457.309 Hydraulics and Laboratory

.05.03 Uncertainty Analysis (3)

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Today's objectives

- Final data handling and presentation without extra uncert ainties.

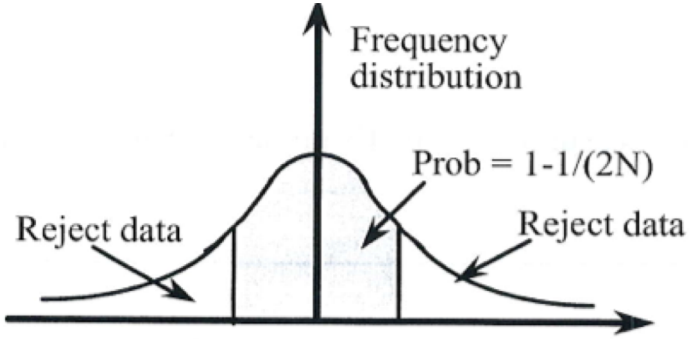
Statistical rejection of "Bad" Data: *Chauvenet's Criterion*

- Occasionally, when a sample of N measurements of a variable is obtained, there may be one or more that appear to differ markedly from the others.
- If some extraneous influence or mistake in experimental technique can be identified, these "bad data" or "wild points" can simply be discarded.
- More difficult is the common situation in which no explanation is readily available. In such situations, the experimenter may be tempted to discard the values on the basis that something must surely have gone wrong.
- However, this temptation must be resisted, since such data may be significant either in terms of the phenomena being studied or in detecting flows in the experimental technique.

Statistical rejection of “Bad” Data: *Chauvenet’s Criterion*

- On the other hand, one does not want tn erroneous value to bias the results. In this case, a *statistical* criterion must be used to identify poi nts that can be considered for rejection. There is no other justifiable method to “throw away” data points.
- One method that has gained wide acceptanace is *Chauvenet’s criter ion;* this technique defins an acceptable scatter, in a statiscal sense, around the mean value from a given sample of N measruments.
- The criterion states that all data points should be retained that fall wi thin a band around the mean that corresponds to a probability of 1-1 /(2N). In other words, data points can be considered for rejection onl y if the probability of obtaining their deviation from the mean is less t han 1/(2N).

Statistical rejection of "Bad" Data: *Chauvenet's Criterion*



- The probability 1-1/(2N) for retention of data distributed about the mean can be related to a maximum deviation from the mean, d_{max}, using the Gaussian probabilities.
- For the given probability, the nondimensional maximum deviation τ_{max} can be from the table where

$$\tau_{max} = \frac{\left|\left(X_i - \bar{X}\right)\right|_{max}}{S_X} = \frac{d_{max}}{S_X}$$

and S_X is the precision index of the sample.

Statistical rejection of "Bad" Data: *Chauvenet's Criterion*

- Therefore, all measurements that deviate from the mean by more th an $\tau_{max} S_X$ can be rejected. A new mean value and a new precision in dex can then be calculated from the remaining measurements.
- No further application of the criterion to the sample is allowed; Chau venet's criterion may be applied only once to a given sample of readi ngs.

Chauvenet's Criterion for Rejecting a Reading

Number of Readings (N)	Ratio of Maximum Acceptable Deviation to Precision Index (d_{max}/S_X)
3	1.38
4	1.54
5	1.65
6	1.73
7	1.80
8	1.87
9	1.91
10	1.96
15	2.13
20	2.24
25	2.33
50	2.57
100	2.81
300	3.14
500	3.29
1,000	3.48

Determine using Gaussian probability

Example

Example. For the 10 temperature measurements in the first example, determine if any should be rejected by Chauvenet's criterion.

Solution) Inspecting the data from the first example, the fifth reading of T=98.5°F appears to deviate substantially from the others and is therefore a candidate for rejection. From the table above for N=10, the maximum dimensionless deviation is d_{max}/S_X=1.96. Since S_X~0.49°F in this case, $d_{max}=1.96S_X$=(1.96)(0.49)=0.96°F. The deviation for the fifth reading is 1.125°F so it can indeed be rejected, although no others can. Eliminating this point and recalculating the mean and precision index results in

$$\bar{X} = 97.25°\text{F}$$

$$S_X = 0.31°\text{F}$$

Comparing these values with those calculated in the first example, $\bar{X}$ is decreased by only about 0.13% while S_X is decreased by over 37%.

