Example 4.17

Consider an integrator—that is, an LTI system specified by the equation

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau.$$

The impulse response for this system is the unit step $u(t)$, and therefore, from Example 4.11 and eq. (4.33), the frequency response of the system is

$$H(j\omega) = \frac{1}{j\omega} + \pi\delta(\omega).$$

Then using eq. (4.56), we have

$$\begin{aligned} Y(j\omega) &= H(j\omega)X(j\omega) \\ &= \frac{1}{j\omega}X(j\omega) + \pi X(j\omega)\delta(\omega) \\ &= \frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega), \end{aligned}$$

which is consistent with the integration property of eq. (4.32).

Example 4.18

As we discussed in Section 3.9.2, frequency-selective filtering is accomplished with an LTI system whose frequency response $H(j\omega)$ passes the desired range of frequencies and significantly attenuates frequencies outside that range. For example, consider the ideal lowpass filter introduced in Section 3.9.2, which has the frequency reponse illustrated in Figure 4.20 and given by

$$H(j\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}. \tag{4.63}$$

Now that we have developed the Fourier transform representation, we know that the impulse response $h(t)$ of this ideal filter is the inverse transform of eq. (4.63). Using the result in Example 4.5, we then have

$$h(t) = \frac{\sin \omega_c t}{\pi t}, \tag{4.64}$$

which is plotted in Figure 4.21.

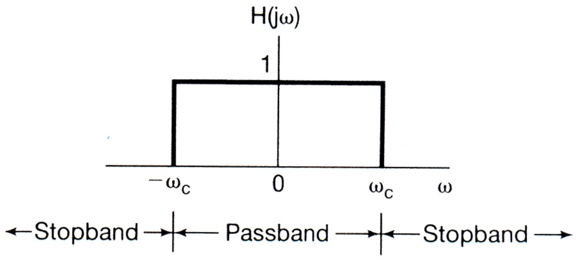


Figure 4.20 Frequency response of an ideal lowpass filter.

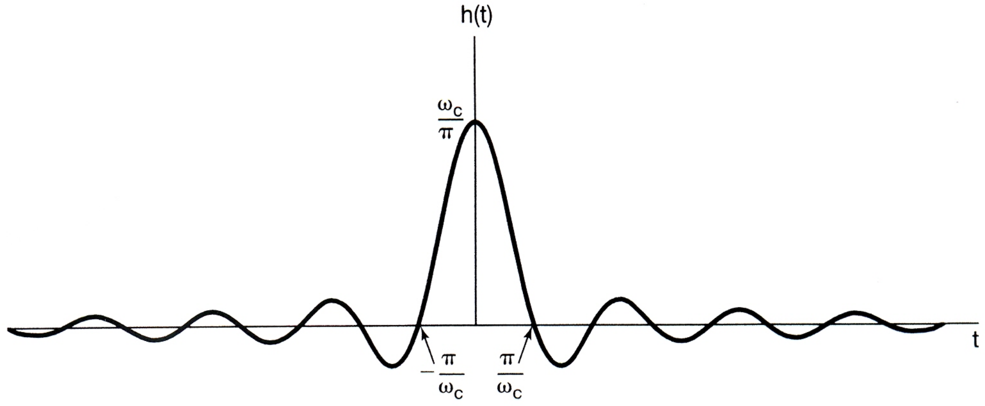


Figure 4.21 Impulse response of an ideal lowpass filter.

From Example 4.18, we can begin to see some of the issues that arise in filter design that involve looking in both the time and frequency domains. In particular, while the ideal lowpass filter does have perfect frequency selectivity, its impulse response has some characteristics that may not be desirable. First, note that $h(t)$ is not zero for $t < 0$. Consequently, the ideal lowpass filter is not causal, and thus, in applications requiring causal systems, the ideal filter is not an option. Moreover, as we discuss in Chapter 6, even if causality is not an essential constraint, the ideal filter is not easy to approximate closely, and non-ideal filters that are more easily implemented are typically preferred. Furthermore, in some applications (such as the automobile suspension system discussed in Section 6.7.1), oscillatory behavior in the impulse response of a lowpass filter may be undesirable. In such applications the time domain characteristics of the ideal lowpass filter, as shown in Figure 4.21, may be unacceptable, implying that we may need to trade off frequency-domain characteristics such as ideal frequency selectivity with time-domain properties.

