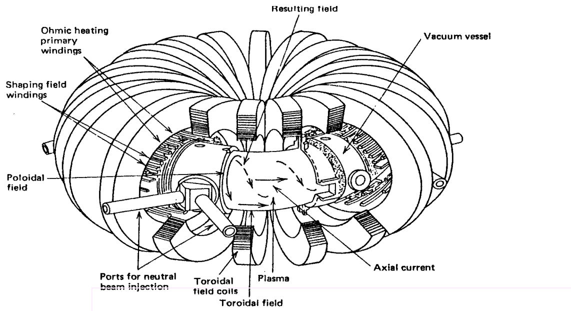
Chapter 3. Closed Magnetic Confinement Systems

Reading assignments: **Harms Chap. 10,**
Stacey Chap. 4,



J.M. Rawls, DOE/ER-0034 (1979), Fig. 1-1

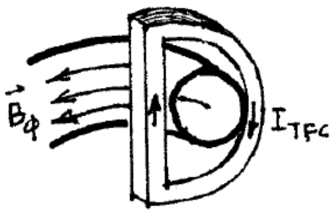
1. Tokamak system

Russian: **TO**roidalnaya **KA**mera **MA**gnit **K**atushka
(English: Toroidal Chamber Magnetic Coil)

A. Features

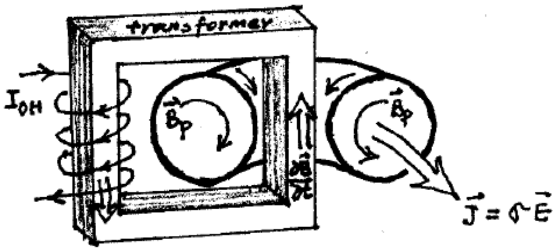
Magnetic fields :

∗ ***Toroidal field*** $\boldsymbol{B}_\phi$ produced by TFC
(toroidal field coils) around a torus



+

∗ ***Polodal field*** $\boldsymbol{B}_p$ produced by plasma
current $\boldsymbol{j}_\phi$ induced by transformer

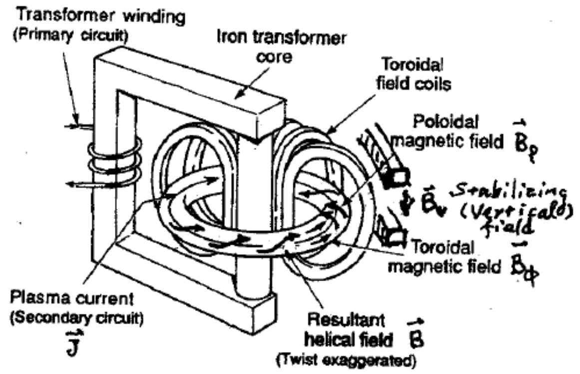


$$(-\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \boldsymbol{E} \quad \Rightarrow \quad -\frac{d\Psi_{trans}}{dt} = \oint \boldsymbol{E}\cdot d\boldsymbol{l}$$

$$\Rightarrow \quad E_\phi = -\frac{1}{2\pi R}\frac{d\Psi_{trans}}{dt}$$

$$\Rightarrow \quad \boldsymbol{j}_\phi = \sigma \boldsymbol{E} = \frac{\boldsymbol{E}}{\eta}$$

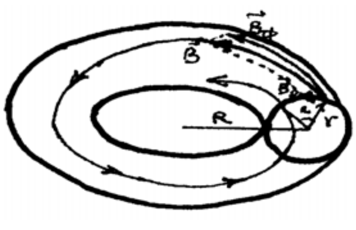
⇓

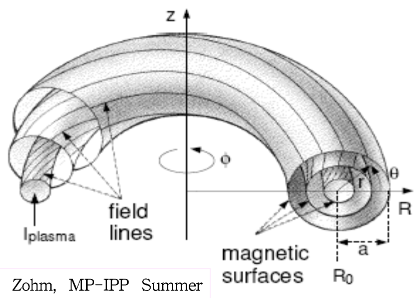


∗ ***Helical field lines B***
with small pitch
or ***rotational transform*** (ι)

$$\frac{\iota r}{2\pi R} = \frac{B_p}{B_\phi}$$

$$\Rightarrow \quad \iota = \frac{2\pi}{(rB_\phi / RB_p)} \equiv \frac{2\pi}{q}$$





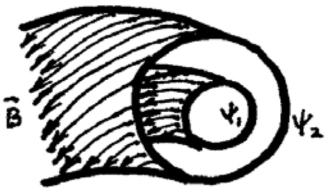
H. Zohm, MP-IPP Summer Univ. (2005), Fig. 6.3

Safety factor $q \equiv \frac{2\pi}{\iota} = \frac{r}{R}\frac{B_\phi}{B_p}$ (1)

= # of transits around the torus when the field lines go around 2π in the poloidal angle

Magnetic ***shear*** S :

$$S \propto \frac{\iota'}{\iota} \propto \frac{q'}{q}$$



Magnetic surface with a constant ψ covered with ***ergodic field lines.***

OHC(ohmic heating coils) produce self-consistent plasma current $\boldsymbol{j}_\phi$, and thereby poloidal magnetic field $\boldsymbol{B}_p$ or $\boldsymbol{B}_\theta$, and ohmic heating power P_{OH}.

$$\frac{\partial I_{OHC}}{\partial t} \Rightarrow \frac{\partial \boldsymbol{B}}{\partial t} \Rightarrow \boldsymbol{E}_\phi(r) \Rightarrow \boldsymbol{J}_\phi(r) \Rightarrow \boldsymbol{B}_p(r)$$

$$\int_{volume} \eta J^2 d^3r = P_{OH} \Rightarrow T_e^{\uparrow}(r)$$: Ohmic heating limited by $\eta^{\downarrow} \propto T_e^{\uparrow\,-3/2}$

VFC(vertical field coils) or EFC(equilibrium field coils) produce ***equilibrium field*** $\boldsymbol{B}_V$ to prevent a toroidal plasma column from moving toward the outboard side of torus

