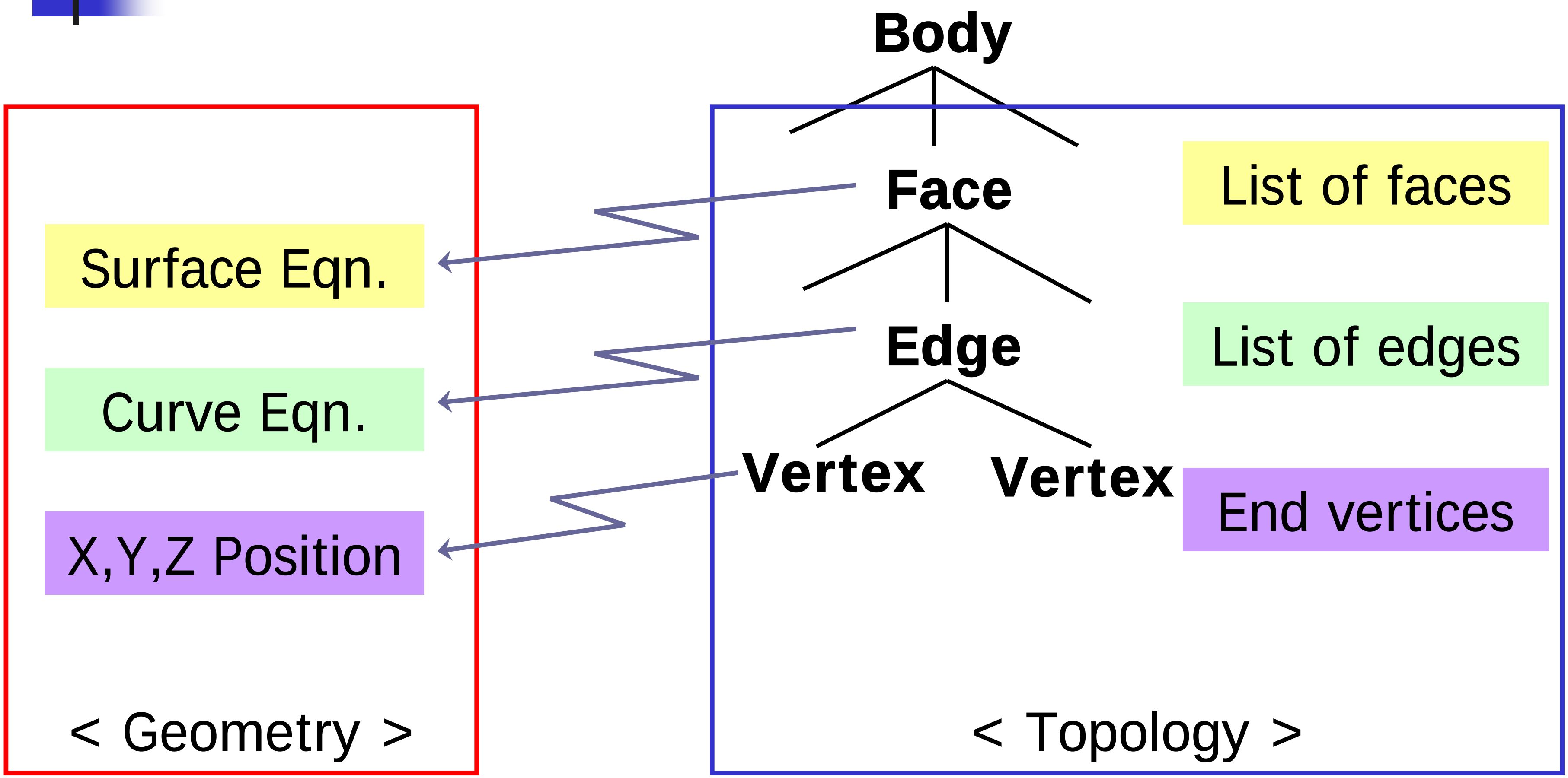
Representation and manipulation of curves

B-Rep Structure – cont'