For example, consider the LTI system with impulse response

$$h(t) = e^{-t}u(t). \tag{4.65}$$

: causal

The frequency response of this system is

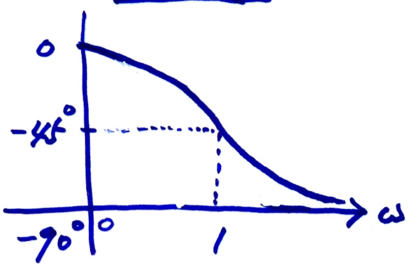
$\Downarrow$ $H(s)\big|_{s=j\omega}$

$$H(j\omega) = \frac{1}{j\omega + 1}. \tag{4.66}$$

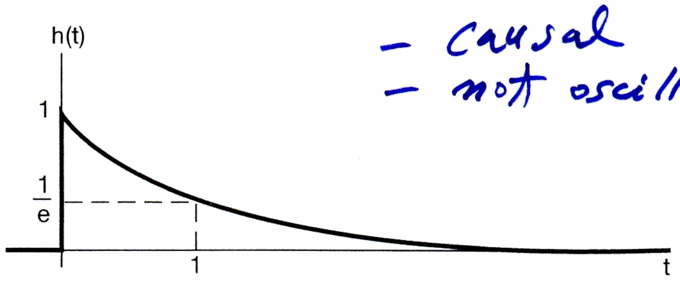
Comparing eqs. (3.145) and (4.66), we see that this system can be implemented with the simple RC circuit discussed in Section 3.10. The impulse response and the magnitude of the frequency response are shown in Figure 4.22. While the system does not have the strong frequency selectivity of the ideal lowpass filter, it is causal and has an impulse response that decays monotonically, i.e., without oscillations. This filter or somewhat more complex ones corresponding to higher order differential equations are quite frequently preferred to ideal filters because of their causality, ease of implementation, and flexibility in allowing trade-offs, among other design considerations such as frequency selectivity and oscillatory behavior in the time domain. Many of these issues will be discussed in more detail in Chapter 6.

The convolution property is often useful in evaluating the convolution integral—i.e., in computing the response of LTI systems. This is illustrated in the next example.

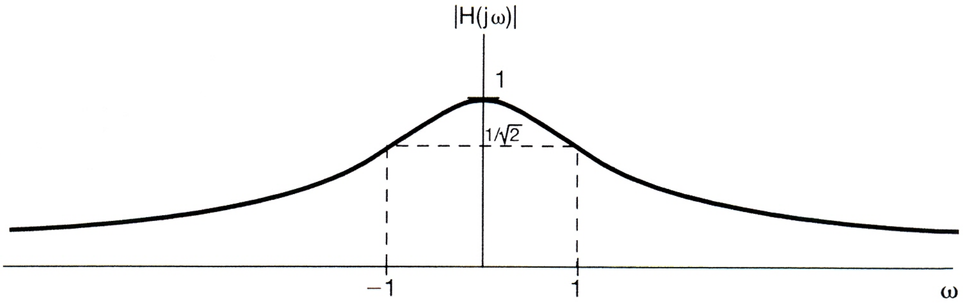
$\angle H(j\omega) \neq 0$



- causal
- not oscillatory



(a)



(b)

Figure 4.22 (a) Impulse response of the LTI system in eq. (4.65); (b) magnitude of the frequency response of the system.

Example 4.19 (Use Laplace Transform)

Consider the response of an LTI system with impulse response

$$h(t) = e^{-at}u(t), \quad a > 0, \qquad \text{: causal}$$

to the input signal $\quad (\dot{y} + ay = x, \ y(0) = 0)$

$$x(t) = e^{-bt}u(t), \quad b > 0.$$

$$\begin{cases} Y(s) = H(s)X(s) \\ H(s) = \dfrac{1}{s+a} \\ X(s) = \dfrac{1}{s+b} \end{cases}$$

