Surfaces

- Parametric eq.

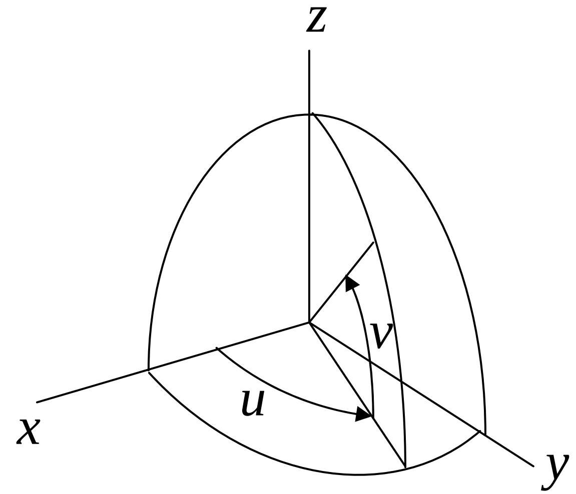
$$\mathbf{P}(u,v) = R\cos v \cos u\,\mathbf{i} + R\cos v \sin u\,\mathbf{j} + R\sin v\,\mathbf{k}$$
$$(0 \le u \le 2\pi, \quad 0 \le v \le 2\pi)$$

- Implicit eq.

$$x^2 + y^2 + z^2 - R^2 = 0$$

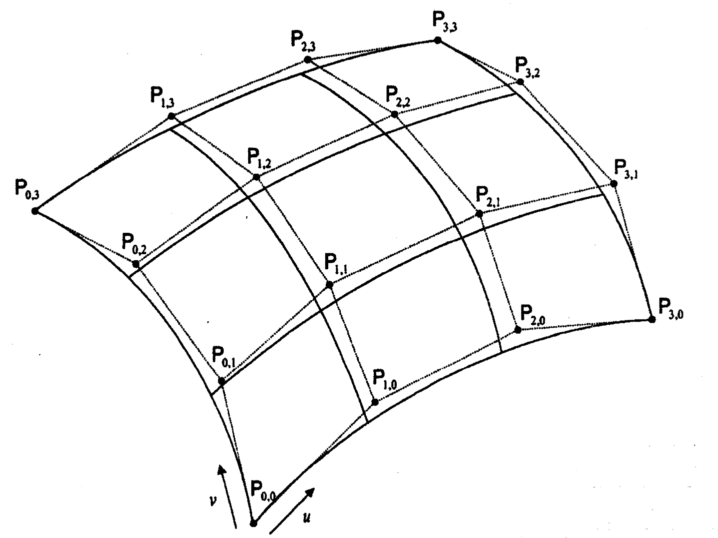
- Explicit eq.

$$\mathrm{z} = \pm\sqrt{\mathrm{R}^2 - x^2 - y^2}$$



Bezier surface

$$\mathbf{P}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} \mathbf{P}_{i,j} B_{i,n}(u) B_{j,m}(v) \qquad 0 \le u \le 1, \qquad 0 \le v \le 1$$



$$= \sum_{i=0}^{n} \left(\mathbf{P}_{i,0} B_{0,m}(v) + \mathbf{P}_{i,1} B_{1,m}(v) + \cdots + \mathbf{P}_{i,m} B_{m,m}(v)\right) B_{i,n}(u)$$

Bezier curve

Bezier surface – cont'

- Surface obtained by blending (n+1) Bezier curves

  (or by blending (m+1) Bezier curves)

- Four corner points on control polyhedron lie on surface

Bezier surface equation

$$\mathbf{P}(0,0) = \sum_{i=0}^{n} \sum_{j=0}^{m} \mathbf{P}_{i,\,j} B_{i,\,n}(0) B_{j,\,m}(0)$$

$$= \sum_{i=0}^{n} \underbrace{\left[\sum_{j=0}^{m} \mathbf{P}_{i,\,j} B_{j,\,m}(0) \right]}_{P_{i,0}} B_{i,\,n}(0)$$