Features

$\boldsymbol{B}_p$ by internal plasma current

Pulsed and complicated operation

Complex coil geometry ($\boldsymbol{B}_\phi > \boldsymbol{B}_p > \boldsymbol{B}_V$)

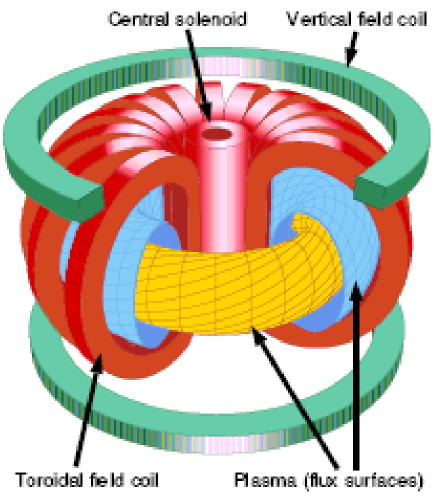
Difficult engineering design

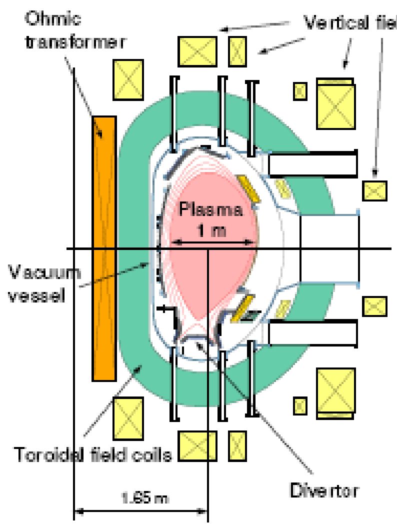
Large minimum unit size ($n\tau_E \propto I^a \times V^b$)

High $n\tau_E$

Flexibility of heating(NBI, RFs, Compression, ...) and refueling(gas puffing, pellet, ...)

B. Main components and their functions





ASDEX-U at Garching

W. Suttrop, MP-IPP Summer Univ. (2005), Figs. 7.3-7.4

1) Toroidal vacuum vessel

- High strength & resistance material (S.S. – low η)
  with bellows (high η, 14 in TFTR ⇒ ~3 *mΩ*) sections
  (or ceramic breaks + insulated slit)
- Base pressure ~ $< 10^{-7}$ *torr* (clean, no leaks, no gassy matter)
  Bake out (~150° C), Gas discharge cleaning
- Main turbopumps (2,000 *l/s* SNUT-79, 10,000 *l/s* TFTR)
  Auxiliary pump (Rotary 1,200 *l/m* SNUT-79 ,
  T_i gettering, ZrAl getter panel in TFTR $\sim 10^{-9}$ *torr*)
- Internal structures : Plasma facing components (PFC)
  Limiters (C tile : good refractive, thermal, electrical conductor, low Z)
  Diverter plates
  Antennas
  Protective plates (C tile) : protect "shine thru" of NB
  Bellows covers : protect runaway electrons
- Manhole access ports for diagnostics, heating, vacuum

2) Fuel systems

Gas puffing ($\sim 10^{-3}$ *torr*)

Pellets (frozen fuel) : 2 *mm* × 4 *mm,* 10^3 *m/s* in TFTR

3) Toroidal field coils + power supply

For toroidally axisymmetric tokamaks,

$$\boldsymbol{B} = B_\Phi(r,\Theta)\,\hat{\Phi}$$

$$\nabla\times\boldsymbol{B} = \mu_o \boldsymbol{j}$$

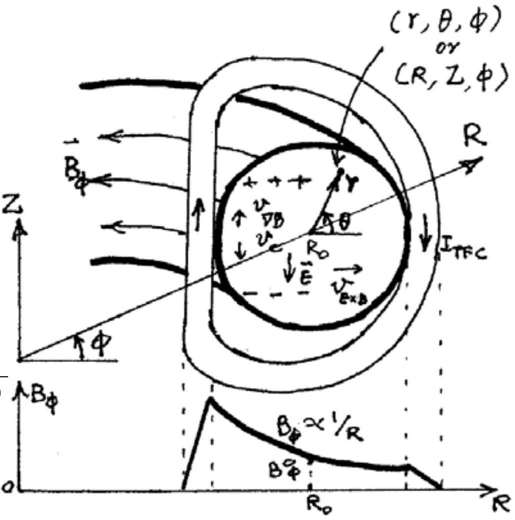
$$\oint \boldsymbol{B}_\Phi \cdot d\boldsymbol{l} = \mu_o I_c$$

$$2\pi R B_\Phi = \mu_o I_c$$

$$B_\Phi(R) = \frac{\mu_o I_c}{2\pi}\frac{1}{R} = \frac{B_\Phi^o R_o}{R}$$

$$= \frac{B_\Phi^o R_o}{R_o + r\cos\Theta} = \frac{B_\Phi^o}{1+(r/R_o)\cos\Theta}$$

$$= \frac{B_\Phi^o}{1+\varepsilon\cos\Theta} = B_\Phi(r,\Theta) \quad (2)$$



Drift motions in the simple toroidal magnetic field :

$$\boldsymbol{v}_D = \boldsymbol{v}_c + \boldsymbol{v}_{\nabla B} = \frac{m}{qB^2}\left[v_\parallel^2 \frac{\boldsymbol{R}\times\boldsymbol{B}}{R^2} + \left(\frac{v_\perp^2}{2}\right)\left(\frac{\boldsymbol{B}}{B}\times\nabla B\right)\right] = \frac{m}{q}\frac{1}{R_o B_\Phi^o}\left[v_\parallel^2 + \frac{v_\perp^2}{2}\right]\hat{\boldsymbol{z}}$$

⇒ charge separation of $q = e \,\&\, i$ ⇒ $\boldsymbol{E}$ ⇒ $\boldsymbol{v}_{E\times B} = \frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2} = \frac{E}{B_\Phi^o}\frac{\boldsymbol{R}}{R_o}$

Discrete structure of TF coils

→ ripple ($\triangle B_\phi / B_\phi^o$ < 1%) → $B_\phi(r, \Theta, \phi)$ for nonaxisymm.

