FUNDAMENTAL CONCEPTS OF RADIATION

- **Radiation Intensity**

  Emission, Irradiation, Radiosity

- **Blackbody Radiation**

  Planck Distribution
  Wien's Displacement Law
  Stefan-Boltzmann Law

- **Surface Radiation Properties**

  Emissivity, Absorptivity, Reflectivity
  Kirchhoff's Law

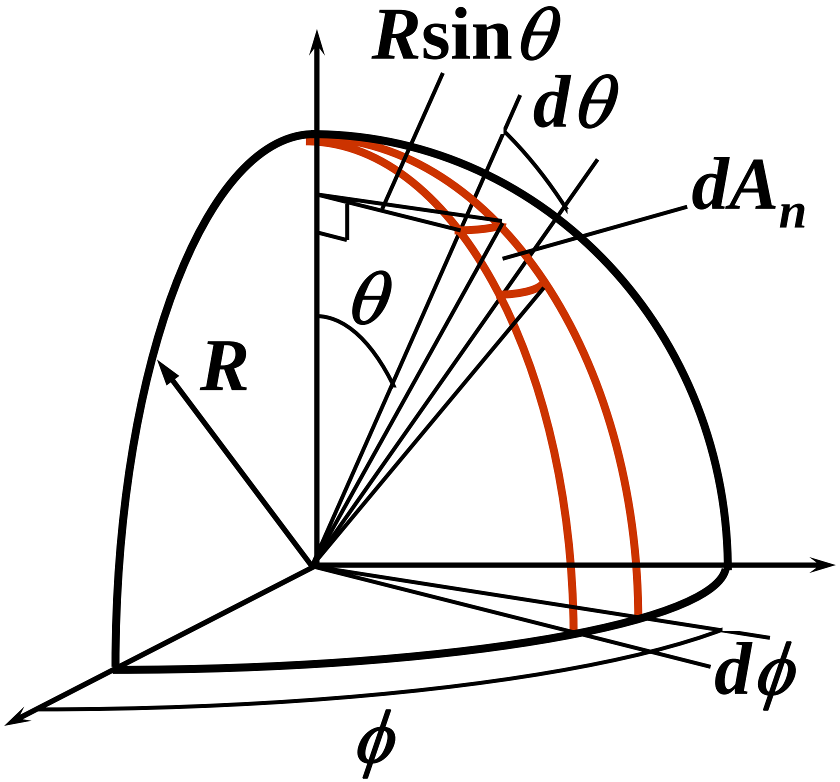
- **Environmental Radiation**

Radiation Intensity

Intensity (spectral)

the amount of radiation energy streaming out through a unit area perpendicular to the direction of propagation $\hat{\Omega}$, per unit solid angle around the direction ω, per unit wavelength around λ, and per unit time about t.

solid angle: a region between the rays of a sphere and measured as the ratio of the element area dA_n on the sphere to the square of the sphere's radius



$$d\omega = \frac{dA_n}{R^2} \quad \text{(steradian, sr)}$$

$$dA_n = (R\sin\theta d\phi)(Rd\theta)$$

$$= R^2 \sin\theta d\theta d\phi$$

$$d\omega = \sin\theta d\theta d\phi$$

ex) hemisphere:

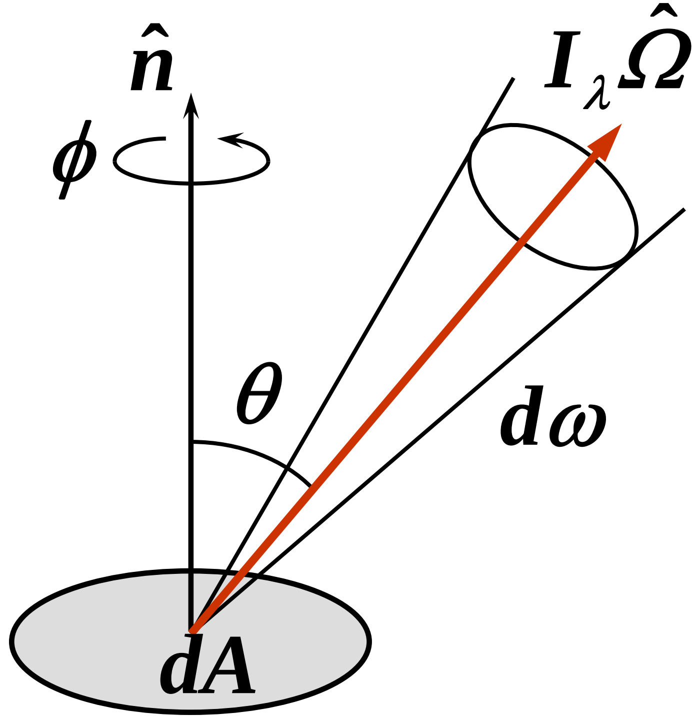
$$\omega = \int_{\cap} d\omega = \int_0^{2\pi}\int_0^{\pi/2} \sin\theta d\theta d\phi = 2\pi \ \text{(sr)}$$

spectral intensity

$$I_\lambda = \frac{d^4Q}{dA\cos\theta d\omega d\lambda dt}$$

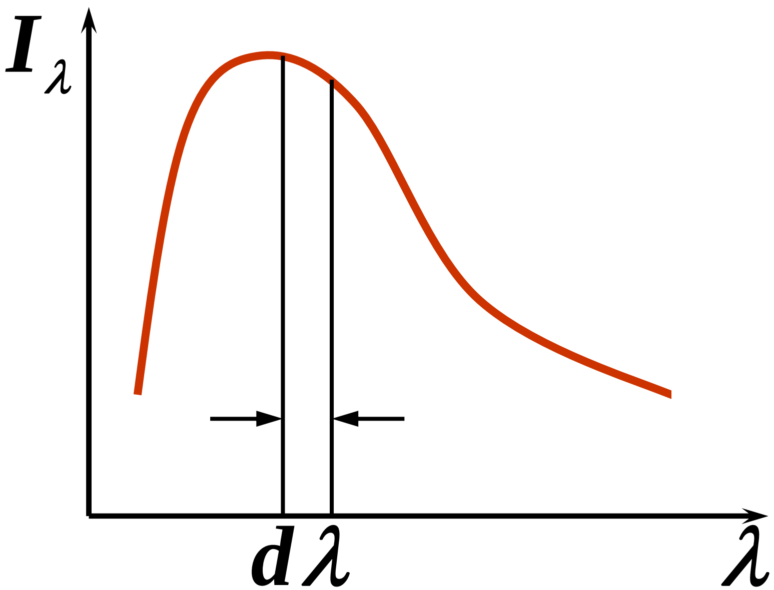
$$[\mathrm{J/m^2 \cdot sr \cdot \mu m \cdot s}]$$

$$I_\lambda(\vec{r},\hat{\Omega}) = I_\lambda(x,y,z,\theta,\phi)$$



total intensity: $I = \int_0^\infty I_\lambda d\lambda$

$$d^4Q = I_\lambda \cos\theta dA d\omega d\lambda dt \ [\mathrm{J}]$$



$$d^3q = \frac{d^4Q}{dt}$$

$$= I_\lambda dA\cos\theta d\omega d\lambda \ [\mathrm{W}]$$

$$d^2q'' = \frac{d^4Q}{dAdt} = I_\lambda \cos\theta d\omega d\lambda \ \ [\mathrm{W/m^2}]$$

$$dq''_\lambda = \frac{d^4Q}{dAdtd\lambda} = I_\lambda \cos\theta d\omega \ \ [\mathrm{W/m^2} \cdot \mu\mathrm{m}]$$

spectral radiative heat flux:

$$q''_\lambda = \int_\cap I_\lambda \cos\theta d\omega = \int_\cap I_\lambda \hat{\Omega} \cdot \hat{\mathbf{n}} d\omega$$

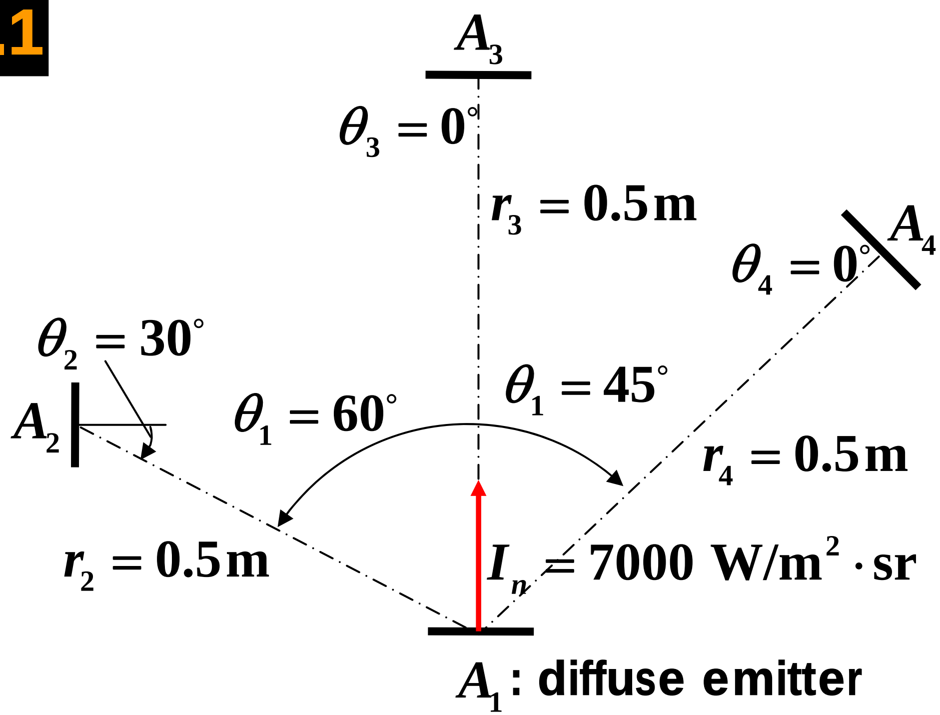
$$= \int_0^{2\pi} \int_0^{\pi/2} I_\lambda \cos\theta \sin\theta d\theta d\phi \ \ [\mathrm{W/m^2} \cdot \mu\mathrm{m}]$$

total radiative heat flux:

$$q'' = \int_0^\infty q''_\lambda d\lambda = \int_0^\infty \int_\cap I_\lambda \cos\theta d\omega d\lambda$$

$$= \int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} I_\lambda \cos\theta \sin\theta d\theta d\phi d\lambda \ \ [\mathrm{W/m^2}]$$

Example 12.1



$$A_1 = A_2 = A_3 = A_4 = 10^{-3}\,\mathrm{m}^2$$

Find:

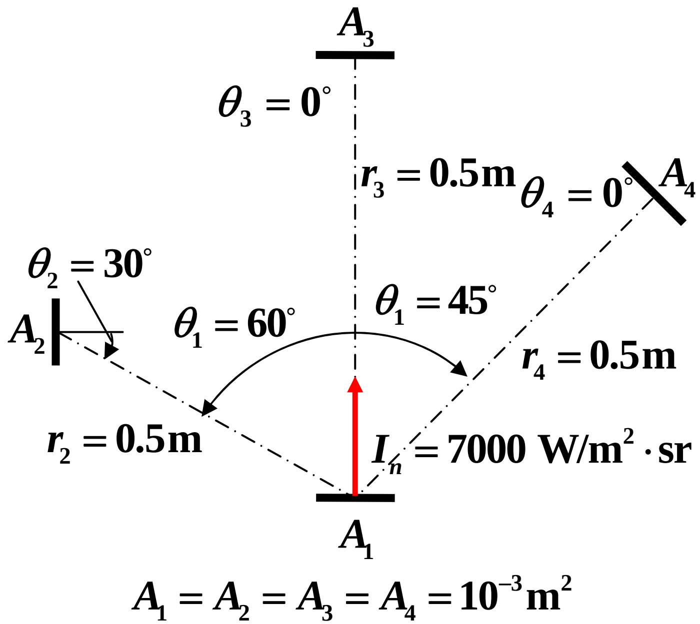
1) Intensity of emission in each of the three directions
2) Solid angles subtended by the three surfaces
3) Rate at which radiation is intercepted by the three surfaces

Assumption:

A_1, A_2, A_3, and A_4 → differential surfaces, $(A_j / r_j^2) << 1$

1) Diffusely emitted intensity: independent of direction

$$I = I_n = 7000 \ \mathrm{W/m^2 \cdot sr}$$



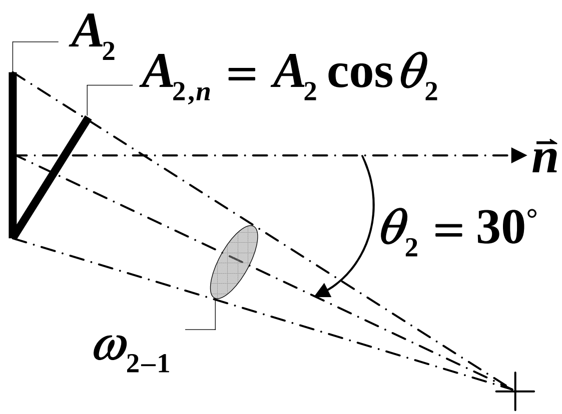
2) Solid angles

$$d\omega \equiv \frac{dA_n}{r^2}, \qquad dA_{n,j} = dA_j \times \cos\theta_j$$

$$\omega_{2-1} \approx \frac{A_2 \times \cos\theta_2}{r_2^2} = 3.46 \times 10^{-3}\mathrm{sr}$$

$$\omega_{3-1} \approx \frac{A_3 \times \cos\theta_3}{r_3^2} = 4.00 \times 10^{-3}\mathrm{sr}$$

$$\omega_{4-1} \approx \frac{A_4 \times \cos\theta_4}{r_4^2} = 4.00 \times 10^{-3}\mathrm{sr}$$



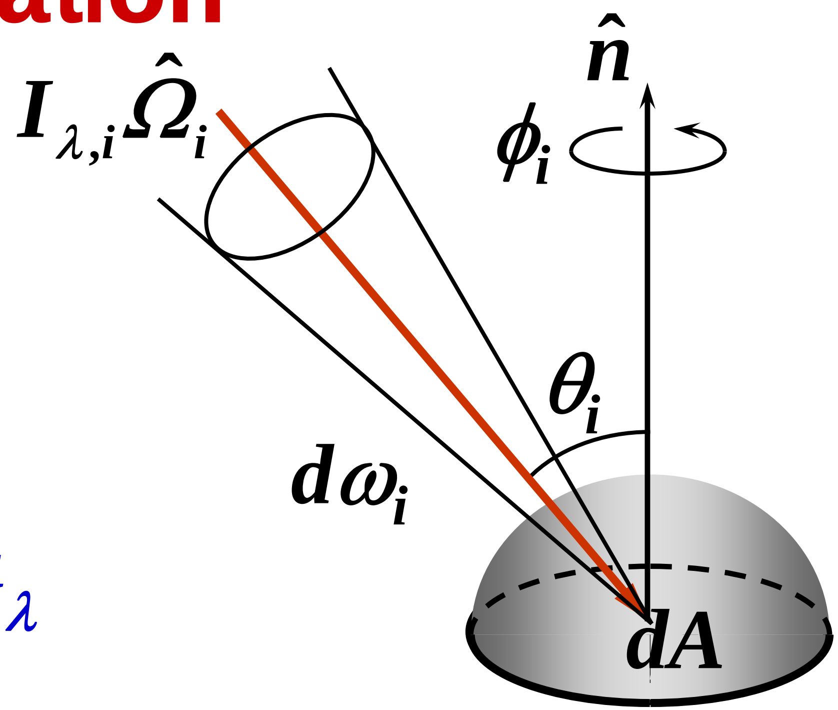
3) Radiation rate on each surface

$$q_{1-j} \approx I_n \times A_1 \cos\theta_1 \times \omega_{j-1}$$

$$q_{1-2} \approx 12.1 \times 10^{-3}\,\mathrm{W}, \quad q_{1-3} \approx 28.0 \times 10^{-3}\,\mathrm{W}, \quad q_{1-4} \approx 19.8 \times 10^{-3}\,\mathrm{W}$$

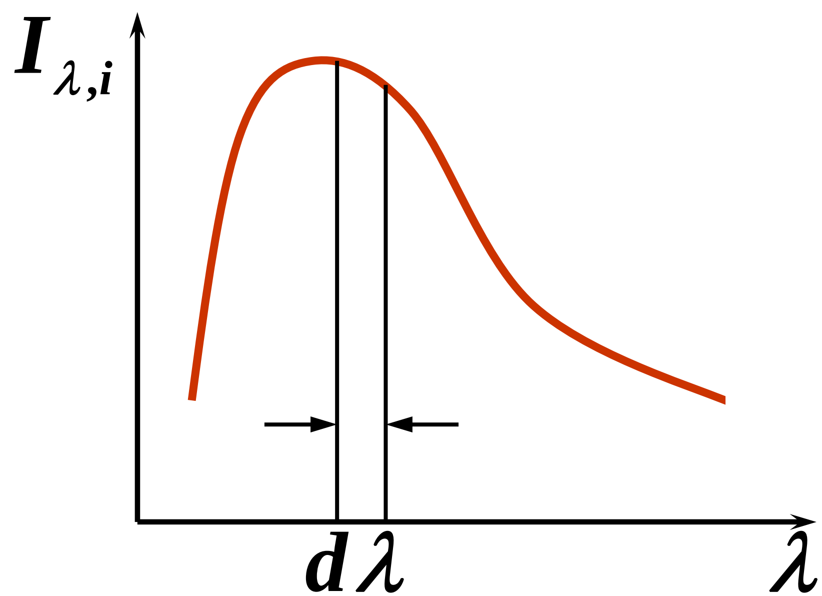
Irradiation

Irradiation:
all incident radiative heat flux through the control surface



spectral irradiation: G_λ

$$G_\lambda = \int_\cap I_{\lambda,i}\cos\theta_i d\omega_i$$



total irradiation: G

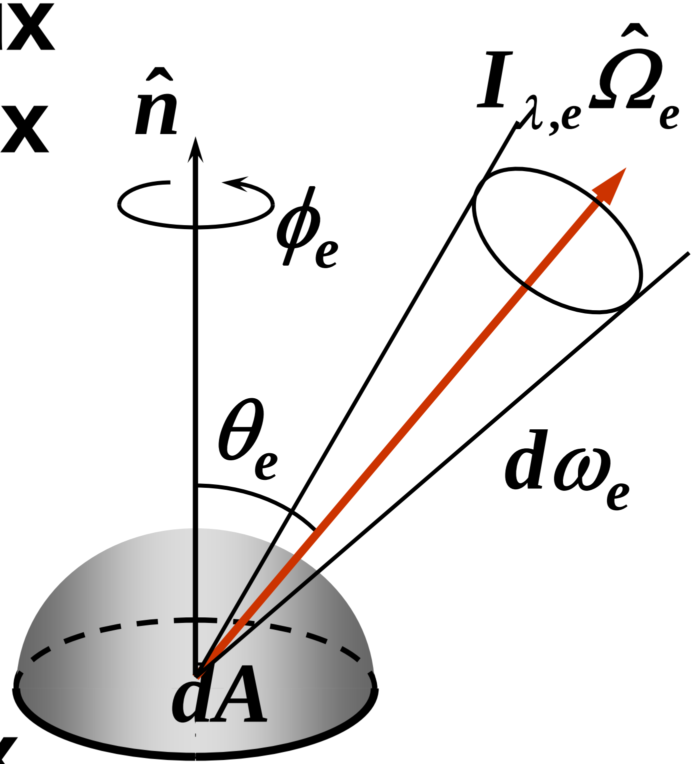
$$G = \int_0^\infty G_\lambda d\lambda$$

$$= \int_0^\infty \int_\cap I_{\lambda,i}\cos\theta_i d\omega_i d\lambda$$

Radiosity

Radiosity: all out-going radiative heat flux through the control surface

radiosity = emitted heat flux + reflected heat flux



spectral radiosity: J_λ

spectral emitted heat flux (**spectral emissive power**)

$$q''_{\lambda,e} = \int_{\cap} I_{\lambda,e} \cos\theta_e \, d\omega_e$$

spectral reflected heat flux

$$q''_{\lambda,r} = \int_{\cap} I_{\lambda,r} \cos\theta_r \, d\omega_r$$

spectral radiosity:

$$J_\lambda = q''_{\lambda,e} + q''_{\lambda,r} = \int_\cap I_{\lambda,e}\cos\theta_e d\omega_e + \int_\cap I_{\lambda,r}\cos\theta_r d\omega_r$$

$$= \int_\cap \left(I_{\lambda,e} + I_{\lambda,r}\right)\cos\theta d\omega = \int_\cap I_{\lambda,e+r}\cos\theta d\omega$$

total radiosity: J

total emitted heat flux (total emissive power)

$$q''_e = \int_0^\infty q''_{\lambda,e} d\lambda = \int_0^\infty \left(\int_\cap I_{\lambda,e}\cos\theta_e d\omega_e\right) d\lambda$$

total reflected heat flux

$$q''_r = \int_0^\infty q''_{\lambda,r} d\lambda = \int_0^\infty \int_\cap \left(I_{\lambda,r}\cos\theta_r d\omega_r\right) d\lambda$$

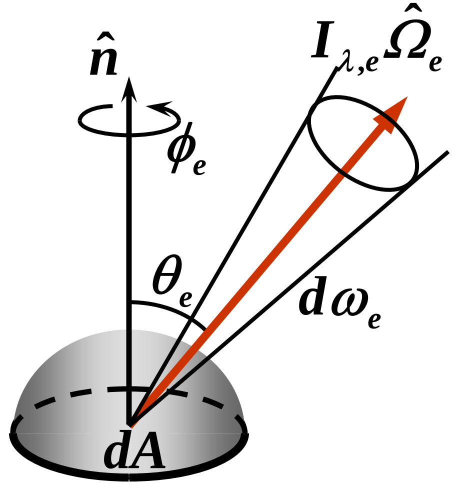
total radiosity: $J = \int_0^\infty J_\lambda d\lambda = q''_e + q''_r$

$$= \int_0^\infty \int_\cap I_{\lambda,e}\cos\theta_e d\omega_e d\lambda + \int_0^\infty \int_\cap I_{\lambda,r}\cos\theta_r d\omega_r d\lambda$$

