Multi View Geometry (Spring '08)

Structure Computation

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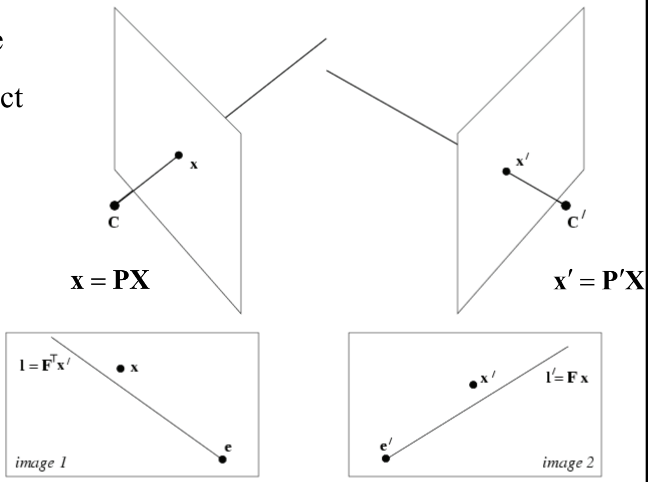
Point reconstruction

- Given (**P**, **P**’, **x**, **x**’), how to find **X**?
- Measurement noise
- Rays do not intersect



- Measured points do not lie on corresponding epipolar lines

Which 3D point to select?

- Mid-point of common perpendicular to the rays
  - ✓ Meaningless under projective distortion
- Weighted point on common perpendicular, weighted by distance from camera center
  - ✓ Distance is also undefined concept
- Some algebraic distance
  - ✓ Minimizes nothing meaningful

Linear triangulation method

- Given equations $\mathbf{x} = \mathbf{PX}$ $\quad \mathbf{x}' = \mathbf{P}'\mathbf{X}$

$$\mathbf{x} \times \mathbf{PX} = 0 \Rightarrow \begin{array}{l} x(\mathbf{p}^{3T}\mathbf{X}) - (\mathbf{p}^{1T}\mathbf{X}) = 0 \\ y(\mathbf{p}^{3T}\mathbf{X}) - (\mathbf{p}^{2T}\mathbf{X}) = 0 \\ x(\mathbf{p}^{2T}\mathbf{X}) - y(\mathbf{p}^{1T}\mathbf{X}) = 0 \end{array} \Rightarrow \begin{bmatrix} x\mathbf{p}^{3T} - \mathbf{p}^{1T} \\ y\mathbf{p}^{3T} - \mathbf{p}^{2T} \\ x'\mathbf{p}'^{3T} - \mathbf{p}'^{1T} \\ y'\mathbf{p}'^{3T} - \mathbf{p}'^{2T} \end{bmatrix} \mathbf{X} = 0$$

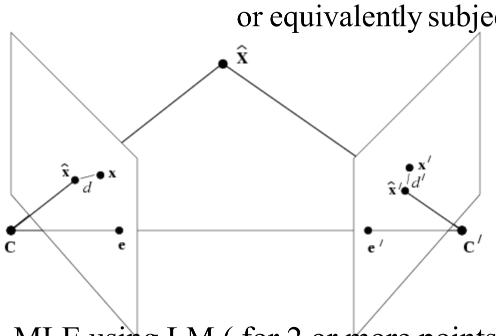
- Solve for **X**
- Homogeneous method (DLT) — invariance
- Inhomogeneous method — $(\mathbf{AH}^{-1})(\mathbf{HX}) = \mathbf{e}$
- No physical meaning, not optimal

Minimizing Geometric error

- Find **X** that minimizes difference between projected and measured points
- Cost function: $C(\mathbf{x},\mathbf{x}') = d(\mathbf{x},\hat{\mathbf{x}})^2 + d(\mathbf{x}',\hat{\mathbf{x}}')^2$

  subject to $\hat{\mathbf{x}}'^{\mathrm{T}}\mathbf{F}\hat{\mathbf{x}} = 0$

  or equivalently subject to $\hat{\mathbf{x}} = \mathbf{P}\hat{\mathbf{X}}$ and $\hat{\mathbf{x}}' = \mathbf{P}'\hat{\mathbf{X}}$