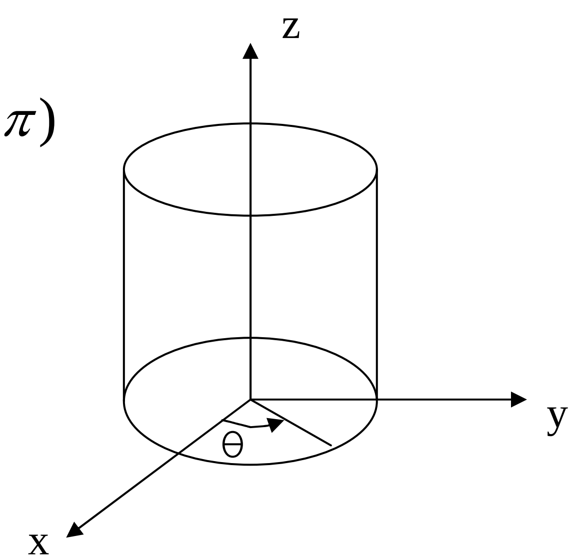
Topology Vs. Geometry

Types of curve equations

- Parametric equation

$$x = x(t), \quad y = y(t), \quad z = z(t)$$

$$Ex)\, x = R\cos\theta, \quad y = R\sin\theta, \quad z = 0 \quad (0 \le \theta \le 2\pi)$$



- Implicit nonparametric

$$x^2 + y^2 - R^2 = 0, \quad z = 0$$

$$F(x, y, z) = 0, \quad G(x, y, z) = 0$$

Intersection of two surfaces
Ambiguous independent parameters

- Explicit nonparametric

$$y = \pm\sqrt{R^2 - x^2}, \quad z = 0$$

Should choose proper neighboring point during curve generation

Conic curves

- Curves obtained by intersecting a cone with a plane
- Circle (circular arc), ellipse, hyperbola, parabola

  Ex) Circle (circular arc)

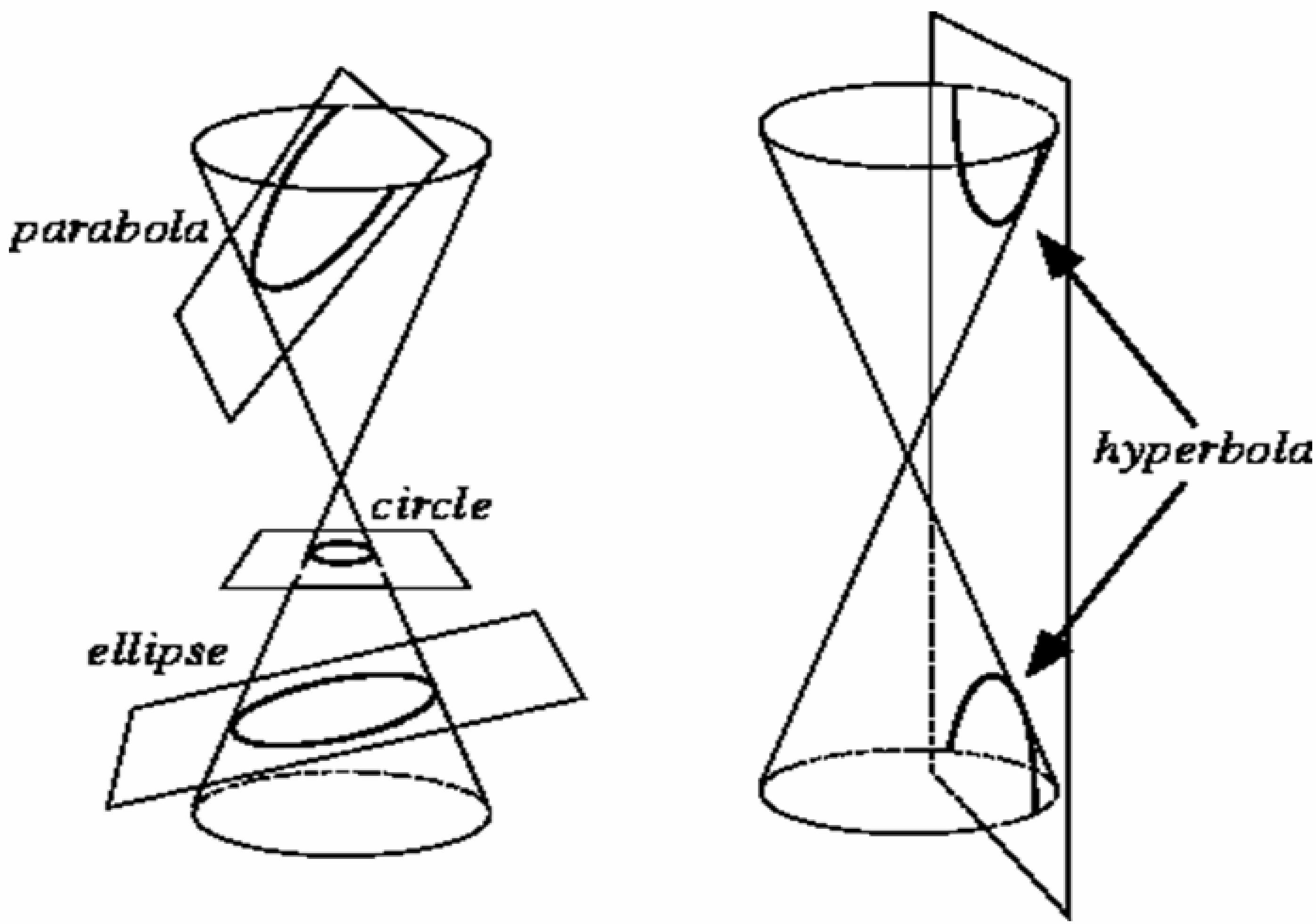
  Circle in xy-plane with center (x_c, y_c) and radius R