D-shaped pure tension coil

4) Poloidal field coils + power supply

a. Ohmic heating coils w/ air-core or ion-core

Provides ohmic current → ι + shear + heating

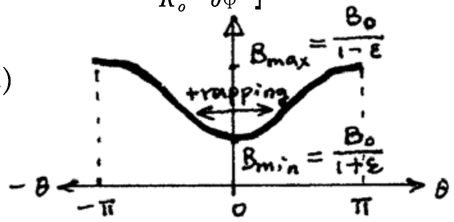
$$\boldsymbol{j}_\phi = j_\phi(r,\Theta)\,\hat{\Theta} \Rightarrow B_\Theta(r) = \frac{\mu_o I_\phi(r)}{2\pi r}$$

$\nabla \times \boldsymbol{B} = \mu_o \boldsymbol{j}$

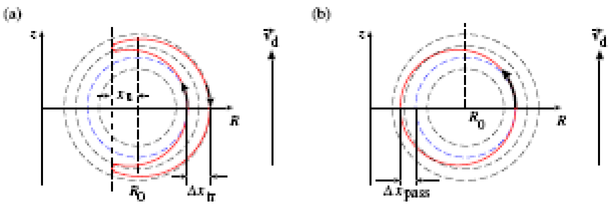
or $\nabla \cdot \boldsymbol{B} = 0 \Rightarrow \frac{1}{1+\varepsilon\cos\Theta}\left[\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{1}{r}\frac{\partial}{\partial \Theta}((1+\varepsilon\cos\Theta)B_\Theta) + \frac{1}{R_o}\frac{\partial B_\phi}{\partial \phi}\right] = 0$

$$\Rightarrow B_\Theta(r,\Theta) = \frac{B_\Theta^o(\Theta=0)}{1+\varepsilon\cos\Theta} \quad (3)$$

$$\boldsymbol{B}(r,\theta) = B_\theta(r,\theta)\,\hat{\theta} + B_\phi(r,\theta)\,\hat{\phi} = \frac{B_o}{1+\epsilon\cos\theta}$$



W. Suttrop, MP-IPP Summer Univ. (2005), Fig. 7.2

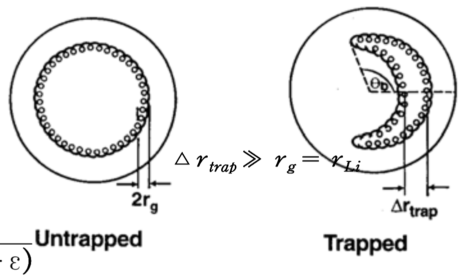


Effects of B_p or B_Θ :

i) Rotational transform ι ⇒ canceling $\boldsymbol{v}_D$ ⇒ confinement

ii) Particle trapping by magnetic mirrors

Trapped particles with banana orbits
Untrapped particles with circular orbits



Trapped fraction :

$$f_{trap} = \sqrt{1-1/R_m} = \sqrt{1-R_{min}/R_{max}}$$
$$= \sqrt{1-(1-\varepsilon)/(1+\varepsilon)} = \sqrt{2\varepsilon/(1+\varepsilon)}$$

(e.g.) For a typical tokamak, $\varepsilon \equiv a/R_o \approx 1/3$ ⇒ $f_{trap} \approx 70\%$

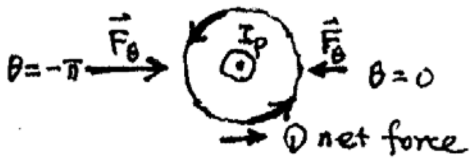
Collisional excursion across flux surfaces :

Untrapped particles = $2r_g = 2r_{Li}$
Trapped particles = $\triangle r_{trap} \gg 2r_g$ ⇒ Enhanced banana particle losses

$\boldsymbol{E}_\phi \times B_\Theta$ = radially inward drifts of banana ptcls ⇒ **Ware-pinch** effect

iii) Force imbalance : ① Hoop force

② Tire-tube force



③ Cetrifugal force by rotating plasma

b. Vertical field (or Equilibrium field) coils

Correct the loss of equilibrium due to loop force

- **<u>Copper stabilizing shell</u>** (old way)

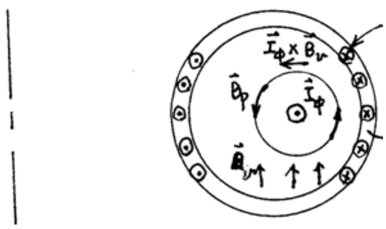


image current

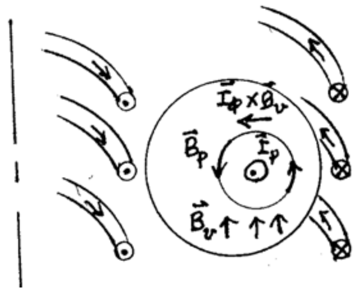
Cu shell :

finite thickness or hole on it

$\Rightarrow \quad \tau_m = \mu_o \sigma (\triangle r)^2 \le 10\ ms$

no use in 100 ms ~ 10 s

- **<u>Vertical field coils</u>** (present active control)

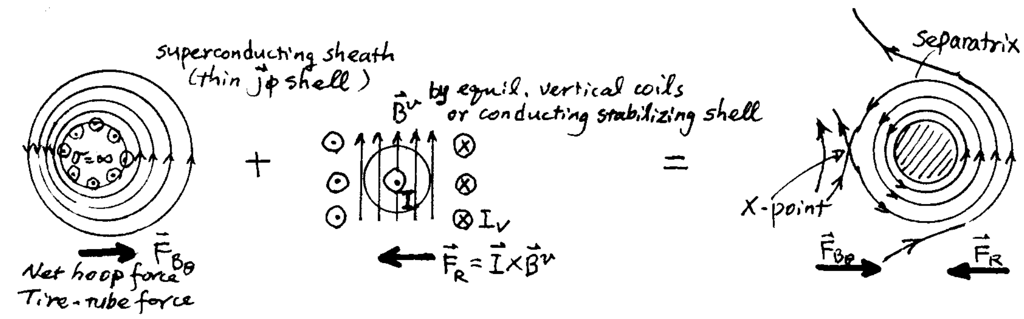


Helmholtz coil arrangement :