Rather than computing $y(t) = x(t) * h(t)$ directly, let us transform the problem into the frequency domain. From Example 4.1, the Fourier transforms of $x(t)$ and $h(t)$ are

$$X(j\omega) = \frac{1}{b + j\omega}$$

and

$$H(j\omega) = \frac{1}{a + j\omega}.$$

$$\Rightarrow Y(s) = H(s)X(s) = \frac{1}{(s+a)(s+b)}$$

Therefore,

$$Y(j\omega) = \frac{1}{(a + j\omega)(b + j\omega)}. \tag{4.67}$$

$$\Rightarrow y(t) = \mathcal{L}^{-1}[Y(s)]$$

To determine the output $y(t)$, we wish to obtain the inverse transform of $Y(j\omega)$. This is most simply done by expanding $Y(j\omega)$ in a partial-fraction expansion. Such expansions are extremely useful in evaluating inverse transforms, and the general method for performing a partial-fraction expansion is developed in the appendix. For this

Example 4.20 (steady-state response)

not causal : $x(t) = \frac{\sin \omega_i t}{\pi t}$, $h(t) = \frac{\sin \omega_c t}{\pi t}$, $-\infty < t < \infty$

The filter output $y(t)$ will therefore be the convolution of two sinc functions, which, as we now show, also turns out to be a sinc function. A particularly convenient way of deriving this result is to first observe that

$$Y(j\omega) = X(j\omega)H(j\omega),$$

where

$$X(j\omega) = \begin{cases} 1 & |\omega| \leq \omega_i \\ 0 & \text{elsewhere} \end{cases}$$

and

$$H(j\omega) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & \text{elsewhere} \end{cases}.$$

Therefore,

$$Y(j\omega) = \begin{cases} 1 & |\omega| \leq \omega_0 \\ 0 & \text{elsewhere} \end{cases},$$

where ω_0 is the smaller of the two numbers ω_i and ω_c. Finally, the inverse Fourier transform of $Y(j\omega)$ is given by

$$y(t) = \begin{cases} \dfrac{\sin \omega_c t}{\pi t} & \text{if } \omega_c \leq \omega_i \\ \dfrac{\sin \omega_i t}{\pi t} & \text{if } \omega_i \leq \omega_c \end{cases}$$

That is, depending upon which of ω_c and ω_i is smaller, the output is equal to either $x(t)$ or $h(t)$.

4.5 THE MULTIPLICATION PROPERTY

The convolution property states that convolution in the *time* domain corresponds to multiplication in the *frequency* domain. Because of duality between the time and frequency domains, we would expect a dual property also to hold (i.e., that multiplication in the time domain corresponds to convolution in the frequency domain). Specifically,

$$\boxed{r(t) = s(t)p(t) \longleftrightarrow R(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S(j\theta)P(j(\omega-\theta))d\theta} \quad (4.70)$$

This can be shown by exploiting duality as discussed in Section 4.3.6, together with the convolution property, or by directly using the Fourier transform relations in a manner analogous to the procedure used in deriving the convolution property.

Multiplication of one signal by another can be thought of as using one signal to scale or *modulate* the amplitude of the other, and consequently, the multiplication of two signals is often referred to as *amplitude modulation*. For this reason, eq. (4.70) is sometimes

Proof of (4.70)

$$\left.\begin{aligned} &\mathcal{F}\left[\tfrac{1}{2\pi}X(-jt)\right] = x(\omega) \\ &\mathcal{F}[x(t)] = X(j\omega) \\ &\mathcal{F}[s(t) * p(t)] = S(j\omega)P(j\omega) \end{aligned}\right\} \begin{aligned} &\Rightarrow \mathcal{F}[S(-jt)P(-jt)/2\pi] = s(\omega) * p(\omega) \\ &\Rightarrow \mathcal{F}\left[\tfrac{1}{2\pi}S(-jt)\,\tfrac{1}{2\pi}P(-jt)\right] = \tfrac{1}{2\pi}\, s(\omega) * p(\omega) \\ &\Rightarrow \mathcal{F}[s(t)p(t)] = \tfrac{1}{2\pi}\, S(j\omega) * P(j\omega) \quad \square \end{aligned}$$

referred to as the *modulation property.* As we shall see in Chapters 7 and 8, this property has several very important applications. To illustrate eq. (4.70), and to suggest one of the applications that we will discuss in subsequent chapters, let us consider several examples.