$$x = R\cos\theta + x_c$$

$$y = R\sin\theta + y_c$$

$$z = 0$$

- Points on the circle are generated by incrementing θ by $\triangle\theta$ from 0, points are connected by line segments
- Equation of a circle lying on an arbitrary plane can be derived by transformation


parabola
circle
ellipse
hyperbola

Hermite curves

- Parametric eq. is preferred in CAD systems
  - Polynomial form of degree 3 is preferred :

    C2 continuity is guaranteed when two curves are connected

$$\therefore P(u) = [x(u)\, v(u)\, z(u)] = a_0 + a_1\ u + a_2\ u^2 + a_3\, u^3 \qquad (1)$$

$$(0 \le u \le 1) : \text{algebraic eq.}$$

- Impossible to predict the shape change from change in coefficients ⇒ not intuitive

  ⇒Bad for interactive manipulation

Hermite curves – cont’

- Apply Boundary conditions to replace algebraic coefficients
- use $P_{(0)}, P_{(1)}, P_{(0)}', P_{(1)}' \Rightarrow$ Substitute in Eq(1)

$P_0, \quad P_1, \quad P_0', \quad P_1'$

$$
\left.
\begin{aligned}
P_{(0)} &= P_0 = a_0 \\
P_{(1)} &= P_1 = a_0 + a_1 + a_2 + a_3 \\
P_{(0)}' &= P_0' = a_1 \\
P_{(1)}' &= P_1' = a_1 + 2a_2 + 3a_3
\end{aligned}
\right] \qquad (2)
$$

Hermite 곡선 방정식 – cont'

- Solve for a_0, a_1, a_2, a_3 in Eq (2)

$$\left.\begin{aligned} a_0 &= P_0 \\ a_1 &= P_0' \\ a_2 &= -3P_0 + 3P_1 - 2P_0' - P_1' \\ a_3 &= 2P_0 - 2P_1 + P_0' - P_1' \end{aligned}\right] \qquad (3)$$

Hermite 곡선 방정식 – cont’

- Substitute (3) into (1)

$$p(u) = \begin{bmatrix} 1-3u^2+2u^3 & 3u^2-2u^3 & u-2u^2+u^3 & -u^2+u^3 \end{bmatrix} \underbrace{\begin{bmatrix} P_0 \\ P_1 \\ P_0 \\ P_1 \end{bmatrix}}_{\text{geometric coefficientt}}$$

$$\Uparrow$$

Hermite curve equation

- It is possible to predict the curve shape change from the change in P_0, P_1, P_0', P_1' to some extent