Neutrally stable to vertical displacement

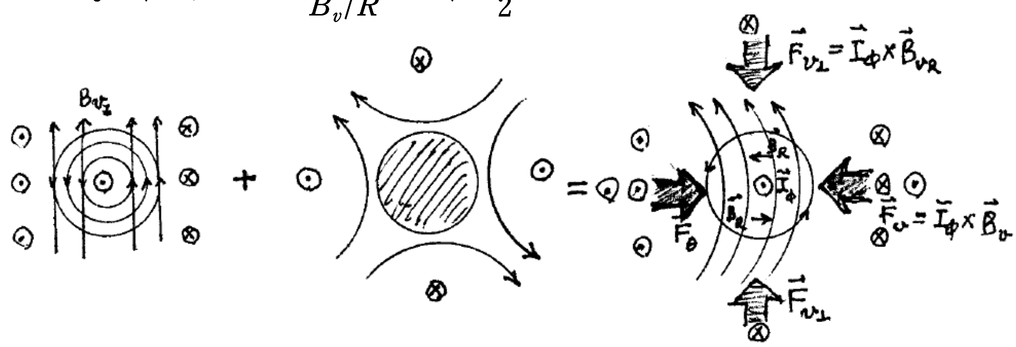
G-S. eq. $\Rightarrow$

$$B_{vz} \approx \frac{\mu_o I_\phi}{4\pi R_o}\left[\ln\left(\frac{8R_o}{a}\right) + \frac{p}{B_o^2/2\mu_o} - \frac{3}{2} + \frac{l_i}{2}\right]$$



Stabilizing condition for vertical and horizontal displacements :

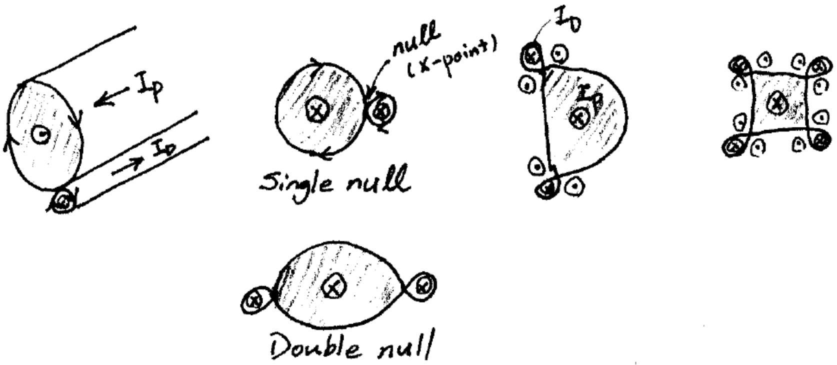
$$0 \;<\; n \equiv -\frac{\partial B_v / \partial R}{B_v / R} \;<\; \frac{3}{2} \tag{4}$$



Helmholtz field　　Quadrupole field　　Equilibrium field

Note) $\quad B_\phi : B_\theta : B_V \approx 1 : \frac{\varepsilon}{q} : \frac{\varepsilon^2}{q}$

c. **Shaping (Divertor) field coils**



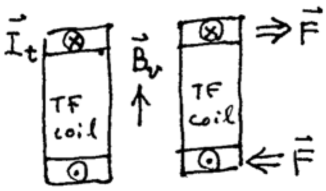
5) Mechanical structure

Torque frame, Coil and buswork supports

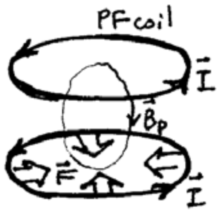
High magnetic forces

Tilting force due to $\vec{I_t} \times \vec{B_v}$

$\approx 10^6\ lbs \approx 500\ tons$

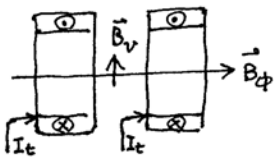


Force on PF coils from their colleague neighbors



Buswork carrying current to coils × ***B***

(e.g.) $20\ kA \times 3\ T = 6{\times}10^4\ N/m = 6\ ton/m$



6) Basic diagnostic systems + Data acquisition & process systems

Principal plasma parameters :

$n_{e,i}(\boldsymbol{r},t)$, $T_{e,i}(\boldsymbol{r},t)$, $Z_{eff}(t)$, $Z_i(\boldsymbol{r},t)$, $P_R(\boldsymbol{r},t)$

Wave active instabilities : $\tilde{T}_e(\boldsymbol{r},t)$, $\tilde{B}(\boldsymbol{r},t)$

Fusion products : $S_n(\boldsymbol{r},t)$, $S_\alpha(\boldsymbol{r},t)$

Not measured well yet for high powered tokamaks :

$j(\boldsymbol{r},t)$, $E_r(\boldsymbol{r},t) \rightarrow \phi(\boldsymbol{r},t)$

7) Auxiliary heating systems

NBI, RF, Adiabatic compression, etc

8) Shielding

X-rays, Neutrons, Scattered γ

C. Tokamak Equilibrium

1) Equilibrium equations in toroidally axisymmetric ($\frac{\partial}{\partial \phi} = 0$) systems

a. Magnetic fluxes

$$\nabla p = \boldsymbol{j} \times \boldsymbol{B} \qquad (5)$$

$$\nabla \times \boldsymbol{B} = \mu_o \boldsymbol{j} \qquad (6)$$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad (7)$$

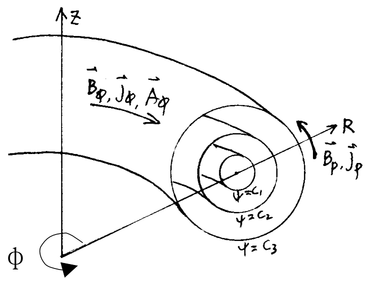
$$\nabla \cdot \boldsymbol{j} = 0 \qquad (8)$$

$(5)_\parallel \Rightarrow \nabla_\parallel p = 0$: p = const along $\boldsymbol{B}$