Example 4.21

Let $s(t)$ be a signal whose spectrum $S(j\omega)$ is depicted in Figure 4.23(a). Also, consider the signal

$$p(t) = \cos \omega_0 t.$$

Then

$$P(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0),$$

as sketched in Figure 4.23(b), and the spectrum $R(j\omega)$ of $r(t) = s(t)p(t)$ is obtained by

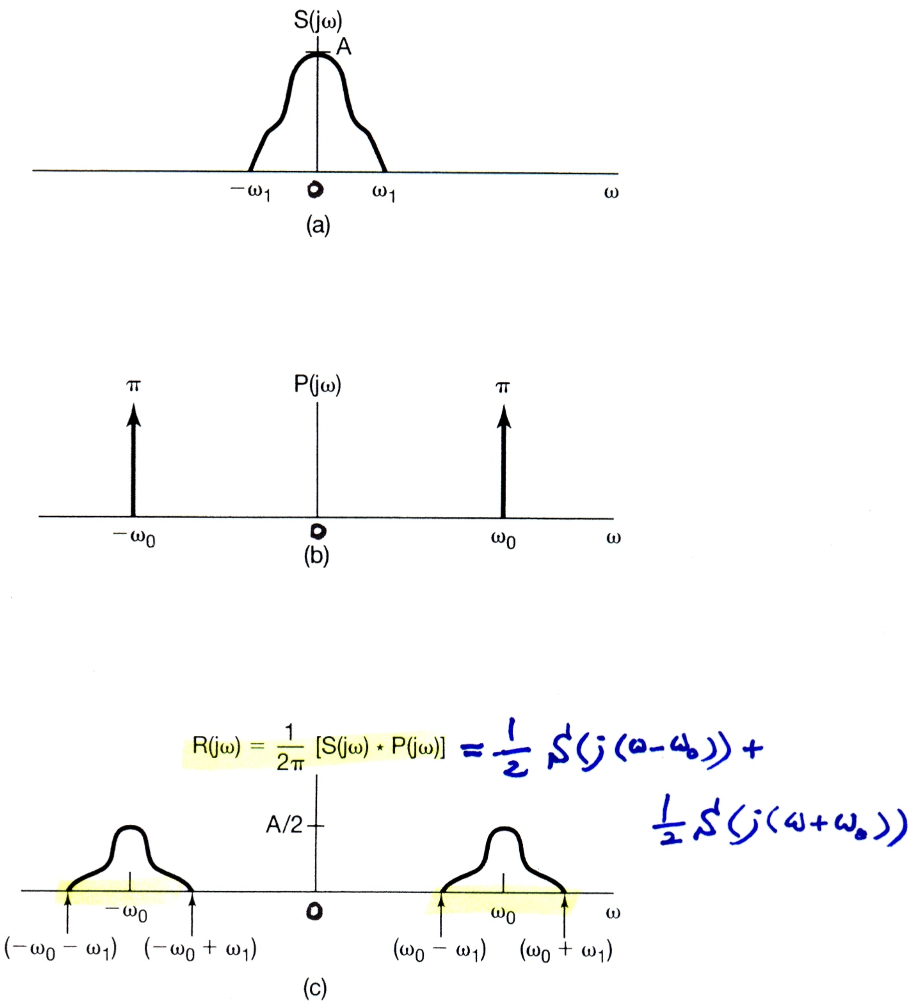


Figure 4.23 Use of the multiplication property in Example 4.21: (a) the Fourier transform of a signal $s(t)$; (b) the Fourier transform of $p(t) = \cos \omega_0 t$; (c) the Fourier transform of $r(t) = s(t)p(t)$.

an application of eq. (4.70), yielding

$$R(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S(j\theta)P(j(\omega - \theta))d\theta$$
$$= \frac{1}{2}S(j(\omega - \omega_0)) + \frac{1}{2}S(j(\omega + \omega_0)), \tag{4.71}$$

which is sketched in Figure 4.23(c). Here we have assumed that $\omega_0 > \omega_1$, so that the two nonzero portions of $R(j\omega)$ do not overlap. Clearly, the spectrum of $r(t)$ consists of the sum of two shifted and scaled versions of $S(j\omega)$.