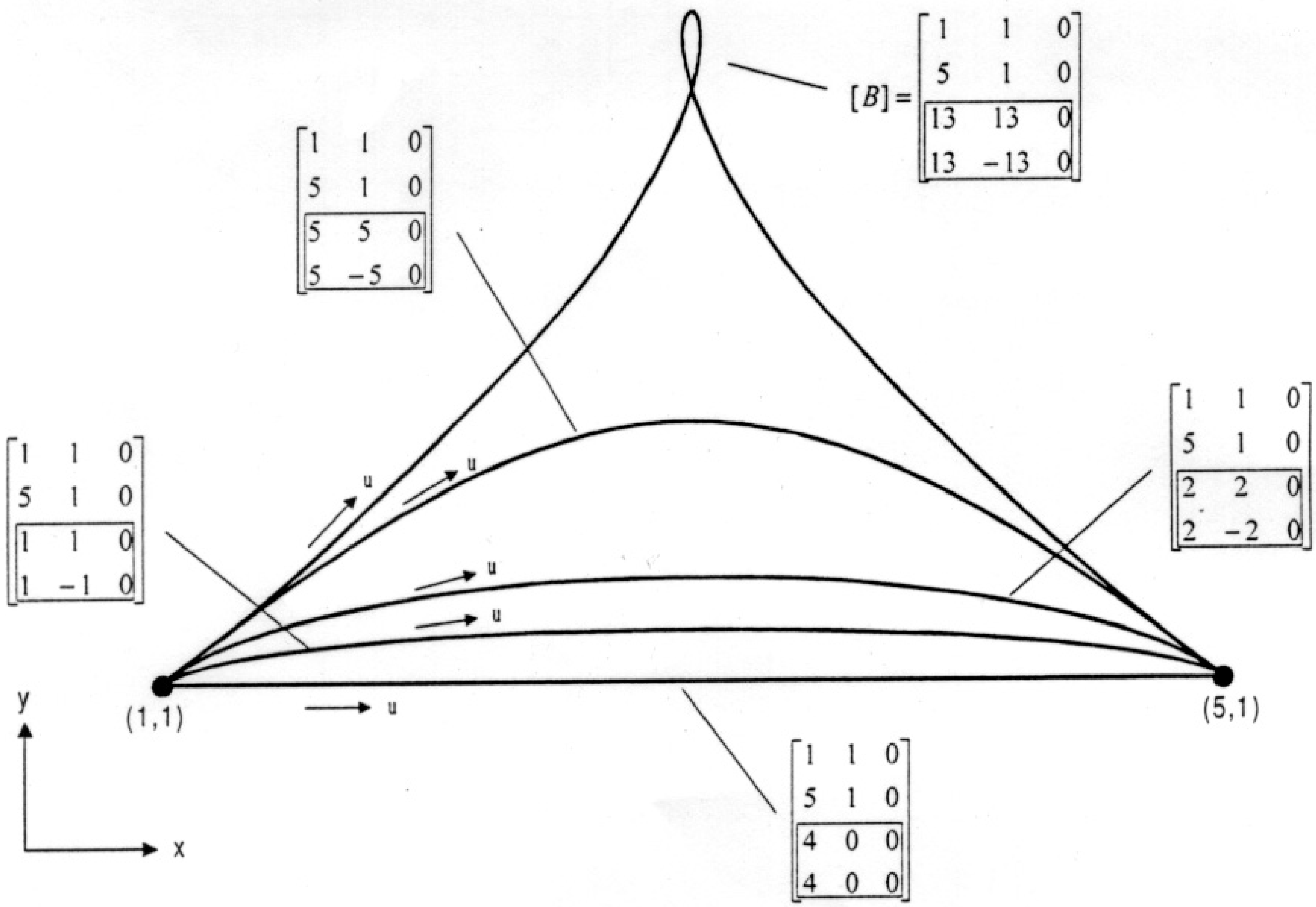


Figure 6.2 Effect of P_0' and P_1' on curve shape

Hermite curves – cont'

- $1-3u^2+2u^3,\ 3u^2-2u^3,\ u-2u^2+u^3,\ -u^2+u^3$ determine the curve shape by blending the effects of P_0, P_1, P_0', P_1'=> blending function

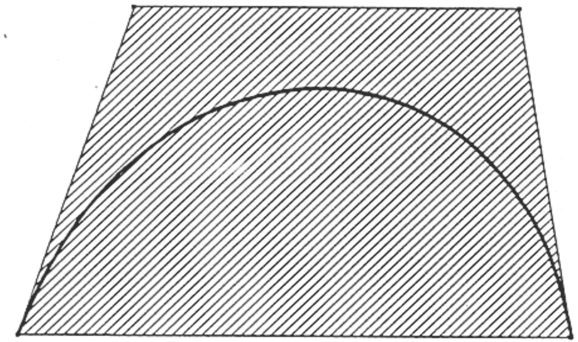
Bezier curves

- It is difficult to realize a curve in one’s mind by changing size and direction of P_0', P_1' in Hermite curves
- Bezier curves
  - Invented by Bezier at Renault
  - Use polygon that enclose a curve approximately