$\boldsymbol{B} \cdot (5) \Rightarrow \boldsymbol{B} \cdot \nabla p = \boldsymbol{B} \cdot (\boldsymbol{j} \times \boldsymbol{B}) = 0$: $\boldsymbol{B} \perp \nabla p$ (9)

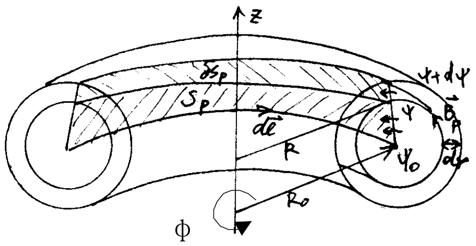
Ch.2(6) $\Rightarrow \boldsymbol{B} \cdot \nabla \psi = 0$: $\boldsymbol{B} \perp \nabla \psi$ (10)

where $2\pi\psi = \int_S \boldsymbol{B} \cdot d\boldsymbol{S}$: magnetic flux (11)



$\Rightarrow$ Magnetic (flux) surface ($\psi = const.$)
= Isobaric surface ($p = const.$)

b. Poloidal magnetic flux



S_p = area of a ribbon obtained by revolving ψ_o and ψ

δS_p = 〃 between ψ and $\psi + d\psi$

Poloidal magnetic flux $2\pi\psi$:

(11) $\Rightarrow$ $2\pi\psi \equiv \int_{S_p} \boldsymbol{B_p} \cdot d\boldsymbol{S_p}$

Stoke's theorem

$$= \int_{S_p} \nabla \times (A_\phi \hat{\Phi}) \cdot d\boldsymbol{S_p} = \oint A_\phi \hat{\Phi} \cdot d\boldsymbol{l}$$

$$= 2\pi (RA_\phi - R_o A_{\phi_o}) \qquad (12)$$

Differential poloidal magnetic flux $2\pi\, d\psi$:

$$2\pi\, d\psi = 2\pi(\psi + d\psi) - 2\pi\psi$$

$$= \int_{S_p + \delta S_p} \boldsymbol{B_p} \cdot d\boldsymbol{S_p} - \int_{S_p} \boldsymbol{B_p} \cdot d\boldsymbol{S_p}$$

$$= \int_{\delta S_p} \boldsymbol{B_p} \cdot d\boldsymbol{S_p} \approx B_p 2\pi R\, dr \qquad (13)$$

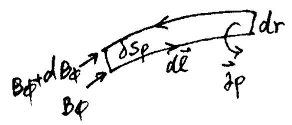
$$\Rightarrow \quad RB_p = \frac{d\psi}{dr} = |\nabla\psi| \qquad (14)$$

$$\Rightarrow \quad \boldsymbol{B_p} = -\frac{1}{R} \hat{\Phi} \times \nabla\psi \qquad (15)$$

c. Poloidal current function $F(\psi)$

$$\nabla \times \boldsymbol{B}_\Phi = \mu_o \boldsymbol{j}_p \qquad (B_\Phi + dB_\Phi)2\pi R - B_\Phi 2\pi R$$

$$\int_{\delta S_p} \nabla \times \boldsymbol{B}_\Phi \cdot d\boldsymbol{S}_p = \oint \boldsymbol{B}_\Phi \cdot d\boldsymbol{l} = 2\pi\, d(RB_\Phi) = 2\pi\, d\psi \frac{\partial (RB_\Phi)}{\partial \psi}$$



$$\int_{\delta S_p} \mu_o \boldsymbol{j}_\Phi \cdot d\boldsymbol{S}_p = \mu_o j_p 2\pi R dr$$

$$\Rightarrow \quad \frac{\partial (RB_\Phi)}{\partial \psi} = \frac{\mu_o j_p R}{(d\psi / dr)}$$

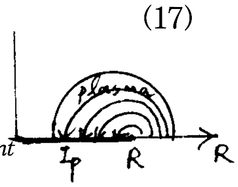
$$\Rightarrow \quad R j_p = \frac{\partial F}{\partial \psi}\frac{\partial \psi}{\partial r} = \frac{\partial F}{\partial \psi} | \nabla \psi | = \frac{\partial F}{\partial r} = | \nabla F(\psi) | \qquad (16)$$

$$\Rightarrow \quad \boldsymbol{j}_p = -\frac{1}{R}\hat{\Phi} \times \nabla F \qquad (17)$$

where $\qquad \nabla \times \boldsymbol{B}_\Phi = \mu_o \boldsymbol{j}_p \Rightarrow 2\pi R B_\Phi = \mu_o I_p$

$$F(\psi) \equiv \frac{RB_\Phi}{\mu_o} = \frac{I_p(\psi)}{2\pi} \qquad (18)$$

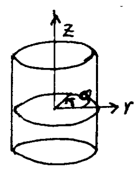
total poloidal current



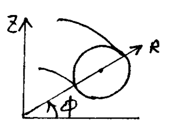
Note) Symmetry of $\boldsymbol{B}$ and $\boldsymbol{j}$ in (5)(7)(8) $\Rightarrow$ (14)(15) $\leftrightarrow$ (16)(17)

d. Magnetic fields and Pressure

In cylindrical coordinates for toroidally axisymmetric fields ($\partial / \partial \Phi = 0$),



(7)(10) :
$$\begin{cases} \dfrac{1}{R}\dfrac{\partial}{\partial R}(RB_R) + \dfrac{\partial B_z}{\partial z} = 0 \\ B_R \dfrac{\partial \psi}{\partial R} + B_z \dfrac{\partial \psi}{\partial z} = 0 \end{cases}$$



$$\Rightarrow \quad \begin{cases} B_R = -\dfrac{1}{R}\dfrac{\partial \psi}{\partial z} \\ B_z = \dfrac{1}{R}\dfrac{\partial \psi}{\partial R} \end{cases} \qquad (19)$$