$$= \sum_{i=0}^{n} \mathbf{P}_{i,0} B_{i,n}(0) = \mathbf{P}_{0,0}$$

Bezier surface

- Boundary curves are Bezier curves defined by associated control points

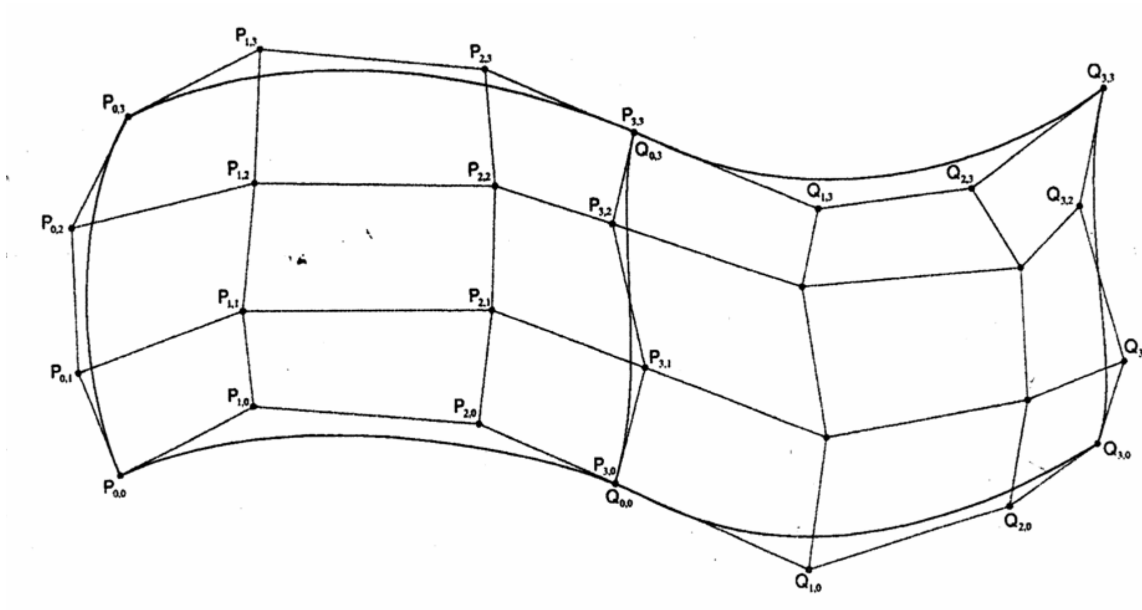
$$\mathbf{P}(0,v) = \sum_{i=0}^{n}\sum_{j=0}^{m}\mathbf{P}_{i,j}B_{i,n}(0)B_{j,m}(v)$$

$$= \sum_{j=0}^{m}\underbrace{\left[\sum_{i=0}^{n}\mathbf{P}_{i,j}B_{i,n}\right]_{u=0}}_{P_{0,j}} B_{j,m}(v)$$

$$= \underbrace{\sum_{j=0}^{m}\mathbf{P}_{0,j}B_{j,m}(v)}$$

Bezier curve defined by $\mathbf{P}_{0,0},\ \mathbf{P}_{0,1},\ \ldots,\ \mathbf{P}_{0,m}$

Bezier surface



- When two Bezier surfaces are connected, control points before and after connection should form straight lines to guarantee G1 continuity

B-spline surface

$$\mathbf{P}(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}\mathbf{P}_{i,j}N_{i,k}(v)N_{j,}(v) \qquad s_{k-1}\le u\le s_{n+1} \qquad t_{-1}\le v\le t_{m+1}$$

- $N_{i,k}(u)$ is defined by $s_0, s_1, \ldots, s_{n+k}$
- $N_{j,l}(v)$ is defined by $t_0, t_1, \ldots, t_{l+m}$
- If $k=(n+1)$, $l=m+1$ and non-periodic knots are used, the resulting surface will become Bezier surface가 된다.

B-spline surface – cont'

- Bezier surface is a special case of B-spline surface.
- Boundary curves are B-spline curves defined by associated control points.
- Four corner points of control polyhedron lie one the surface (when non-periodic knots are used)

NURBS surface

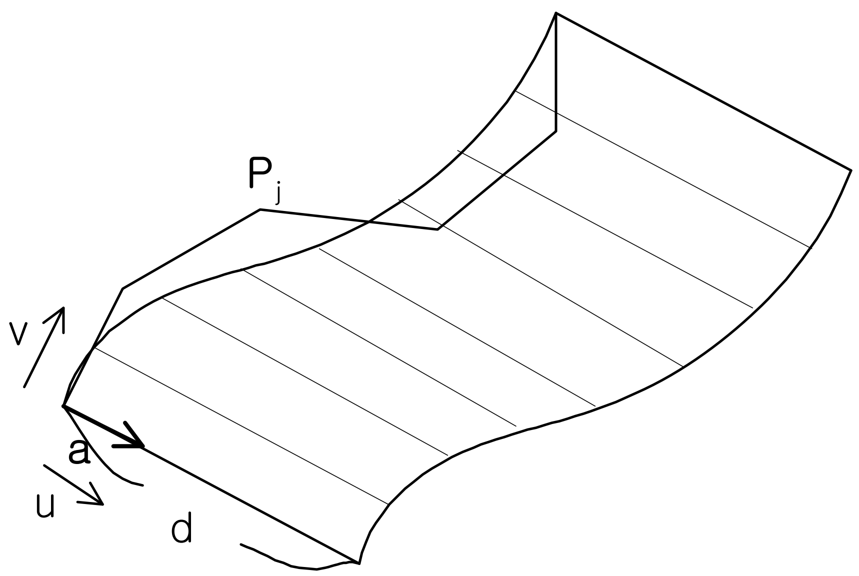
$$\mathbf{P}(u,v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} h_{i,j}\mathbf{P}_{i,j}N_{i,k}(u)N_{j,\ell}(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m} h_{i,j}N_{i,k}(u)N_{j,\ell}(v)}$$