Numerical Precision of Reported Results

- Except for a few constants such as π, all numerical values used in e ngineering computations are know to limited precision.
- Indeed, most engineering measurements are valid to 2, 3, or 4 digits at best, meaning that the digit that should appear in the third, fourth or fifth place, respectively, is somewhat unsure.
- If substantial information is available from which a good estimate of t he uncertainty in a measurement can be obtained, for example from the detailed analysis described in preceding sections, then this unce rtainty should be stated explicitly, together with the confidence level, if available.
- As an example of this approach, the drag coefficient in the previous example was found to be $C_D = 0.510 \pm 0.058$ at the 95% confidence level.

Numerical Precision of Reported Results

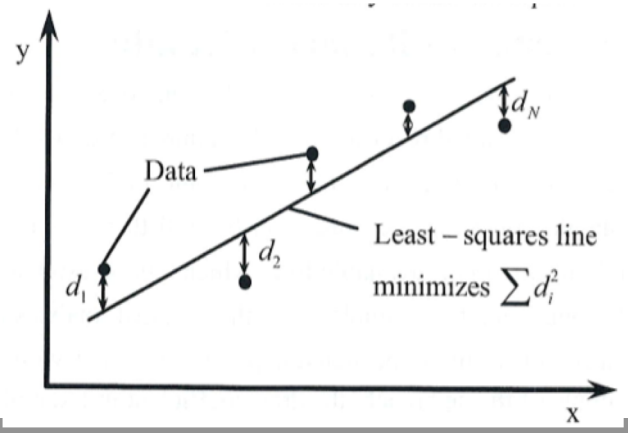
- Lacking such explicit detailed uncertainty specification, then the num ber of significant digit written fro a numerical value implicitly specifie s the uncertainty.
- For example, a length measurement stated as 10.2 m generally impl ies that the value is believed to be between 10.15 and 10.25 m. Ther efore, the number of significant digits used in reporting numerical val ues should be limited to that which is truly reflective of the uncertaint y in the value.
- The common practice of writing 10 significant digits for a value result ing from a series of calculations just because they appear on a calcu lator display is generally totally unfounded; the values in most engin eering applications are not known with anywhere near this precision.

Numerical Precision of Reported Results

- Another rule-of-thumb to keep in mind is that *the number of significant digits used for the result of a calculation should not exceed the number of significant digits for the least precise quantity used in the calculation.*

Least Squares Curve Fitting

- Supposed that a set of data in two variables x and y has been obtained where x is the independent variable and y is the dependent variable.
- It is desired to obtain a simple analytical expression (or correlation equ ation) to represent these data. The question arises, " what is the 'best' equation or 'best' curve through the data?" The criterion used most ofte n to determine the "best fit" is the least-squared criterion.
- The least-squares method derives its name from the fat that the sum of the squares of the deviations between the data and the curve is minimi zed, as shown below.



Least Squares Curve Fitting

- In the following discussion it is assumed that the “true” relationship bet ween x and y is linear, i.e. the best fit straight line through the data is so ught. Although this is a common situation, the analysis below can also be extended easily to more complex curves.
- In addition, it is assumed that all of the uncertainty in the measurement s is concentrated in the dependent variable y and that the uncertainties for all measured y values are equal.
- An equation of the form y=ax+b is sought which minimized the sum of t he squared deviations

$$S = \sum_{i=1}^{N} d_i^2 = \sum_{i=1}^{N} \left[y_i - \left(a x_i + b \right) \right]^2$$

Least Squares Curve Fitting

- In order to accomplish this, the partial derivatives are set to zero. T his leads to

$$\frac{\partial S}{\partial a} = 0 \quad \Rightarrow \quad a\sum x_i^2 + b\sum x_i = \sum x_i y_i$$

$$\frac{\partial S}{\partial b} = 0 \quad \Rightarrow \quad a\sum x_i + bN = \sum y_i$$

- Solving for a and b simultaneously yields (all summations run from i =1 to N)

$$a = \frac{N\sum x_i y_i - \left(\sum x_i\right)\left(\sum y_i\right)}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2} \quad \text{and} \quad b = \frac{\left(\sum x_i^2\right)\left(\sum y_i\right) - \left(\sum x_i\right)\left(\sum x_i y_i\right)}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2}$$

- $y=ax^2+b$ can be treated in the same manner by regarding x^2 as the new variable. sin(x) or exp(x), etc. can be used similarly.