Emissive Power

directional spectral emissive power

$$E'_\lambda = I_{\lambda,e}\cos\theta_e = I_{\lambda,e}\hat{\Omega}_e \cdot \hat{n}$$



hemispherical spectral emissive power

$$E_\lambda = q''_{\lambda,e} = \int_\cap I_{\lambda,e}(\vec{r},\hat{\Omega})\cos\theta_e d\omega_e$$

$$= \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\vec{r},\hat{\Omega})\cos\theta_e \sin\theta_e d\theta_e d\phi_e$$

hemispherical total emissive power

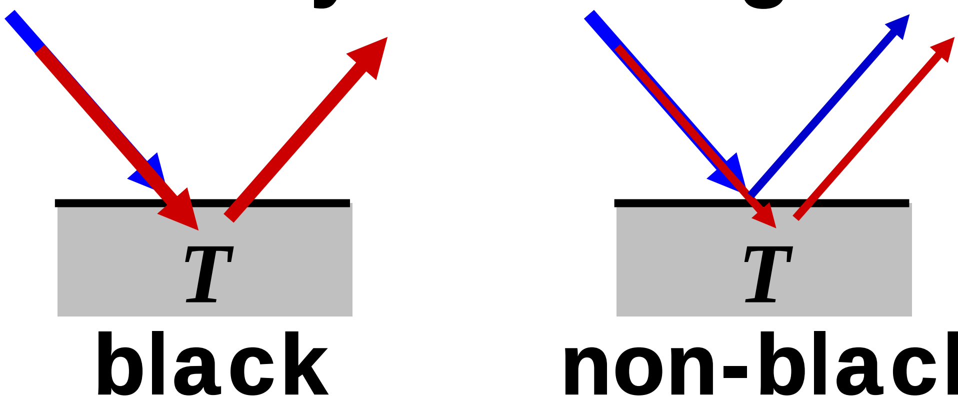
$$E = q''_e = \int_0^\infty E_\lambda d\lambda$$

$$= \int_0^\infty \int_\cap I_{\lambda,e}(\vec{r},\hat{\Omega})\cos\theta_e d\omega_e d\lambda$$

Blackbody Radiation

a) blackbody: a perfect absorber for all incident radiation

b) Maximum emitter in each direction and at every wavelength



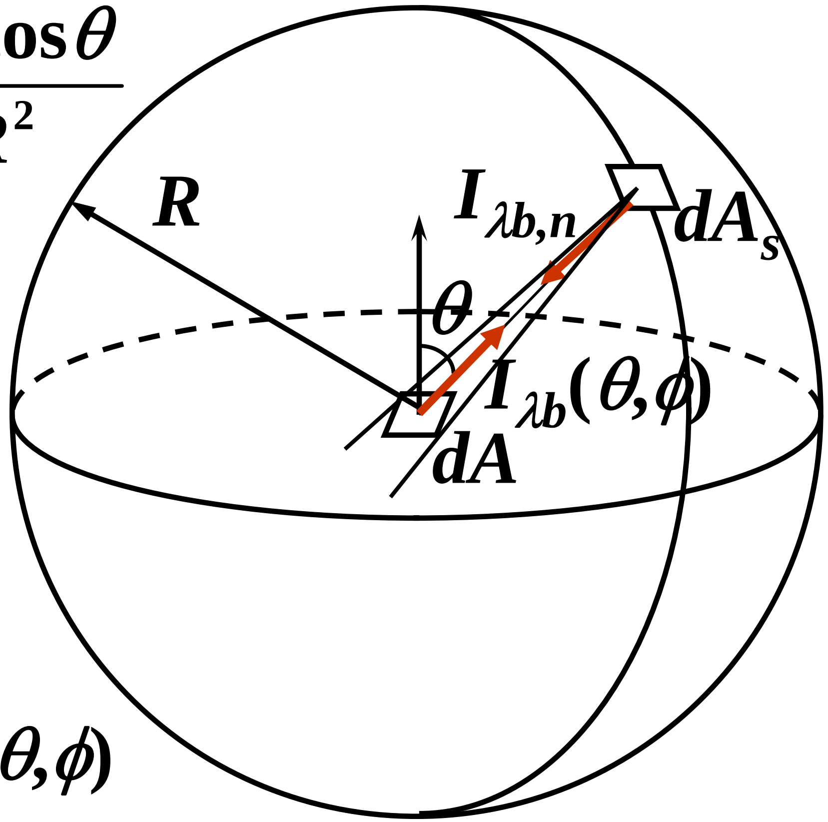
black: termed based on the visible radiation so not a perfect description

c) Emitted intensity from a blackbody: invariant with emission angle

Proof) consider energy exchange between an element on a spherical black enclosure, dA_s and an element at the center of the enclosure, dA. Both elements are in thermal equilibrium.

energy absorbed by dA

$$I_{\lambda b,n} dA_s d\lambda d\omega_s = I_{\lambda b,n} dA_s d\lambda \frac{dA\cos\theta}{R^2}$$



energy absorbed by dA_s

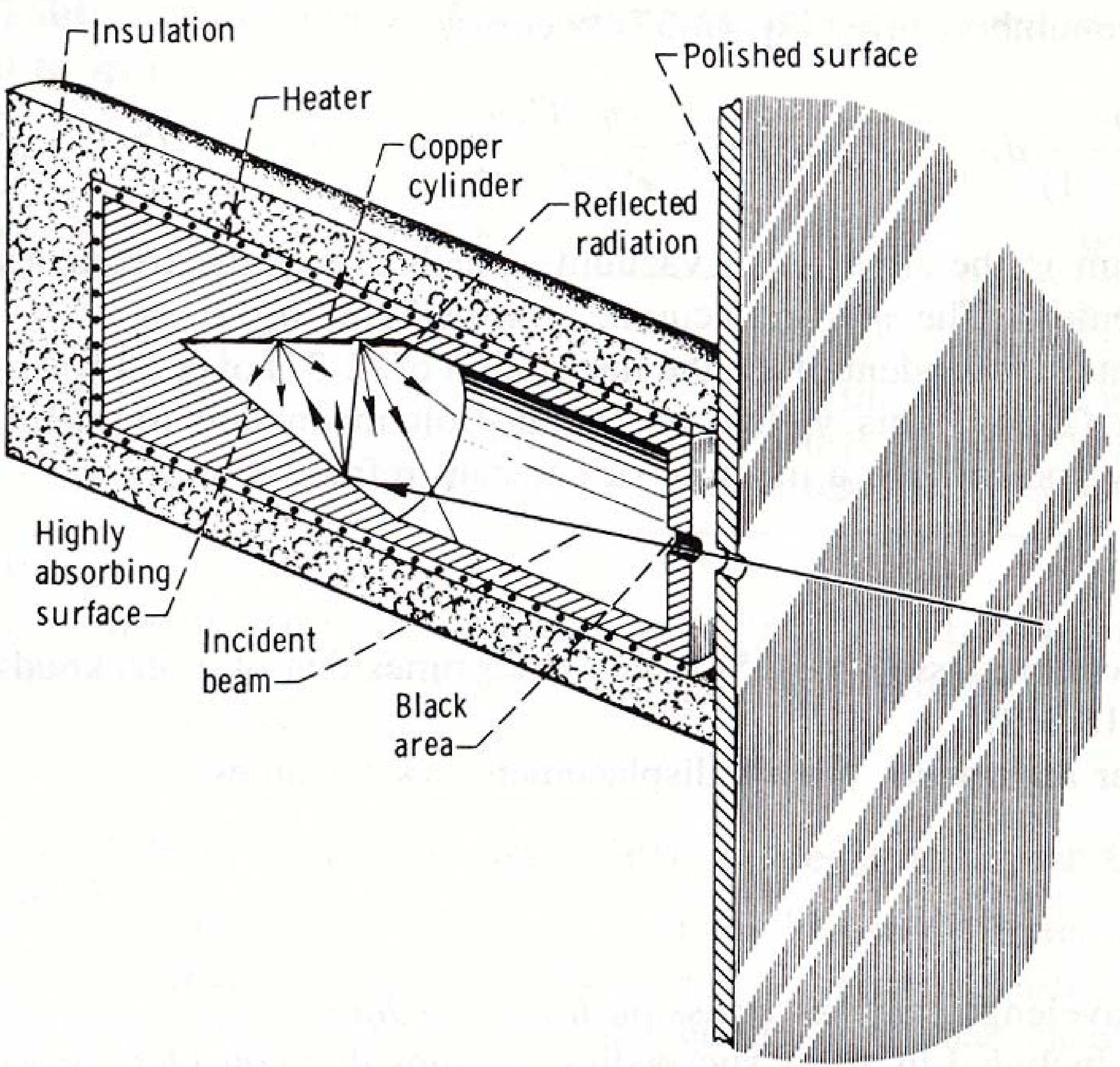
$$I_{\lambda b}(\theta,\phi) dA\cos\theta d\lambda d\omega$$

$$= I_{\lambda b}(\theta,\phi) dA\cos\theta d\lambda \frac{dA_s}{R^2}$$

in equilibrium

$$I_{\lambda b}(\theta,\phi) = I_{\lambda b,n} \neq \text{function of } (\theta,\phi)$$

and since max at a given temperature



Simulated blackbody

Planck Law

blackbody hemispherical spectral emissive power

$$E_{\lambda b} = q''_{\lambda b,e} = \int_{\cap} I_{\lambda b}(\vec{r})\cos\theta\, d\omega$$

$$= \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda b}(\vec{r})\cos\theta \sin\theta\, d\theta\, d\phi$$

$$= I_{\lambda b}(\vec{r})\int_0^{2\pi}\int_0^{1}\cos\theta\, d(\cos\theta)\, d\phi$$

$$= \pi I_{\lambda b}(\vec{r})$$

Planck's law

(The Theory of Heat Radiation, Max Planck, 1901)

spectral distribution of hemispherical emissive power of a blackbody in vacuum

$$E_{\lambda b} = \pi I_{\lambda b} = \frac{2\pi C_1}{\lambda^5 \left(e^{C_2/\lambda T} - 1\right)}, \quad C_1 = hC_0^2, \; C_2 = hC_0/k$$

h: Planck constant, k: Boltzmann constant

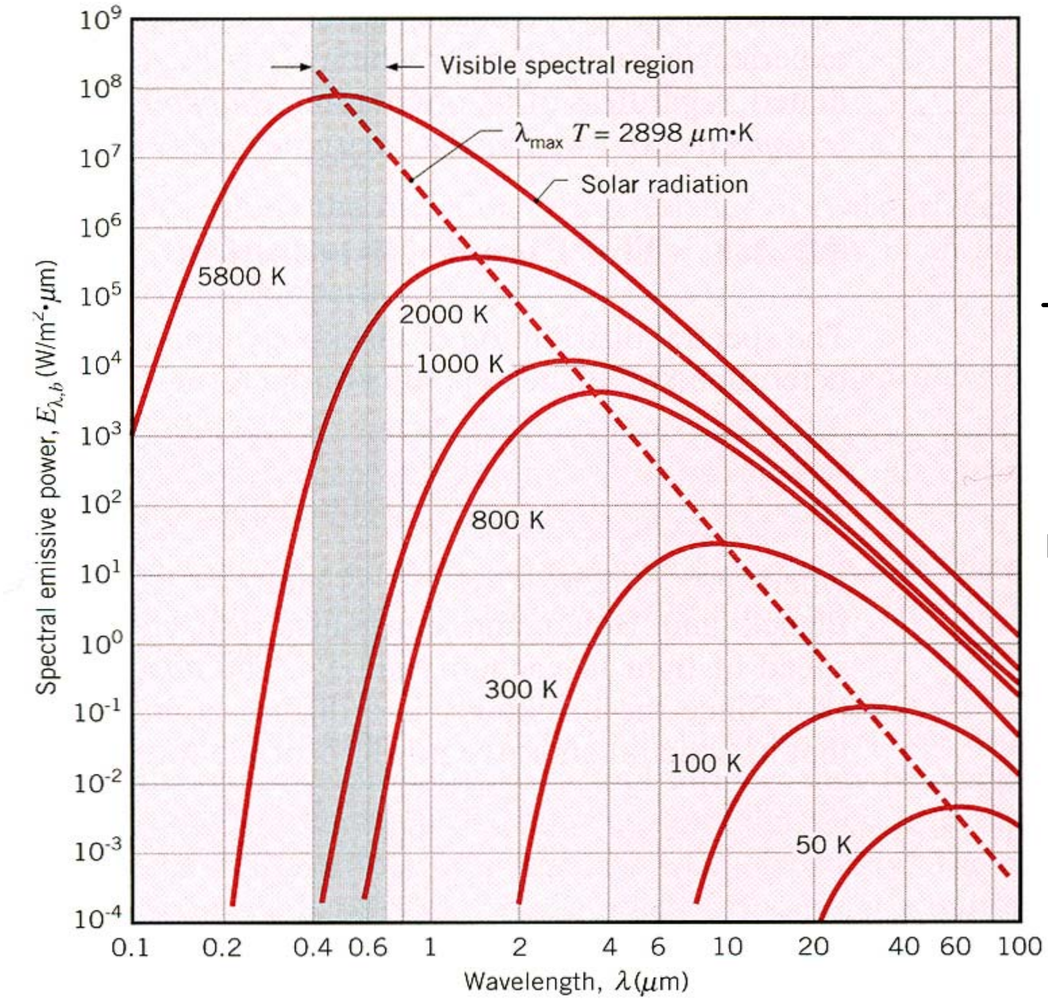
in a medium with a refractive index n:

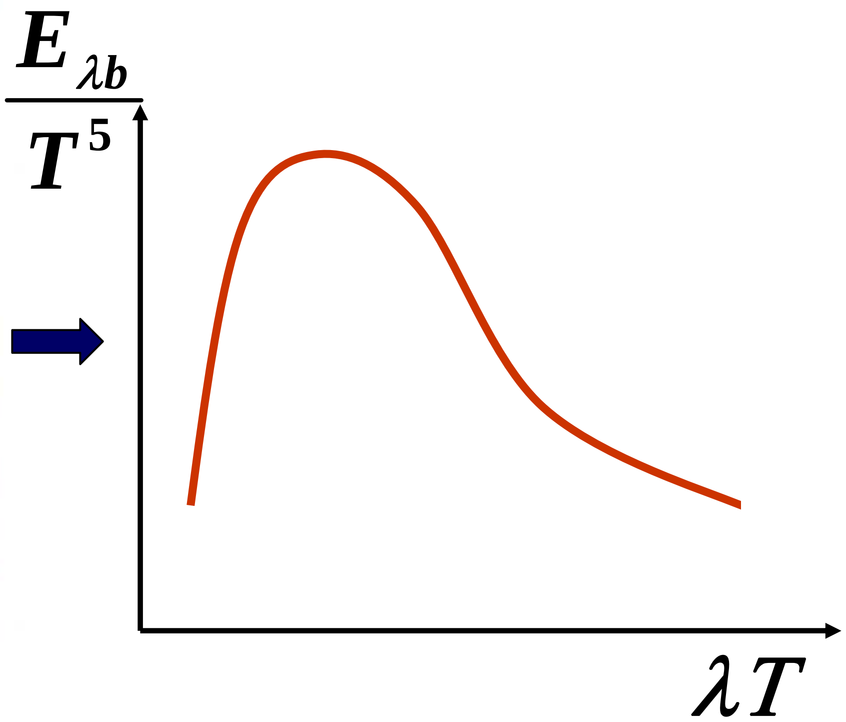
$$E_{\lambda b} = \pi I_{\lambda b} = \frac{2\pi C_1}{n^2 \lambda^5 \left(e^{C_2/\lambda n T} - 1\right)}$$

$n = 1$ in vacuum and $n = 1.00029$ in air at room temperature over the visible spectrum

$$E_{\lambda b} = \frac{2\pi C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)} \qquad \frac{E_{\lambda b}(\lambda, T)}{T^5} = \frac{2\pi C_1}{(\lambda T)^5\left(e^{C_2/\lambda T} - 1\right)}$$

$$\equiv E(\lambda T)$$





Blackbody spectral emissive power

Wien's Displacement law (1891)

$\lambda_{\max}$: the wavelength at which $E_{\lambda b}(\lambda,T)$ is maximum

$$\frac{E_{\lambda b}(\lambda,T)}{T^5}=\frac{2\pi C_1}{(\lambda T)^5\left(e^{C_2/\lambda T}-1\right)}$$

$$\frac{d}{d(\lambda T)}\left(\frac{E_{\lambda b}}{T^5}\right)=0 \;\rightarrow \lambda_{\max}T=\frac{C_2}{5}\frac{1}{1-e^{-C_2/\lambda_{\max}T}}$$

$$\lambda_{\max}T=C_3=2897.8\ \mu\mathrm{m}\cdot\mathrm{K}$$

Stefan-Boltzmann's Law

blackbody total intensity and total emissive power

$$I_b = \int_0^\infty I_{\lambda b} d\lambda = \int_0^\infty \frac{2C_1}{\lambda^5 \left(e^{C_2/\lambda T} - 1\right)} d\lambda$$

$$= \frac{2C_1 T^4}{C_2^4} \int_0^\infty \frac{\zeta^3}{e^\zeta - 1} d\zeta = \frac{2C_1 T^4}{C_2^4} \frac{\pi^4}{15} \equiv \frac{\sigma}{\pi} T^4$$

$$\sigma = 2C_1 \pi^5 / 15 C_2^4 = 5.6696 \times 10^{-8} \ \mathrm{W/m^2 \cdot K^4}$$

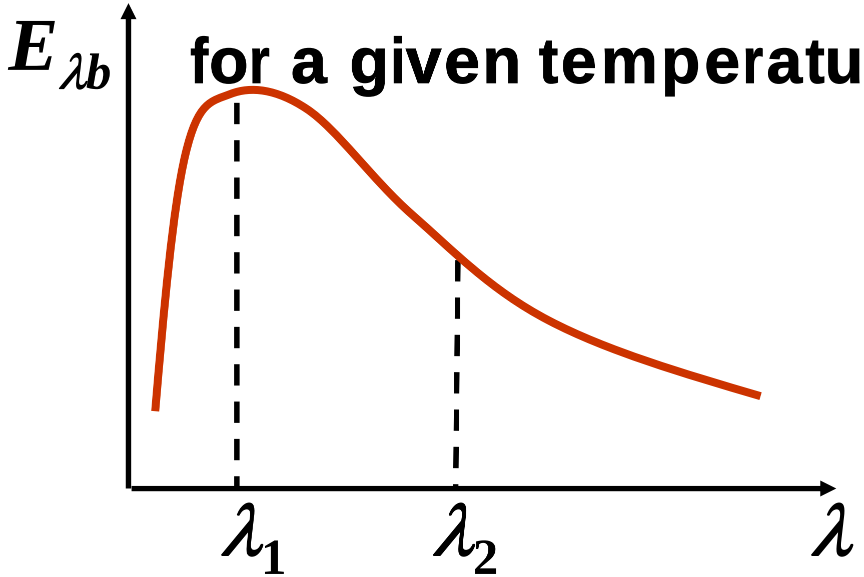
Stefan-Boltzmann's law:

$$E_b = q''_{b,e} = \int_0^\infty E_{\lambda b} d\lambda = \pi I_b = \sigma T^4 \ [\mathrm{W/m^2}]$$

Stefan by experiment (1879): $E_b \sim T^4$

Boltzmann by theory (1884): $E_b = \sigma T^4$

Blackbody radiation in a wavelength interval

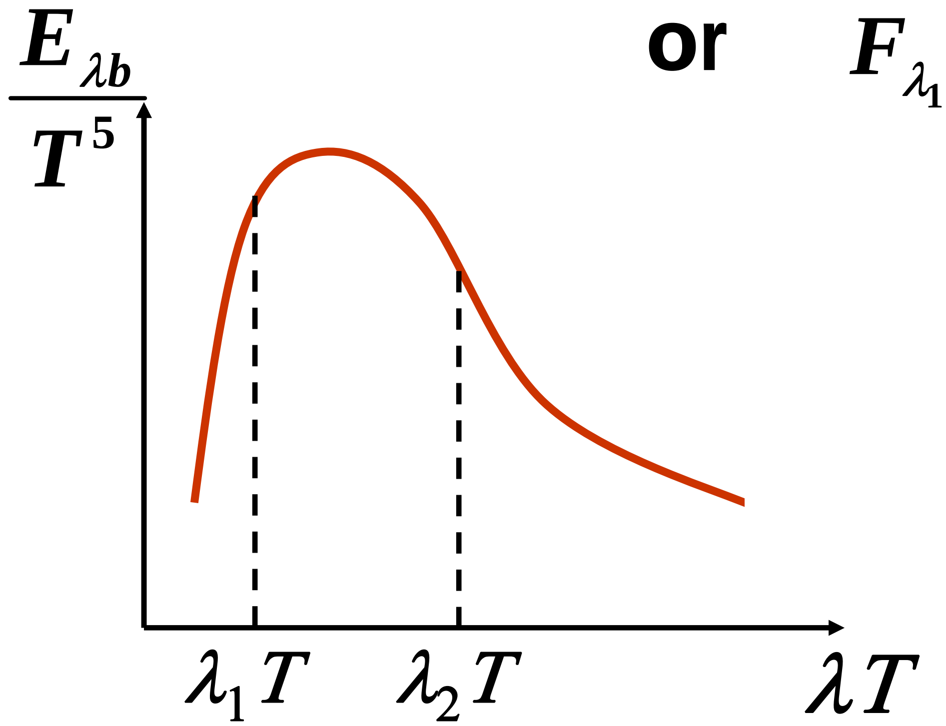


$$F_{\lambda_1-\lambda_2} = \frac{\int_{\lambda_1}^{\lambda_2} E_{\lambda b}(\lambda,T)d\lambda}{\int_0^{\infty} E_{\lambda b}(\lambda,T)d\lambda}$$

$$= \frac{1}{\sigma T^4}\int_{\lambda_1}^{\lambda_2} E_{\lambda b}(\lambda,T)d\lambda$$

$$= \frac{1}{\sigma T^4}\left[\int_0^{\lambda_2} E_{\lambda b}(\lambda,T)d\lambda - \int_0^{\lambda_1} E_{\lambda b}(\lambda,T)d\lambda\right]$$

$$= F_{0-\lambda_2} - F_{0-\lambda_1}$$



or

$$F_{\lambda_1-\lambda_2} = \frac{1}{\sigma T^4}\int_{\lambda_1}^{\lambda_2} E_{\lambda b}(\lambda, T)\, d\lambda$$

$$= \frac{1}{\sigma}\int_{\lambda_1 T}^{\lambda_2 T} \frac{E_{\lambda b}(\lambda, T)}{T^5}\, d(\lambda T)$$

$$\equiv F_{\lambda_1 T-\lambda_2 T}$$

$$= F_{0-\lambda_2 T} - F_{0-\lambda_1 T}$$

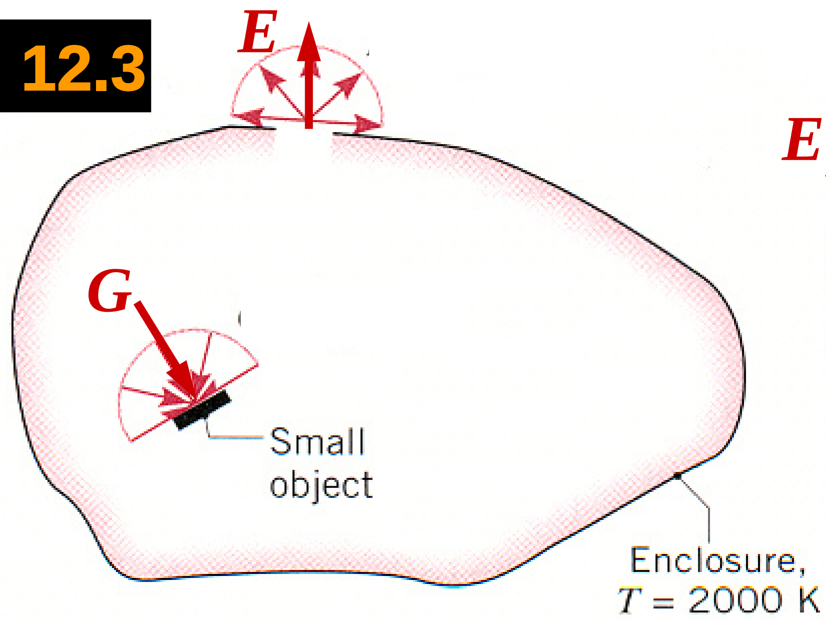
TABLE 12.1 Blackbody Radiation Functions