$$s_{k-1} \le u \le s_{n+1}$$
$$t_{\ell-1} \le v \le t_{m+1}$$

- If $h_{i,j}=1$, B-spline surface is obtained
- Represent quadric surface(cylindrical, conical, spherical, paraboloidal, hyperboloidal) exactly

NURBS surface – cont’

- Represent a surface obtained by sweeping a curve



NURBS surface – cont'

- 주어진 곡선을 곡면의 v방향으로 가정
- v방향 knot order는 곡선의 knot 사용
- u 방향order는 2 (1차식이면 되므로)
- control point: 우측 경계에만 있으면 됨

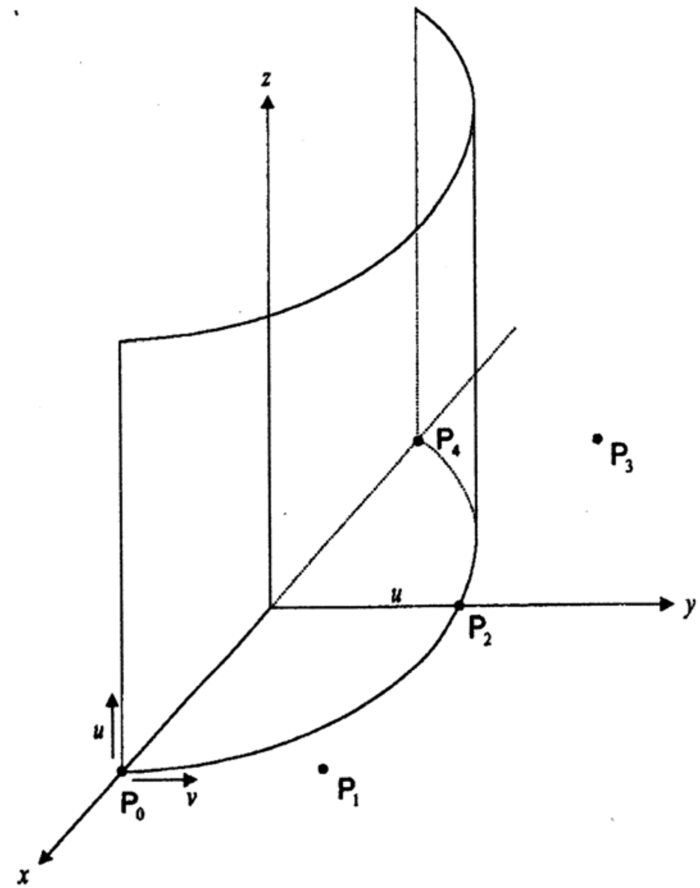
  u 방향 knot 0 0 1 1

  $\mathbf{P}_{0,j} = \mathbf{P}_j$

  $\mathbf{P}_{1,j} = \mathbf{P}_j + d\,\mathbf{a}$

  $h_{0,j} = h_{1,j} = h_j$ from the given curve

Ex) half circle 을 translate해서 cylinder를 생성



Ex) translate half circle to make cylinder

$\mathbf{P}_0$=(1, 0, 0) h_0=1 $\mathbf{P}_1$=(1, 1, 0) $h_1 = \frac{1}{\sqrt{2}}$

$\mathbf{P}_2$=(0, 1, 0) h_2=1 $\mathbf{P}_3$=(-1, 1, 0) $h_3 = \frac{1}{\sqrt{2}}$

P_4=(-1, 0, 0) h_4=1

$\mathbf{P}_{0,0} = \mathbf{P}_0$, $\mathbf{P}_{1,0} = \mathbf{P}_0 + H\mathbf{k}$ $h_{0,0} = h_{1,0} = 1$

$\mathbf{P}_{0,1} = \mathbf{P}_1$, $\mathbf{P}_{1,1} = \mathbf{P}_1 + H\mathbf{k}$ $h_{0,1} = h_{1,1} = \frac{1}{\sqrt{2}}$

$\mathbf{P}_{0,2} = \mathbf{P}_2$, $\mathbf{P}_{1,2} = \mathbf{P}_2 + H\mathbf{k}$ $h_{0,2} = h_{1,2} = 1$