(19) in (9) :

$$-\frac{1}{R}\frac{\partial \psi}{\partial z}\frac{\partial p}{\partial R} + \frac{1}{R}\frac{\partial \psi}{\partial R}\frac{\partial p}{\partial z} = 0$$

$$\Rightarrow \quad p = p(\psi) \qquad (20)$$

$$\textit{since} \quad -\frac{\partial \psi}{\partial z}\frac{\partial p(\psi)}{\partial \psi}\frac{\partial \psi}{\partial R} + \frac{\partial \psi}{\partial R}\frac{\partial p(\psi)}{\partial \psi}\frac{\partial \psi}{\partial z} = 0$$

e. Current density

(6): $\boldsymbol{j} = \dfrac{1}{\mu_o}\nabla \times \boldsymbol{B}$

$$j_\Phi = \frac{1}{\mu_o}\left(\frac{\partial B_R}{\partial z} - \frac{\partial B_z}{\partial R}\right) \overset{(19)}{=} -\frac{1}{\mu_o}\left[\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \psi}{\partial R}\right) + \frac{1}{R}\frac{\partial^2 \psi}{\partial z^2}\right] \equiv -\frac{1}{\mu_o R}\triangle^* \psi \qquad (21)$$

$$j_z = \frac{1}{\mu_o}\frac{1}{R}\frac{\partial}{\partial R}(RB_\Phi) \overset{(18)}{=} \frac{1}{R}\frac{\partial F(\psi)}{\partial R} \qquad (22)$$

f. Grad(-Schlueter)-Shafranov equation

= force balance equation in terms of flux function

(5): $\nabla p = \boldsymbol{j} \times \boldsymbol{B}$

$$\nabla p = (\boldsymbol{j}_\Phi + \boldsymbol{j}_p) \times (\boldsymbol{B}_\Phi + \boldsymbol{B}_p)$$

$$= \left(\boldsymbol{j}_\Phi - \frac{1}{R}\hat{\Phi} \times \nabla F\right) \times \left(\boldsymbol{B}_\Phi - \frac{1}{R}\hat{\Phi} \times \nabla \Psi\right)$$

$$= -\boldsymbol{j}_\Phi \times \left(\frac{1}{R}\hat{\Phi} \times \nabla \Psi\right) - \left(\frac{1}{R}\hat{\Phi} \times \nabla F\right) \times \boldsymbol{B}_\Phi$$

$\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{C} \times \boldsymbol{B}) \times \boldsymbol{A} = (\boldsymbol{A} \cdot \boldsymbol{C})\boldsymbol{B} - (\boldsymbol{A} \cdot \boldsymbol{B})\boldsymbol{C}$

$$= \frac{j_\Phi}{R}\nabla\Psi - \frac{B_\Phi}{R}\nabla F$$

$$\nabla p(\Psi) = \frac{j_\Phi}{R}\nabla\Psi - \frac{B_\Phi}{R}\nabla F(\Psi) \tag{23}$$

(21)(18) $\quad \frac{\partial F(\Psi)}{\partial \Psi}\nabla\Psi$

$$\frac{\partial p(\psi)}{\partial \psi}\nabla\psi = -\frac{\triangle^*\psi}{\mu_o R^2}\nabla\psi - \frac{\mu_o F}{R^2}\nabla F$$

$$\Rightarrow \quad \triangle^*\Psi = -\mu_o R^2 \frac{dp}{d\Psi} - \mu_o^2 F \frac{dF}{d\Psi} \tag{24}$$

(nonlinear elliptic PDE)

$\boldsymbol{j}_\Phi \times \boldsymbol{B}_p$ $\qquad \nabla p$ $\qquad \boldsymbol{j}_p \times \boldsymbol{B}_\Phi$

where $\triangle^*\Psi \equiv \left[R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}\right) + \frac{\partial^2}{\partial z^2}\right]\Psi = \left(\frac{\partial^2}{\partial R^2} - \frac{1}{R}\frac{\partial}{\partial R} + \frac{\partial^2}{\partial z^2}\right)\Psi$

or from (23) & (21)

$$\mu_o j_\Phi = \mu_o R\frac{dp}{d\Psi} + \frac{\mu_o^2}{R}F\frac{dF}{d\Psi} = -\frac{1}{R}\triangle^*\Psi \tag{24*}$$

Notes)

i) $p(\Psi)$: plasma load on a magnetic flux surface

$F(\Psi)F'(\Psi) = \frac{1}{2}(F^2(\Psi))'$: strength of B_Φ

$\triangle^*\Psi$: strength of B_p

⇒ G-S Eq.(24) = Nonlinear elliptic PDE for Ψ describing how much plasma can be supported by ***B***.

ii) Ideally, LHS of G-S Eq. = 0

$$\Rightarrow \quad R^2 p' = \mu_o FF' = \frac{\mu_o}{2}(F^2)' = \frac{R^2}{2\mu_o}(B_\Phi^2)'$$

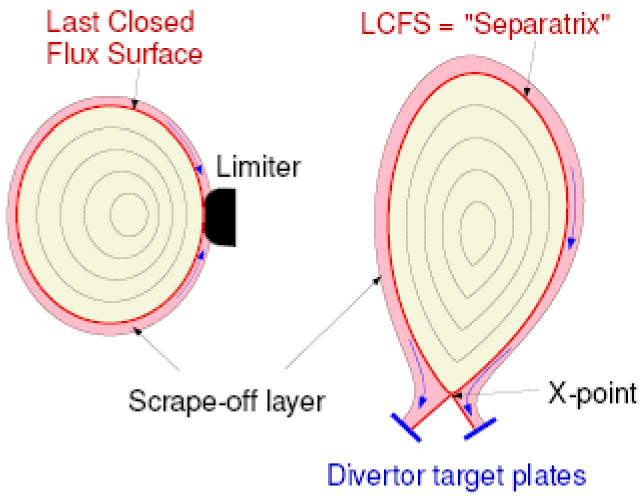
$$\Rightarrow \quad p = \frac{B_\Phi^2}{2\mu_o} \quad \Rightarrow \quad \beta \equiv \frac{p}{B_\Phi^2/2\mu_o} = 1 \quad : \quad \text{not realizable}$$

In reality, $B_p(a)$ supports $\frac{a}{R_o}$ of $\langle p \rangle$

$$\Rightarrow \quad \beta_p \equiv \frac{\langle p \rangle}{B_p^2(a)/2\mu_o} \leq \frac{R_o}{a} \equiv A \text{ : aspect ratio} \tag{25}$$

iii) If $B_p^\uparrow$ for balancing p, then $q^\downarrow = \frac{r}{R}\frac{B_\Phi}{B_p^\uparrow}$ → kink instability

For reasons of stability, plasma confinement by large B_Φ for $q^\uparrow$.