From eq. (4.71) and from Figure 4.23, we see that all of the information in the signal $s(t)$ is preserved when we multiply this signal by a sinusoidal signal, although the information has been shifted to higher frequencies. This fact forms the basis for sinusoidal amplitude modulation systems for communications. In the next example, we learn how we can recover the original signal $s(t)$ from the amplitude-modulated signal $r(t)$.

Example 4.22

Let us now consider $r(t)$ as obtained in Example 4.21, and let

$$g(t) = r(t)p(t),$$

where, again, $p(t) = \cos\omega_0 t$. Then, $R(j\omega)$, $P(j\omega)$, and $G(j\omega)$ are as shown in Figure 4.24.

From Figure 4.24(c) and the linearity of the Fourier transform, we see that $g(t)$ is the sum of $(1/2)s(t)$ and a signal with a spectrum that is nonzero only at higher frequen-

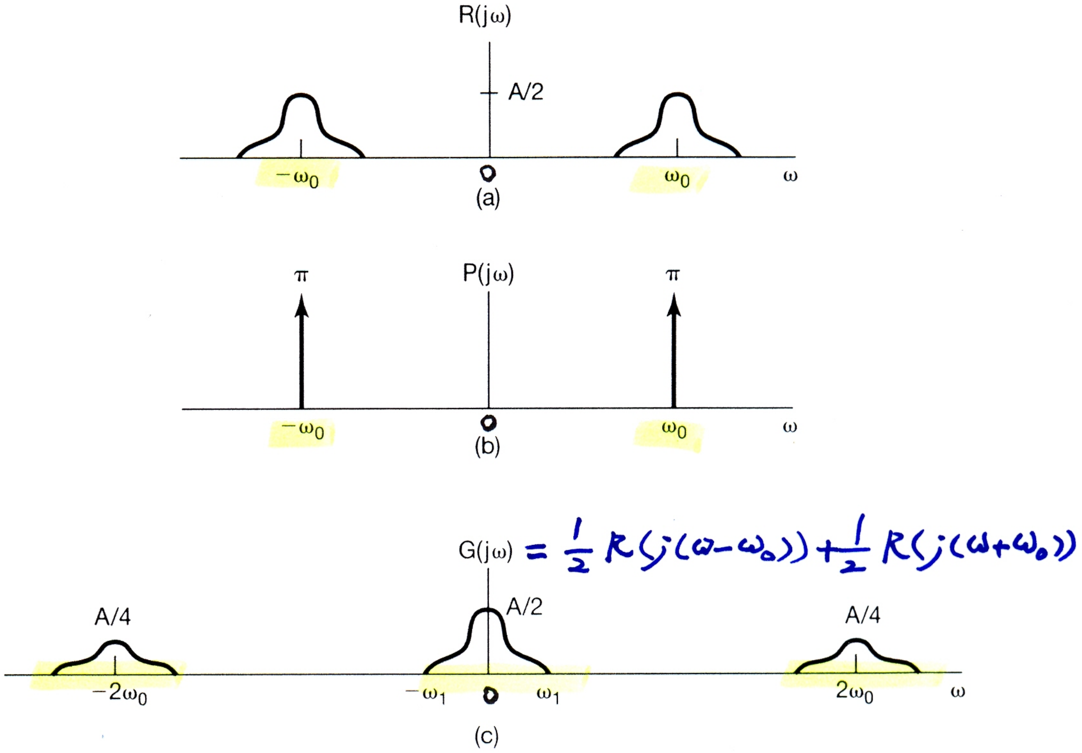


Figure 4.24 Spectra of signals considered in Example 4.22: (a) $R(j\omega)$; (b) $P(j\omega)$; (c) $G(j\omega)$.

§4.5.1 Frequency - Selective Filtering with Variable Center Frequency

shift the spectrum of the signal appropriately, using the principles of sinusoidal amplitude modulation.

For example, consider the system shown in Figure 4.26. Here, an input signal $x(t)$ is multiplied by the complex exponential signal $e^{j\omega_c t}$. The resulting signal is then passed through a lowpass filter with cutoff frequency ω_0, and the output is multiplied by $e^{-j\omega_c t}$. The spectra of the signals $x(t)$, $y(t)$, $w(t)$, and $f(t)$ are illustrated in Figure 4.27.