$\mathbf{P}_{0,3} = \mathbf{P}_3$, $\mathbf{P}_{1,3} = \mathbf{P}_3 + H\mathbf{k}$ $h_{0,3} = h_{1,3} = \frac{1}{\sqrt{2}}$

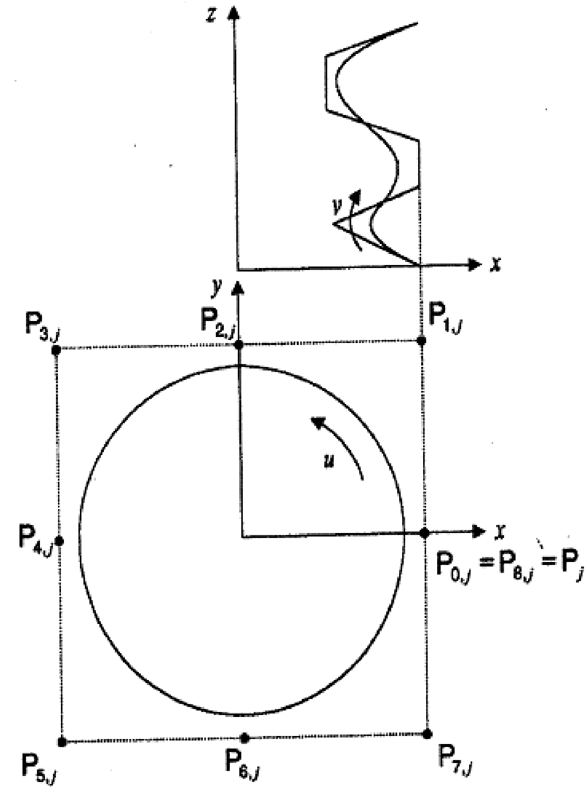
$\mathbf{P}_{0,4} = \mathbf{P}_4$, $\mathbf{P}_{1,4} = \mathbf{P}_4 + H\mathbf{k}$ $h_{0,4} = h_{1,4} = 1$

Knots for v 0 0 0 1 1 2 2 2

Knot for u 0 0 1 1

Ex) Surface obtained by revolution

- Curve
  - order ℓ, knot t_p ($p=0,1,\ldots,m+\ell$)
  - control points P_j, h_j ($j=0,1,\ldots,m$)
- Original control points needs to be split into 9.



Ex) Surface obtained by revolution – cont’

$\mathbf{P}_{0,j} = \mathbf{P}_j \qquad h_{0,j} = h_j$

$\mathbf{P}_{1,j} = \mathbf{P}_{0,j} + x_j\,\mathbf{j} = \mathbf{P}_j + x_j\,\mathbf{j} \qquad h_{1,j} = h_j \cdot \frac{1}{\sqrt{2}}$

$\mathbf{P}_{2,j} = \mathbf{P}_{1,j} - x_j\,\mathbf{i} = \mathbf{P}_j - x_j\,(\mathbf{i}-\mathbf{j}) \qquad h_{2,j} = h_j$

$\mathbf{P}_{3,j} = \mathbf{P}_{2,j} - x_j\,\mathbf{i} = \mathbf{P}_j - x_j\,(2\mathbf{i}-\mathbf{j}) \qquad h_{3,j} = h_j \cdot \frac{1}{\sqrt{2}}$

$\mathbf{P}_{4,j} = \mathbf{P}_{3,j} - x_j\,\mathbf{j} = \mathbf{P}_j - 2x_j\,\mathbf{i} \qquad h_{4,j} = h_j$

$\mathbf{P}_{5,j} = \mathbf{P}_{4,j} - x_j\,\mathbf{j} = \mathbf{P}_j - x_j\,(2\mathbf{i}+\mathbf{j}) \qquad h_{5,j} = h_j \cdot \frac{1}{\sqrt{2}}$

$\mathbf{P}_{6,j} = \mathbf{P}_{5,j} + x_j\,\mathbf{i} = \mathbf{P}_j - x_j\,(\mathbf{i}+\mathbf{j}) \qquad h_{6,j} = h_j$

$\mathbf{P}_{7,j} = \mathbf{P}_{6,j} + x_j\,\mathbf{i} = \mathbf{P}_j - x_j\,\mathbf{j} \qquad h_{7,j} = h_j \cdot \frac{1}{\sqrt{2}}$

$\mathbf{P}_{8,j} = \mathbf{P}_{0,j} = \mathbf{P}_j \qquad h_{8,j} = h_j$

u방향 order 3

u방향 knot 0 0 0 1 1 2 2 3 3 4 4 4

4개의 quarter circle을 합성