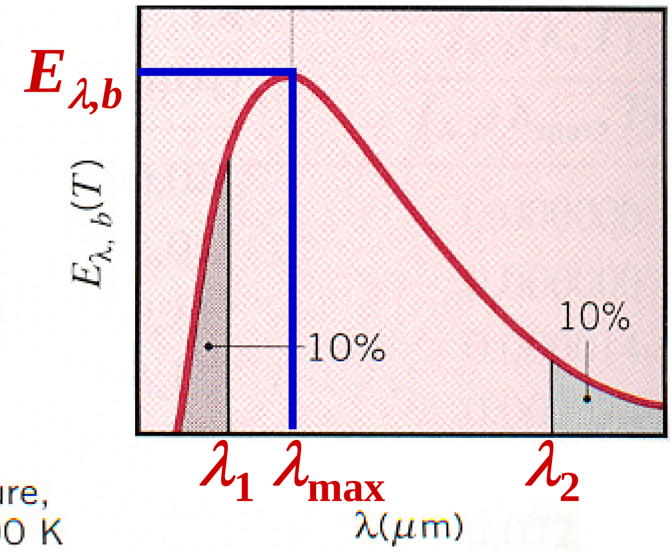
λT (μm $\cdot$ K)	$F_{(0 \rightarrow \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ (μm $\cdot$ K $\cdot$ sr)$^{-1}$	$\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551

TABLE 12.1 *Continued*

λT ($\mu m \cdot K$)	$F_{(0 \rightarrow \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
9,500	0.903085	0.765338	0.105956
10,000	0.914199	0.653279×10^{-5}	0.090442
10,500	0.923710	0.560522	0.077600
11,000	0.931890	0.483321	0.066913
11,500	0.939959	0.418725	0.057970
12,000	0.945098	0.364394×10^{-5}	0.050448
13,000	0.955139	0.279457	0.038689
14,000	0.962898	0.217641	0.030131
15,000	0.969981	0.171866×10^{-5}	0.023794
16,000	0.973814	0.137429	0.019026
18,000	0.980860	0.908240×10^{-6}	0.012574
20,000	0.985602	0.623310	0.008629
25,000	0.992215	0.276474	0.003828
30,000	0.995340	0.140469×10^{-6}	0.001945
40,000	0.997967	0.473891×10^{-7}	0.000656
50,000	0.998953	0.201605	0.000279
75,000	0.999713	0.418597×10^{-8}	0.000058
100,000	0.999905	0.135752	0.000019

Example 12.3





Find:

1) Emissive power of a small aperture on the enclosure
2) Wavelengths below which and above which 10% of the radiation is concentrated
3) Spectral emissive power and wavelength associated with maximum emission
4) Irradiation on a small object inside

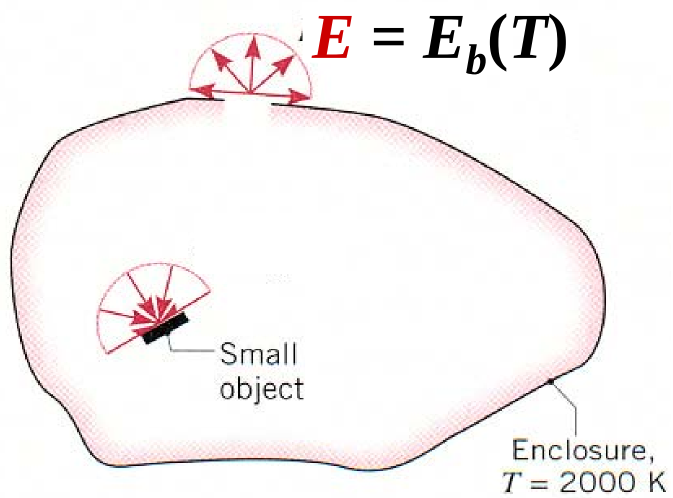
Assumption:

Areas of aperture and object are very small relative to enclosure surface.

1) Emission from the aperture

$$E = E_b(T) = \sigma T^4$$

$$= 5.67 \times 20^4 = 9.07 \times 10^5 \ \mathrm{W/m^2}$$



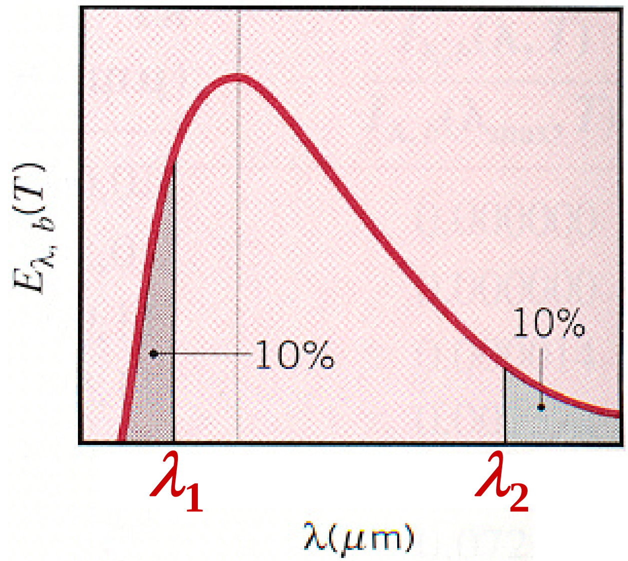
2) $F_{(0\to\lambda_1)} = 0.1$

From Table 12.1

$$\lambda_1 T \approx 2200\mu\mathrm{m}\cdot\mathrm{K}\ ,\ \lambda_1 = 1.1\mu\mathrm{m}$$

$$F_{(0\to\lambda_2)} = 0.9$$

$$\lambda_2 T \approx 9382\mu\mathrm{m}\cdot\mathrm{K}\ ,\ \lambda_2 = 4.69\mu\mathrm{m}$$



3) Spectral emissive power and wavelength associated with maximum emission

Wien's law, $\lambda_{\max} T = 2898\,\mu\mathrm{m}\cdot\mathrm{K}$

$$\lambda_{\max} = 2898/2000 = 1.45\ \mu\mathrm{m}$$

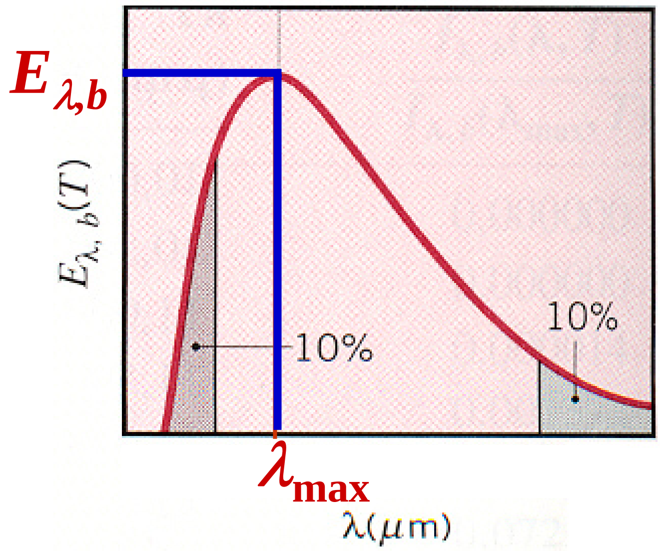
$$E_{\lambda b} = \pi I_{\lambda b}$$

From Table 12.1

$$I_{\lambda b}(1.45\mu\mathrm{m}, T) = 0.722\times10^{-4}\,\sigma T^5$$

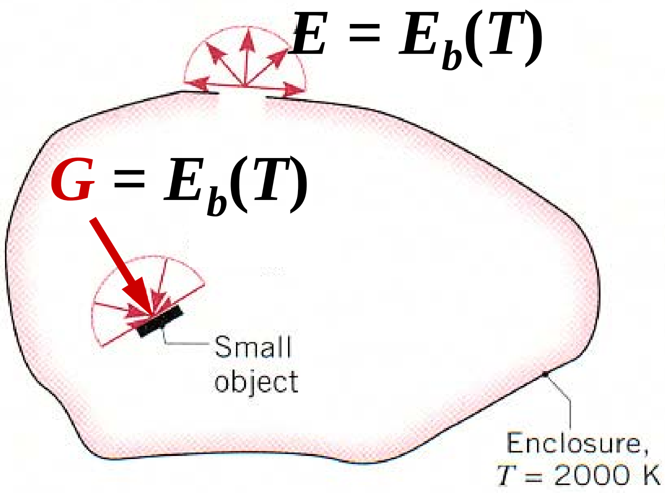
$$= 1.31\times10^5\,\mathrm{W/m^2\cdot sr\cdot\mu m}$$

$$E_{\lambda b} = \pi I_{\lambda b} = 4.12\times10^5\ \mathrm{W/m^2\cdot\mu m}$$

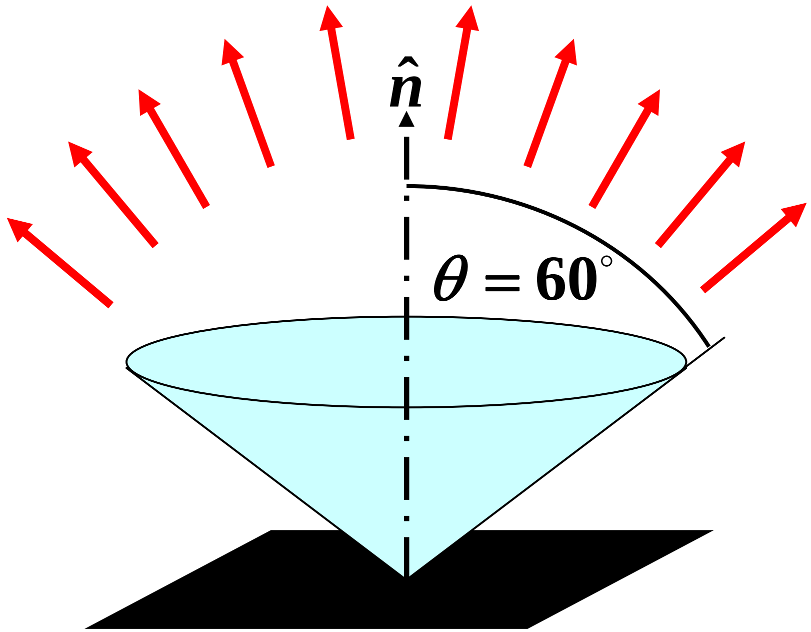


4) Irradiation of any small object inside the enclosure

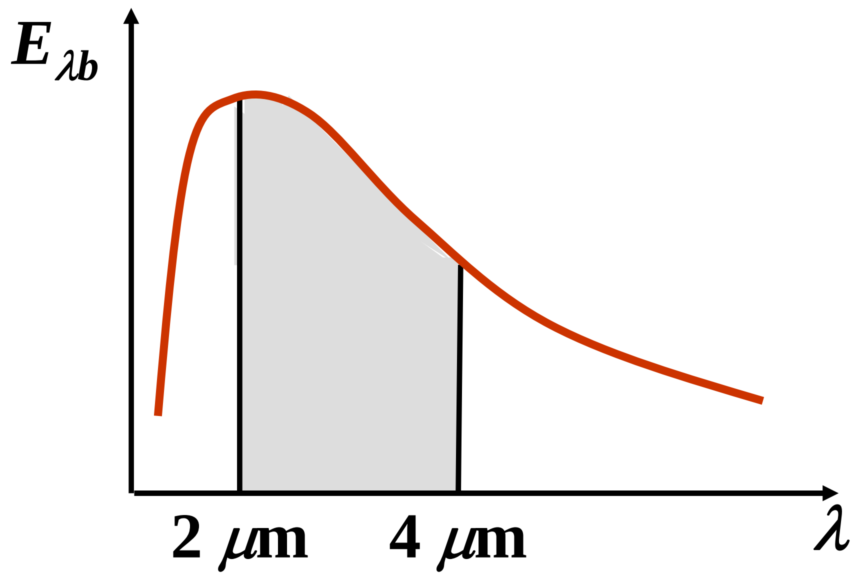
$$G = E_b(T) = 9.07\times10^5\,\mathrm{W/m^2}$$



Example 12.4



Blackbody at 1500 K



Find:

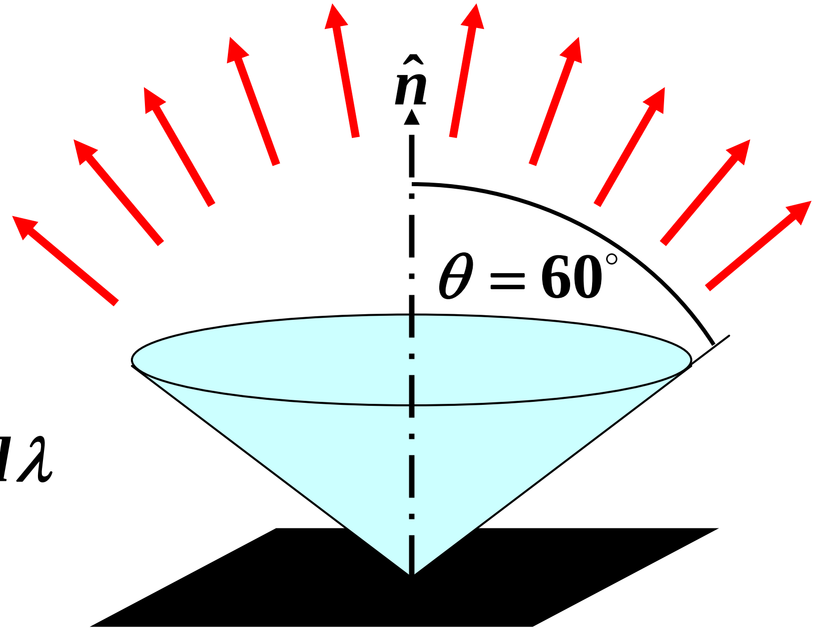
Rate of emission per unit area in directions $0^\circ \le \theta \le 60^\circ$, and in a wavelength interval $2\mu\text{m} \le \lambda \le 4\mu\text{m}$

$$0^\circ \le \theta \le 60^\circ, \quad 2\mu\text{m} \le \lambda \le 4\mu\text{m}$$

$$I_{\lambda b}\cos\theta\, d\omega\, d\lambda, \quad \int_{\omega} I_{\lambda b}\cos\theta\, d\omega\, d\lambda$$

$$\Delta E = \int_{\lambda_1}^{\lambda_2}\int_{\omega} I_{\lambda b}\cos\theta\, d\omega\, d\lambda$$

$$\Delta E = \int_2^4\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/3} I_{\lambda b}\cos\theta\sin\theta\, d\theta\, d\phi\, d\lambda$$



Blackbody at 1500 K

$I_{\lambda b}$: **independent of direction**

$$\Delta E = \int_2^4 I_{\lambda b}\left(\int_0^{2\pi}\int_0^{\pi/3}\cos\theta\sin\theta\, d\theta\, d\phi\right) d\lambda$$

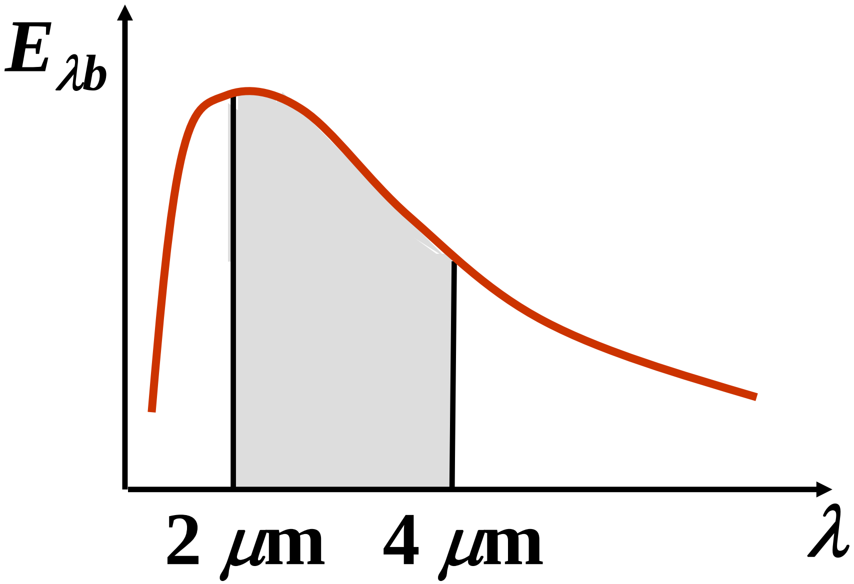
$$\Delta E = \int_2^4 I_{\lambda b}\left(2\pi \left.\frac{\sin^2\theta}{2}\right|_0^{\pi/3}\right) d\lambda = 0.75\int_2^4 \pi I_{\lambda b}\, d\lambda$$

$$= 0.75\int_2^4 E_{\lambda b}\, d\lambda$$

$$\Delta E = 0.75\int_2^4 E_{\lambda b} d\lambda$$

$$\Delta E = 0.75 E_b \int_2^4 \frac{E_{\lambda,b}}{E_b} d\lambda$$

$$\Delta E = 0.75 E_b \left[F_{(0\rightarrow 4\,\mu\text{m})} - F_{(0\rightarrow 2\,\mu\text{m})} \right]$$



From Table 12.1

$$\lambda_1 T = 2\mu\text{m} \times 1500\,\text{K} = 3000\,\mu\text{m}\cdot\text{K} \rightarrow F_{(0\rightarrow 2\,\mu\text{m})} = 0.273232$$

$$\lambda_2 T = 4\mu\text{m} \times 1500\,\text{K} = 6000\,\mu\text{m}\cdot\text{K} \rightarrow F_{(0\rightarrow 4\,\mu\text{m})} = 0.737818$$

$$E_b = \sigma T^4$$

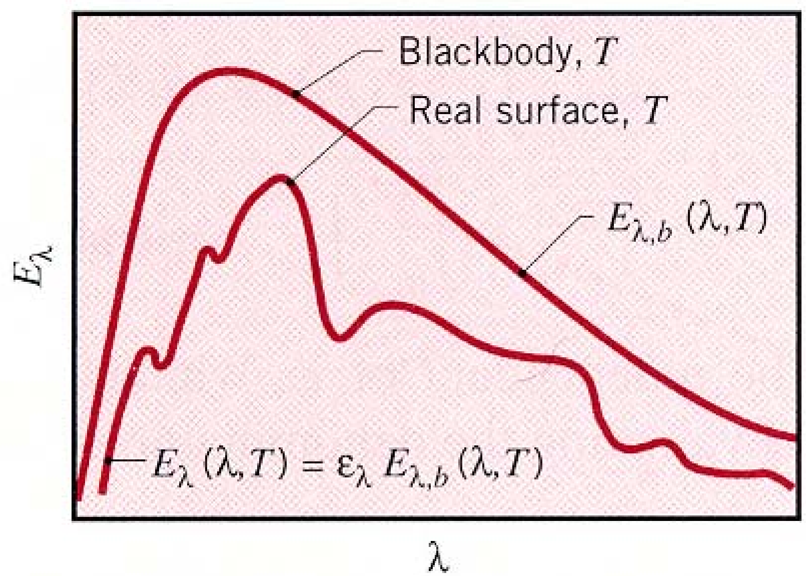
$$\Delta E = 0.75\sigma T^4 \left[F_{(0\rightarrow 4\,\mu\text{m})} - F_{(0\rightarrow 2\,\mu\text{m})} \right]$$

$$= 0.75 \times 5.67 \times 15^4 \times \left(0.737818 - 0.273232\right) = 10^5\,\text{W/m}^2$$