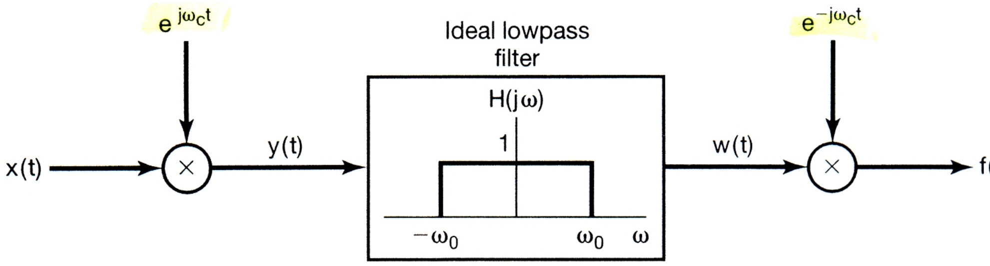
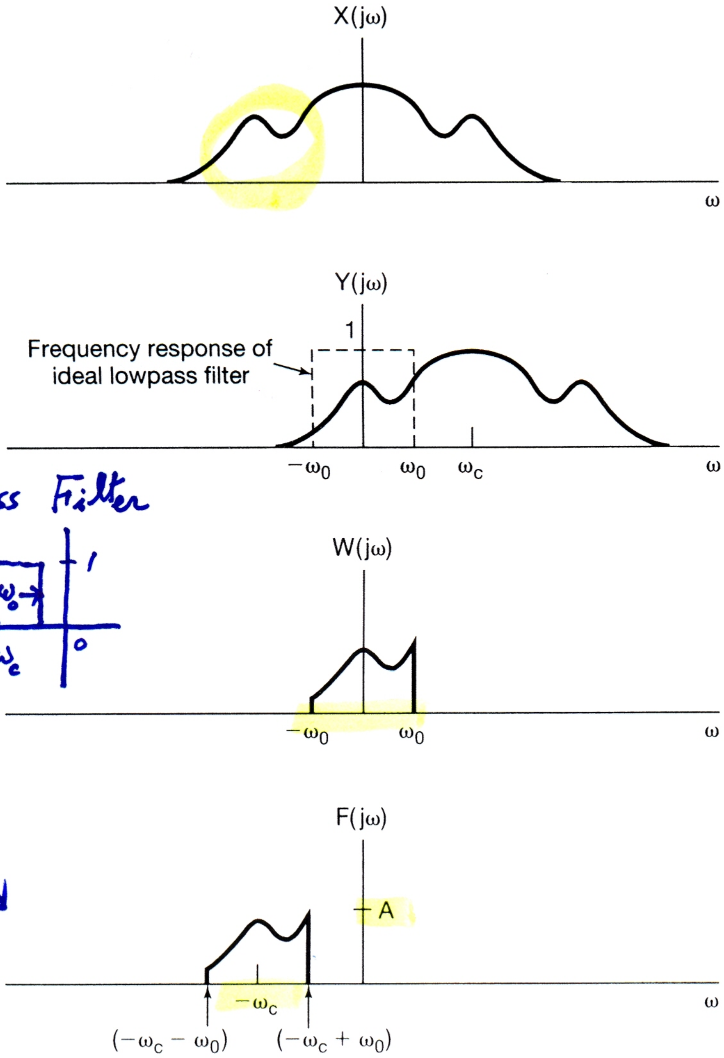


Figure 4.26 Implementation of a bandpass filter using amplitude modulation with a complex exponential carrier.



Ideal Bandpass Filter

Figure 4.27 Spectra of the signals in the system of Figure 4.26.

Specifically, from either the multiplication property or the frequency-shifting property it follows that the Fourier transform of $y(t) = e^{j\omega_c t}x(t)$ is

$$Y(j\omega) = \int_{-\infty}^{+\infty} \delta(\theta - \omega_c)X(\omega - \theta)d\theta$$

so that $Y(j\omega)$ equals $X(j\omega)$ shifted to the right by ω_c and frequencies in $X(j\omega)$ near $\omega = \omega_c$ have been shifted into the passband of the lowpass filter. Similarly, the Fourier transform of $f(t) = e^{-j\omega_c t}w(t)$ is

$$F(j\omega) = W(j(\omega + \omega_c)),$$

so that the Fourier transform of $F(j\omega)$ is $W(j\omega)$ shifted to the left by ω_c. From Figure 4.27, we observe that the overall system of Figure 4.26 is equivalent to an ideal bandpass filter with center frequency $-\omega_c$ and bandwidth $2\omega_0$, as illustrated in Figure 4.28. As the frequency ω_c of the complex exponential oscillator is varied, the center frequency of the bandpass filter varies.