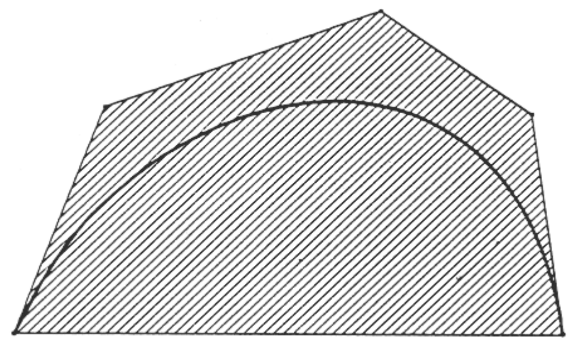
control polygon, control point

Bezier curves – cont'

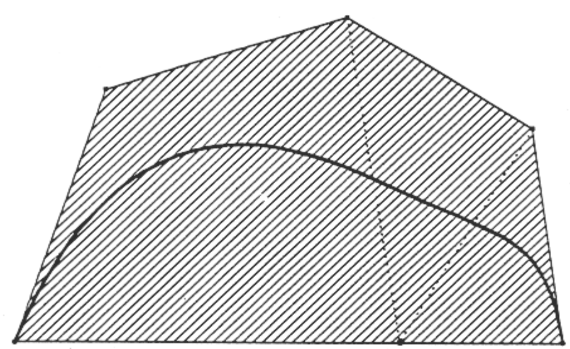
- Passes through 1st and last vertex of control polygon
- Tangent vector at the starting point is in the direction of 1st segment of control polygon
- Tangent vector at the ending point is in the direction of the last segment
  - Useful feature for smooth connection of two Bezier curves
- The n-th derivative at starting or ending point is determined by the first or last (n+1) vertices of control polygon
- Bezier curve resides completely inside its convex hull
  - Useful property for efficient calculation of intersection points



(a)



(b)



(c)

Bezier curves – cont'

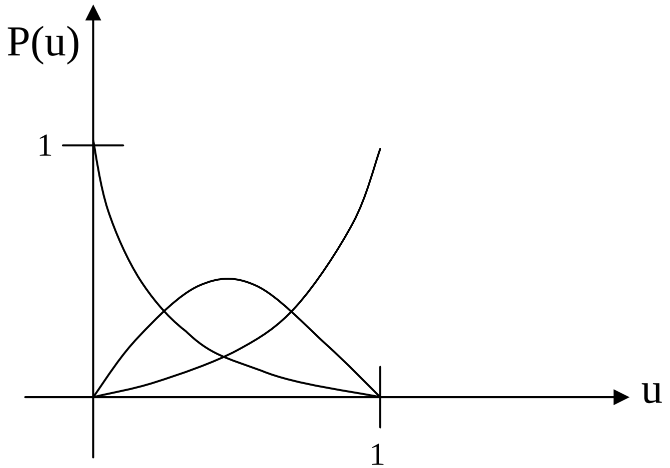
$$p(u) = \sum_{i=0}^{n} \binom{n}{i} u^i (1-u)^{n-i} \mathbf{P}_i \quad (0 \le u \le 1)$$

↑ ***control point***

$$P(u) = (1-u)P_0 + uP_1$$: *Straight line from P_0 to P_1 satisfies the desired qualities including convex hull property*

$$P(u) = (1-u)^2 P_0 + 2(1-u)uP_1 + u^2 P_2 \;\Rightarrow\; (1-u)^2 + 2(1-u)u + u^2 = 1$$

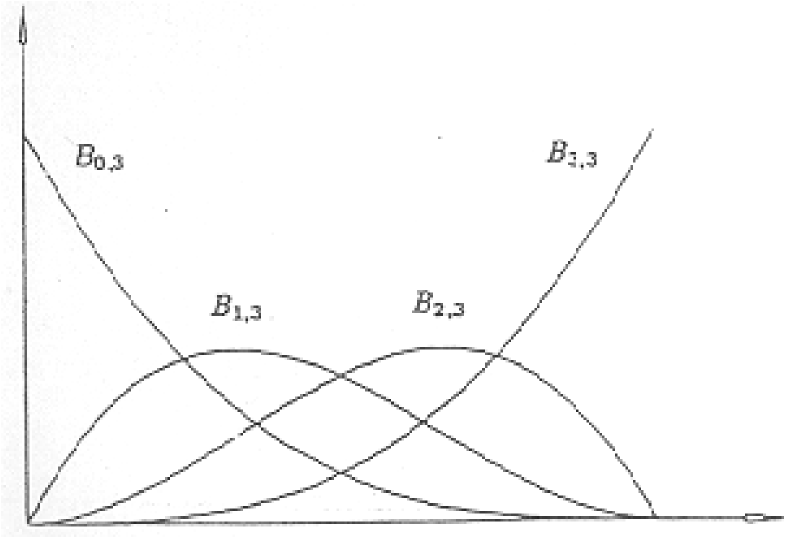
satisfies the desired qualities



Bezier curves – cont'