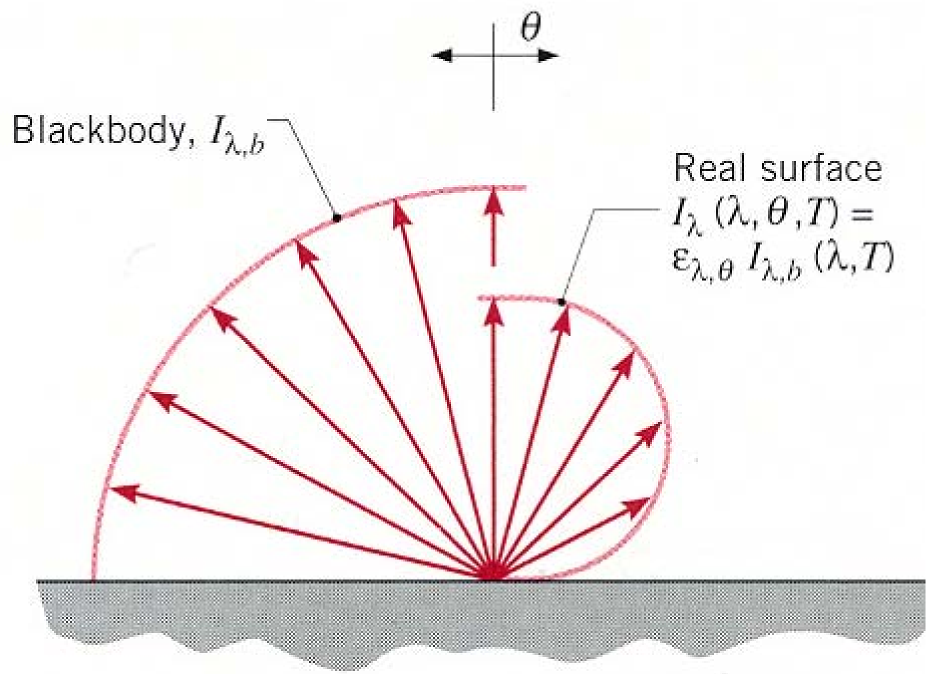
$$0.75 \times 0.4645686 = 0.3484395$$

Surface Radiation Properties

Definition of Properties for Non-black Opaque Surfaces



spectral distribution



directional distribution

- ity : intensive, theoretical
- ance : extensive, experimental

Emissivity

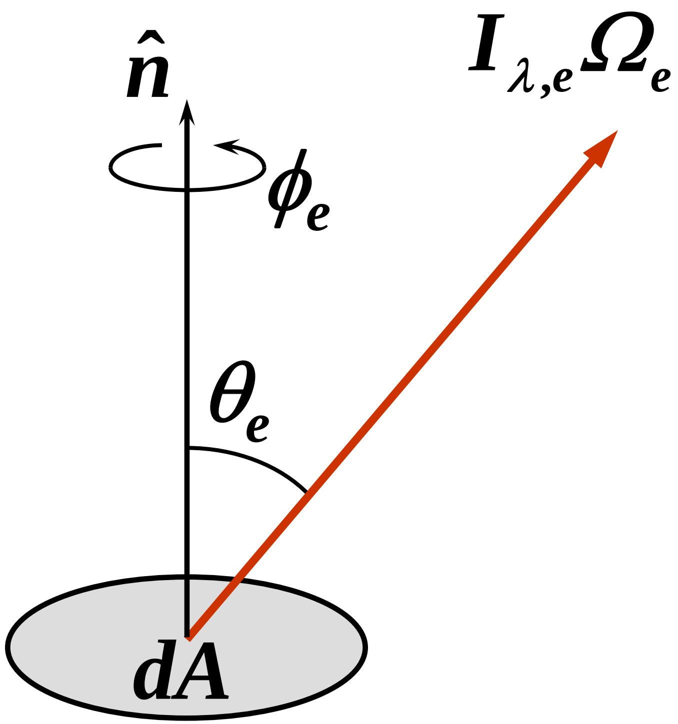
Directional spectral emissivity $\varepsilon'_\lambda(\lambda,\theta_e,\phi_e,T)$

$$\varepsilon'_\lambda = \frac{I_{\lambda,e}\cos\theta_e}{I_{\lambda b,e}\cos\theta_e}$$

$$= \frac{E'_\lambda}{E'_{\lambda b}} = \frac{I_{\lambda,e}}{I_{\lambda b,e}}$$

$$I_{\lambda,e} = \varepsilon'_\lambda I_{\lambda b,e}$$

$$E'_\lambda = \varepsilon'_\lambda E'_{\lambda b}$$



Directional total emissivity $\varepsilon'(\theta_e, \phi_e, T)$

$$\varepsilon' = \frac{\int_0^\infty I_{\lambda,e} \cos\theta_e d\lambda}{\int_0^\infty I_{\lambda b,e} \cos\theta_e d\lambda} = \frac{\int_0^\infty I_{\lambda,e} d\lambda}{\int_0^\infty I_{\lambda b,e} d\lambda} = \frac{I_e}{I_{b,e}} = \frac{E'}{E'_b}$$

$$I_e = \varepsilon' I_{b,e}, \quad E' = \varepsilon' E'_b$$

or $I_{\lambda,e} = \varepsilon'_\lambda I_{\lambda b,e}$

$$\varepsilon' = \frac{\int_0^\infty \varepsilon'_\lambda I_{\lambda b,e} d\lambda}{\int_0^\infty I_{\lambda b,e} d\lambda} \qquad \left(E_b = \pi I_{b,e} = \sigma T^4\right)$$

$$= \frac{\pi \int_0^\infty \varepsilon'_\lambda I_{\lambda b,e} d\lambda}{\sigma T^4} = \frac{\int_0^\infty \varepsilon'_\lambda E_{\lambda b} d\lambda}{\sigma T^4}$$

Hemispherical spectral emissivity $\varepsilon_\lambda(\lambda,T)$

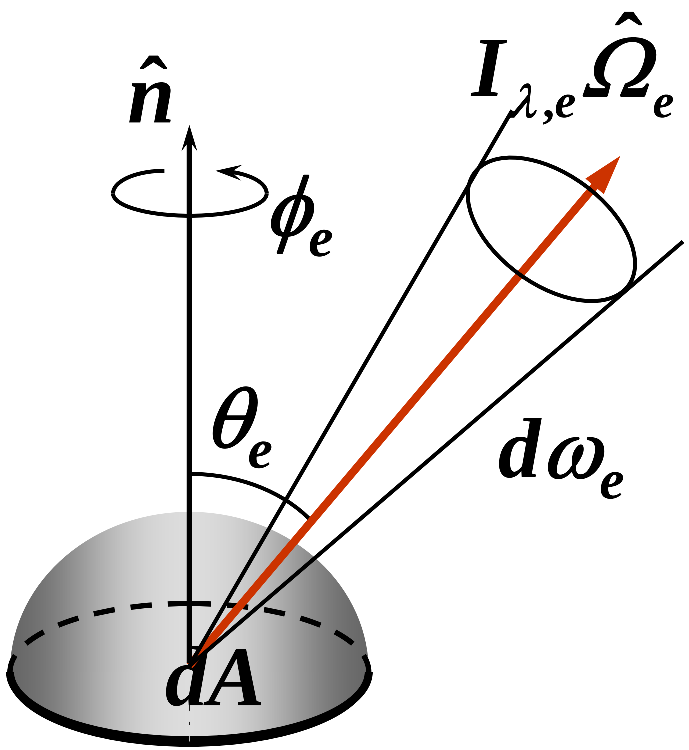
$$\varepsilon_\lambda = \frac{\int_\cap I_{\lambda,e}\cos\theta_e d\omega_e}{\int_\cap I_{\lambda b,e}\cos\theta_e d\omega_e}$$

$$= \frac{\int_\cap I_{\lambda,e}\cos\theta_e d\omega_e}{\pi I_{\lambda b,e}}$$

$$\left(I_{\lambda,e} = \varepsilon'_\lambda I_{\lambda b,e}\right)$$

$$= \frac{\int_\cap \varepsilon'_\lambda I_{\lambda b,e}\cos\theta_e d\omega_e}{\pi I_{\lambda b,e}}$$

$$= \frac{1}{\pi}\int_\cap \varepsilon'_\lambda \cos\theta_e d\omega_e$$



Hemispherical total emissivity $\varepsilon(T)$

$$\varepsilon = \frac{\int_0^\infty \int_\cap I_{\lambda,e} \cos\theta_e \, d\omega_e \, d\lambda}{\int_0^\infty \int_\cap I_{\lambda b,e} \cos\theta_e \, d\omega_e \, d\lambda} = \frac{E}{E_b}$$

$$= \frac{\int_0^\infty \left[\int_\cap \varepsilon'_\lambda I_{\lambda b,e} \cos\theta_e \, d\omega_e \right] d\lambda}{\sigma T^4} \qquad \left(I_{\lambda,e} = \varepsilon'_\lambda I_{\lambda b,e} \right)$$

$$= \frac{\int_0^\infty \pi I_{\lambda b,e} \left[\frac{1}{\pi} \int_\cap \varepsilon'_\lambda \cos\theta_e \, d\omega_e \right] d\lambda}{\sigma T^4}$$

$$= \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda b} \, d\lambda}{\sigma T^4}$$

or $$\varepsilon = \frac{\int_{\cap}\int_0^{\infty} I_{\lambda,e}\cos\theta_e d\lambda d\omega_e}{\int_{\cap}\int_0^{\infty} I_{\lambda b,e}\cos\theta_e d\lambda d\omega_e} \qquad \left(I_{\lambda,e} = \varepsilon'_\lambda I_{\lambda b,e}\right)$$

$$= \frac{\int_{\cap}\cos\theta_e\left[\int_0^{\infty}\varepsilon'_\lambda I_{\lambda b,e} d\lambda\right]d\omega_e}{\sigma T^4}$$

$$= \frac{1}{\pi}\frac{\int_{\cap}\cos\theta_e\left[\int_0^{\infty}\varepsilon'_\lambda \pi I_{\lambda b} d\lambda\right]d\omega_e}{\sigma T^4}$$

$$= \frac{1}{\pi}\int_{\cap}\cos\theta_e\left[\frac{\int_0^{\infty}\varepsilon'_\lambda \pi I_{\lambda b} d\lambda}{\sigma T^4}\right]d\omega_e$$

$$= \frac{1}{\pi}\int_{\cap}\varepsilon'\cos\theta_e d\omega_e$$

Summary

Directional spectral emissivity $\varepsilon'_\lambda(\lambda,\theta_e,\phi_e,T)$

$$\varepsilon'_\lambda = \frac{I_{\lambda,e}}{I_{\lambda b,e}} = \frac{E'_\lambda}{E'_{\lambda b}} \quad \rightarrow I_{\lambda,e} = \varepsilon'_\lambda I_{\lambda b,e},\; E'_\lambda = \varepsilon'_\lambda E'_{\lambda b}$$

Directional total emissivity $\varepsilon'(\theta_e,\phi_e,T)$

$$\varepsilon' = \frac{\int_0^\infty \varepsilon'_\lambda E_{\lambda b} d\lambda}{\sigma T^4}$$

Hemispherical spectral emissivity $\varepsilon_\lambda(\lambda,T)$

$$\varepsilon_\lambda = \frac{1}{\pi}\int_\cap \varepsilon'_\lambda \cos\theta_e d\omega_e$$

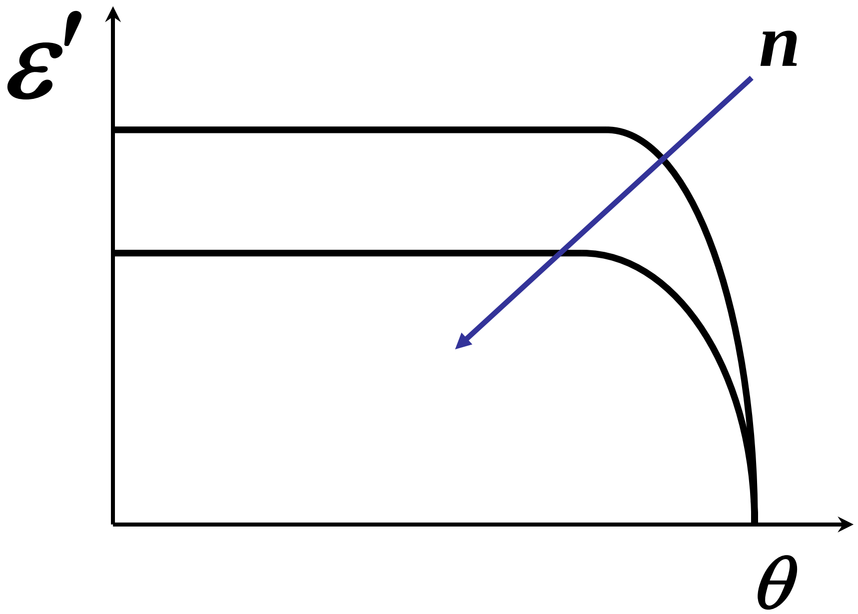
Hemispherical total emissivity $\varepsilon(T)$

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda b} d\lambda}{\sigma T^4} \quad \text{or} \quad \varepsilon = \frac{1}{\pi}\int_\cap \varepsilon' \cos\theta_e d\omega_e$$

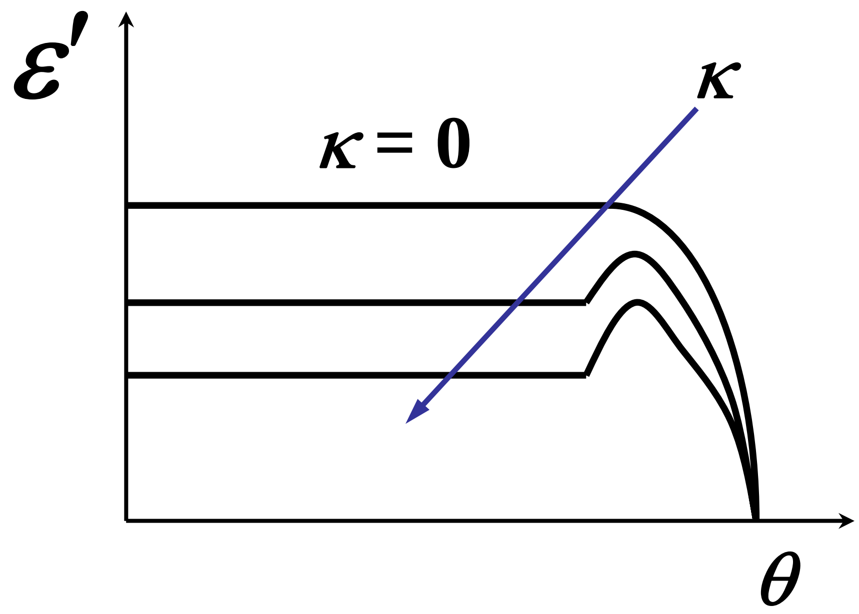
Theoretical trend: Maxwell's equation

directional dependence

1) Dielectrics



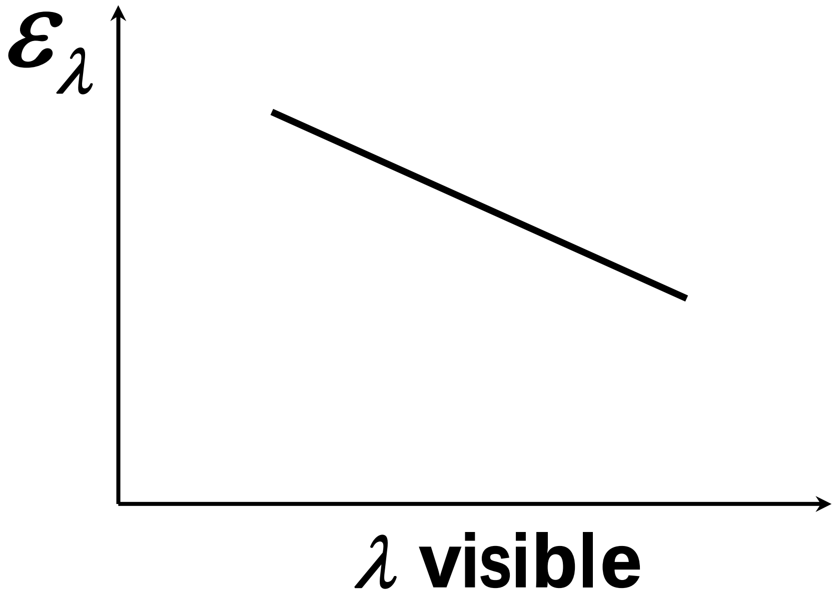
2) Conductors



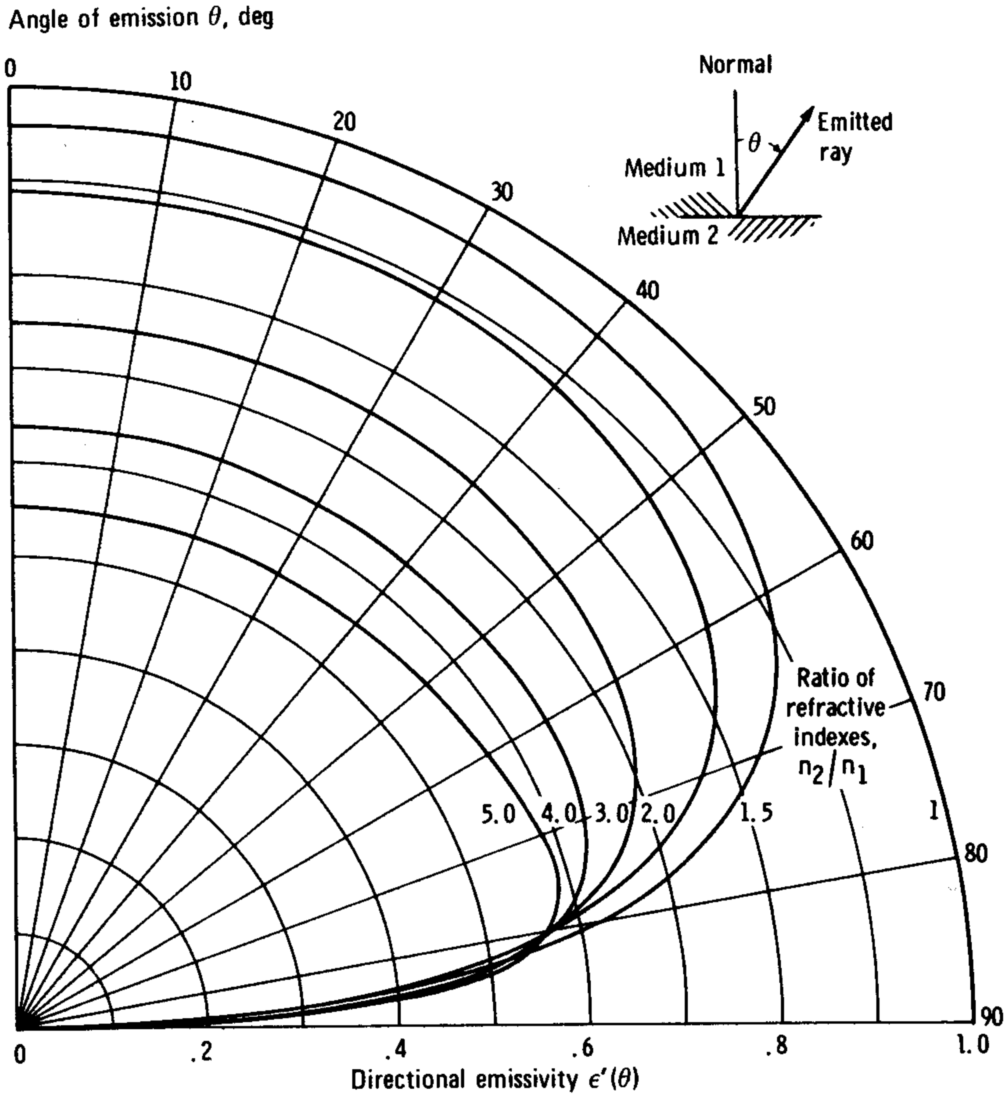
spectral dependence

1) Dielectrics

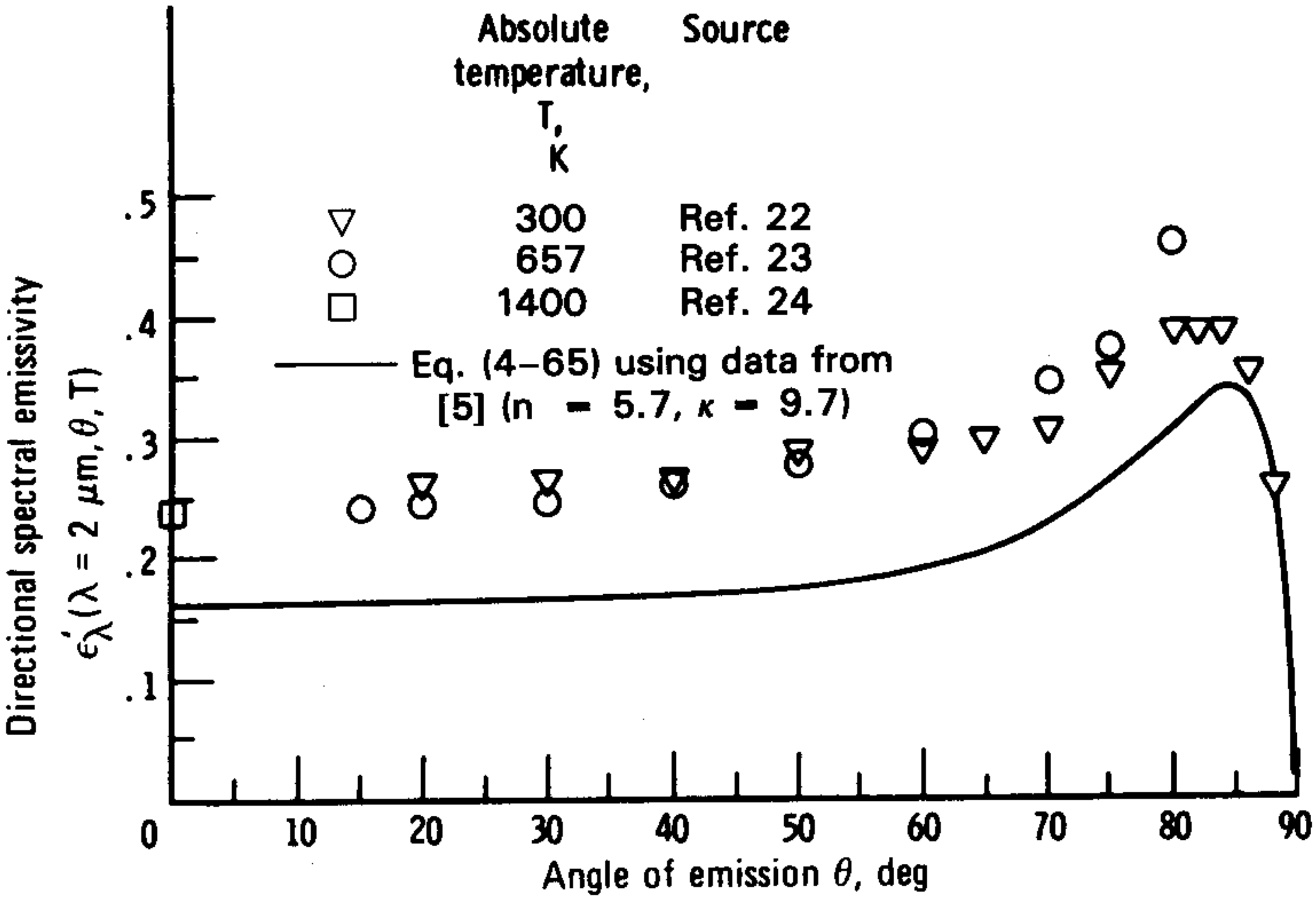
2) Conductors



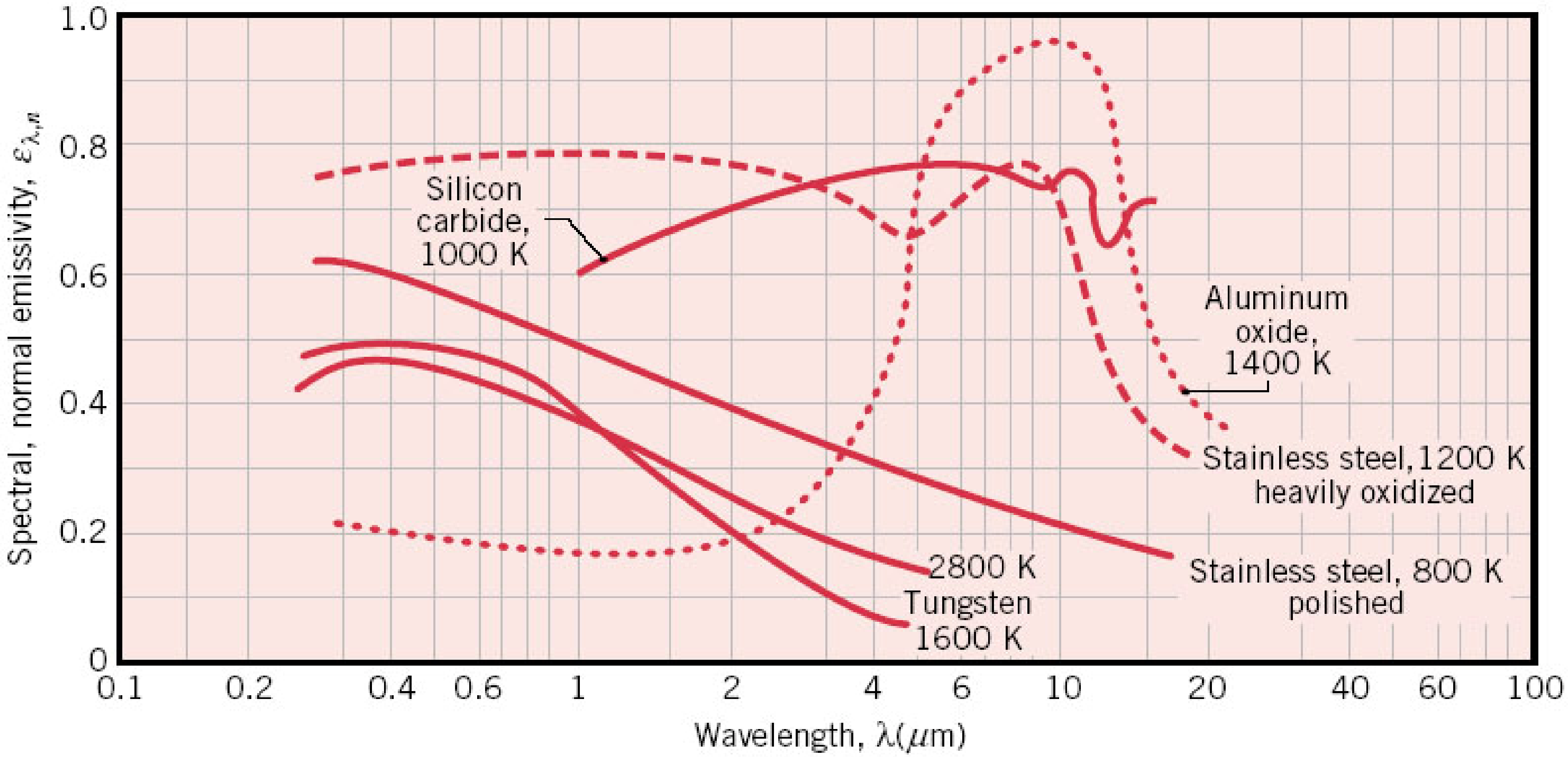
Directional emissivity of ideal dielectrics predicted by EM theory