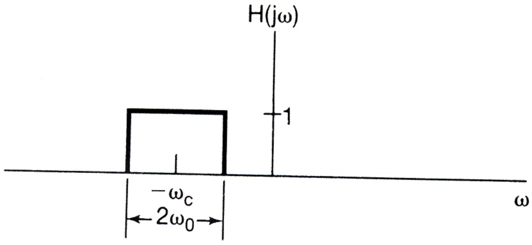


Figure 4.28 Bandpass filter equivalent of Figure 4.26.

In the system of Figure 4.26 with $x(t)$ real, the signals $y(t)$, $w(t)$, and $f(t)$ are all complex. If we retain only the real part of $f(t)$, the resulting spectrum is that shown in Figure 4.29, and the equivalent bandpass filter passes bands of frequencies centered around ω_c and $-\omega_c$, as indicated in Figure 4.30. Under certain conditions, it is also possible to use sinusoidal rather than complex exponential modulation to implement the system of the latter figure. This is explored further in Problem 4.46.

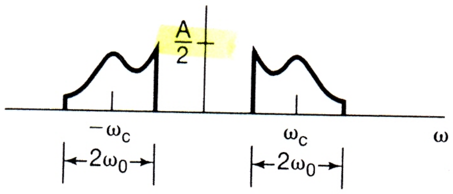


Figure 4.29 Spectrum of $\mathcal{R}e\{f(t)\}$ associated with Figure 4.26.

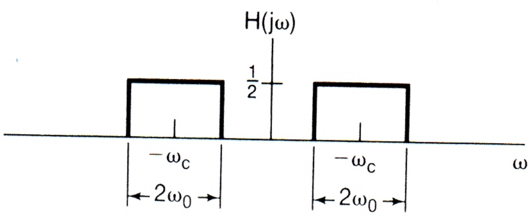


Figure 4.30 Equivalent bandpass filter for $\mathcal{R}e\{f(t)\}$ in Figure 4.29.

Recall : $\dfrac{f(t)+f^*(t)}{2} \xrightarrow{\mathcal{F}} \dfrac{1}{2}\{F(j\omega) + F^*(-j\omega)\}$

4.6 TABLES OF FOURIER PROPERTIES AND OF BASIC FOURIER TRANSFORM PAIRS

In the preceding sections and in the problems at the end of the chapter, we have considered some of the important properties of the Fourier transform. These are summarized in Table 4.1, in which we have also indicated the section of this chapter in which each property has been discussed.

In Table 4.2, we have assembled a list of many of the basic and important Fourier transform pairs. We will encounter many of these repeatedly as we apply the tools of

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$ $y(t)$	$X(j\omega)$ $Y(j\omega)$
4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t} x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{\|a\|} X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)Y(j(\omega-\theta))d\theta$
(*) 4.3.4	Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
(*)' 4.3.6	Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \Re e\{X(j\omega)\} = \Re e\{X(-j\omega)\} \\ \Im m\{X(j\omega)\} = -\Im m\{X(-j\omega)\} \\ \|X(j\omega)\| = \|X(-j\omega)\| \\ \sphericalangle X(j\omega) = -\sphericalangle X(-j\omega) \end{cases}$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real] $x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real]	$\Re e\{X(j\omega)\}$ $j\Im m\{X(j\omega)\}$

4.3.7 Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$

(*) Valid if x has no jump discontinuity

(*)' $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \Rightarrow \frac{dX(j\omega)}{d\omega} = -j\int_{-\infty}^{\infty} t x(t) e^{-j\omega t} dt$

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$, otherwise
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, otherwise
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$, otherwise
$x(t) = 1$	$2\pi\,\delta(\omega)$	$a_0 = 1, \quad a_k = 0, \ k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$)
Periodic square wave $x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & T_1 < \lvert t\rvert \le \frac{T}{2} \end{cases}$ and $x(t + T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k
$x(t)\begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & \lvert t\rvert > T_1 \end{cases}$	$\frac{2 \sin \omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert < W \\ 0, & \lvert\omega\rvert > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega} + \pi\,\delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at}u(t), \mathcal{Re}\{a\} > 0$	$\frac{1}{a + j\omega}$	—
$te^{-at}u(t), \mathcal{Re}\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!} e^{-at} u(t)$, $\mathcal{Re}\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	—