- Highest term is u^n for the curve defined by (n+1) control points
  - Polynomial of degree n
- Degree of curve is determined by number of control points
- Large number of control points are needed to represent a curve of complex shape -> high degree is necessary.
  - Heavy computation, oscillation
  - Better to connect multiple Bezier curves
- Global modification property (not local modification)
  - Difficult to result a curve of desired shape by modifying portions

Blending functions in Bezier curve



for degree 3

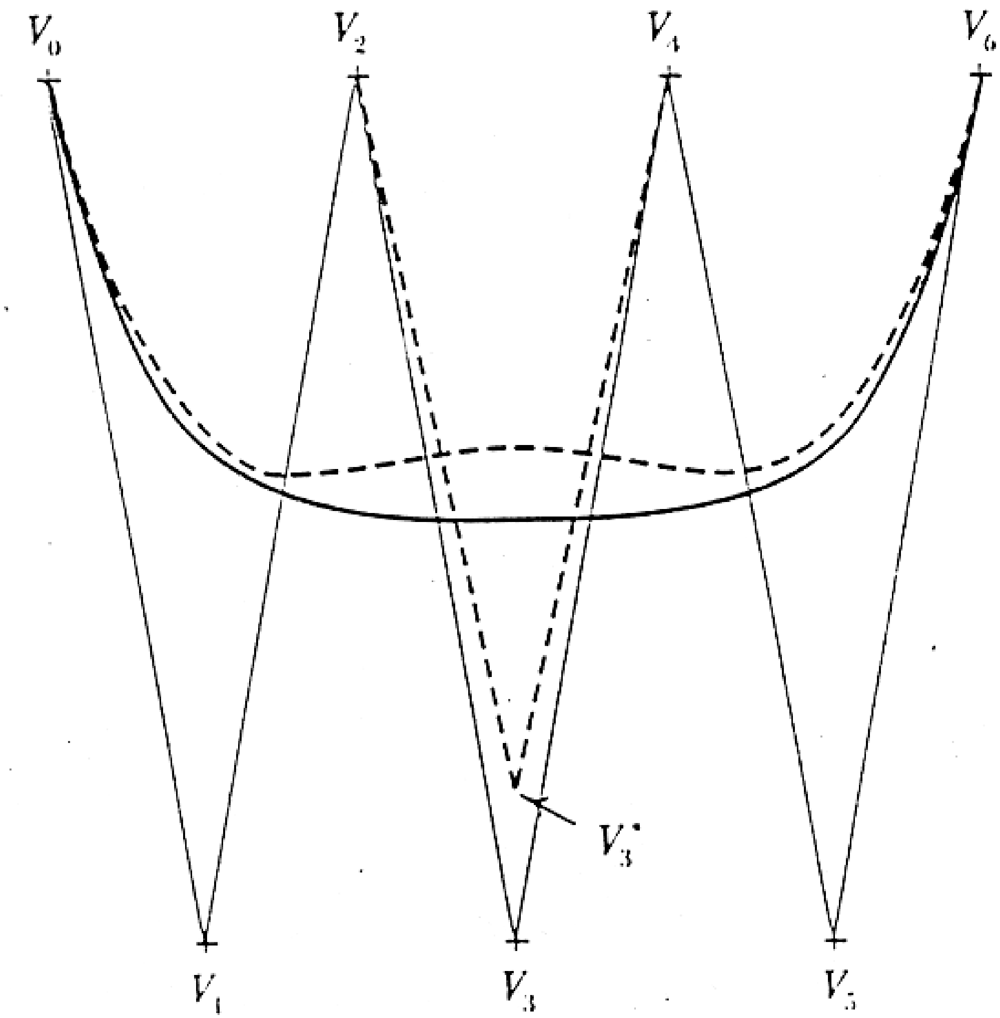


그림 10.22 베지에르 곡선은
국부 조정 특성이 없음.