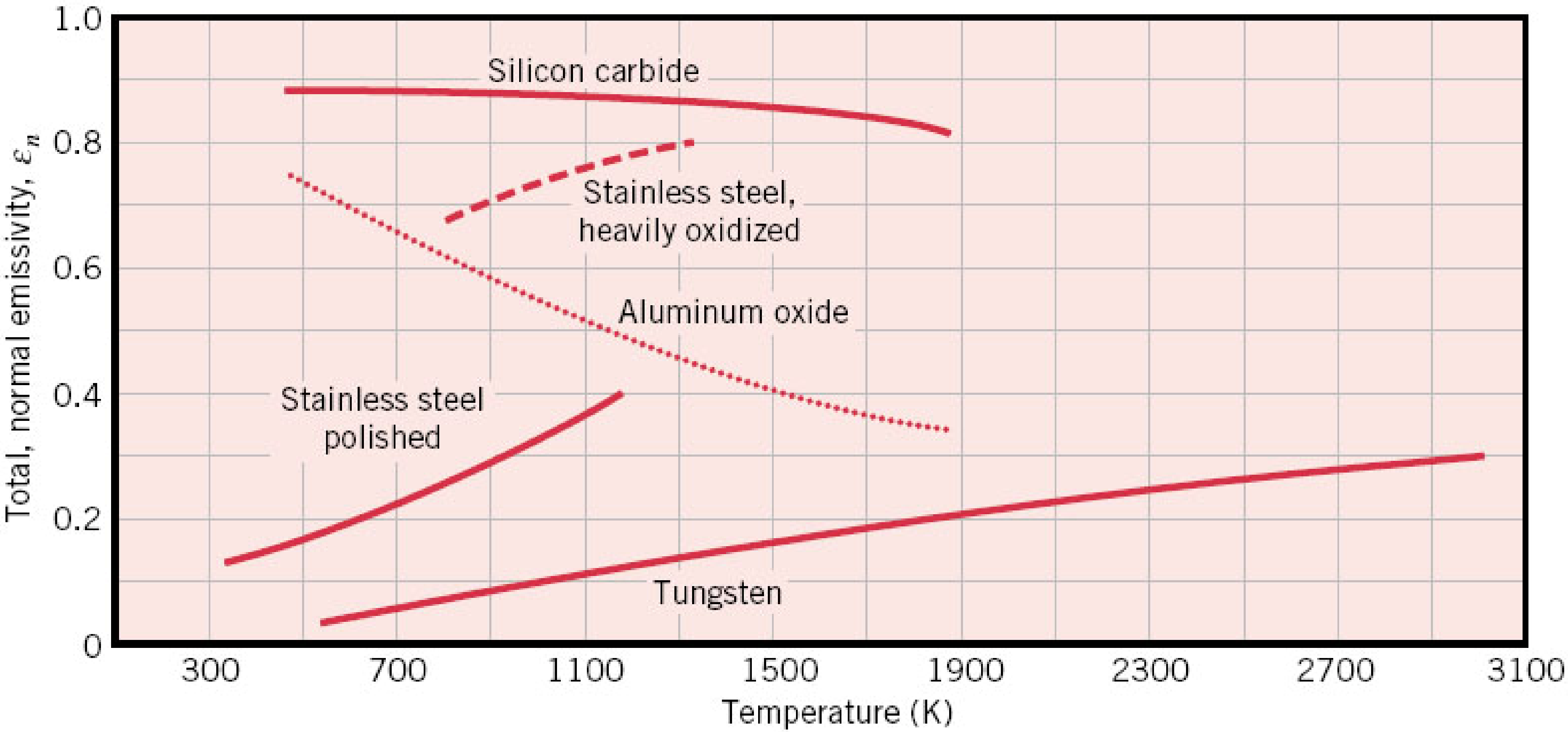
Directional spectral emissivity of platinum at wavelength $\lambda = 2\ \mu m$



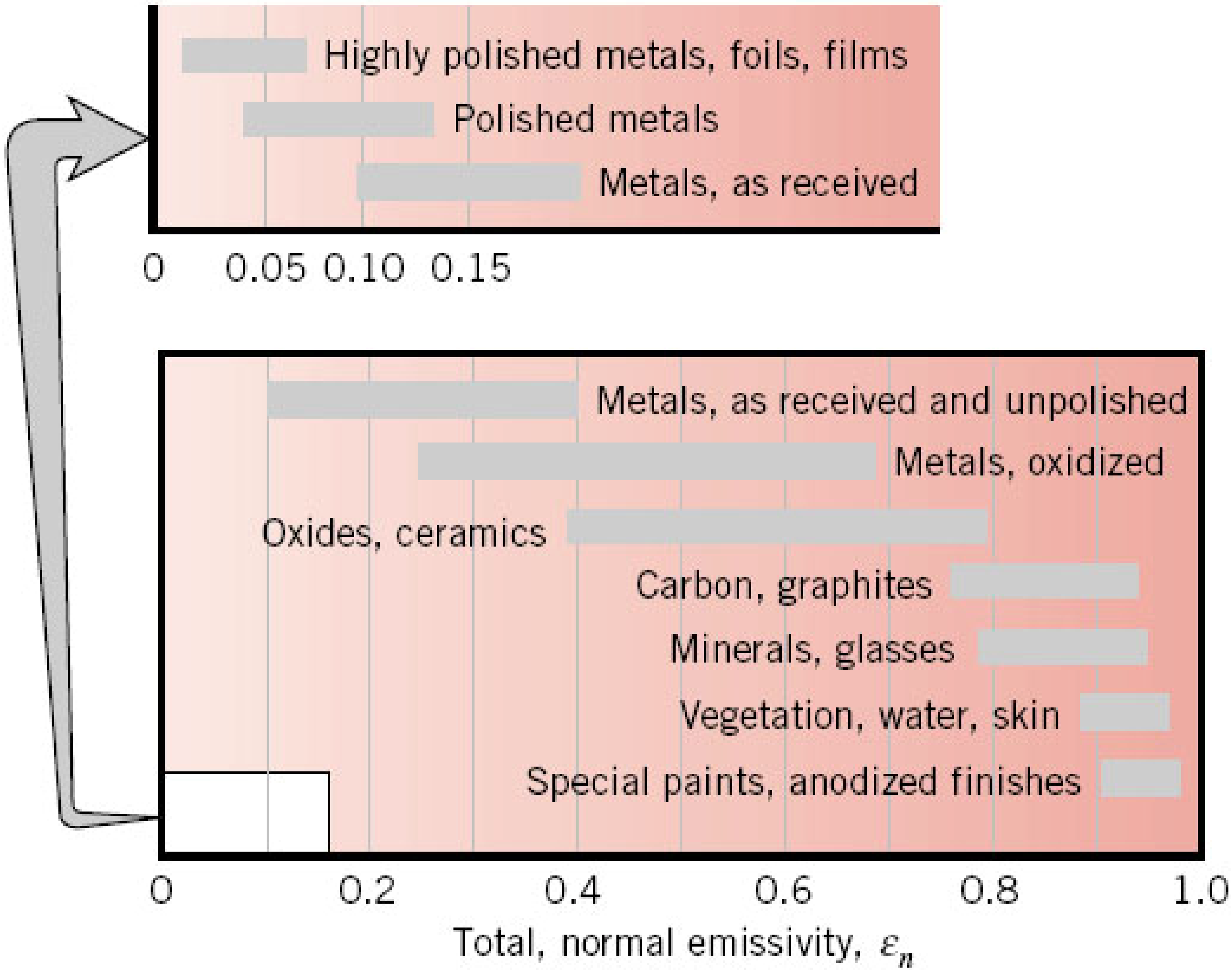
Spectral dependence of the spectral, normal emissivity $\varepsilon_{\lambda,n}$ of selected materials



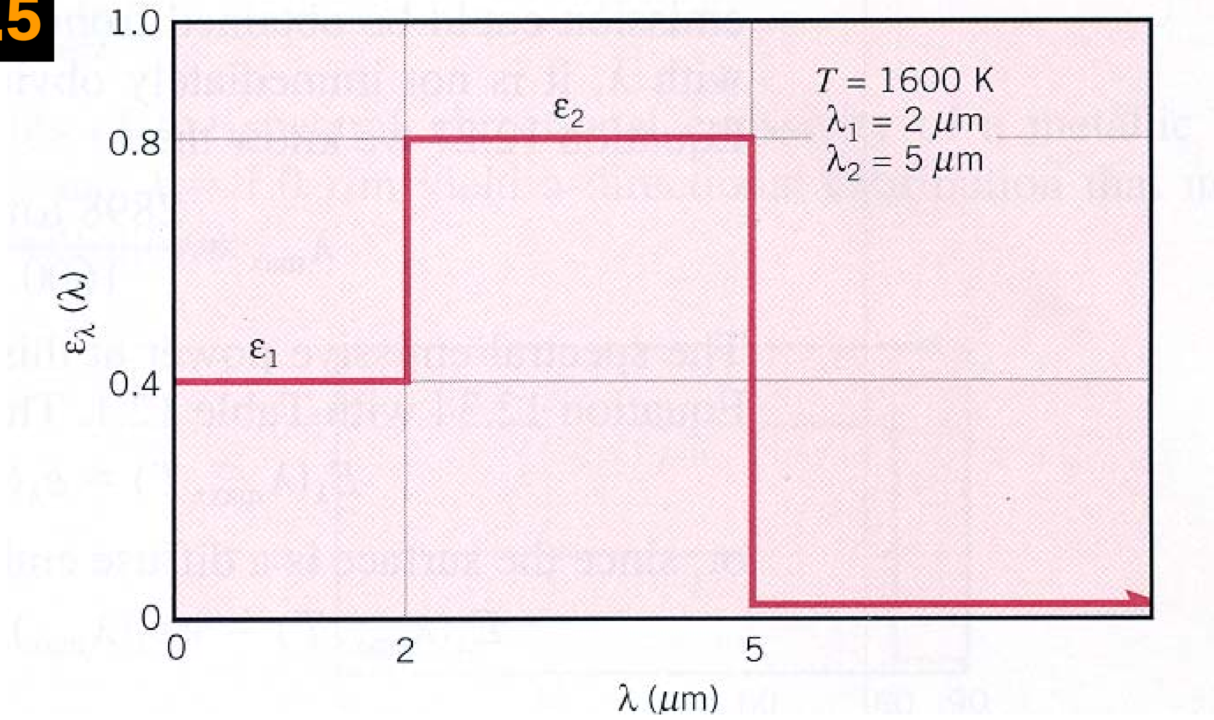
Temperature dependence of the total, normal emissivity ε_n of selected materials



Representative values of total, normal emissivity ε_n



Example 12.5



Find:

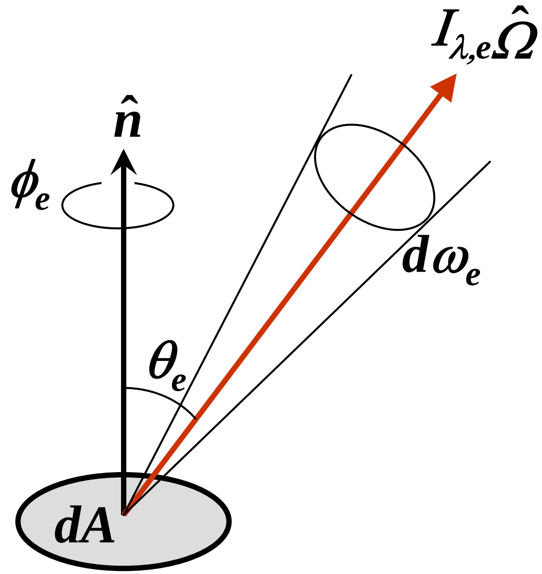
1) **Hemispherical total emissivity, ε**
2) **Total emissive power, E**
3) **Wavelength at which spectral emissive power will be a max**

Assumption:

Surface is a diffuse emitter.

1) Hemispherical total emissivity

$$\varepsilon'_\lambda = \frac{I_{\lambda,e}\cos\theta_e}{I_{\lambda b,e}\cos\theta_e} = \frac{I_{\lambda,e}}{I_{\lambda b,e}}$$

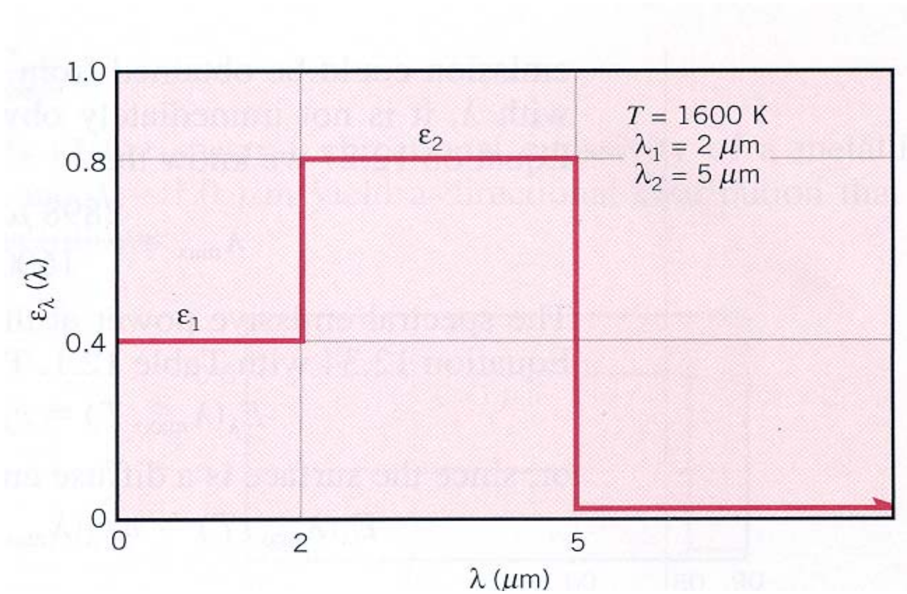


$$\varepsilon_\lambda = \frac{\int_\cap I_{\lambda,e}\cos\theta_e d\omega_e}{\int_\cap I_{\lambda b,e}\cos\theta_e d\omega_e} = \frac{\int_\cap \varepsilon'_\lambda I_{\lambda b,e}\cos\theta_e d\omega_e}{E_{\lambda b}}$$

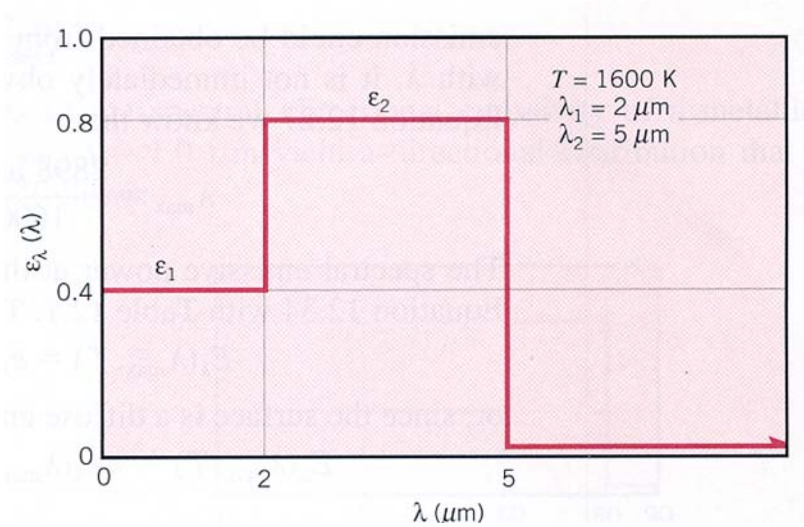
$$\varepsilon = \frac{\int_0^\infty \left(\frac{1}{\pi}\int_\cap \varepsilon'_\lambda E_{\lambda b}\cos\theta_e d\omega_e\right) d\lambda}{\int_0^\infty E_{\lambda b} d\lambda}$$

$$= \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda b} d\lambda}{\sigma T^4}$$

$$= \frac{\varepsilon_1 \int_0^2 E_{\lambda,b} d\lambda}{\sigma T^4} + \frac{\varepsilon_2 \int_2^5 E_{\lambda,b} d\lambda}{\sigma T^4}$$



$$\varepsilon = \frac{\varepsilon_1 \int_0^2 E_{\lambda,b} d\lambda}{\sigma T^4} + \frac{\varepsilon_2 \int_2^5 E_{\lambda,b} d\lambda}{\sigma T^4}$$



$$\varepsilon = \varepsilon_1 F_{(0\to 2\mu\mathrm{m})} + \varepsilon_2 \left[F_{(0\to 5\mu\mathrm{m})} - F_{(0\to 2\mu\mathrm{m})} \right]$$

From Table 12.1

$$\lambda_1 T = 2\mu\mathrm{m} \times 1600\,\mathrm{K} = 3200\,\mu\mathrm{m}\cdot\mathrm{K} \quad \to F_{(0\to 2\mu\mathrm{m})} = 0.318102$$

$$\lambda_2 T = 5\mu\mathrm{m} \times 1600\,\mathrm{K} = 8000\,\mu\mathrm{m}\cdot\mathrm{K} \quad \to F_{(0\to 5\mu\mathrm{m})} = 0.856288$$

$$\varepsilon = 0.4 \times 0.318102 + 0.8 \times (0.856282 - 0.318102)$$

$$= 0.4 \times 0.318102 + 0.8 \times 0.53818 = 0.558$$

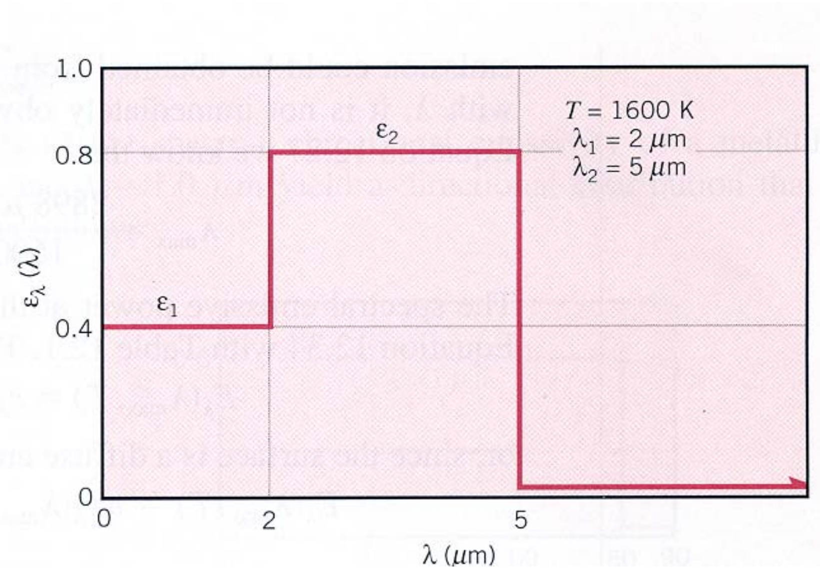
2) Total emissive power

$$E = \varepsilon E_b = \varepsilon \sigma T^4 = 0.558 \times 5.67 \times 16^4 = 207\ \mathrm{kW/m^2}$$

3) Wavelength at which spectral emissive power will be a max

$$\varepsilon_\lambda = \frac{\int_\cap I_{\lambda,e}\cos\theta_e d\omega_e}{\int_\cap I_{\lambda b,e}\cos\theta_e d\omega_e} = \frac{E_\lambda}{E_{\lambda b}}$$

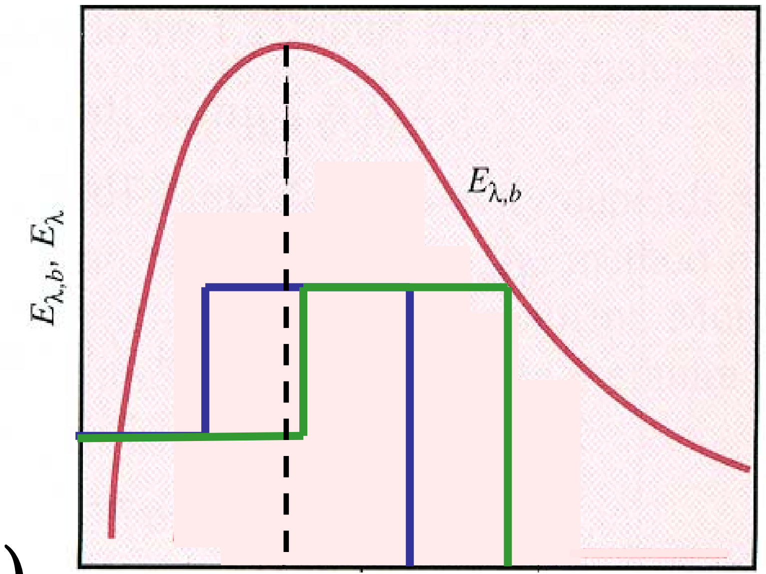
$$E_\lambda = \varepsilon_\lambda E_{\lambda b}$$



Maximum E_λ may occur

in $0 < \lambda \le 2\ \mu m$ or $2 < \lambda \le 5\ \mu m$.

First check where maximum $E_{\lambda b}$ occurs.