Typical flux configurations of limiter and divertor tokamaks

(W. Suttrop, MP-IPP Summer Univ. (2005), Fig. 7.5)

2) Pressure balance

Averaging G-S Eq. over a flux surface ψ , i.e., $\langle (24) \rangle_\psi \equiv \dfrac{\oint_\psi (24)\, dl_p/B_p}{\oint_\psi dl_p/B_p}$:

$$\langle p \rangle = p(\psi_a) + \frac{1}{2\mu_o}\left[\langle B_p^2 \rangle_{\psi(a)} + \langle B_\phi^2 \rangle_{\psi(a)} - \langle B_\phi^2 \rangle \right] \tag{26}$$

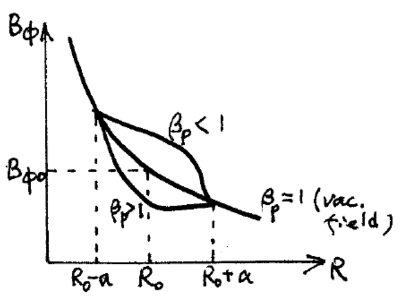
where $\langle p \rangle$ and $\langle B_\phi^2 \rangle$ are volume-average values ($\left(\int_{\psi(0)}^{\psi(a)} (\ )\, d^3r \Big/ \int_{\psi(0)}^{\psi(a)} d^3r \right)$

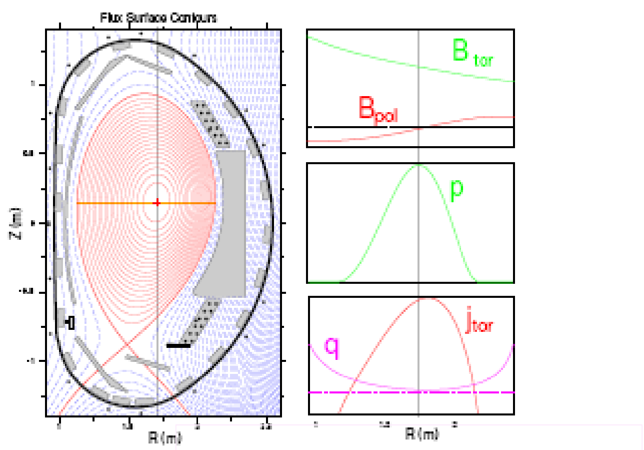
Poloidal beta β_p :

(26) $/ \langle B_p^2 \rangle_{\psi(a)} / 2\mu_o \quad \Rightarrow$

$$\beta_p \equiv \frac{\langle p \rangle}{\langle B_p^2 \rangle_{\psi(a)} / 2\mu_o} \approx 1 + \frac{\langle B_\phi^2 \rangle_{\psi(a)} - \langle B_\phi^2 \rangle}{\langle B_p^2 \rangle_{\psi(a)}} \tag{27}$$

	Paramag.	Diamag.
$\langle B_\phi^2 \rangle_{\psi(a)} - \langle B_\phi^2 \rangle$	< 0	> 0
β_p	< 1	> 1





W. Suttrop, MP-IPP Summer Univ. (2005), 7.8

Toroidal beta : $\quad \beta_t \equiv \dfrac{\langle p \rangle}{B_\phi^2/2\mu_o} = \beta_p \dfrac{B_p^2}{B_\phi^2}$ (28)

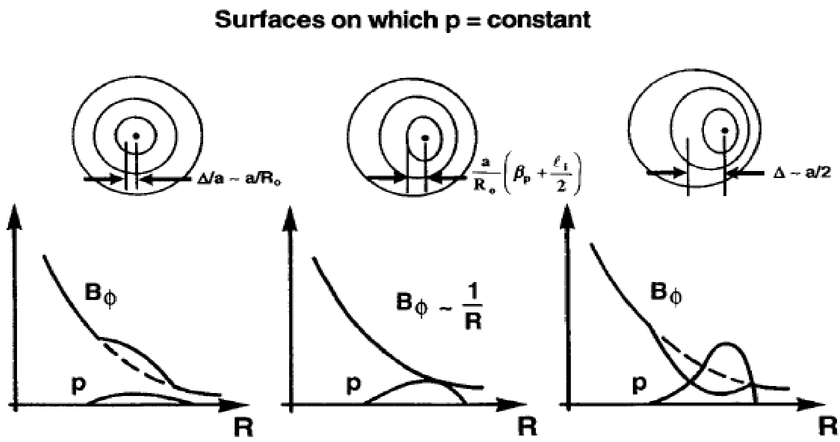
Total beta : $\quad \beta \equiv \dfrac{\langle p \rangle}{B^2/2\mu_o} = \beta_p \dfrac{B_p^2}{B_p^2 + B_\phi^2} = \dfrac{\beta_p}{1 + B_\phi^2 / B_p^2}$ (29)

Equilibrium characteristics

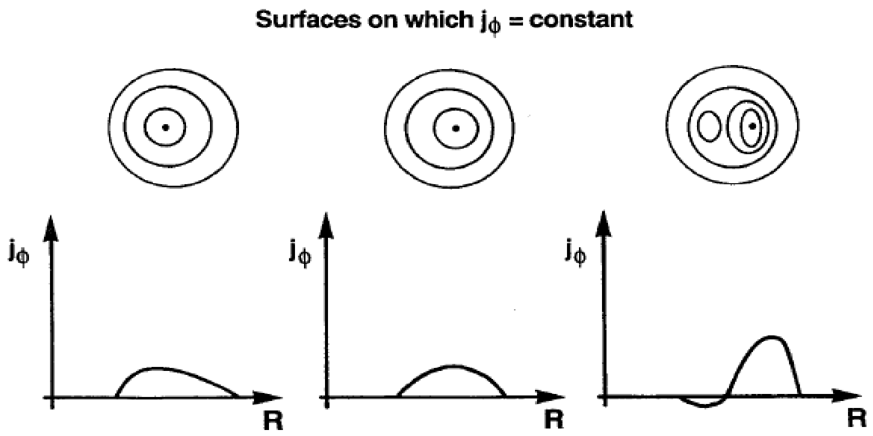
① *Low pressure*(low β) ② *Medium pressure*(typical) ③ *High pressure*(high β)