Fourier analysis in our examination of signals and systems. All of the transform pairs, except for the last one in the table, have been considered in examples in the preceding sections. The last pair is considered in Problem 4.40. In addition, note that several of the signals in Table 4.2 are periodic, and for these we have also listed the corresponding Fourier series coefficients.

4.7 SYSTEMS CHARACTERIZED BY LINEAR CONSTANT-COEFFICIENT DIFFERENTIAL EQUATIONS

As we have discussed on several occasions, a particularly important and useful class of continuous-time LTI systems is those for which the input and output satisfy a linear constant-coefficient differential equation of the form

non causal description :

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}. \tag{4.72}$$

In this section, we consider the question of determining the frequency response of such an LTI system. Throughout the discussion we will always assume that the frequency response of the system exists, i.e., that eq. (3.121) converges.

There are two closely related ways in which to determine the frequency response $H(j\omega)$ for an LTI system described by the differential equation (4.72). The first of these, which relies on the fact that complex exponential signals are eigenfunctions of LTI systems, was used in Section 3.10 in our analysis of several simple, nonideal filters. Specifically, if $x(t) = e^{j\omega t}$, then the output must be $y(t) = H(j\omega)e^{j\omega t}$. Substituting these expressions into the differential equation (4.72) and performing some algebra, we can then solve for $H(j\omega)$. In this section we use an alternative approach to arrive at the same answer, making use of the differentiation property, eq. (4.31), of Fourier transforms.

Consider an LTI system characterized by eq. (4.72). From the convolution property,

$$Y(j\omega) = H(j\omega)X(j\omega),$$

or equivalently,

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}, \tag{4.73}$$

where $X(j\omega)$, $Y(j\omega)$, and $H(j\omega)$ are the Fourier transforms of the input $x(t)$, output $y(t)$, and impulse response $h(t)$, respectively. Next, consider applying the Fourier transform to both sides of eq. (4.72) to obtain

$$\mathfrak{F}\left\{\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k}\right\} = \mathfrak{F}\left\{\sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}\right\}. \tag{4.74}$$

From the linearity property, eq. (4.26), this becomes

$$\sum_{k=0}^{N} a_k \mathfrak{F}\left\{\frac{d^k y(t)}{dt^k}\right\} = \sum_{k=0}^{M} b_k \mathfrak{F}\left\{\frac{d^k x(t)}{dt^k}\right\}, \tag{4.75}$$

and from the differentiation property, eq. (4.31),

$$\sum_{k=0}^{N} a_k (j\omega)^k Y(j\omega) = \sum_{k=0}^{M} b_k (j\omega)^k X(j\omega),$$

or equivalently,

$$Y(j\omega)\left[\sum_{k=0}^{N} a_k (j\omega)^k\right] = X(j\omega)\left[\sum_{k=0}^{M} b_k (j\omega)^k\right].$$

Thus, from eq. (4.73),

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{\sum_{k=0}^{M} b_k (j\omega)^k}{\sum_{k=0}^{N} a_k (j\omega)^k}. \tag{4.76}$$

Observe that $H(j\omega)$ is thus a rational function; that is, it is a ratio of polynomials in $(j\omega)$. The coefficients of the numerator polynomial are the same coefficients as those that appear on the right-hand side of eq. (4.72), and the coefficients of the denominator polynomial are the same coefficients as appear on the left side of eq. (4.72). Hence, the frequency response given in eq. (4.76) for the LTI system characterized by eq. (4.72) can be written down directly by inspection.

Remark

Causal description :
$$\begin{cases} \sum_{k=0}^{N} a_k z^{(k)}(t) = x(t) \\ y(t) = \sum_{k=0}^{M} b_k z^{(k)}(t) \end{cases} \qquad (4.72)' \qquad \text{with } M \le N$$

$\Rightarrow$ (4.76)

HW #4

4, 5, 15, 25, 29, 34, 37, 43, 46, 51