$\lambda = 1.81\ \mu m$

From Wien's law

$$\lambda_{max} = \frac{2898\,\mu m \cdot K}{1600K} = 1.81\mu m\,(< 2\mu m)$$

Thus, maximum may occur at $\lambda = 1.81\ \mu m$ or $\lambda = 2\ \mu m$

$$E_\lambda = \varepsilon_\lambda E_{\lambda b} = \varepsilon_\lambda \left(\pi I_{\lambda b}\right) = \pi\varepsilon_\lambda \left(\frac{I_{\lambda b}}{\sigma T^5}\right)\sigma T^5$$

at $\lambda = 1.81\ \mu\text{m}$

From Table 12.1, $\lambda T = 2898\ \mu\text{m}\cdot\text{K} \rightarrow \dfrac{I_{\lambda b}}{\sigma T^5} = 0.722\times 10^{-4}$

$$E_\lambda(1.81\ \mu\text{m}, 1600\ \text{K}) = \pi\times 0.4\times 0.722\times 10^{-4}\times 5.67\times 10^{-8}\times 1600^5$$

$$= 54\ \text{kW/m}^2\cdot\mu\text{m}$$

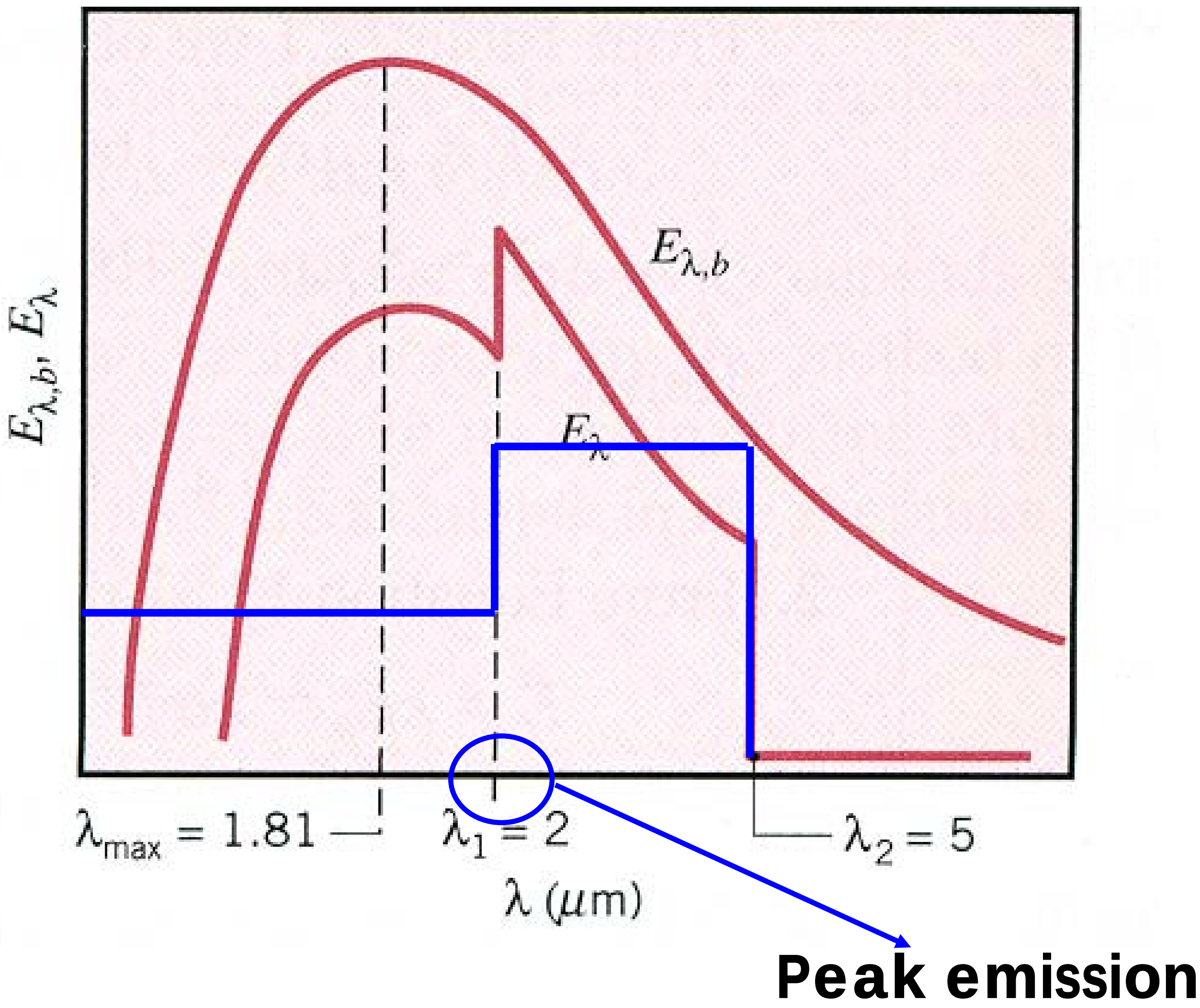
at $\lambda = 2\ \mu\text{m}$

From Table 12.1, $\lambda T = 3200\ \mu\text{m}\cdot\text{K} \rightarrow \dfrac{I_{\lambda b}}{\sigma T^5} = 0.706\times 10^{-4}$

$$E_\lambda(2\ \mu\text{m}, 1600\ \text{K}) = \pi\times 0.8\times 0.706\times 10^{-4}\times 5.67\times 10^{-8}\times 1600^5$$

$$= 105.5\ \text{kW/m}^2\cdot\mu\text{m}$$

Maximum spectral emissive power occurs at $\lambda = 2\ \mu\text{m}$**.**

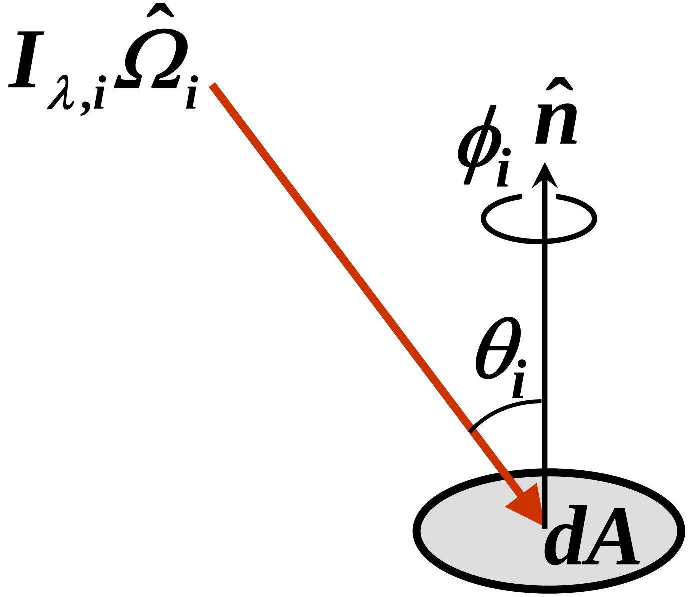


$E_{\lambda,b}$, E_λ
$E_{\lambda,b}$
E_λ
$\lambda_{max} = 1.81$
$\lambda_1 = 2$
$\lambda_2 = 5$
λ (μm)
Peak emission

Absorptivity

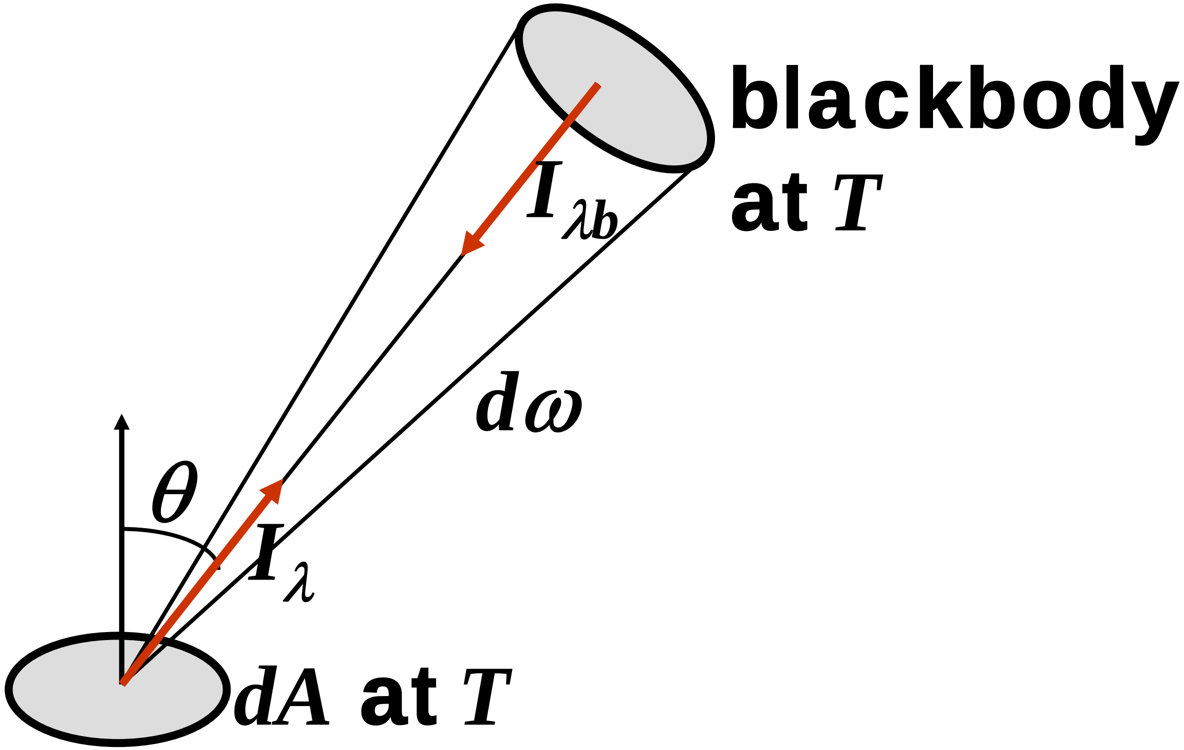
dependence on the directional and spectral distributions of the incident radiation, thus not a material property except α'_λ

Directional spectral absorptivity $\alpha'_\lambda(\lambda, \theta_i, \phi_i, T)$



$$\alpha'_\lambda = \frac{\text{absorbed energy at } \lambda \text{ and } (\theta_i, \phi_i)}{I_{\lambda,i}\cos\theta_i}$$

Kirchhoff's law



absorbed energy $= \alpha'_\lambda I_{\lambda b} dA \cos\theta d\omega d\lambda$

emitted energy $= \varepsilon'_\lambda I_{\lambda b} dA \cos\theta d\omega d\lambda$

in equilibrium $\alpha'_\lambda(\lambda,\theta,\phi,T) = \varepsilon'_\lambda(\lambda,\theta,\phi,T)$

: no restriction

Directional total absorptivity $\alpha'(\theta_i,\phi_i,T)$

absorbed energy at λ and $(\theta_i, \phi_i) = \alpha'_\lambda I_{\lambda,i}\cos\theta_i$

$$\alpha' = \frac{\int_0^\infty \alpha'_\lambda I_{\lambda,i}\cos\theta_i d\lambda}{\int_0^\infty I_{\lambda,i}\cos\theta_i d\lambda} = \frac{\int_0^\infty \alpha'_\lambda I_{\lambda,i} d\lambda}{\int_0^\infty I_{\lambda,i} d\lambda} = \frac{\int_0^\infty \varepsilon'_\lambda I_{\lambda,i} d\lambda}{\int_0^\infty I_{\lambda,i} d\lambda}$$

$$\varepsilon' = \frac{\int_0^\infty \varepsilon'_\lambda E_{\lambda b} d\lambda}{\sigma T^4} = \frac{\int_0^\infty \varepsilon'_\lambda I_{\lambda b} d\lambda}{\int_0^\infty I_{\lambda b} d\lambda}$$

i) when $I_{\lambda,i}(\lambda,\theta_i,\phi_i) = C(\theta_i,\phi_i) I_{\lambda b,i}(\lambda,T)$

$$\rightarrow \alpha' = \varepsilon'$$

ii) when ε'_λ **not function of** $\lambda \rightarrow \varepsilon'_\lambda = \varepsilon'$

$$\rightarrow \alpha' = \varepsilon'$$ **directional-gray surface**

Hemispherical spectral absorptivity $\alpha_\lambda(\lambda, T)$

$$\alpha_\lambda = \frac{\int_\cap \alpha'_\lambda I_{\lambda,i} \cos\theta_i d\omega_i}{\int_\cap I_{\lambda,i} \cos\theta_i d\omega_i} = \frac{\int_\cap \varepsilon'_\lambda I_{\lambda,i} \cos\theta_i d\omega_i}{\int_\cap I_{\lambda,i} \cos\theta_i d\omega_i}$$

$$= \frac{\int_\cap \varepsilon'_\lambda I_{\lambda,i} \cos\theta_i d\omega_i}{G_\lambda}$$

$$\varepsilon_\lambda = \frac{1}{\pi}\int_\cap \varepsilon'_\lambda \cos\theta_e d\omega_e$$

i) when $I_{\lambda,i}(\lambda, \theta_i, \phi_i) = I_{\lambda,i}(\lambda)$ only:

diffuse irradiation $\alpha_\lambda = \varepsilon_\lambda$

ii) when ε'_λ independent of direction $\varepsilon'_\lambda = \varepsilon_\lambda$

$\rightarrow \alpha_\lambda = \varepsilon_\lambda$ **diffuse-spectral surface**

Hemispherical total absorptivity $\alpha(T)$

$$\alpha = \frac{\int_0^\infty \int_\cap \alpha'_\lambda I_{\lambda,i} \cos\theta_i d\omega_i d\lambda}{\int_0^\infty \int_\cap I_{\lambda,i} \cos\theta_i d\omega_i d\lambda} \quad \left(\alpha_\lambda = \frac{\int_\cap \alpha'_\lambda I_{\lambda,i} \cos\theta_i d\omega_i}{\int_\cap I_{\lambda,i} \cos\theta_i d\omega_i} \right)$$

$$= \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{G}$$

$$\alpha = \frac{\int_0^\infty \int_\cap \varepsilon'_\lambda I_{\lambda,i} \cos\theta_i d\omega_i d\lambda}{\int_0^\infty \int_\cap I_{\lambda,i} \cos\theta_i d\omega_i d\lambda}, \quad \varepsilon = \frac{\int_0^\infty \int_\cap \varepsilon'_\lambda I_{\lambda b} \cos\theta_e d\omega_e d\lambda}{\int_0^\infty \int_\cap I_{\lambda b} \cos\theta_e d\omega_e d\lambda}$$

i) when $\varepsilon'_\lambda(\lambda, \theta_e, \phi_e, T) = \varepsilon(T) \rightarrow \alpha = \varepsilon$

: diffuse-gray surface

ii) when $I_{\lambda,i}(\lambda, \theta_i, \phi_i) = C I_{\lambda b,i}(\lambda, T) \rightarrow \alpha = \varepsilon$

iii) when $\varepsilon'_\lambda(\lambda, \theta_e, \phi_e, T) = \varepsilon'(\theta_e, \phi_e, T)$ **and** $I_{\lambda,i}(\lambda, \theta_i, \phi_i) = I_{\lambda,i}(\lambda)$

iv) when $\varepsilon'_\lambda(\lambda, \theta_e, \phi_e, T) = \varepsilon_\lambda(\lambda, T)$ **and** $I_{\lambda,i}(\lambda, \theta_i, \phi_i) = C(\theta_i, \phi_i) I_{\lambda b,i}(\lambda, T)$

Relations among reflectivity, absorptivity, and emissivity

a) $\alpha'_{\lambda}(\lambda,\theta,\phi,T)+\rho'_{\lambda}(\lambda,\theta,\phi,T)=1$

Kirchhoff's law $\alpha'_{\lambda}(\lambda,\theta,\phi,T)=\varepsilon'_{\lambda}(\lambda,\theta,\phi,T)$

$$\varepsilon'_{\lambda}(\lambda,\theta,\phi,T)+\rho'_{\lambda}(\lambda,\theta,\phi,T)=1$$

b) $\alpha'(\theta,\phi,T)+\rho'(\theta,\phi,T)=1$

for a directional-gray surface,

$$\alpha'(\theta,\phi,T)=\varepsilon'(\theta,\phi,T)$$

$$\varepsilon'(\theta,\phi,T)+\rho'(\theta,\phi,T)=1$$

c) $\alpha_\lambda(\lambda,T)+\rho_\lambda(\lambda,T)=1$

for a diffuse-spectral surface,

$$\alpha_\lambda(\lambda,T)=\varepsilon_\lambda(\lambda,T)$$

$$\varepsilon_\lambda(\lambda,T)+\rho_\lambda(\lambda,T)=1$$

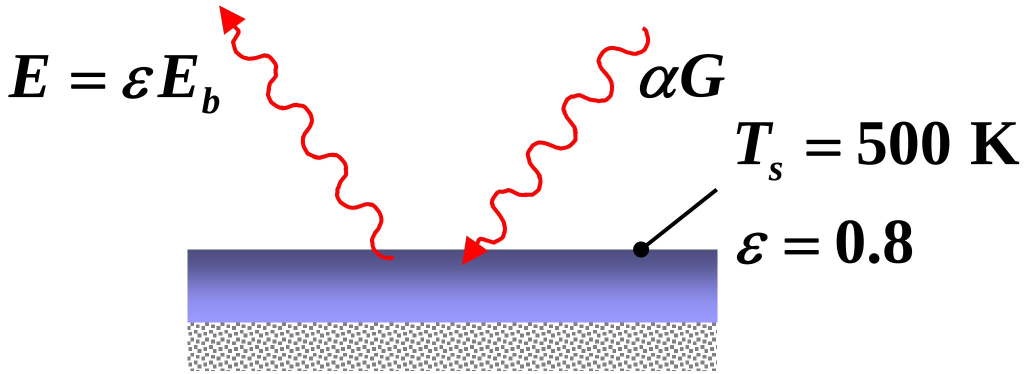
d) $\alpha(T)+\rho(T)=1$

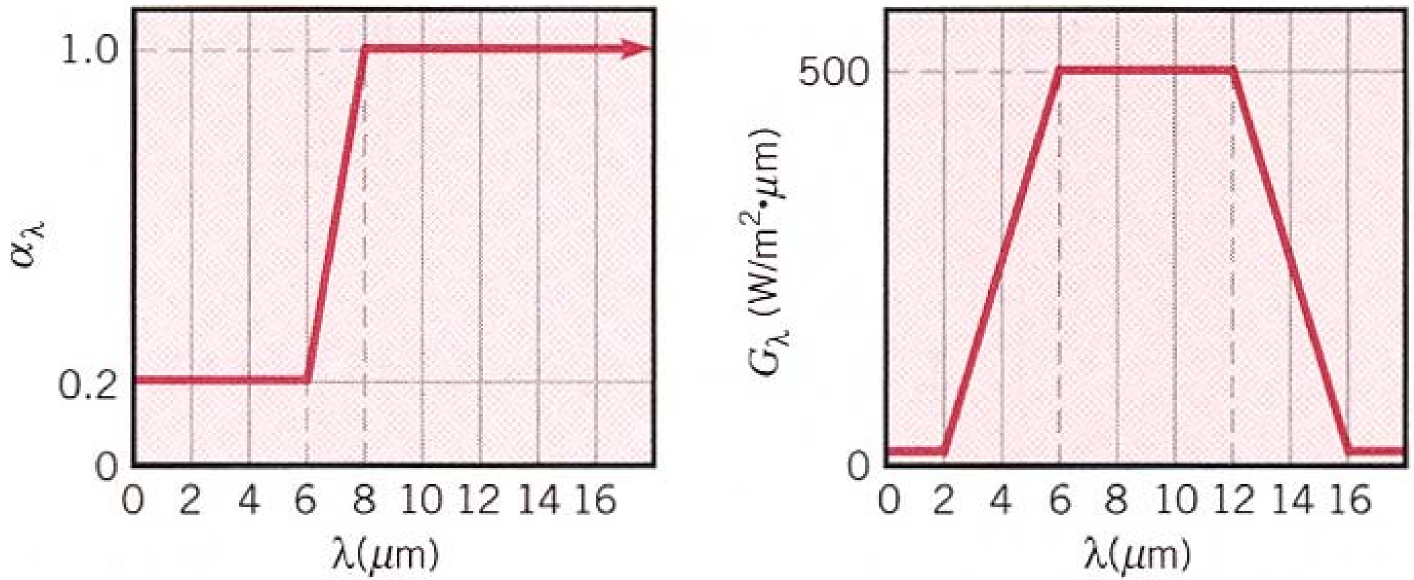
for a diffuse-gray surface,

$$\alpha(T)=\varepsilon(T)$$

$$\varepsilon(T)+\rho(T)=1$$

Example 12.7



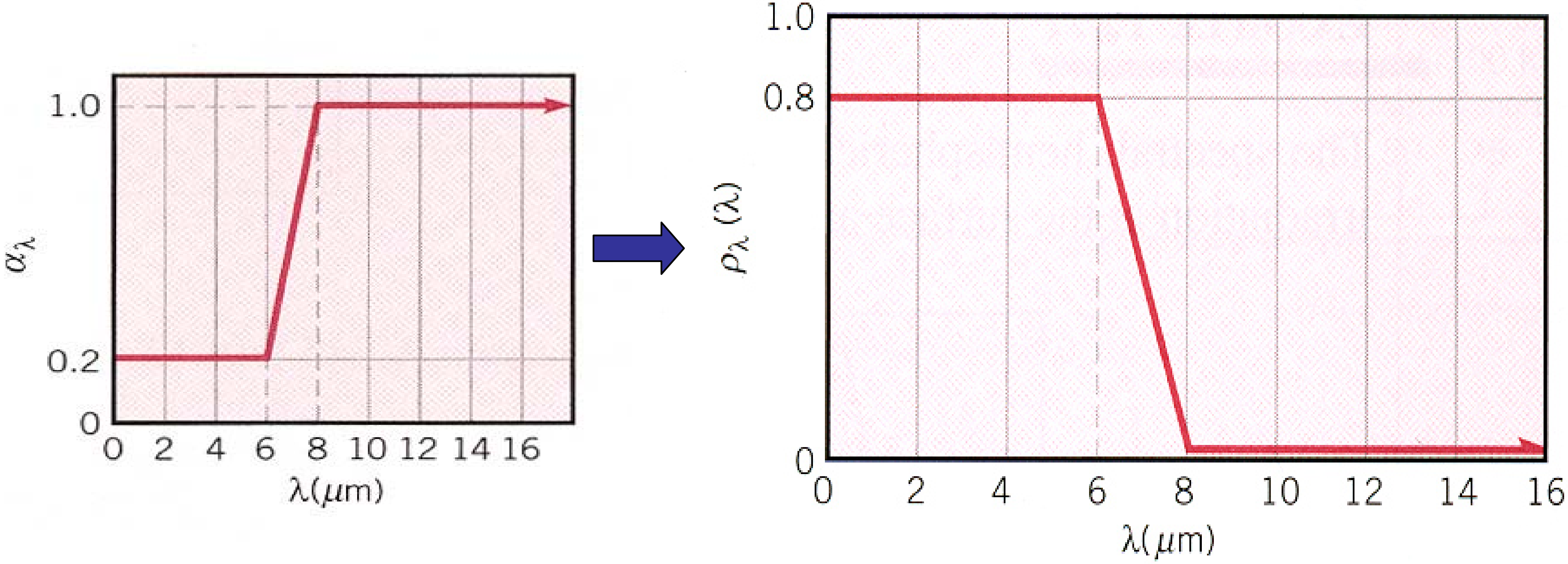


Find:

1) **Spectral distribution of reflectivity**
2) **Hemispherical total absorptivity**
3) **Nature of surface temperature change**

1) Spectral reflectivity

$$\rho_\lambda = 1 - \alpha_\lambda$$

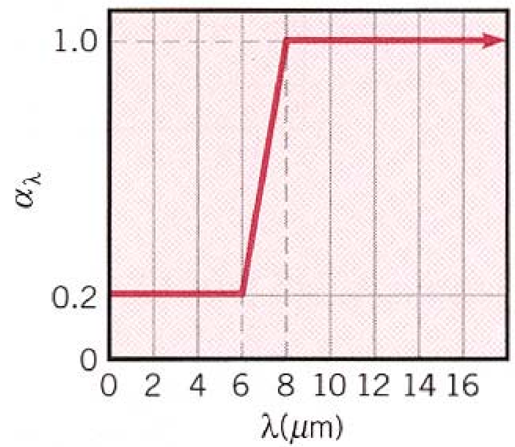


2) Hemispherical total absorptivity

incident radiation in Ω at λ: $I_{\lambda,i}\cos\theta_i$

absorbed energy: $\alpha'_\lambda I_{\lambda,i}\cos\theta_i$

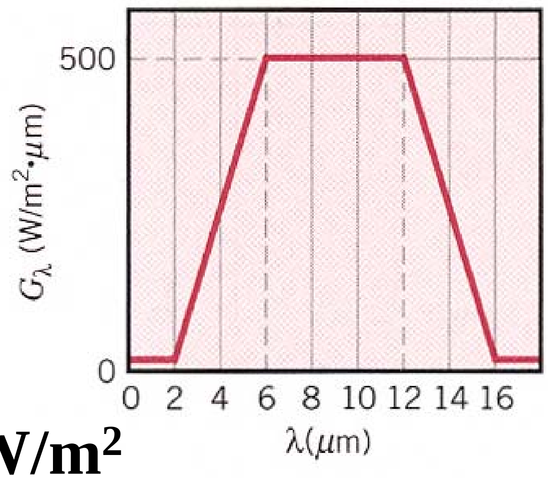
$$\alpha_\lambda = \frac{\int_\cap \alpha'_\lambda I_{\lambda,i}\cos\theta_i d\omega_i}{\int_\cap I_{\lambda,i}\cos\theta_i d\omega_i} = \frac{\int_\cap \alpha'_\lambda I_{\lambda,i}\cos\theta_i d\omega_i}{G_\lambda}$$



$$\alpha = \frac{\int_0^\infty \left(\int_\cap \alpha'_\lambda I_{\lambda,i}\cos\theta_i d\omega_i\right) d\lambda}{\int_0^\infty \int_\cap I_{\lambda,i}\cos\theta_i d\omega_i d\lambda} = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{G}$$

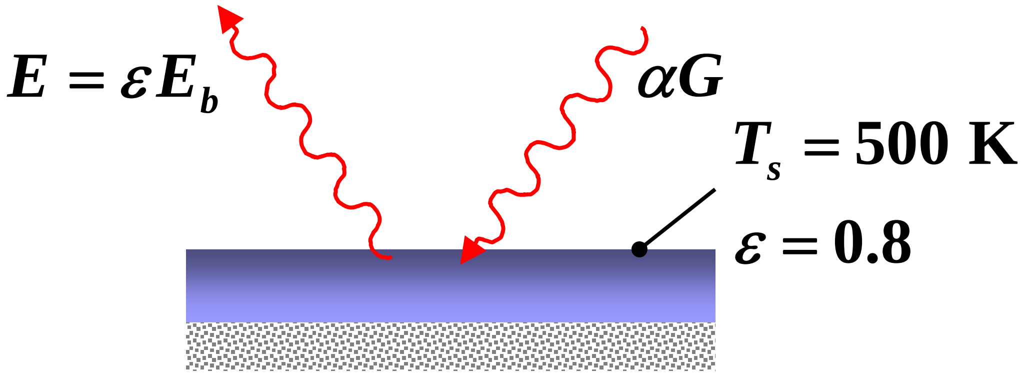
$$\alpha = \frac{0.2\int_2^6 G_\lambda d\lambda + 500\int_6^8 \alpha_\lambda d\lambda + 1.0\int_8^{16} G_\lambda d\lambda}{\int_2^6 G_\lambda d\lambda + \int_6^{12} G_\lambda d\lambda + \int_{12}^{16} G_\lambda d\lambda} = 0.76$$

$= 5000$ W/m^2



Why is the value of α (= 0.76) closer to unity ?

3) Nature of surface temperature change



Does the surface temperature increase or decrease?

$$\dot{E}_{\mathrm{st}} = \dot{E}_{\mathrm{in}} - \dot{E}_{\mathrm{out}} + \dot{E}_{\mathrm{g}}$$

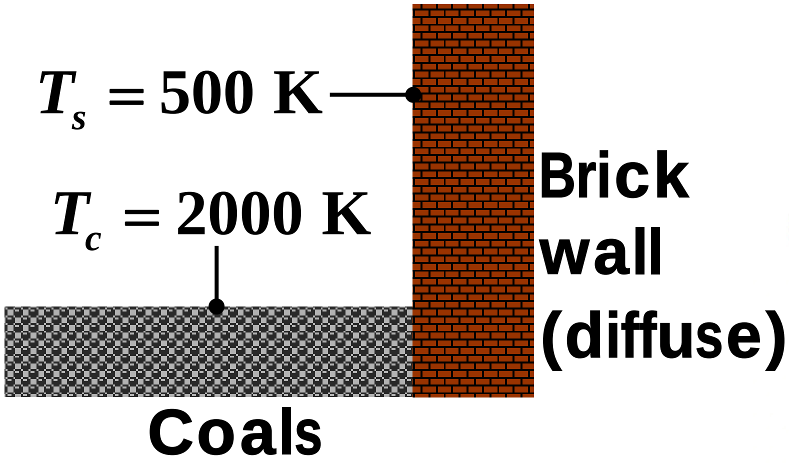
$$= \alpha G - \varepsilon E_b = \alpha G - \varepsilon \sigma T_s^4$$

$$= 0.76 \times 5000 - 0.8 \times 5.67 \times 10^{-8} \times 500^4$$

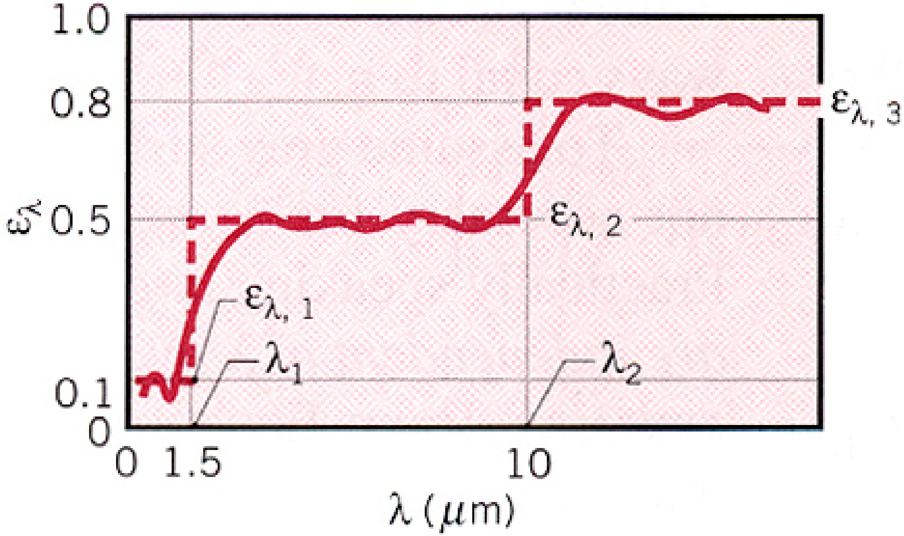
$$= 3800 - 2835 = 965\ \mathrm{W/m^2}$$

Since $\dot{E}_{\mathrm{st}} > 0$, the surface temperature will increase with time.

Example 12.9



spectral emissivity of brick wall



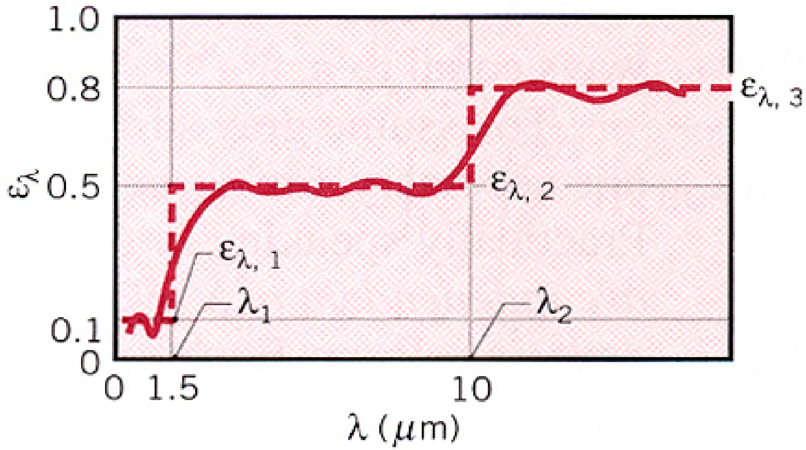
Find:

1) Hemispherical total emissivity of the fire brick wall
2) Total emissive power of the brick wall
3) Absorptivity of the wall to irradiation from the coals

Assumptions:

Spectral distribution of irradiation at the brick wall approximates that due to emission from a blackbody at 2000 K

1) Hemispherical total emissivity



$$\varepsilon(T_s) = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda b} d\lambda}{\sigma T_s^4}$$

$$= \frac{1}{\sigma T_s^4}\left(\varepsilon_{\lambda,1}\int_0^{\lambda_1} E_{\lambda b} d\lambda + \varepsilon_{\lambda,2}\int_{\lambda_1}^{\lambda_2} E_{\lambda b} d\lambda + \varepsilon_{\lambda,3}\int_{\lambda_2}^{\infty} E_{\lambda b} d\lambda\right)$$

$$= \varepsilon_{\lambda,1} F_{(0\to\lambda_1)} + \varepsilon_{\lambda,2}\left[F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)}\right] + \varepsilon_{\lambda,3}\left[1 - F_{(0\to\lambda_2)}\right]$$

From Table 12.1

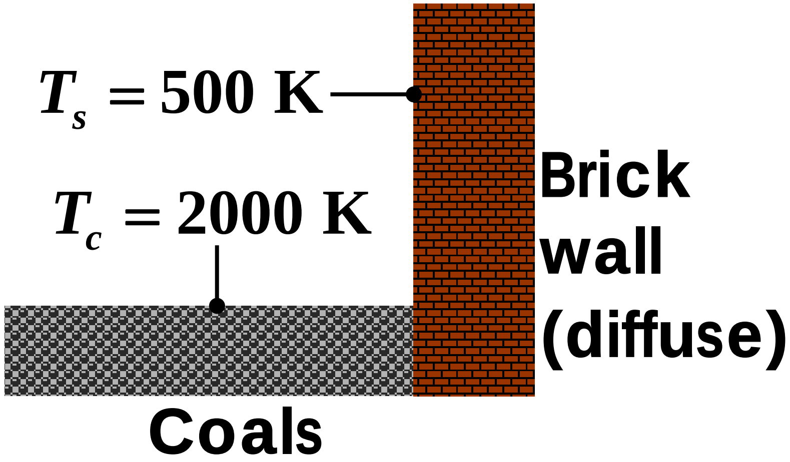
$$\lambda_1 T_s = 1.5\times 500 = 750\ \mu\text{m}\cdot\text{K}: \quad F_{(0\to\lambda_1)} = 0.000$$

$$\lambda_2 T_s = 10\times 500 = 5000\ \mu\text{m}\cdot\text{K}: \quad F_{(0\to\lambda_2)} = 0.634$$

Hence,

$$\varepsilon(T_S) = 0.1\times 0 + 0.5\times 0.634 + 0.8\left(1 - 0.634\right) = 0.610$$

2) total emissive power



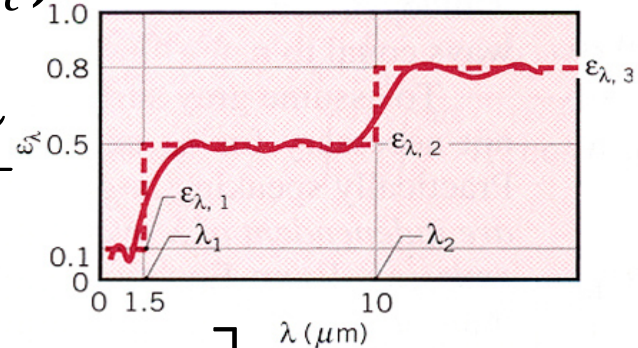
$$E(T_s) = \varepsilon(T_s)E_b(T_s) = \varepsilon(T_s)\sigma T_s^4$$

$$E(T_s) = 0.61 \times 5.67 \times 10^{-8} \times 500^4 = 2161\,\mathrm{W/m^2}$$

3) total absorptivity to irradiation from coal

Since brick wall is diffuse, $\alpha_\lambda(\lambda) = \varepsilon_\lambda(\lambda)$

irradiation from the coal: $G_\lambda(\lambda) \propto E_{\lambda b}(\lambda, T_c)$



$$\alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda) G_\lambda(\lambda)\,d\lambda}{\int_0^\infty G_\lambda(\lambda)\,d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda) E_{\lambda b}(\lambda, T_c)\,d\lambda}{\int_0^\infty E_{\lambda b}(\lambda, T_c)\,d\lambda}$$

$$\alpha = \varepsilon_{\lambda,1} F_{(0\to\lambda_1)} + \varepsilon_{\lambda,2}\left[F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)}\right] + \varepsilon_{\lambda,3}\left[1 - F_{(0\to\lambda_2)}\right]$$

From Table 12.1

$$\lambda_1 T_c = 1.5 \times 2000 = 3000\ \mu\text{m}\cdot\text{K}: \quad F_{(0\to\lambda_1)} = 0.273$$

$$\lambda_2 T_c = 10 \times 2000 = 20{,}000\ \mu\text{m}\cdot\text{K}: F_{(0\to\lambda_2)} = 0.986$$

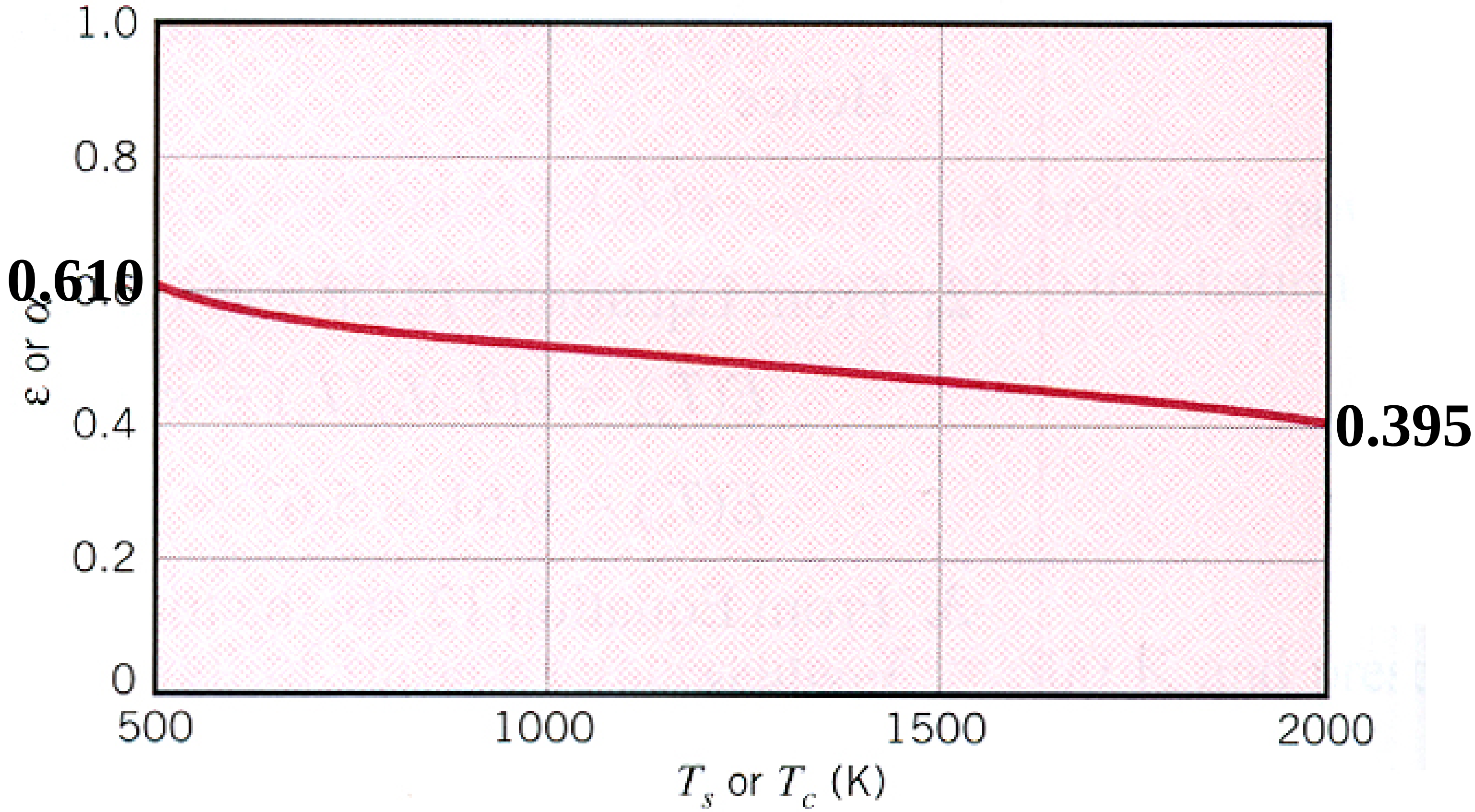
$$\alpha = 0.1 \times 0.273 + 0.5 \times (0.986 - 0.273) + 0.8(1 - 0.986) = 0.395$$

$$\varepsilon = 0.610$$

Thus, the brick wall cannot be regarded as a gray surface.

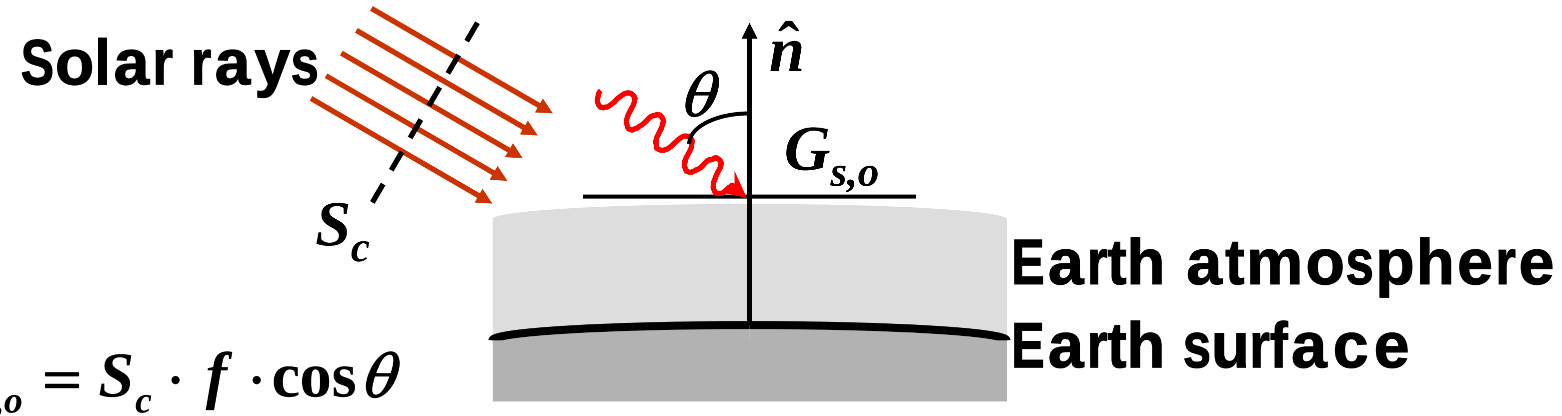
Comments:

The foregoing expressions for ε and α may be used to determine their equivalent variation with T_s and T_c



Environmental Radiation

extraterrestrial solar irradiation



$$G_{s,o} = S_c \cdot f \cdot \cos\theta$$

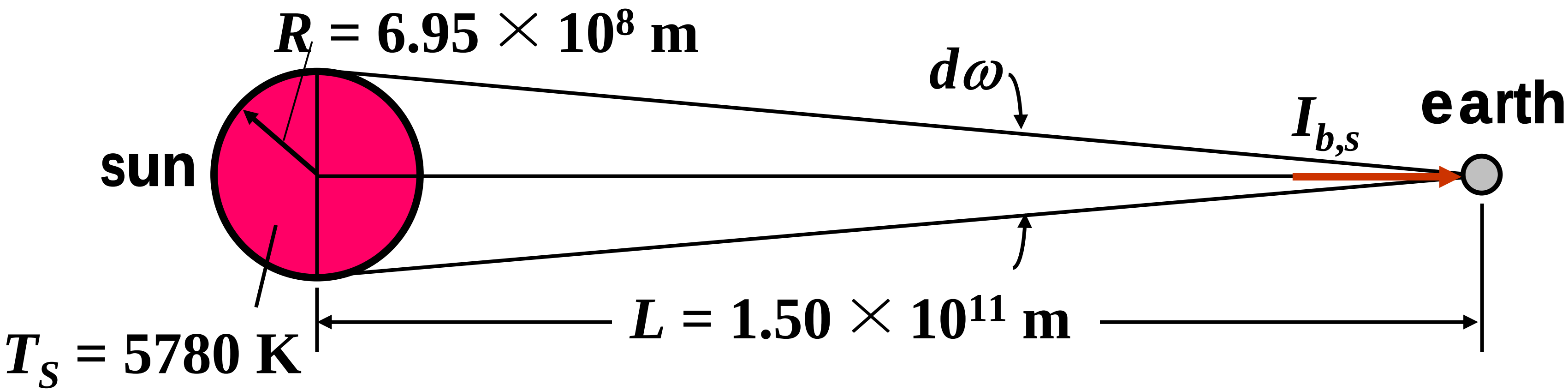
S_c : solar constant (flux of solar energy incident on a surface oriented normal to the sun's rays, when the earth is at its mean distance from the sun)

$$S_c = 1353 \text{ W/m}^2$$

f : correction factor to account for the eccentricity of the earth's orbit about the sun

$$0.97 \le f \le 1.03$$

θ : incident angle of solar irradiation



$$S_c = I_{b,s} \cos\theta d\omega = \frac{\sigma T_S^4}{\pi} \frac{\pi R^2}{L^2} \quad (\cos\theta \approx 1)$$

$$= \frac{5.67 \times (57.8)^4 (6.95)^2 \times 10^{16}}{(1.5)^2 \times 10^{22}} = 1358 \text{ W/m}^2$$