$\beta_p < 1$ $\beta_p \approx 1$ $\beta_p > 1$

Paramagnetic $B_\phi^{\uparrow}$ Almost vacuum field B_ϕ Diamagnetic $B_\phi^{\downarrow}$



Harms, p. 170



3) Low-β tokamak confinement

$F(\psi) \propto RB_\phi = const = 0$, $B_\phi = B_{\phi o} R_o / R$ (vac. field) case : $\beta_p \equiv \dfrac{\langle p \rangle}{B_p^2 / 2\mu_o} = 1$

⇒ $\langle p \rangle$ is supported by B_p only, i.e., $\boldsymbol{j}_\phi \times \boldsymbol{B}_p$ pinching force of self-mag. field induced by plasma current.

If B_Θ is intended to increase for balancing high p,

then, reduced $q(r) \downarrow \equiv (r/R_0)\ (B_{\phi o} / B_\Theta \uparrow)$ result in ideal kink instab.

Limited I_ϕ value or large B_ϕ are needed to maintain a limited q value for stable confinement.

Stability condition for ideal kink modes :

$$q(a) = \frac{a}{R_o}\frac{B_\phi}{B_p(a)} > q_{\min} = \begin{cases} 1 \quad \text{for m = 1 mode (K-S limit)} \\ \text{m/n} \approx 2.5 \quad \text{for higher modes m = 2, 3, ...} \end{cases} \tag{30}$$

$$\Rightarrow \frac{B_p}{B_\phi} < \frac{a/R_o}{q_{\min}} \equiv \frac{\varepsilon}{q_{\min}} \approx O(\varepsilon^2) \tag{31}$$

$$\Rightarrow \frac{\mu_o I_\phi/2\pi a}{B_\phi} < \frac{a/R_o}{q_{\min}} \Rightarrow I_\phi < \frac{2\pi}{\mu_o}\frac{a^2}{R_o q_{\min}} B_\phi \ : \textit{plasma current limit} \tag{32}$$

For low pressure plasmas ($\beta_p < 1$), $q_{\min} \approx 2.5$

$$(28) : \beta_t = \beta_p \frac{B_p^2}{B_\phi^2} < \beta_p \frac{\varepsilon^2}{q_{\min}^2} < \frac{\varepsilon^2}{q_{\min}^2} \approx \frac{\varepsilon^2}{6.25} \tag{33}$$

$$(29) : \beta = \frac{\beta_p}{1 + B_\phi^2/B_p^2} < \frac{\beta_p}{1 + q_{\min}^2/\varepsilon^2} < \frac{\varepsilon^2}{q_{\min}^2 + \varepsilon^2} \approx \frac{\varepsilon^2}{6.25 + \varepsilon^2} \ll 1 \tag{34}$$

2) High-β tokamak confinement

Ideal confinement by toroidal field

$$\Rightarrow \quad p = \frac{B_\phi^2}{2\mu_o} \quad \Rightarrow \quad \beta_t = 1 \quad : \textit{not realizable and unstable}$$

For suppressing ballooning modes, B_p supports more than a fraction a/R_0 of p :

$$\beta_p \equiv \frac{\langle p \rangle}{B_p^2(a)/2\mu_o} \le \frac{R_o}{a} \equiv \frac{1}{\varepsilon} = A > 1 \tag{34}$$

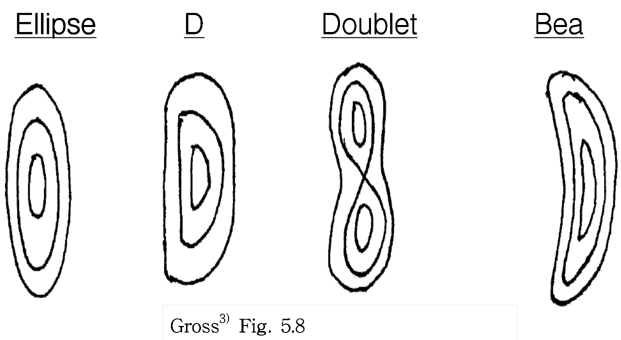
$$(29) : \beta = \frac{\beta_p}{1 + B_\phi^2/B_p^2} < \frac{\beta_p}{1 + q_{\min}^2/\varepsilon^2} < \frac{\varepsilon}{q_{\min}^2 + \varepsilon^2} \approx \frac{\varepsilon}{6.25 + \varepsilon^2} < 1 \tag{35}$$

(e.g.) For R_o/a = 3, $\varepsilon = 1/3$, $q_{\min} = 2.5 \Rightarrow \beta < 5\%$

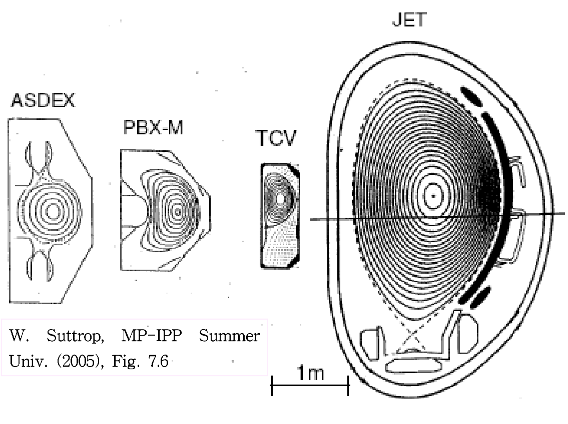
4, 1/4, 2.5 $\Rightarrow \beta < 4\%$

High β for high output power & low cost ⇔ Low β for stable operation

↳ Non-circular plasma cross sections :



Gross[3] Fig. 5.8



W. Suttrop, MP-IPP Summer Univ. (2005), Fig. 7.6