- MLE using LM (for 2 or more points)
- Sampson error (1st order approximation)

Minimizing Geometric error

- Reformulation of the minimizaton problem

$$\min_{\hat{\mathbf{x}}} C = d(\mathbf{x},\hat{\mathbf{x}})^2 + d(\mathbf{x}',\hat{\mathbf{x}}')^2 \Rightarrow \min_{t} C = d(\mathbf{x},\mathbf{l}(t))^2 + d(\mathbf{x}',\mathbf{l}'(t))^2$$

- **l** and **l'** range over all choices of corresponding epipolar lines
- $\hat{\mathbf{x}}\ (\hat{\mathbf{x}}')$ is the closest point on the line **l** (**l'**) to $\mathbf{x}\ (\mathbf{x}')$

Minimizing Geometric error

- Minimization method:

(i) Parameterize the pencil of epipolar lines in the first image by a parameter t. Epipolar line is $\mathbf{l}(t)$.

(ii) Using the fundamental matrix $\mathbf{F}$, compute the corresponding epipolar line $\mathbf{l}'(t)$ in the second image.

(iii) Express the distance function $d(\mathbf{x},\mathbf{l}(t))^2 + d(\mathbf{x}',\mathbf{l}'(t))^2$ explicitly as a function of t.

(iv) Find the value of t that minimizes this function

- Provides optimal solution

Minimizing Method

- Assume points $\mathbf{x}$ and $\mathbf{x}'$ are at the origin $(0,0,1)^T$, and the epipoles are $(1,0,f)^T$ and $(1,0,f')^T$
- Then $\mathtt{F}(1,0,f)^\top = (1,0,f')\mathtt{F} = \mathbf{0}$,

$$\mathtt{F} = \begin{pmatrix} ff'd & -f'c & -f'd \\ -fb & a & b \\ -fd & c & d \end{pmatrix}$$

- The epipolar lines of function t

$$(0,t,1) \times (1,0,f) = (tf, 1, -t) \longrightarrow d(\mathbf{x}, \mathbf{l}(t))^2 = \frac{t^2}{1+(tf)^2}$$

$$\mathbf{l}'(t) = \mathtt{F}(0,t,1)^\top = (-f'(ct+d), at+b, ct+d)^\top \longrightarrow d(\mathbf{x}', \mathbf{l}'(t))^2 = \frac{(ct+d)^2}{(at+b)^2 + f'^2(ct+d)^2}$$

- The total squared distance and the derivative

$$s(t) = \frac{t^2}{1+f^2t^2} + \frac{(ct+d)^2}{(at+b)^2 + f'^2(ct+d)^2}$$

$$s'(t) = \frac{2t}{(1+f^2t^2)^2} - \frac{2(ad-bc)(at+b)(ct+d)}{((at+b)^2 + f'^2(ct+d)^2)^2}.$$

- 6 real roots $\quad s'(t) = 0 \Rightarrow$

$$\begin{aligned} g(t) &= t((at+b)^2 + f'^2(ct+d)^2)^2 \\ &\quad -(ad-bc)(1+f^2t^2)^2(at+b)(ct+d) \\ &= 0. \end{aligned}$$

Minimizing Method

<u>Objective</u>

Given a measured point correspondence $\mathbf{x} \leftrightarrow \mathbf{x}'$, and a fundamental matrix $\mathtt{F}$, compute the corrected correspondences $\hat{\mathbf{x}} \leftrightarrow \hat{\mathbf{x}}'$ that minimize the geometric error (11.1) subject to the epipolar constraint $\hat{\mathbf{x}}'^\top \mathtt{F} \hat{\mathbf{x}} = 0$.

<u>Algorithm</u>

(i) Define transformation matrices

$$\mathtt{T} = \begin{bmatrix} 1 & & -x \\ & 1 & -y \\ & & 1 \end{bmatrix} \text{ and } \mathtt{T}' = \begin{bmatrix} 1 & & -x' \\ & 1 & -y' \\ & & 1 \end{bmatrix}.$$

These are the translations that take $\mathbf{x} = (x, y, 1)^\top$ and $\mathbf{x}' = (x', y', 1)^\top$ to the origin.