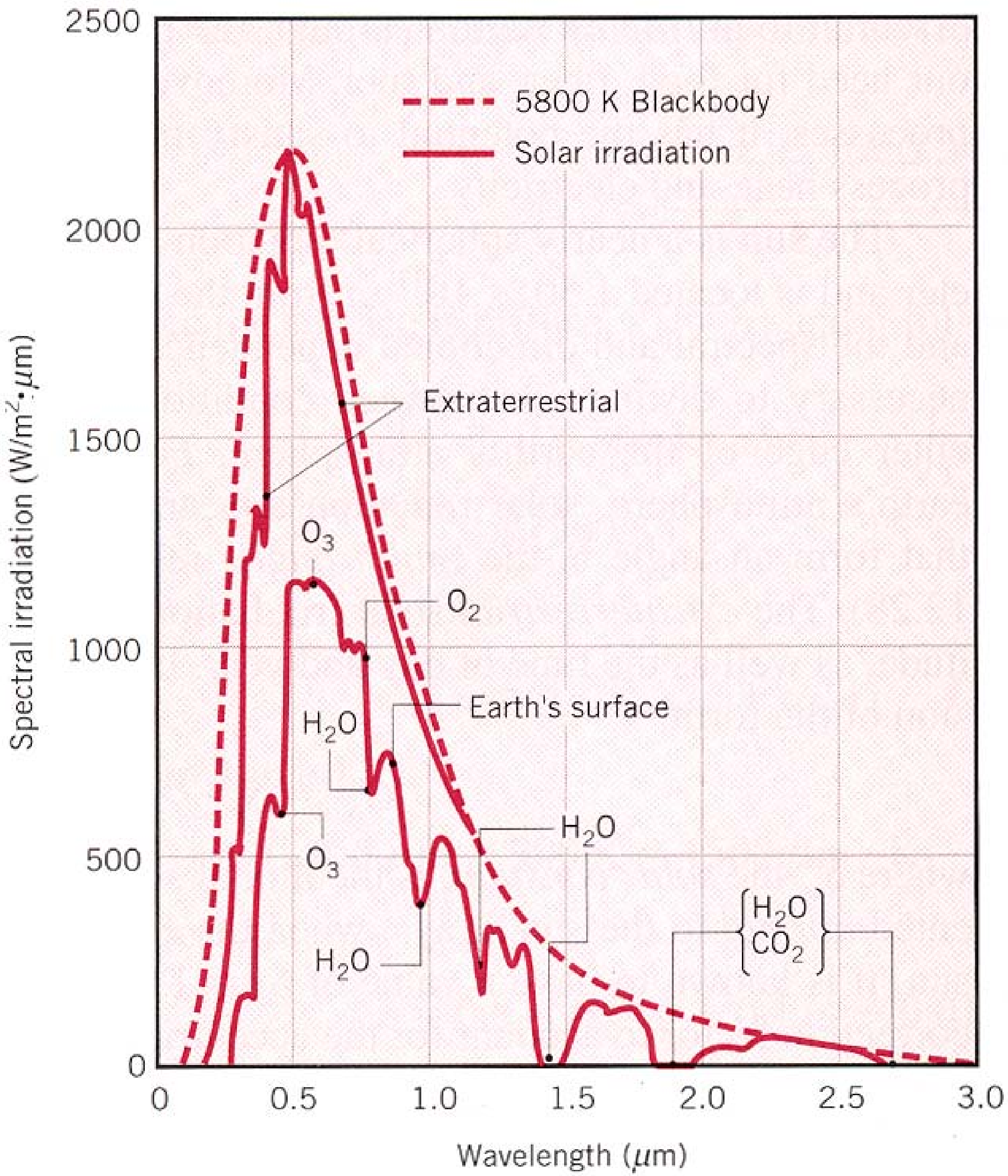
The earth surface can not be considered as gray surface to the solar irradiation

Because: $T_S = 5780K \rightarrow \lambda_{\max} \sim 0.5\mu m$

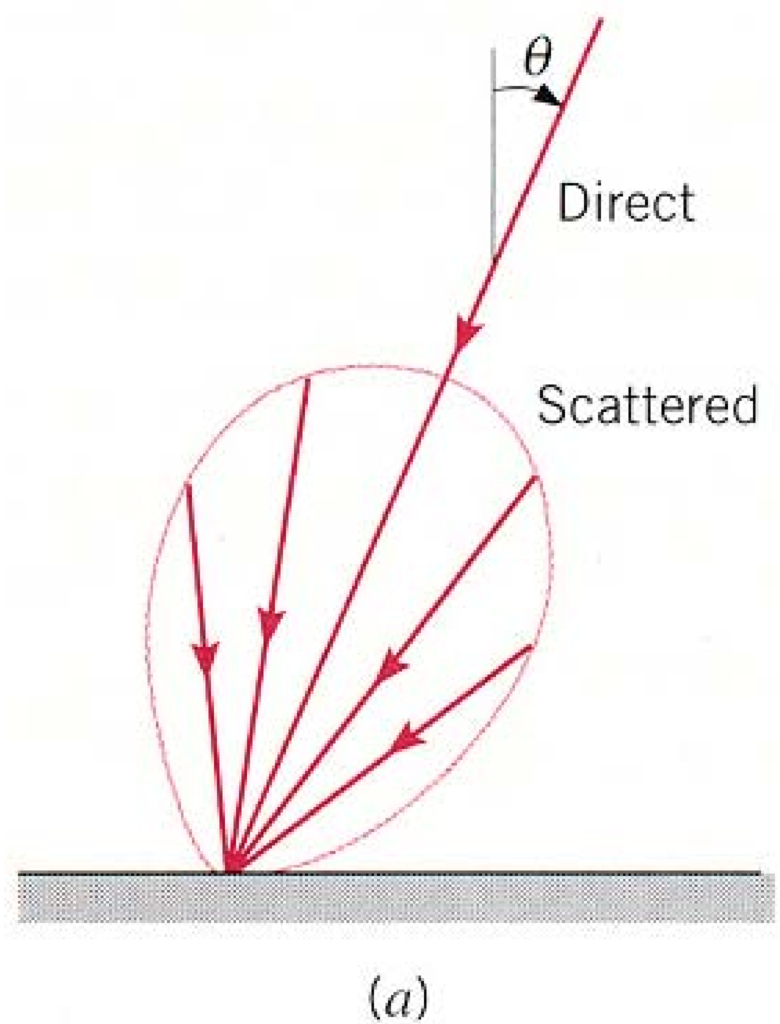
$T_E = 250 \sim 300K \rightarrow \lambda_{\max} \sim 10\mu m$

Earth irradiation due to atmospheric emission: $G_{\text{atm}} = \sigma T_{\text{sky}}^4$

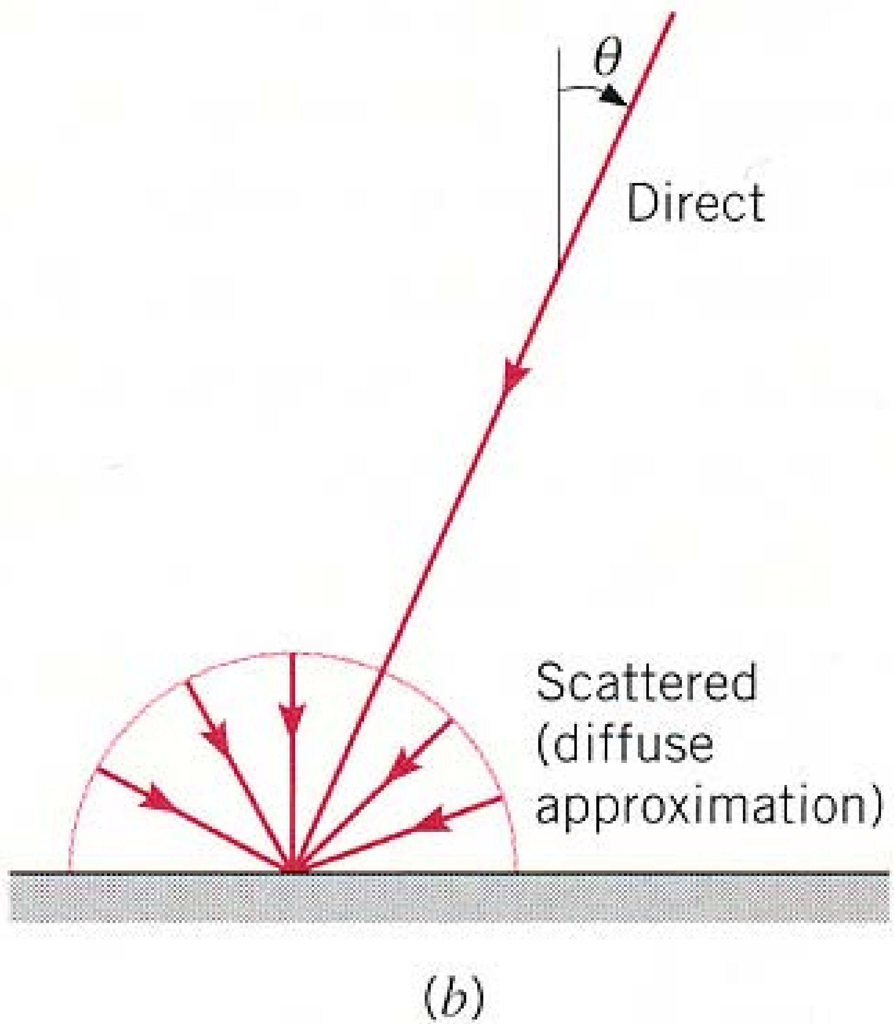
Spectral distribution of solar radiation



Directional distribution of solar radiation at earth's surface

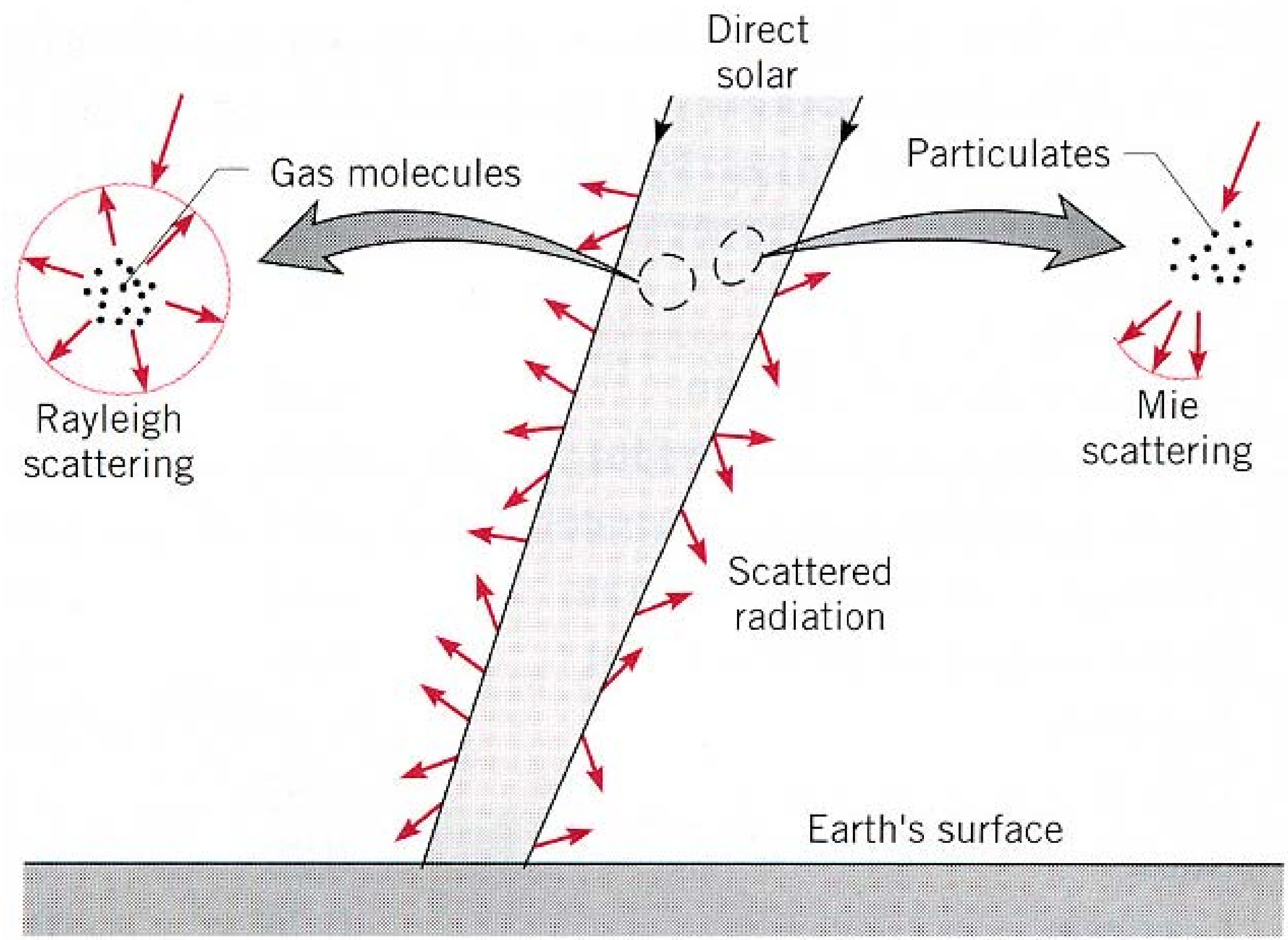


actual

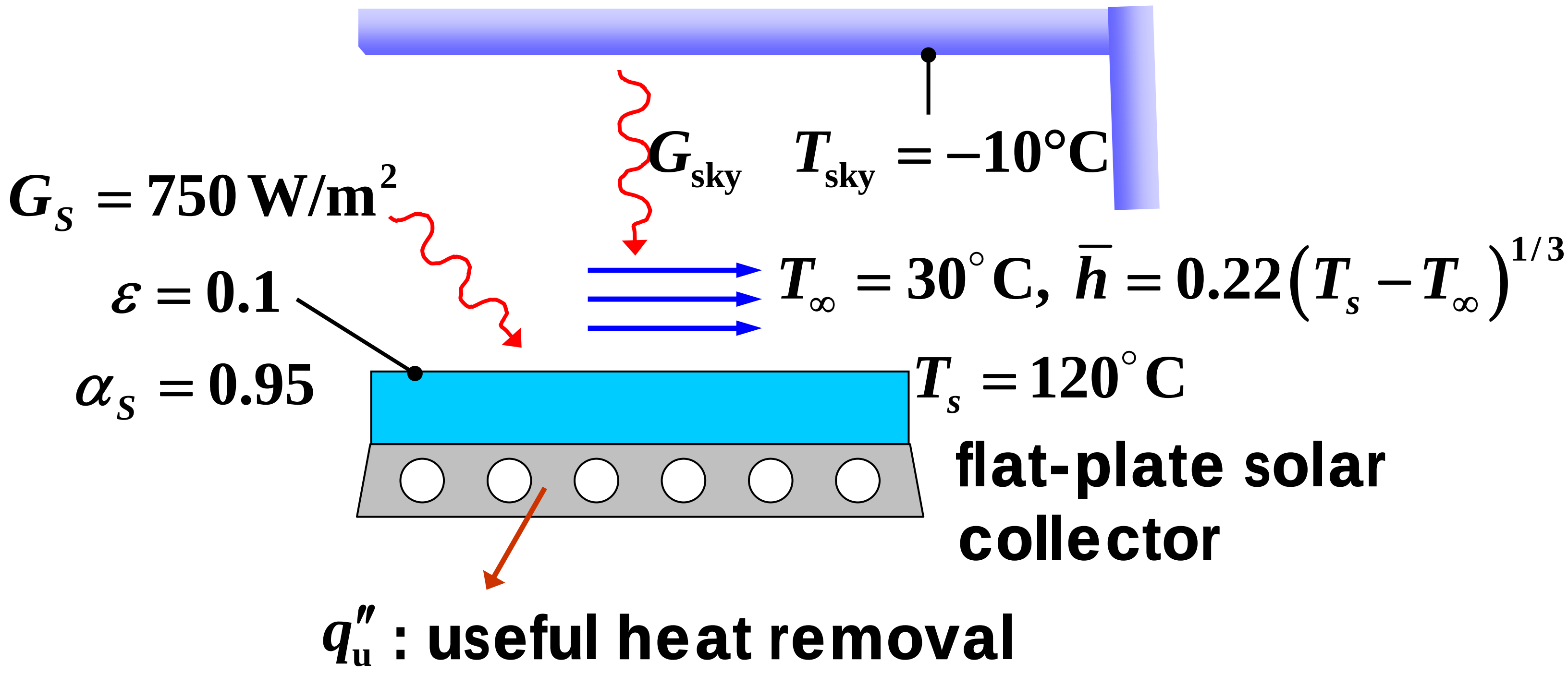


diffuse

Scattering of solar radiation in the earth's atmosphere



Example 12.11



Find:

1) Useful heat removal rate per unit area, $q''_u\,[\mathrm{W/m^2}]$

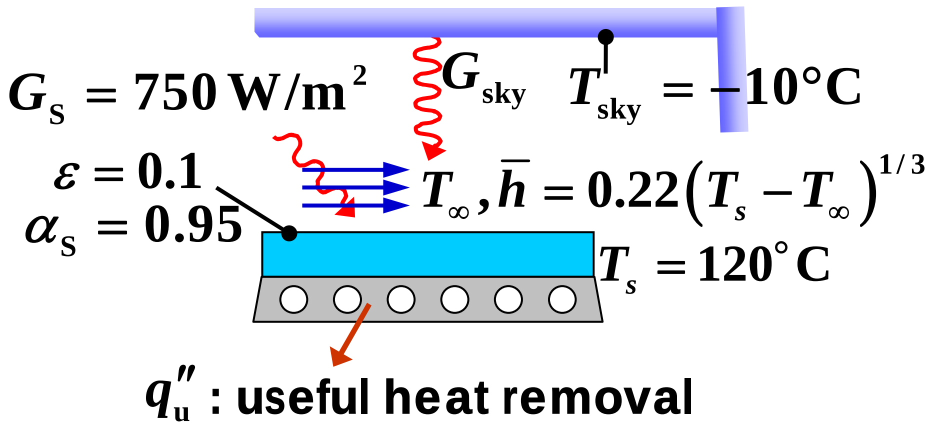
2) Efficiency η of the collector.

Assumptions:

steady-state

absorber surface diffuse

1) Useful heat removal rate



$$\dot{E}_{in} = \dot{E}_{out}$$

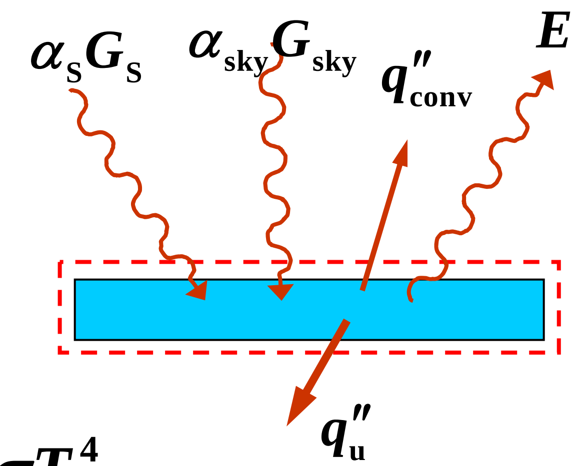
$$\alpha_S G_S + \alpha_{sky} G_{sky} = q''_{conv} + E + q''_u$$

$$q''_u = \alpha_S G_S + \alpha_{sky} G_{sky} - q''_{conv} - E$$

$$G_{sky} = \sigma T_{sky}^4, \quad \alpha_{sky} \approx \varepsilon = 0.1$$

$$q''_{conv} = \bar{h}(T_s - T_\infty) = 0.22(T_s - T_\infty)^{4/3}$$

$$E = \varepsilon \sigma T_s^4$$

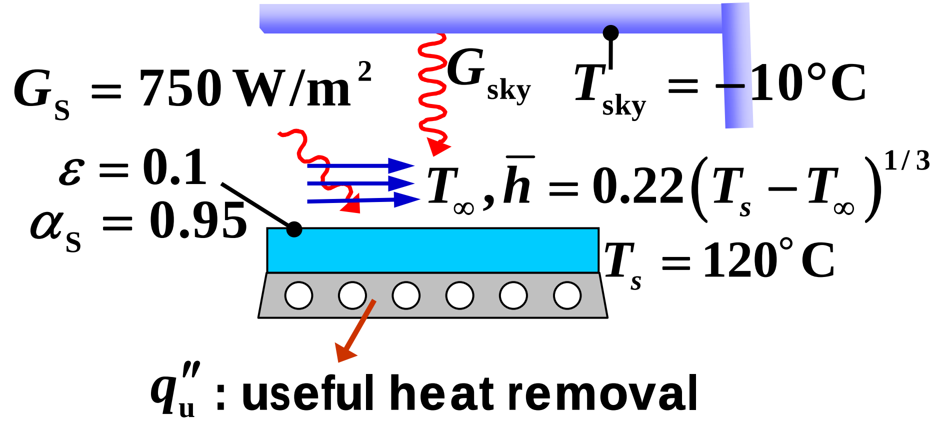


$$q''_u = \alpha_S G_S + \varepsilon\sigma T_{sky}^4 - 0.22(T_s - T_\infty)^{4/3} - \varepsilon\sigma T_s^4$$

$$= 712.5 + 27.1 - 88.7 - 135.2 = 516\ \mathrm{W/m^2}$$

2) The collector efficiency

$$\eta = \frac{q''_u}{G_S} = \frac{516\,\mathrm{W/m^2}}{750\,\mathrm{W/m^2}} = 0.69$$



Comments:

1) Since the spectral range of G_{sky} is entirely different from that of G_S, it would be incorrect to assume that $\alpha_{sky} = \alpha_S$.

2) With a convection coefficient $\bar{h} = 5\ \mathrm{W/m^2 \cdot K}$, the useful heat flux and the efficiency are reduced to $q''_u = 161\,\mathrm{W/m^2}$ and $\eta = 0.21$. A cover plate can contribute significantly to reducing convection (and radiation) heat loss from the absorber plate.

$$\bar{h} = 0.22(T_s - T_\infty)^{1/3} = 0.22(120 - 30)^{1/3} = 0.986\ \mathrm{W/m^2 \cdot K}$$