(ii) Replace $\mathtt{F}$ by $\mathtt{T}'^{-\top}\mathtt{F}\mathtt{T}^{-1}$. The new $\mathtt{F}$ corresponds to translated coordinates.

(iii) Compute the right and left epipoles $\mathbf{e} = (e_1, e_2, e_3)^\top$ and $\mathbf{e}' = (e'_1, e'_2, e'_3)^\top$ such that $\mathbf{e}'^\top \mathtt{F} = \mathbf{0}$ and $\mathtt{F}\mathbf{e} = \mathbf{0}$. Normalize (multiply by a scale) $\mathbf{e}$ such that $e_1^2 + e_2^2 = 1$ and do the same to $\mathbf{e}'$.

(iv) Form matrices

$$\mathtt{R} = \begin{bmatrix} e_1 & e_2 & \\ -e_2 & e_1 & \\ & & 1 \end{bmatrix} \text{ and } \mathtt{R}' = \begin{bmatrix} e'_1 & e'_2 & \\ -e'_2 & e'_1 & \\ & & 1 \end{bmatrix}$$

and observe that $\mathtt{R}$ and $\mathtt{R}'$ are rotation matrices, and $\mathtt{R}\mathbf{e} = (1, 0, e_3)^\top$ and $\mathtt{R}'\mathbf{e}' = (1, 0, e'_3)^\top$.

(v) Replace $\mathtt{F}$ by $\mathtt{R}'\mathtt{F}\mathtt{R}^\top$. The resulting $\mathtt{F}$ must have the form (11.3).

(vi) Set $f = e_3$, $f' = e'_3$, $a = F_{22}$, $b = F_{23}$, $c = F_{32}$ and $d = F_{33}$.

(vii) Form the polynomial $g(t)$ as a polynomial in t according to (11.7). Solve for t to get 6 roots.

(viii) Evaluate the cost function (11.5) at the real part of each of the roots of $g(t)$ (alternatively evaluate at only the real roots of $g(t)$). Also, find the asymptotic value of (11.1) for $t = \infty$, namely $1/f^2 + c^2/(a^2 + f'^2 c^2)$. Select the value $t_{\min}$ of t that gives the smallest value of the cost function.

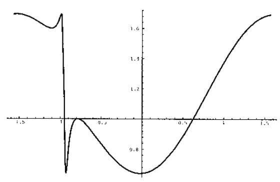
(ix) Evaluate the two lines $\mathbf{l} = (tf, 1, -t)$ and $\mathbf{l}'$ given by (11.4) at $t_{\min}$ and find $\hat{\mathbf{x}}$ and $\hat{\mathbf{x}}'$ as the closest points on these lines to the origin. For a general line (λ, μ, ν), the formula for the closest point on the line to the origin is $(-\lambda\nu, -\mu\nu, \lambda^2 + \mu^2)$.

(x) Transfer back to the original coordinates by replacing $\hat{\mathbf{x}}$ by $\mathtt{T}^{-1}\mathtt{R}^\top\hat{\mathbf{x}}$ and $\hat{\mathbf{x}}'$ by $\mathtt{T}'^{-1}\mathtt{R}'^\top\hat{\mathbf{x}}'$.

(xi) The 3-space point $\widehat{\mathbf{X}}$ may then be obtained by the homogeneous method of section 11.2.

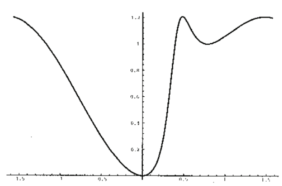
Minimizing Method

- Local minima
  - ✓ Cost function may have local minima



a

a) Three minima

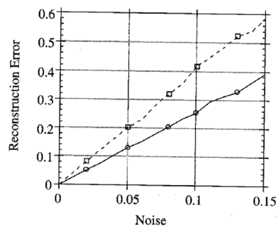


b

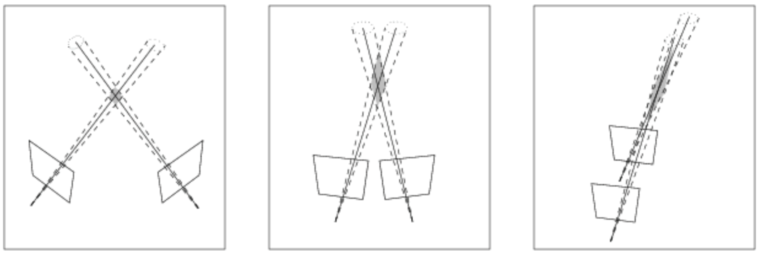
b) Two minima

Uncertainty of reconstruction

- Reconstruction error comparison of triangulation methods



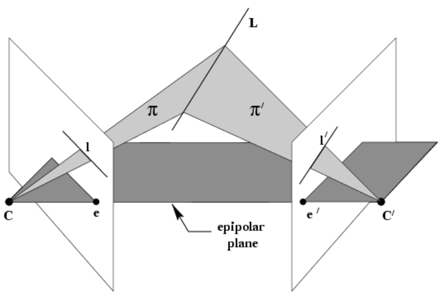
- Uncertainty of reconstruction
  - ✓ The shape of uncertainty depends on the angle between the rays



Line reconstruction

- The planes defined by the lines are $\boldsymbol{\pi} = \mathbf{P}^{\mathsf{T}}\mathbf{l}$ and $\boldsymbol{\pi}' = \mathbf{P}'^{\mathsf{T}}\mathbf{l}'$

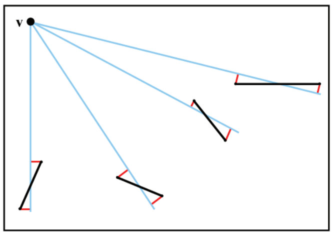
$$\mathbf{L} = \begin{bmatrix} \mathbf{l}^{\mathsf{T}}\mathbf{P} \\ \mathbf{l}'^{\mathsf{T}}\mathbf{P}' \end{bmatrix} \qquad \mathbf{LX} = 0$$

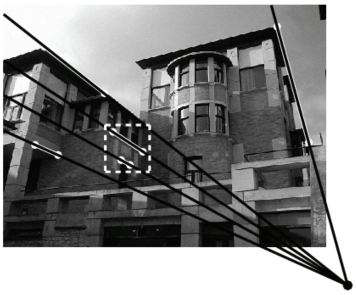


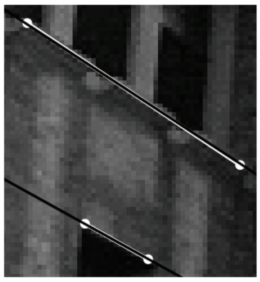
- Degeneracy
  - ✓ Lines intersect the camera baseline can be poorly localized in a reconstruction.

Computing vanishing points

- Estimation of vanishing point
  - ✓ Line intersection problem
  - ✓ Minimize the distances from the endpoints of measured imaged parallel line segments
  - ✓ Maximum likelihood estimate (MLE)
  - ✓ Levenberg-Marquardt algorithm (LM)

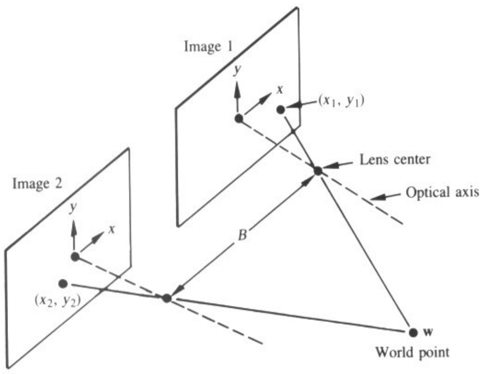






Dense Structure recovery

- Mesh surface modeling
- Dense stereo matching



$$Z = \frac{Bf}{d},$$ where the disparity $d = (x_1 - x_2)$

Dense Structure recovery

image I(x,y) Disparity map d(x,y) image I´(x´,y´)

(x´,y´)=(x+d(x,y),y)